Kinematics of Machines

Prof A K Mallik

Department of Civil Engineering

Indian Institute of Technology, Kanpur

Module - 2 Lecture - 3

Today's topic is range of movement and rotatability. In this topic, we shall discuss the range of movement that a link can perform in a particular linkage which obviously depends on the relative link lines. More specifically, the important point in this area is the complete rotatability of a particular link. Why this is important? Because, as we know most of the machines are driven by an electric motor and the shaft of an electric motor is capable of unidirectional complete rotation. There must be one link which is connected to this motor shaft should be able to rotate completely, that is the presence of all other links should not prevent it from undergoing complete or full rotation. This aspect of range of movement and rotatability is most comprehensively studied for four-link mechanism. In this rotatability condition, most important notion is known as Grashof's criterion.

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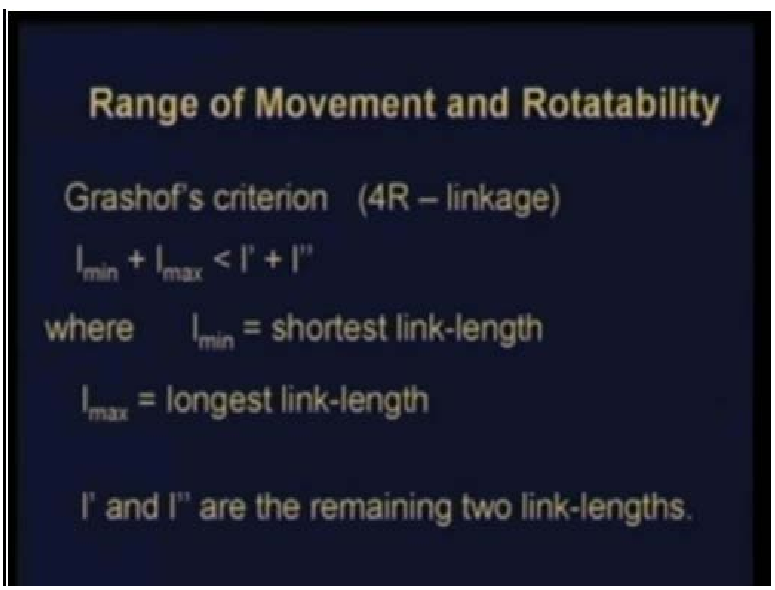

Let us start with a 4R - linkage. That means, there are 4 links which are connected by four revolute joins which we normally called a four-bar linkage. There are four kinematic dimensions in such a 4R – linkage, which we call the link-length. That is the distance between successive kinematic pairs.

Let us say, l_{min} is the shortest link-length and l_{max} is the longest link-length, where as l prime and l double prime are the remaining two link-lengths. Grashof's criterion says l_{min} plus l_{max} is less than the l prime plus l two prime. If the chain satisfies this Grashof's criterion then we call it a Grashof chain.

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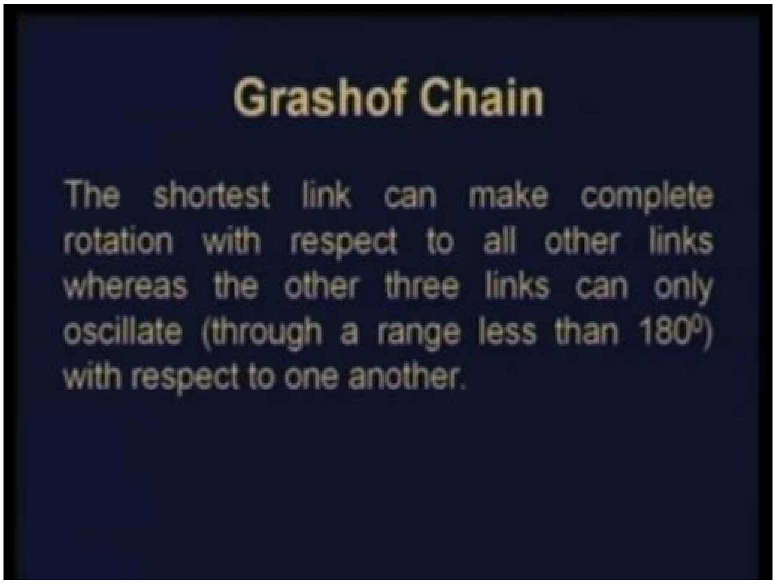

In a Grashof chain, the shortest link can always make complete rotation with respect to all other links, whereas the other three links that is except the shortest link can only oscillate with respect to one another and this amount of oscillation is always less than 180 degree. If in a Grashof chain the shortest link decides how the kinematic behavior is?

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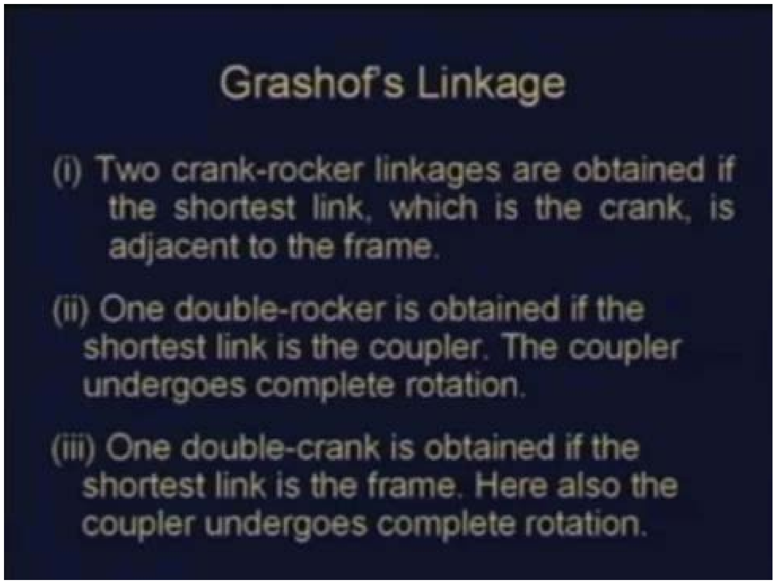

In a Grashof linkage, we can see two crank-rocker linkages will be obtained if the shortest link which is the crank is adjacent to the frame. That means a shortest link which has two adjacent links. If I make any of this adjacent links as the frame, then the shortest link will be able to make complete rotation with respect all other links that means, even with respect to the frame. Thus the shortest link becomes the crank. Because there are the two adjacent links to the shortest link it is connected on two sides to two different links, any one of these if I keep fixed or keep it as the frame then we get two crank-rockers, because all other links can only oscillate with respect to the frame.

If I make a kinematic inversion from a Grashof chain, we can also get a double-rocker linkage. If the shortest link is the coupler, that is the link which is opposite to the shortest link that means, which is not directly connected to the shortest link if I hold that link fixed or use that as the frame then the coupler the shortest link undergoes complete rotation but the links which are connected to the frame they can only oscillate so we get a double-rocker mechanism.

If I make another kinematic inversion from the same Grashof chain, we can get a double-crank. A double-crank linkage will be obtained if the shortest link is the frame, because all the other three links can rotate completely with respect to the shortest link because, it is a relative motion that matters, if the shortest link can make complete rotation with respect to all other links then all other links can make complete rotation with respect to the shortest link. If the shortest link is held fixed of the frame, then all the other three links can make complete rotation. We get a double crank even the coupler is also capable of making complete rotation.

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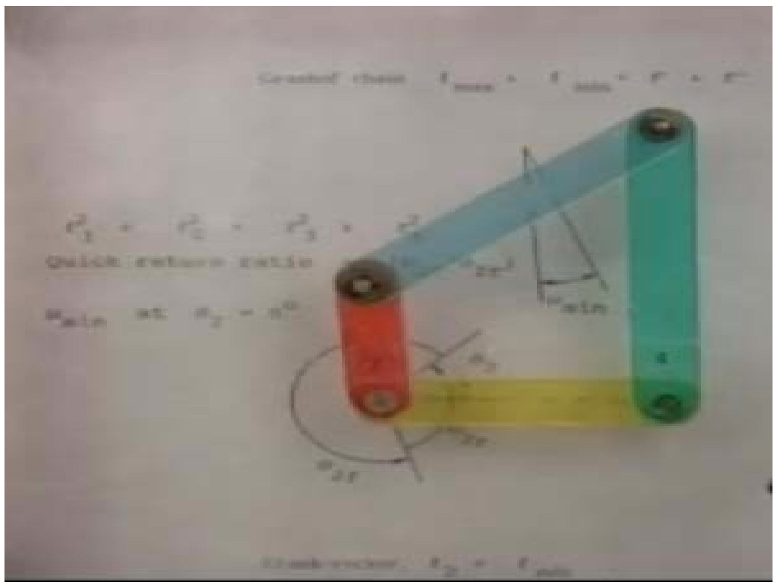

Let us look at the model of this 4R-chain where the red link is the shortest link that is l_{min} and this green link is the longest link that is l_{max}. If the adjacent link to the shortest link this yellow one is held fixed and this is a Grashof chain that is l_{min} plus l_{max} is less than l prime plus l double prime. In this Grashof chain, if I hold one of the links which is adjacent to the shortest links that is this yellow one fixed, then we gets a crank-rocker because the shortest link is able to make complete rotation with respect to this link, whereas this green link only oscillates with respect to this link. We see that the red link is rotating completely, whereas the green link is only performing oscillatory motion and we get a crank-rocker linkage. Another inversion, if I hold this link adjacent to the shortest link held fixed, then also we will get a crank-rocker because this link will also make complete rotation in that case and this green link will only oscillate with respect to this link.

Let us now use the same chain but with a kinematic inversion. The shortest link is the coupler and the link opposite to the shortest link that is this green link is held fixed. If I move this link we will see that these two links which are input and output may be connected to the frame can perform only oscillatory motion, whereas this coupler is undergoing complete rotation, because the shortest link in a Grashof chain can always make complete rotation with respect to all the other links.

Let us see we start from this horizontal position for the coupler that is link 3. As we see, it has rotated 90 degree in the clock wise direction, it has rotated 180 degree, it has rotated 270 degree and it has performed complete rotation. From the same Grashof chain, by making a kinematic inversion earlier we had a crank-rocker linkage and now we had a double-rocker linkage. The thing to note is the extreme positions of various links, this is I call link number 4, this is link 2, this is link number 3 and this is link number 1, which is the fixed link.

Let us see why link 4 oscillates? As we see, during this movement link 2 and 3 have become collinear and this point cannot go any further as a result link 4 now has to return. It cannot go in the clock wise direction anymore it has to move anti-clock wise direction, so the link oscillates. Again in this position, link 2 and link 3 have become collinear and this is the extreme position of link 4 it has to now rotate clock wise. The same thing is true for link number 2 when link 3 and 4 become collinear link 2 occupies one of its extreme positions. The other extreme position is again when link 2, link 3 and link 4 have become collinear and link 2 occupies an extreme position. We note also this important thing which will be used later on for displacement analysis that the extreme position of a link is taken, when the two other links become collinear like this or like this (Refer Slide Time: 09:14).

Let us now look at another kinematic inversion from the same Grashof chain. Here, the red link that is the shortest link is held fixed that is acting as the frame. All these three links can rotate completely with respect to this shortest link. Consequently we will get, what is known as a double-crank linkage. That means, all the links connected to the frame including even the coupler are able to make complete rotation. We see that if it is a

Grashof's chain, then by kinematic inversions it is possible to get all the three possible varieties of linkages namely crank-rocker, double-rocker and double-crank. Two of the inversions resulted in crank-rocker when the shortest link is the frame. One inversion when the shortest link is the coupler we get a double-crank and when the shortest link is the fixed link, we get a double-crank. We see that, if it is a Grashof chain then by kinematic inversion we can get all the three possible varieties of linkages namely double-crank, crank-rocker and double-rocker. We get two crank-rockers if the shortest link is adjacent to the frame. We get a double-rocker, when the shortest link is the coupler and we get a double-crank when the shortest link is the frame. We have seen that in a Grashof chain it is the position of the shortest link which decides the characteristics of the linkage, whether is a crank-rocker or double-crank or double-rocker.

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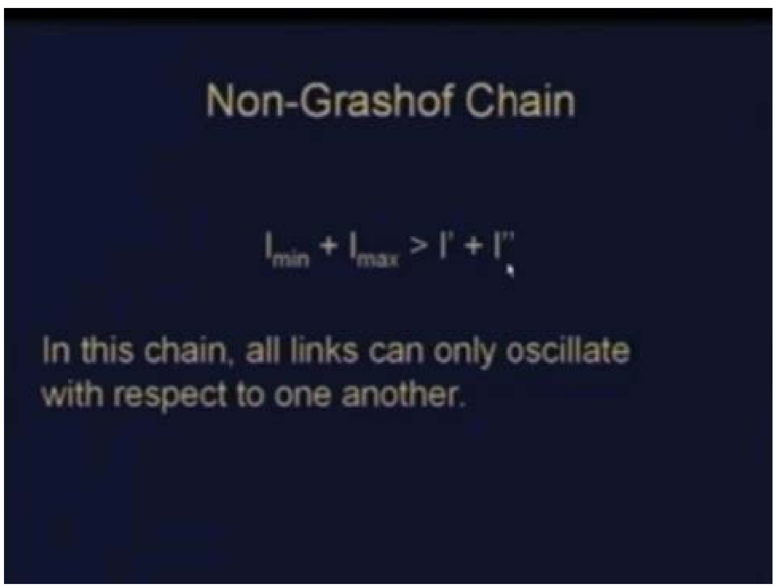

Let me now discuss in a Non-Grashof chain. Non-Grashof chain is defined as l_{min} plus l_{max} is greater than l prime plus l two prime, where l_{min} is the shortest link-length and l_{max} is the longest link-length and the other two links-lengths are l pirme and l two prime. In this Non-Grashof chain whatever may be the inversion all links can only oscillate with respect to one another.

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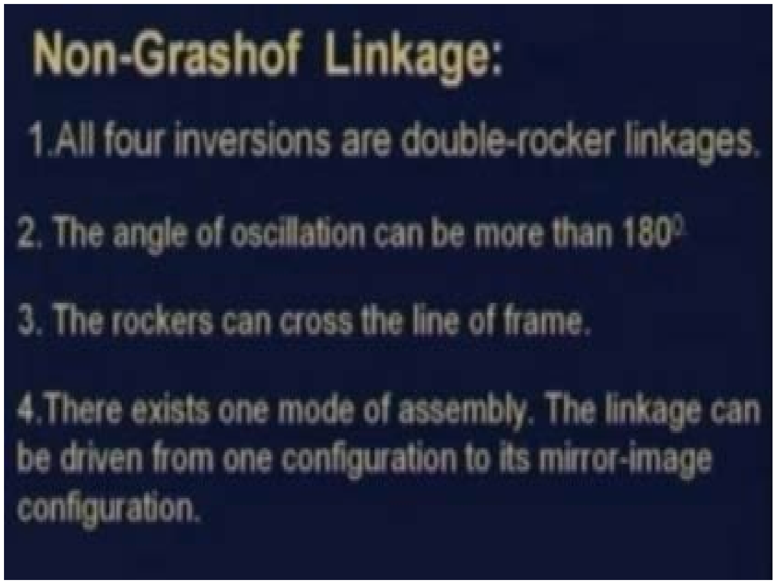

In a Non-Grashof linkage, all four inversions are double-rocker linkages, because all the links can only oscillate with respect to one another. However the angle of oscillation can be more than 180 degree, whereas in a Grashof chain angle of oscillation was always less than 180 degree. The rockers can cross the line of frame if it is a Non-Grashof linkage, whereas in a Grashof linkage one can show that the rocker can never cross the line of frame. If it remains above the line of frame in one configuration then during the entire movement in a Grashof's double-rocker or the crank-rocker, the rocker can never cross the line of frame, it will be either above the line of frame or below the line of frame. Whereas in a Grashof linkage which is always a double-rocker, the rockers can cross the line of frame that means, it can come from above to below or below to above of the line of frame.

We explain another thing that in a Non-Grashof linkage there exists only one mode of assembly. The linkage can be driven from one configuration to its mirror image configuration, this needs a further explanation as we shall do right now.

We have just now talked off mode of assembly, let me explain what do we mean by mode of assembly?

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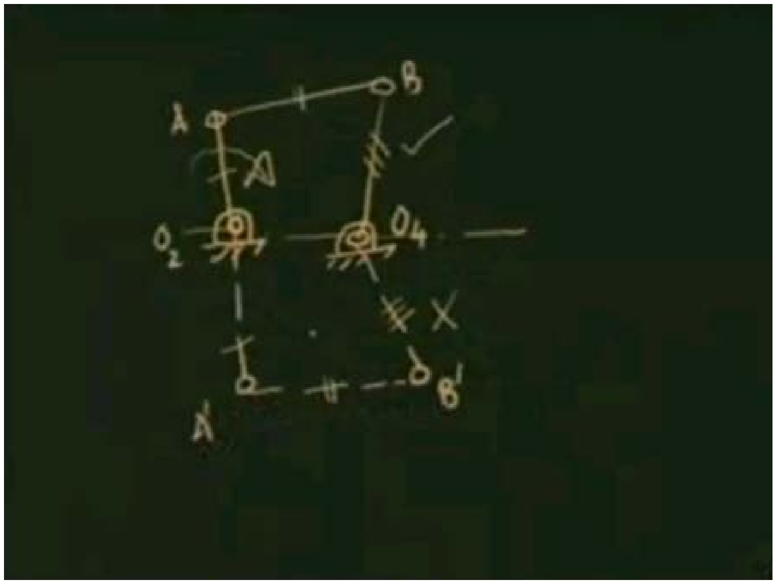

For given link-lengths I can have a 4-bar linkage, in this configuration this O_2, A, B and O_4. With the same link-lengths I could have also assembled this link just in its mirror image configuration, which let me call O_2, A prime, B prime, O_4. This link-length is exactly same as this link-length, this link-length is same as this link-length and this link-length is same as this link length. O_2, A, B, O_4 and O_2, A prime, B prime, O_4 are two modes of assembly depending on whether assembled above the line of frame or below the line of frame. It is needless to say that, these two configurations are mirror image of each other with the mirror placed along this line of frame that is O_2, O_4.

If this chain is a Grashof linkage, then we can never drive from one assembly to the other. That means, if I assembled it in this configuration then by driving this linkage that is by moving this link I can never occupy this mirror image configuration. Whereas, in a Non-Grashof linkage, if I start from one configuration then as I drive the linkage there will be an instant where this mirror image configuration will be taken up by the linkage. These points will be further explained with the help of models.

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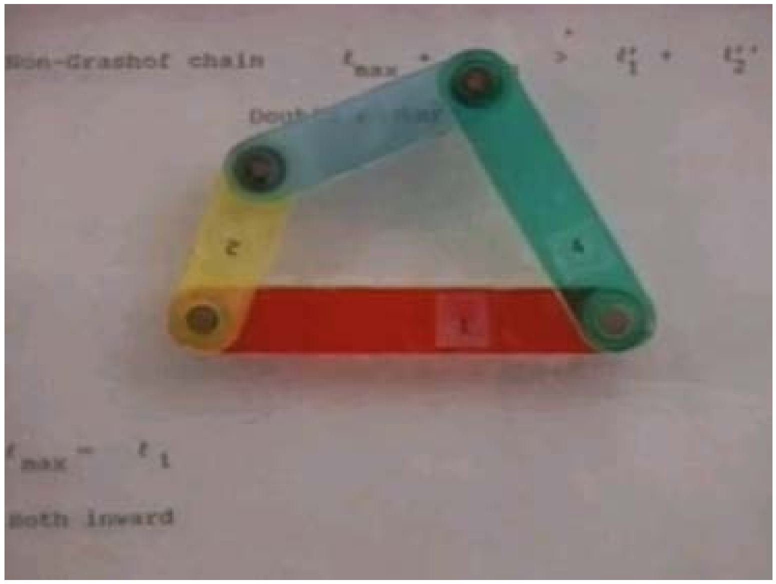

Let us now look at the model of this Non-Grashof chain. Here, the red link is the longest link that is l_{max}, this yellow link is the shortest link that is l_{min} and these two are l prime and l double prime the remaining two link-lengths. It is easily seen in this model l_{min} plus l_{max} is more than the sum of l prime and l double prime. It is a Non-Grashof chain and independent of which link we hold fixed, we always get a double rocker-linkage. In this particular situation this longest link the red link has been held fixed.

We see, all the links are only performing oscillatory motion they are unable to make complete rotation, consequently we get a double-rocker linkage. The thing to note, that in this Non-Grashof double-rocker linkage this rocking links is crossing the line of frame that is this line. It is now above, now it has crossed it is below the line of frame. It is true for this rocker it was above the line of frame, in this configuration it is below the line of frame. Whereas, in a Grashof double-rockers, the rockers could never cross the line of frame, we could never get from above to below.

Let me demonstrate that the mirror image configurations are taken by the same assembly. Suppose, this is one assembly mode and we can imagine what will be the mirror assembly mode that will be like this. If I drive this mechanism, one can see that it has occupied the mirror image configuration. Thus, in a Non-Grashof linkage one can drive the linkage from one mode of assembly to the other, which was not possible in a Grashof linkage. That I will demonstrate with a different model.

Let us consider another kinematic inversion of the same Non-Grashof chain. If we remember in the previous model this longest link was the frame. Here, the longest link has been connected to the frame. This is the frame the fixed link. Here again, even in this inversion we will get a double-rocker linkage. As we see that, this is one extreme position of this red link and these two links have become collinear and this is another extreme position of this red link when these two links have become collinear. We have also seen that both the rockers could cross the line of frame. And one can easily see that mirror image configurations are taken by the same mechanism, same assembly can be driven to the mirror image configuration. For this mirror image configuration if we imagine will be something like this. The mechanism could be driven from one mode of assembly to the other, as I said earlier is not possible in a Grashof linkage.

Let us now consider another inversion of the same Non-Grashof chain. Here, as we see the longest link that is this red link is the coupler, opposite to this link that is the frame. Even in this inversion this is a double -rocker. The rocking angle is very large. But still all these links are unable to make complete rotation. The nature of oscillation of this rockers, that is here as we see this is crossing the line of frame in this direction, whereas this rocker is crossing the line of frame in this direction, it is not crossing in this direction.

We see both these rockers are crossing the line of frame in the outward direction not in this inward direction; such a rocker is called both -outward. Similarly, other inversions from same chain though have double-rockers but the rocking movements are different either inward-outward or both inward. That depends on the position of this longest link. With the longest link as the coupler we got both outward oscillations. Whereas longest link connected to the frame we will get inward-outward and longest link as the frame will get both inward.

We will see all the three models from the same Non-Grashof chain, link-lengths in all these models are equal, only thing that longest link is position differently, here it is frame, here it is coupler, here it is connected to the frame and all these inversions have produced as we have seen are double-rocker linkages. When these two links become collinear, this gets its extreme position. Similarly, when these two links become collinear, this gets its one extreme position, same is true here these two links are collinear and this gets its extreme position.

So we have seen different inversion from the same Non-Grashof chain always in double-rocker linkages. As a result, such a Non-Grashof linkage is not much useful in real life, because if it has to be driven by a motor then there has to be a crank, whereas no crank exists in a Non-Grashof links. Thus, only the Grashof linkage is useful in practice if it has to be driven by a motor, and the shortest link must be connected to the motor shaft.

Let me now go back to the model of a Grashof linkage and to show that in a Grashof linkage one mode of assembly cannot be driven to the other mode of assembly which is the mirror image configuration with the mirror placed along the line of frame. For example, this is one mode of assembly. If we dismantle all these revolute joins, I could have assembled it in the mirror image configuration with these two links vertical but below this line of frame. As we see if we drive this mechanism, this line has become mirror image of its previous configuration, but this link has not because, this is a Grashof linkage and if it is assembled it one more, it can never be driven to the other mode of assembly. A Grashof linkage has two distinct modes of assembly, whereas a Non-Grashof linkage has a single mode of assembly. One more important thing is to see that, in a Grashof linkage it is the position of the shortest link that decides the movement characteristics depending on where the shortest link is it may be a crank-rocker, it may be a double rocker, it may be a double crank. Whereas, in a Non-Grashof linkage it is always double-rocker independent of any kinematic inversion, however it is the position of the longest link that decides the rocking characteristics whether it will be both inward or both outward or inward-outward.

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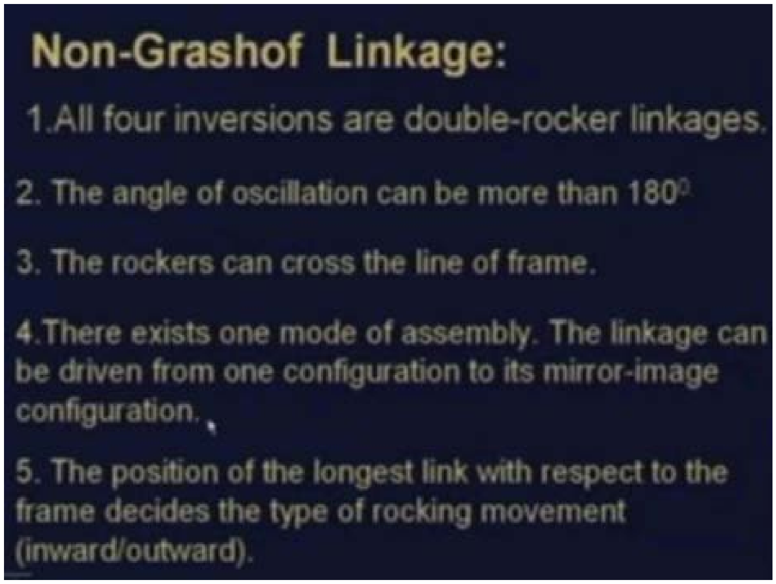

Let me now summarize what we have seen so far.

1. In a Non-Grashof linkage we have seen all four inversions are double-rocker linkages.
2. The angle of oscillation in a Non-Grashof linkage can be more than 180 degree.

3. The rockers can cross the line of frame.

4. There exists only one mode of assembly the linkage can be driven from one configuration to its mirror image configuration.

5. The position of the longest link with respect to the frame decided the type of rocking movement that is whether inward-outward or outward-outward or both outward or both inward or inward and outward.

For a Grashof linkage let me go through the similar points. What we have seen, for a Grashof linkage all the three varieties of linkages can be obtained from the same chain by kinematic inversion, two inversions give crank-rocker linkages which are most useful, one inversion gives a double-rocker linkage and last inversion gives a double-crank linkage. The angle of oscillation of the rocking links can never be more than 180 degree, it has to be less than 180 degree. The rockers of a Grashof linkage can never cross the line of frame. There exists two distinct modes of assembly that is the two mirror image configuration and the linkage can never be driven from one configuration to its mirror image configuration. And lastly, it is the position of the shortest link with respect to the frame that decides the type of movement, that is if the shortest link is frame then it is double crank, if the shortest link is the coupler then it is double-rocker, if the shortest link is connected to the frame then it is a crank-rocker with shortest link as the crank. We have done with both Grashof and Non-Grashof chain. Let me talk of with boundary between Grashof and Non-Grashof which is known as transition linkage.

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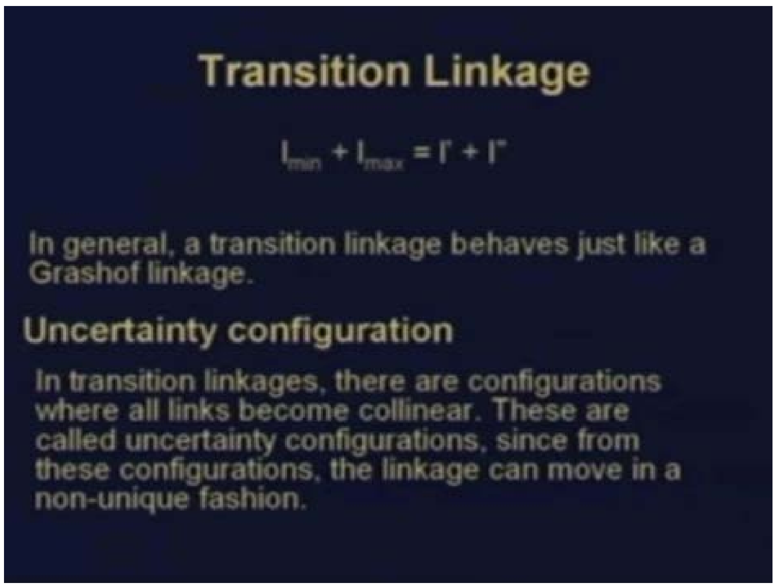

In the transition linkage, the sum of the longest link and the shortest link is exactly equal to the sum of the remaining two links, that is l_{min} plus l_{max} is equal to l prime plus l double prime. In general, a transition linkage behaves just like a Grashof linkage, that is if the shortest link is the frame then we get a double crank, if the shortest link is the coupler then we get a double-rocker, if the shortest link is connected to the frame then we get crank-rocker. However, in this transition linkage where l_{min} plus l_{max} is exactly equal to l prime plus l double prime then it is obvious that there will be configurations when all the links become collinear. This collinear configuration is called uncertainty configuration. In these transition linkages, there are configurations where all links become collinear which are called uncertainty configurations. From these configurations, the linkage can move in a non-unique fashion as we shall demonstrate later with a model.

We have discussed in a general what happens in a transition linkage but now we have to discuss special cases of transition chain.

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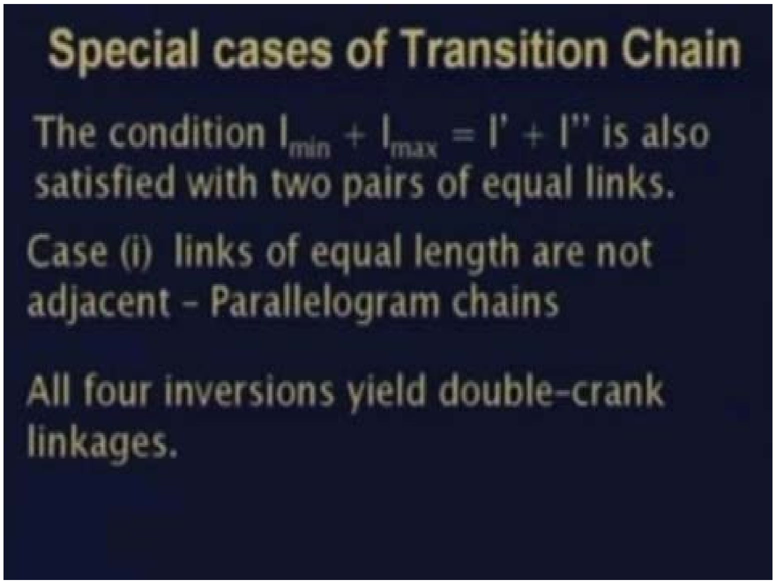

As we know the condition l_{min} plus l_{max} equal to l prime plus l double prime is also satisfied with two pairs of equal links. That means there are two pairs, one pair of l_{min} and the other pair is l_{max}. In this special case there are two varieties,

Case (i) when the links of equal length are not adjacent, that means links of equal length are opposite to each other when we call it a parallelogram chain. In a parallelogram chain all four inversions are double-crank. So, all four inversions of parallelogram chain in double-cranks linkages of course with uncertainty configuration where the parallelogram linkage can flip into anti-parallelogram configuration as we shall see just now.

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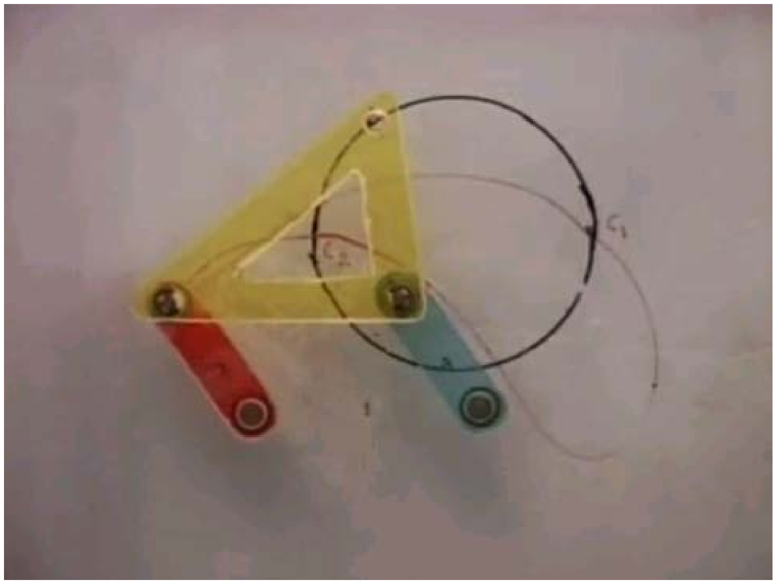

Let us now look at the model of this transition, linkage that is the special situation of the transition linkage. Here these two links are of same lengths which are opposite to each other and this coupler link is same as the frame link. That is, we have a pair of l_{min} and a pair of $l_{max.}$ However, because these two equal lengths are not connected directly they are the opposite sides it forms a parallelogram and we call it a parallelogram linkage. As this parallelogram linkage moves, it is easy to see that there will be instance where all the 4 revolute pairs have become collinear. As a result, the linkage is passing through its uncertainty configuration and from here the non-unique movement is possible. If sufficient care is taken, we can maintain the parallelogram configuration. From this uncertainty configuration, it can also flip back to anti-parallelogram configuration and it is no longer a parallelogram. The two opposite sides are equal but it is in the closed configuration this is called anti-parallelogram.

At this uncertainty configuration, the linkage becomes uncertain whether to maintain the parallelogram or to flip back into anti-parallelogram configuration. This uncertainty configuration is true for all types of transition linkages whenever l_{min} plus l_{max} is l prime plus l double prime. Again this is another uncertainty configuration, we can either maintain the parallelogram or it can flip back to anti-parallelogram configuration. To overcome this uncertainty configuration in a parallelogram linkage, we can use an extra coupler a redundant coupler which we have seen earlier and I will show it to you again.

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Let us again look at this parallelogram linkage where this length is equal to this length and this coupler length is equal to the frame length. This is a parallelogram linkage. This parallelogram linkage has a redundant or extra coupler which is of same length as this original coupler. As a result when these four revolute pairs become collinear apparently this parallelogram linkage is passing through uncertainty configuration. This extra coupler which is not passing through uncertainty configuration ensures that, the parallelogram is always maintained it can never feed back to anti-parallelogram configuration. The parallelogram linkage is very useful because, it maintains unit angular velocity ratio, this crank and the follower are always parallel, so it transmit unit angular velocity ratio from the input to the output link. But to ensure that it remains a parallelogram and it does not flip back to anti-parallelogram configuration at the uncertainty configuration we must have this extra or redundant coupler.

Let me now summarize what we have just seen for a transition linkage with opposite sides of equal link lengths.

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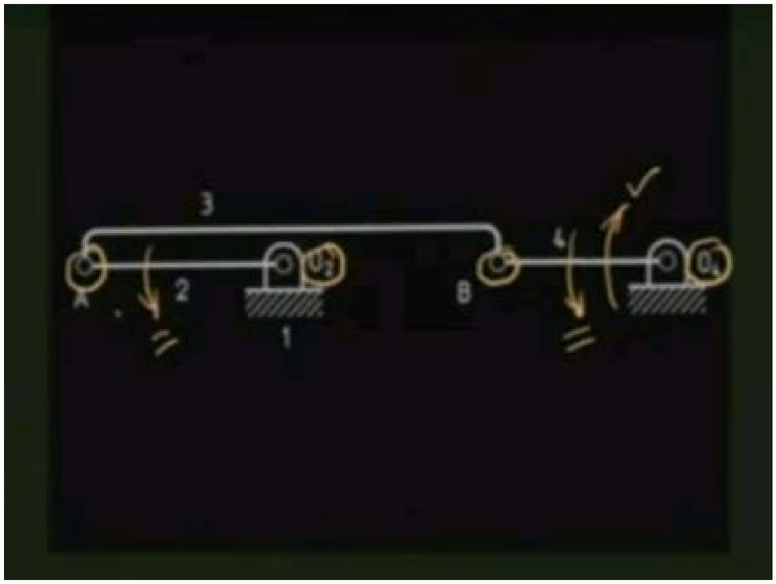

Here as we see, the four revolute pairs namely O_2, A, B and O_4 have all become collinear. As a result, all the links become collinear and from this configuration onwards the linkage moves in a non-unique fashion. If O_2A is driven in this direction, O_4B can move in this direction or can flip back in the opposite direction. If it moves in the same direction then it maintains the parallelogram, whereas if it moves in the opposite direction then it flips into the anti-parallelogram configuration.

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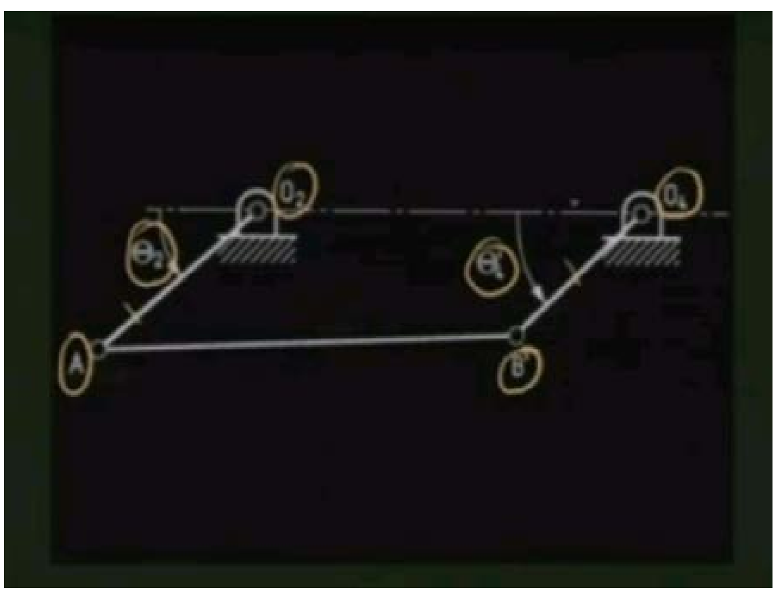

Here we show that, the parallelogram linkage with revolute pair at O_2, A, B and $O_{4.}$ What see that, because this is a parallelogram this angle theta$_2$ is always same as theta$_4$ and it maintains unique angular velocity ratio between the input and the output link. At the uncertainty configuration A, B prime O_2, O_4 everything becomes collinear.

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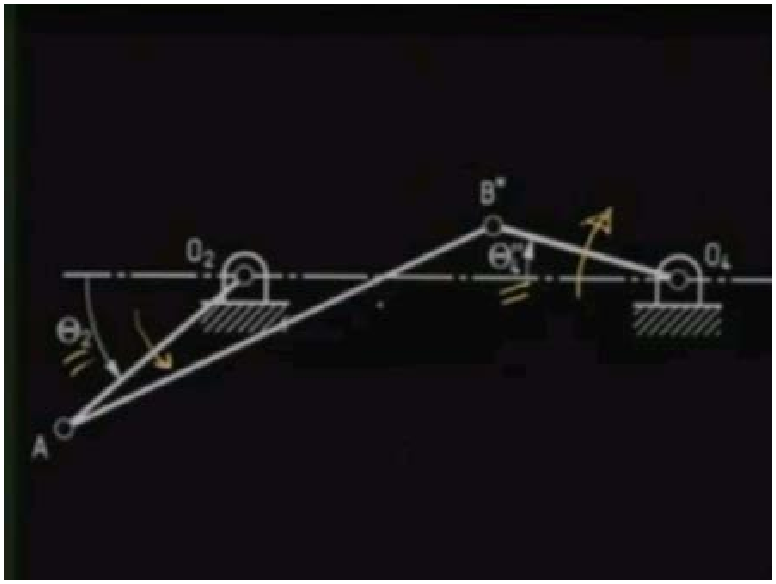

And it can flip back into this anti-parallelogram configuration. O_2A is still moving in the counter-clock wise direction from the uncertainty configuration, whereas O_4B has flip back and moving in the clockwise direction and theta$_2$ and theta$_4$ prime that is the anti parallelogram configuration they are not equal. Always the parallelogram configuration, that is to avoid this anti- parallelogram configuration after crossing the uncertainty position we need to have the extra redundant coupler as we explained with the help of a model.

Let us now discuss the second case of this special situation of a transition linkage when we have two pairs of equal link-lengths.

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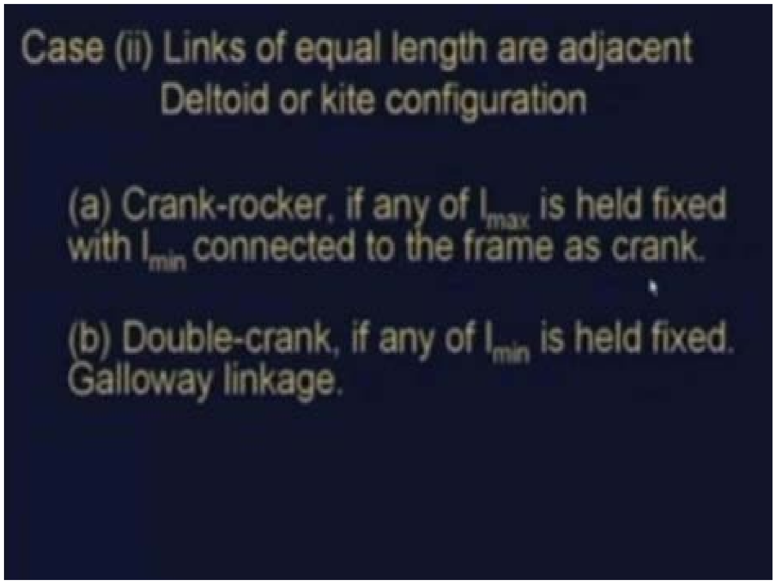

Unlike in a parallelogram situation, here the links of equal length are adjacent, not opposite to each other and this configuration where the links of equal lengths are adjacent are called deltoid or kite configuration. From this deltoid or kite configuration, there are two different possibilities. We will get a crank-rocker if any of the l_{max} that is any of the longer links is held fixed and the connected l_{min} will be the crank. Whereas, we get a double-crank if any of the l_{min} that any of the shortest links is held fixed. Such a linkage when we have double-crank is called Galloway linkage. I will explain both this deltoid configuration with the help of a model.

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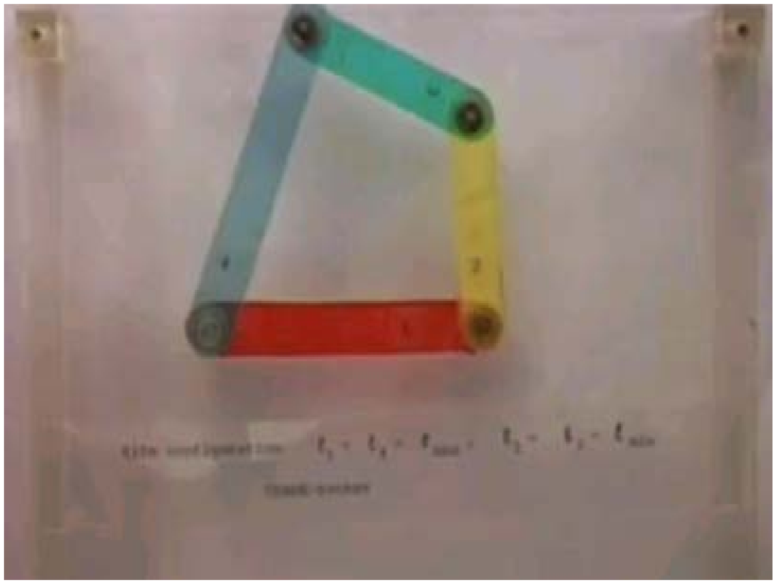

Let us look at one kinematic inversion from this kite configuration. Here as we see these two links are of equal link-lengths and these two links is another pair of equal link-lengths. Unlike in a parallelogram configuration here the equal link-lengths are adjacent to each other rather than opposite of each other. These two links of equal lengths are adjacent, these two links another pair of equal lengths are adjacent. So this is the kite configuration.

We are considering a kinematic inversion where one of the l_{max} that is one of the longest links is held fixed. As a result we will get crank-rocker with the shorter link which is connected to this fixed link will be the crank and the longer link will be the rocker. As we see, we start from here, the shorter link can rotate completely whereas the longer link is only oscillating. Here of course because it is a transitional linkage, there will be uncertainty configuration when all the link-lengths become collinear. Here we see, there is loss of unique movement the linkage can move like this which is no motion transmission or if care is taken it can be driven as linkage with positive motion transmission. Here we get a crank-rocker kinematic inversion with the longer link of this kite configuration held fixed. Next we will see the model from the same chain where one of the shorter links will be held fixed.

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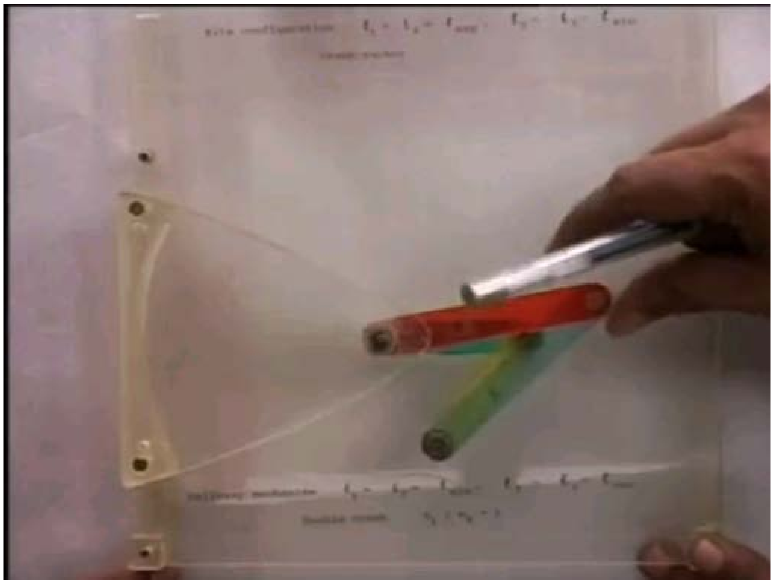

Let us now look at another kinematic inversion from the same kite configuration. Here again one pair of longer links, one pair of shorter links. But one of the shorter links is held fixed, previously we have seen one of the longer links which was held fixed. In this kinematic inversion we will get a double-crank that means both the yellow link, the shorter link and this red link will be able to make complete rotation. As we saw, both the red link and the yellow were able to perform complete rotation. So this is a double crank.

There exists, a very fundamental difference between these double-crank and the double-crank that we got earlier from a Grashof linkage or a parallelogram linkage. There one rotation of the crank was also causing one full rotation of the follower. But here, we must have noticed that it is two revolutions of the shorter crank, as we see the shorter crank has already made one complete revolution, the longer crank is yet to make its complete revolution. If I rotate the shorter crank one more revolution then the longer crank is completing its full rotation. Thus, two revolutions of the shorter crank are generating one full revolution of the longer crank. Such a mechanism is called a Galloway mechanism.

In fact, we can see that for this configuration of the shorter crank with the same link-lengths, I could have added another configuration of this linkage. I can draw a circle with this point as centre and this as radius, this point as centre and this as radius. These two circles can intersect either here or at another point. Because two circles normally intersect at two points. After one full revolution of the shorter crank, this point is going to the other points of intersection of these two circles with this point as center and this length as radius, this point as center and this as radius. It is a quite different type of double-crank than the normal double-crank that we have encountered so far and this has a special name as I said earlier is called a Galloway linkage.

Another trivial situation of a transition linkage occurs when all the link-lengths are equal. That is l_{min} plus l_{max} is l prime plus l double prime, because all the four link lengths are equal. With such equal link-lengths we get what is known as a rhombus linkage.

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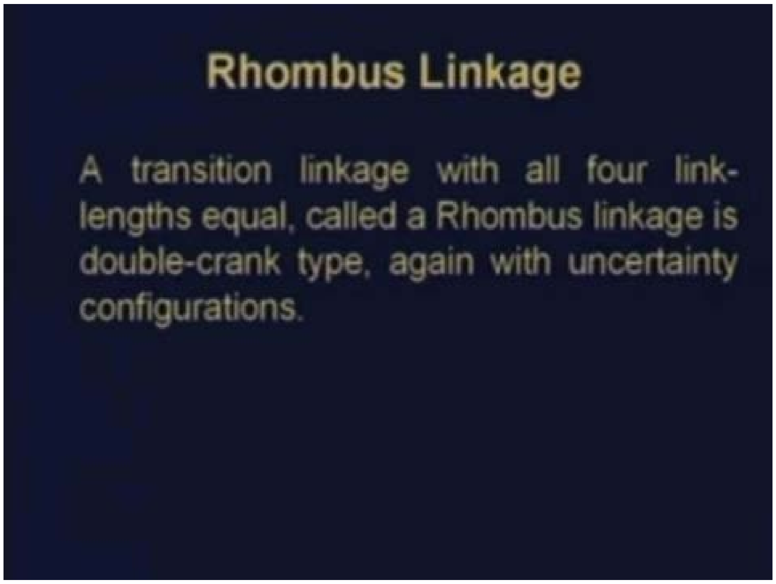

In a rhombus linkage, whatever may be the kinematic inversion, just like a parallelogram linkage we will get double crank type linkages, of course only when uncertainty configurations are avoided. Here again all the link lengths will become collinear at various configurations and as we will see the linkage will move in an uncertain manner at this uncertainty configurations.

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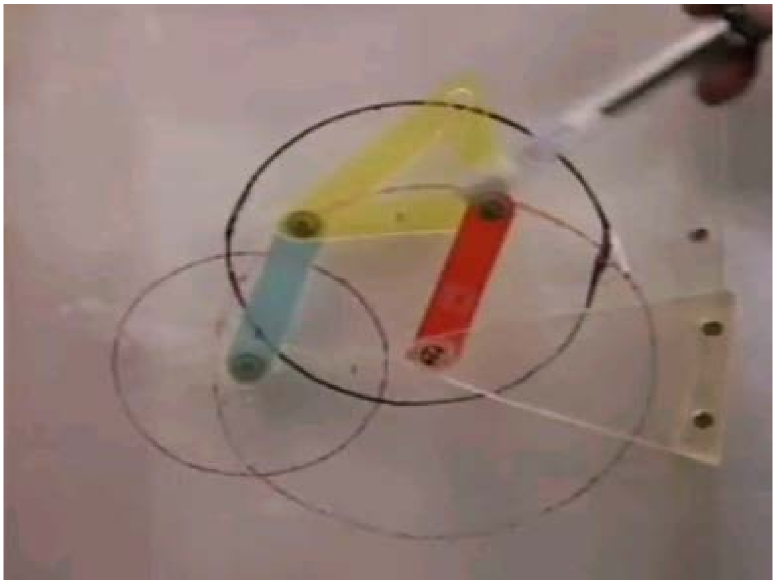

Let us now look at the model of this rhombus linkage. Here all the link-lengths are equal that is this length is equal to the coupler link, is equal to the follower link and also the frame link. All these four link lengths are equal as a result we get a rhombus. From this rhombus linkage, all four kinematic inversions will give double prime, just like a parallelogram. In this rhombus linkage also, as we see there are uncertainty configurations where all the four revolute pairs become collinear and at these uncertainty configurations the linkage moves in a non-unique fashion. If we maintain the rhombus it moves with a positive transmission from input to the output, whereas at this uncertainty configuration that linkage moves in a different way there is no transmission from this link 2 to link 4.

Again here, we get uncertainty configuration and there is no transmission from input to the output link. However one can maintain the rhombus and get positive transmission. This behavior is a very similar to the parallelogram linkage. We have discussed all types of 4R-linkages. Let us see how we can extend Grashof's criteria that are Grashof like criteria for 3R-1P linkage.

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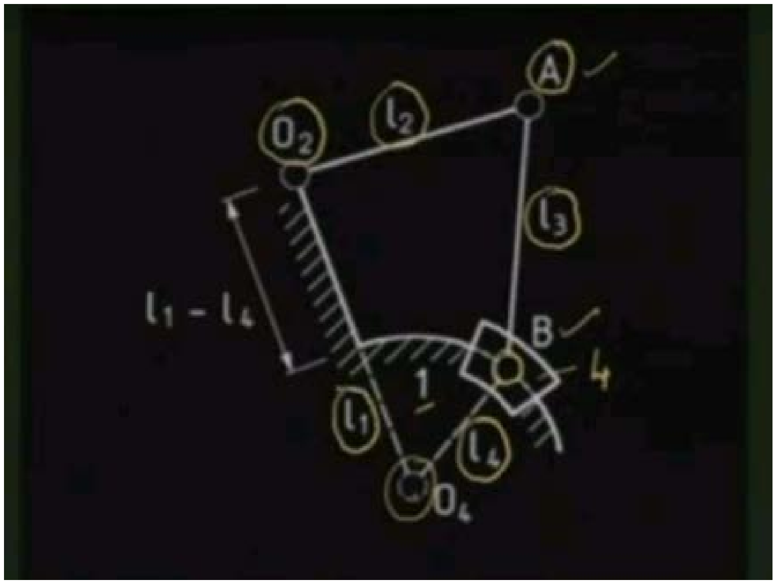

Let us recall that a car slider is nothing but a revolute pair. Look at this figure we have a revolute pair at O_2, a revolute pair at A and a revolute pair at B and a car slider between this link 4 and the fixed link that is link 1. If the centre of this circle of this car slider is at O_4 then this linkage is nothing but a 4R-linkage. With a revolute pair at O_2, A, B and O_4, we can see the kinematic dimensions, l_2 is a link-length which is obvious, l_3 is a link-length which is obvious and the other two links-lengths are O_4B which we call l_4 and O_2O_4 which we call l_1.

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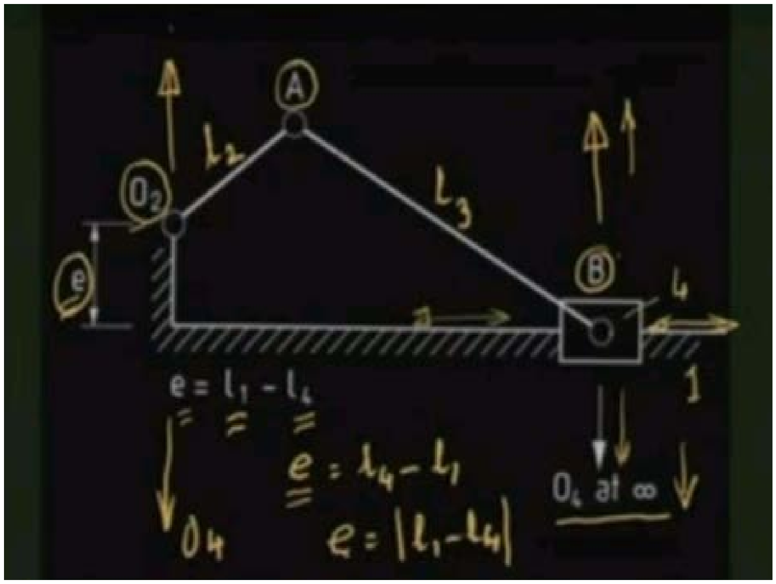

If we come to this 3R-1P linkage we have a revolute pair at O_2, we have a revolute pair at A and a revolute pair at B. Whereas between link 4 and 1 we have a horizontal prismatic pair. We can imagine, this 3R-1P linkage is equivalent to having a revolute pair O_4 at infinity in a direction perpendicular to the direction of sliding which is horizontal. We can think of a 4R-linkage O_2, A, B and O_4 where O_4 is at infinity.

Let us look at the kinematic dimensions, here we have l_2 the link-length O_2A and l_3 that is the link-length AB, whereas the offset which is this e, that is the perpendicular distance of O_2 from the direction of relative sliding passing through B which is this line, this we called offset which is e. Considering O_4 at vertical infinity, say in this direction or in this direction, because all these vertical lines meet at infinity then this e the offset is standing out to be O_2O_4 minus O_4B. If we call O_2O_4 as the l_1 and O_4B as l_4 then this offset is nothing but l_1 minus l_4. I could have considered this O_4 at infinity in the upward direction that is O_4 is at vertical infinity in the upward direction. Then we see that O_2O_4 which is l_1 and it is this O_4B which is l_4 I would have got e equal to l_4 minus l_1.

For 3R-1P mechanism I see there are two link-lengths namely l_1 and l_4 which are infinite. Difference of these two infinities either l_1 minus l_4 or l_4 minus l_1 is the other kinematic dimension which we call offset e. We can write e as the modulus of l_1 minus l_4 depending on whether I am considering O_4B in the vertically upward direction or vertically downward direction which will decide whether l_4 is more than l_1 or l_1 is more than l_4, the difference of these two is the offset e. Keeping this in mind, we can decide the Grashof like criterion, we see as we said e equal to l_4 minus l_1.

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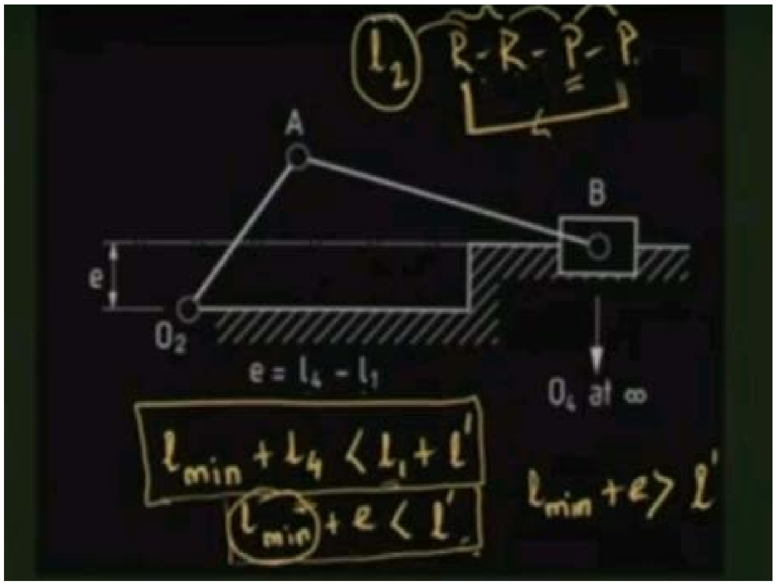

We say the Grashof condition turns out to be the shorter link-length l_{min} plus the longest link-length l_4 is less than the other two link-lengths that is the l_1 plus the other link-length which I for the time being, write l prime. l_4 minus l_1 is e this equation I can write l_{min} plus e less than l prime. l_4 the infinite link-length, l_1 is the another infinite link length but I have assumed l_4 to be more than l_1 so l_4 becomes l_{max}. l_{min} plus l_{max} less than l_1 plus l prime is what we called equivalent Grashof's criteria for 3R-1P linkage and that I can convert to l_{min} plus e less than l prime, Where e is the amount of offset l_{min} is the shorter link-length and l prime is the other link-length.

If the Grashof's condition is satisfied then the shorter link that is l_{min} can make complete rotation with respect to all other links and we can get a slider crank mechanism. Whereas, if it is a Non-Grashof slider crank that is l_{min} plus e is greater than the other link-length l prime, then no link can make complete rotation and we will unable to get slider crank mechanism, we will get a slider rocker mechanism.

To conclude in today's lecture, what we have seen the rotatability of 4R-linkage is most comprehensibly summarized by Grashof's criterion. When we apply it to a 4R-linkage we have seen that, l_{min} plus l_{max} less than l prime plus l double prime satisfies the Grashof's criteria and from a Grashof's linkage by kinematic inversion we can get all kinds of linkages. Then we have seen the motion characteristics of Non-Grashof linkage when Grashof condition is violated. We have also seen the boundary between the Grashof and Non-Grashof linkage, which we called transition linkages. Then we have seen special cases of transition linkages where the chain consists of two pairs of equal link lengths. At the end, we have also seen how we can modify the Grashof's criterion for a 3P-1P linkage and we got that l_{min} plus e less than l prime this is the equivalent Grashof's condition for a 3 R_1 P chain and if this Grashof's condition is satisfied then this shortest link can make complete revolution with respect to all other link and the shortest link can act as the crank of a slider crank mechanism.

I leave the students with a little problem, can we extend this Grashof criterion for an R-R-P-P type 4R-links. There is a link between these two revolute pairs and there are links between this revolute and prismatic, prismatic and prismatic and prismatic and this revolute.

If we recall, we had Scotch Yoke mechanism of this type and then elliptic trammel was a mechanism of this type and Oldham's coupling is a mechanism of this type. In such a linkage, as we see because there are prismatic pairs the link between connecting these R and P pair is of infinite length, because the equivalent revolute pair corresponding to this prismatic pair is at infinity. Similarly, this link which has both prismatic pair at its end is also of infinite length and this link connecting P and R pair is also infinite link-length. There is only one kinematic dimension, between these two revolute pairs let me call that is l_2. There is one kinematic dimension that is one link-length connecting two revolute pairs and all other link-lengths are of infinite length. Consequently, Grashof's condition is always satisfied as a result the shortest link l_2 will be able to make complete rotation. As we have seen, in the Scotch Yoke mechanism, the crank was always able to rotate completely or the other two links we find in the elliptic trammel and Oldham's coupling. The shortest link was able to make complete rotation with respect to all other links.