Kinematics of Machines

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Module - 2 Lecture - 1

The topic of today's lecture is mobility analysis. By mobility analysis, we obtain the degrees of freedom of a given mechanism. This is accomplished by the counting number of links and the number of different types of kinematic pairs those are used to connect these links.

Let me now elaborate, how we carry out this mobility analysis for planar mechanisms. It is worthwhile to recall that in a planar mechanism each link has 3 degrees of freedom: 2 of which are translational in the plane of motion and 1 is rotational about an axis perpendicular to this plane of motion.

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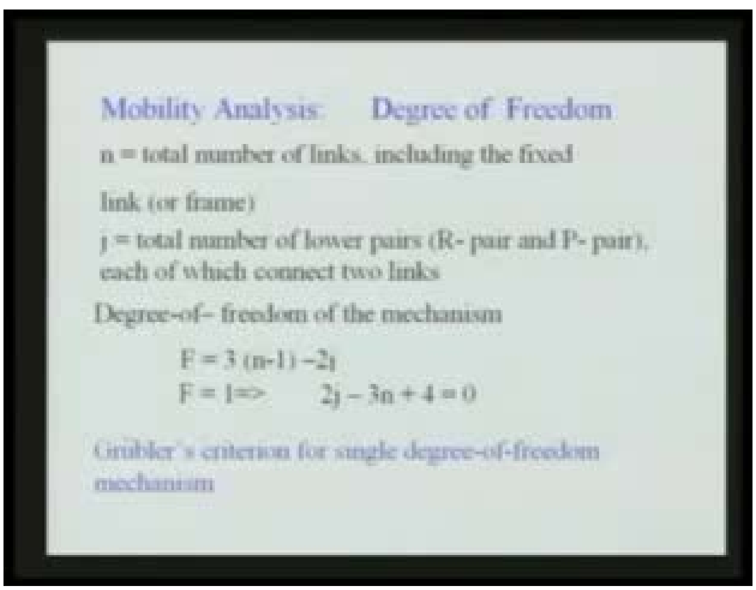

Let there be n number of total links in a mechanism, which includes the fixed link of the frame that means there are n minus 1 moving links. When these links are not connected by any kinematic pair then the total degrees of freedom is obviously 3 times n minus 1. For each of these n minus 1 links there are 3 degrees of freedom, so the total degree of freedom of the system is 3 times n minus 1. Let these n links be connected by j number of lower pairs. By lower pair in a planar mechanism, we can mean either a revolute pair or a prismatic pair and each of these kinematic pairs connects only 2 links. We also recall that whether it is a revolute pair or a lower pair, at each of these pairs, 2 degrees of freedom is cuttled and only 1 out of 3 is maintained. If there j number of total kinematic pairs 2 times j numbers of degrees of freedom are cuttled.

The effective degree of freedom of the mechanism is reduced to f, which are the degrees of freedom of the mechanism is 3 times n minus 1 minus 2 times j. Let us consider a constant mechanism with a single degree of freedom; that is, there exist a unique input-output relationship, where the degree of freedom of the mechanism F is 1. Substituting F equal to 1 in the above equation, we get 2j minus 3n plus 4 is equal to 0.

For a single degree of freedom mechanism, maintaining a unique input-output relationship, the number of links and the number of lower pairs must be related to this equation that is: 2j minus 3n plus 4 equal to 0. This equation is called Grubler's criterion for single degree of freedom mechanism. While deriving this Grubler's criterion, we assume that each of these lower pairs is connecting only 2 links. However, due to practical considerations some times more than 2 links can be connected at a particular hinge. As an example of different types of kinematic pairs which, connects more than 2 links let us consider this figure.

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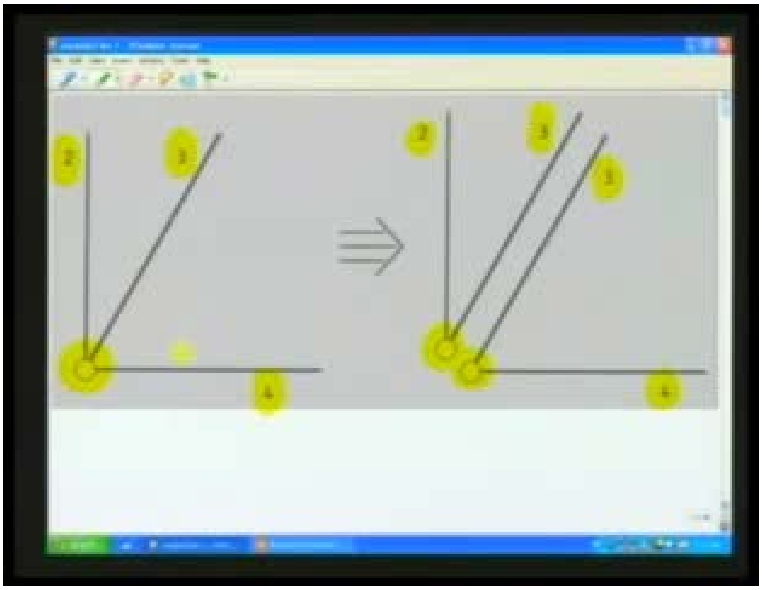

Here 3 links namely 2, 3 and 4 are connected by a single hinge at this location. Such hinges are called compound hinges or higher order hinges. This particular compound hinge is equivalent to two simple hinges as explained in the adjoining figure. For example, this particular hinge can be thought of as 2 hinges. One connecting link number 2 and link number 3, whereas another hinge connects link number 3 and link number 4.

Thus, a hinge which connects 3 different links is equivalent to 2 simple hinges. This way we can think of another type of hinge where 4 links are connected and such a hinge will obviously be equivalent to 3 simple hinges. Maintaining this equivalent between higher order hinges and simple hinges, we would like to modify the equation for calculating the degrees of freedom of a mechanism as follows. When higher order hinges are present, the symbol j in the equation, we would like to modify as follows.

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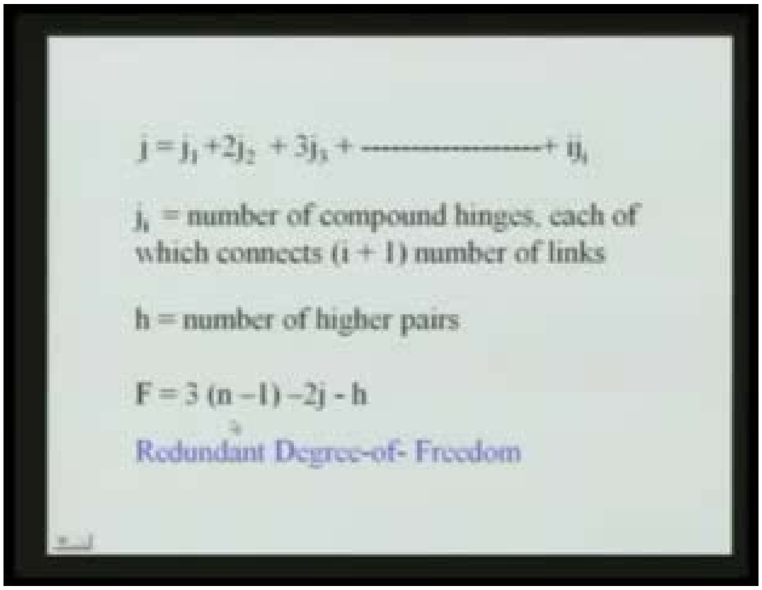

j is equal to j_1, which represents the number of simple hinges, which connects only 2 links plus $2j_2$, where j_2 is the number of hinges to each one of which connects 3 links and so on; that is j_3 represents the number of hinges each one of which connects 3 plus 1, that is 4 links and so on up to j_i. j_i is the number of compound hinges each of which connects i plus 1 number of links. In a mechanism, there can be higher pair as well and as we recall, if there is a higher pair then at each higher pair only 1 translational degree of freedom is cuttled that is along the common normal to the point or line of contact. Two other degrees of freedom can be retained. Consequently, at each higher pair only 1 degree of freedom is cuttled.

I would like to modify the equation, the degrees of freedom of a mechanism F is equal to 3 times n minus 1 minus 2j minus h, where h represents the number of higher pairs, j represents the number of equivalent simple hinges and n represents the number of total links.

Sometimes there can be some redundant degree of freedom of a mechanism. What do we mean by a redundant degree of freedom? Due to some typical kinematic pairs and their placement, we may find that in a mechanism a particular link may be moved without transmitting any motion to any other link. Such a degree of freedom is referred to as redundant degree of freedom.

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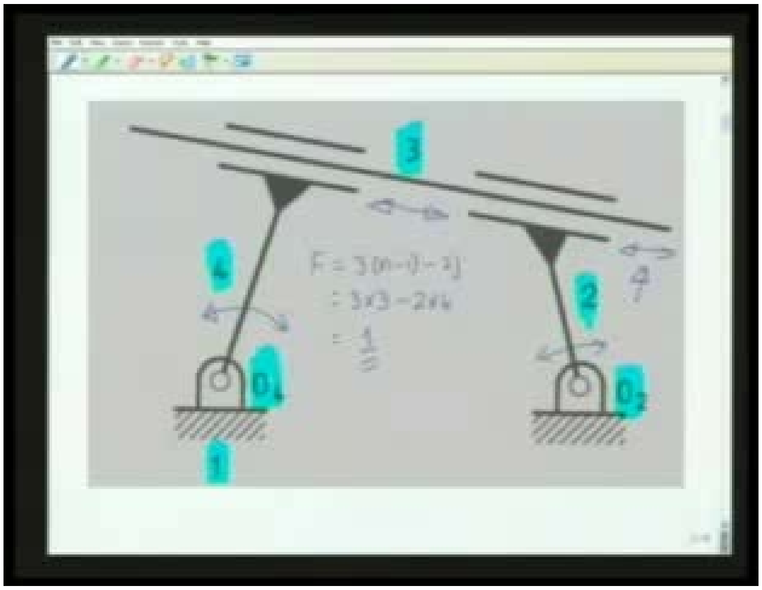

Let me now explain some redundant degrees of freedom and how to take care of that in the equation so that we get the effective degrees of freedom. As an example of a redundant degree of freedom, let us look at this 4-link mechanism, where we have link number 1 which is the fixed link; link number 2 which is connected to link number 1 through this revolute pair at O_2. There is link number 4, which is connected to link number 1 through this revolute pair at $O_{4.}$ Link number 3, has 2 prismatic pairs connecting it to link number 2 and link number 4.

The thing to be noted is that the direction of this revolute pair is same; both this prismatic pair the direction is along this link 3. Consequently, link 3 can be dragged along this direction without transferring any motion either to link 2 or to link 4. Consequently, this constitutes a redundant degree of freedom. If we apply the formula bluntly, that is F is equal to 3 times n minus 1 minus 2j, we get, there are 4 links, so this is 3 into 4 minus 1 three minus 2. There are 4 kinematic pairs - 2 revolute and 2 prismatic. So 2 into 4 is equal to 1. It appears according to the formula that this is a single degree freedom mechanism implying unique input-output relationship. However, this link 2 or link 4 cannot be moved at all. This is permanently locked; so this acts like a structure. What is this degree of freedom 1? That is nothing but, this redundant degree of freedom of the link 3 along this direction of the prismatic pairs.

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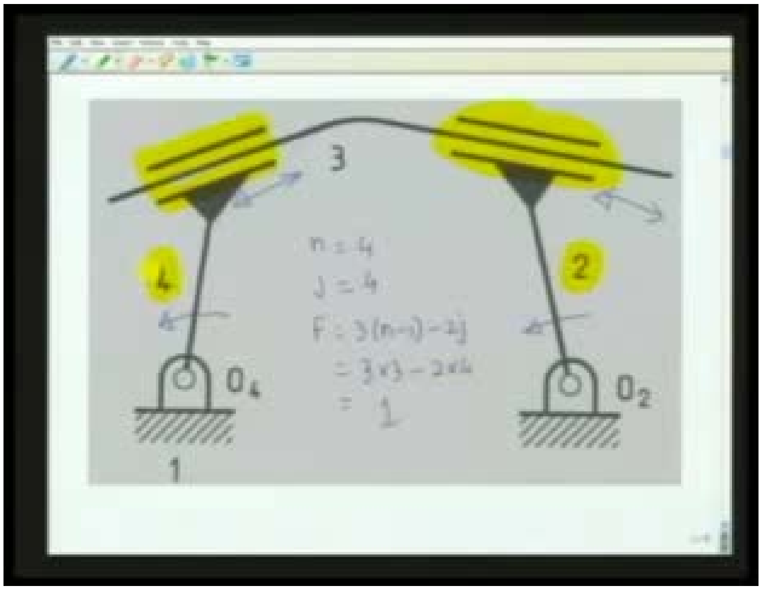

It may be interesting to see what happens if the directions of the 2 prismatic pairs are different. Between 3 and 4, it is in this direction; whereas, between 3 and 2 it is along a different direction. Consequently, here the formula will work perfectly, because, there is no redundant degree of freedom. We cannot move link 3 without transferring motions to links 2 and 4. So, here n is 4, j is 4 as we obtained earlier and F is equal to 3 times n minus 1 minus 2j is again 3 into 3 minus into 2 into 4 which is equal to 1.

Actually, here link number 2 can be moved to transmit motion to link number 4. A little thought would convince that the rotation of link 2 and link 4 must be identical. Let me explain why. As we see, link 2 and link 3 has a prismatic pair here, which means there is no relative notation between link number 2 and 3. Similarly, there is a prismatic pair here between link 3 and link 4. So there cannot be any relative rotation between link number 3 and link number 4. Consequently, there cannot be any relative rotation between link number 2 and link number 4, both of which are in translation with respect to link number 3. What is the implication? That there is no relative rotation between links 2 and 4. Both of them rotate but they rotate by the same amount, so that, there is no relative rotation.

Let me now take another example of a redundant degree of freedom which is very commonly seen.

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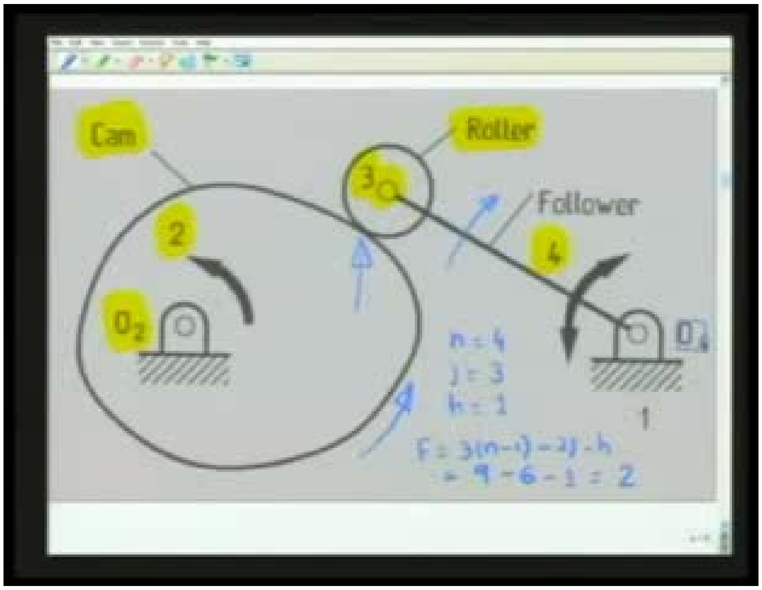

In this figure, we see what is known as a cam follower mechanism and we have a roller follower. Cam is this input link which is number 2, which is hinged to link number 1 the fixed link at this revolute joint at O_2. Follower that is link number 4 is hinged to roller at this revolute pair; roller is the link number 3. It is intuitively pretty obvious that if we move link number 2, say I give it a rotation then the follower will also have a rotation in this direction.

There exists a unique input-output relationship, unique rotations of link 2 causes unique rotation of link 4. Let me calculate the degree of freedom. As we have seen, there is a unique input-output relationship depending on the shape of the cam profile, so the degree of freedom should turn out to be 1. But, let me do it by counting according to our formula. We have already seen that there are 4 links; so n is 4. There are three revolute pairs: one between 1 and 2, one between 1 and 4 at O_4 and one between 3 and 4, at the roller centre; so j is 3.

There is a higher pair between link number 2 and 3 at this point; so h is 1. If we calculate the degree of freedom F, which is 3 times n minus 1 minus 2j minus h, which is 3 into 3 is 9 minus 2 into j is 6, minus h, that is 1, which gives 2. So, the degree of freedom according to the formula is standing out to be true, because, there is a redundant degree of freedom and that is, roller 3 can be rotated about this revolute pair without transferring any motion either to link 2 or link 4. So, that is the redundant degree of freedom. So F_r if we call as the redundant degree of freedom, F_r

is 1. In view of this redundant degree of freedom, let us modify our equation which we obtained earlier.

Now that we have seen there can be some redundant degrees of freedom, let us now modify the formula in view of this.

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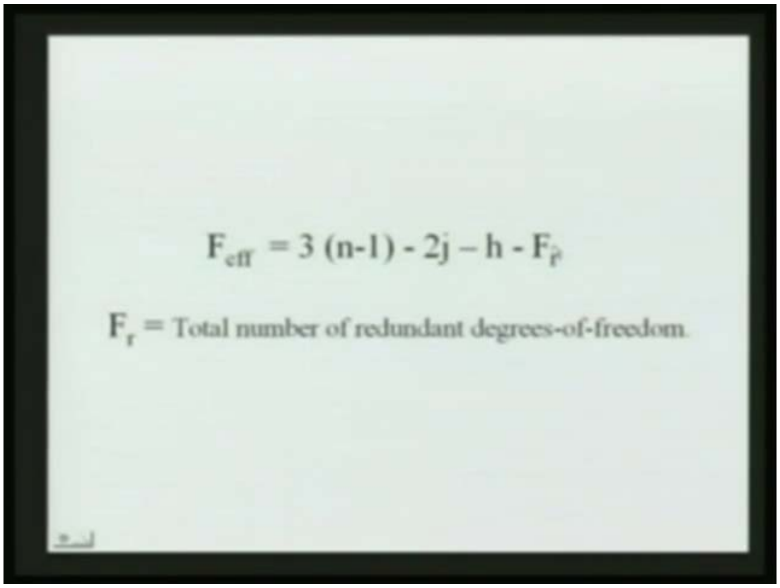

F_{eff} that is the really the input-output relationship is governed by F_{eff} is given by 3 times n minus 1 minus 2j minus h minus F_r, where F_r is the total number of redundant degrees of freedom. Sometimes due to some other practical considerations, a mechanism may have some redundant kinematic pairs, which means, those kinematic pairs are not kinematically important, but they may be required due to some other considerations. The simplest example is a shaft is normally mounted on 2 bearings, but both the bearings act as 1 revolute pair permitting rotation about the same axis. By counting we may call it 2, but kinematically, that is only 1 revolute pair. Let me show an example of such redundant kinematic pair.

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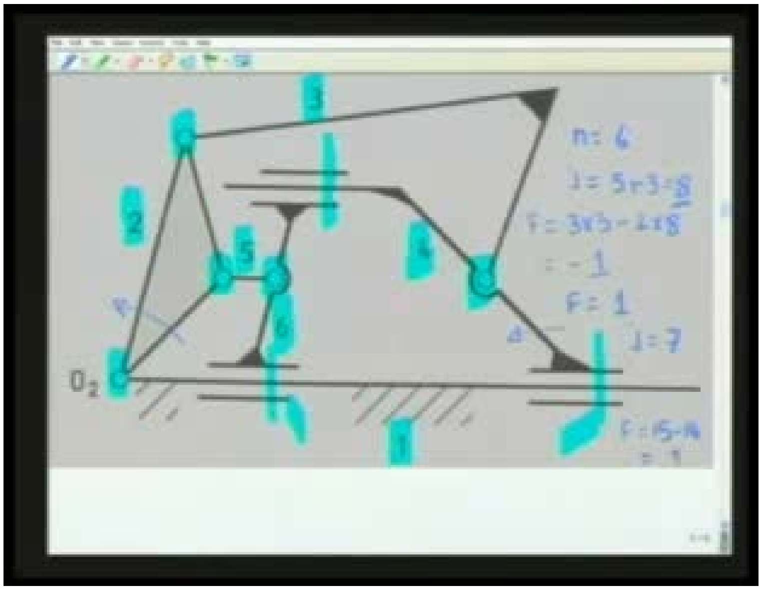

Here, we consider a 6-mechanism. This is link number 1 which is fixed, which is connected to link number 2 through a revolute pair at O_2. This is link number 3 having a revolute pair between 2 and 3 here. 4 is the next link which is connected to 3 by this revolute pair. 5 is this link which is connected to link number 2 by this revolute pair. 5 is connected to 4 by this prismatic pair here. 6 is another link which is connected to link number 5 through this revolute pair. 6 is connected to 4 by this prismatic pair. 6 is connected to 1 by this prismatic pair and 4 is connected to 1 by this prismatic pair.

Let me apply the formula and try to find the degree of freedom of this mechanism. Here, we have n is 6. All these pairs are simple pairs because they connect only 2 links. So j we count there are 1, 2, 3, 4, 5 revolute pairs and 3 prismatic pairs, so j is 8. Consequently, the degree of freedom of the mechanism F is 3 times 6 minus 1 that is 5 minus twice of j that is 2 times 8 and we get 15 minus 16, that is, minus 1. According to the formula, this mechanism is a structure rather a statically indeterminate structure with negative degrees of freedom and no relative motion should be possible between various links. However, this as we see shortly, has degree of freedom 1 and there is a unique input-output relationship that means, if I use link 2 as my input link, I rotate it, link 4, which I may treat as output link will have some motion.

Why is this calculation failing? This is because if we notice these 3 revolute pairs, we should note that all this 3 prismatic pairs are in the same direction. This prismatic pair is allowing horizontal translation between link number 1 and link number 6 (Refer Slide Time: 17:58 min). This prismatic pair here is allowing relative translation in the horizontal direction between link number 1 and link number 4. This prismatic pair which is there to ensure horizontal translation between link 4 and link 6 may be redundant. Even we can replace, we can withdraw, any of this 3 prismatic pairs because all of these are ensuring horizontal translation between links 1, 4 and 6. Thus, j which we counted previously as 8 is actually j is 7 because, kinematically, 1 of these 3 prismatic pairs is redundant. So, I can remove this as a redundant pair and make j equal to 7, which will give me F is equal to 15 minus 2 into 7, 14, which is 1. Now that we have seen there is a possibility in an actual mechanism to have some redundant kinematic pairs, let us rewrite the formula in the light of such redundant kinematic pairs.

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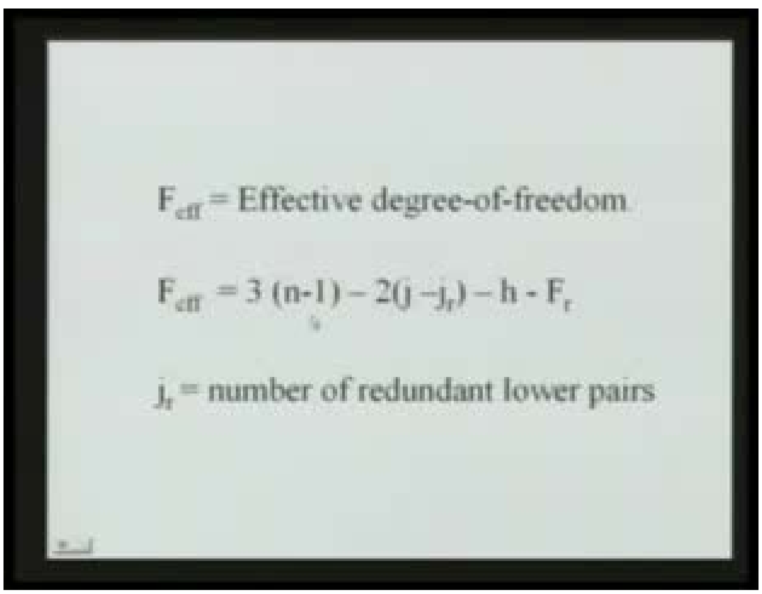

If F_{eff} implies the effective degree of freedom of a mechanism that is given by 3 times n minus 1 minus 2 times j minus j_r minus h minus F_r, where F_r was the redundant degrees of freedom, h was the number of higher pairs, j_r is the number of redundant kinematic pairs, j is the total number of lower pairs and n is the total number of link. Thus, we arrive at a formula by counting the number of links and considering the different types of pairs and redundant degrees of freedom and redundant kinematic pair, we are in a position to calculate the effective degrees of freedom of a planar mechanism. At this stage, I would like to emphasize a very subtle difference between this revolute pairs and prismatic pairs. So far this formula is concerned, we have not made any distinction between a revolute pair and a prismatic pair because, both types of pairs cuttled 2 degrees of freedom and allowed 1 degree of freedom. Let me now point out what is this subtle difference.

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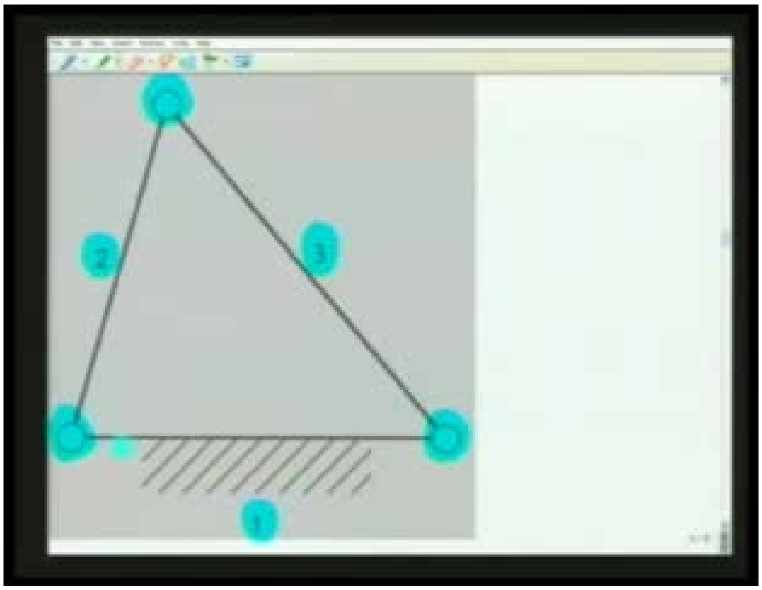

Let us notice this 3-link closed mechanism consisting of only 3 revolute pairs: Link number 1, link number 2 and link number 3 constitutes a closed kinematic chain consisting of 3 revolute pairs. We are already familiar with this and we have seen that this is not a mechanism. It is a structure; no relative motion between various links is possible when all these pairs are revolute pairs. Let us see what happens if all 3 becomes prismatic pairs in different directions.

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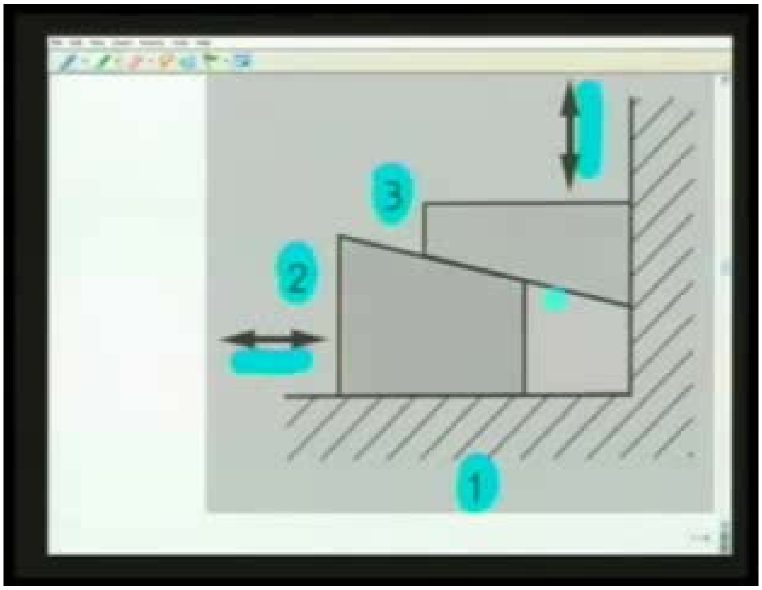

This is again; there are 3 links: link 1, link 2 and link 3. This constitutes a closed kinematic chain and there are 3 prismatic pairs. One in this horizontal direction between link 1 and 2, one in the vertical direction between link 1 and 3 and there is one in this inclined direction between links 2 and 3.

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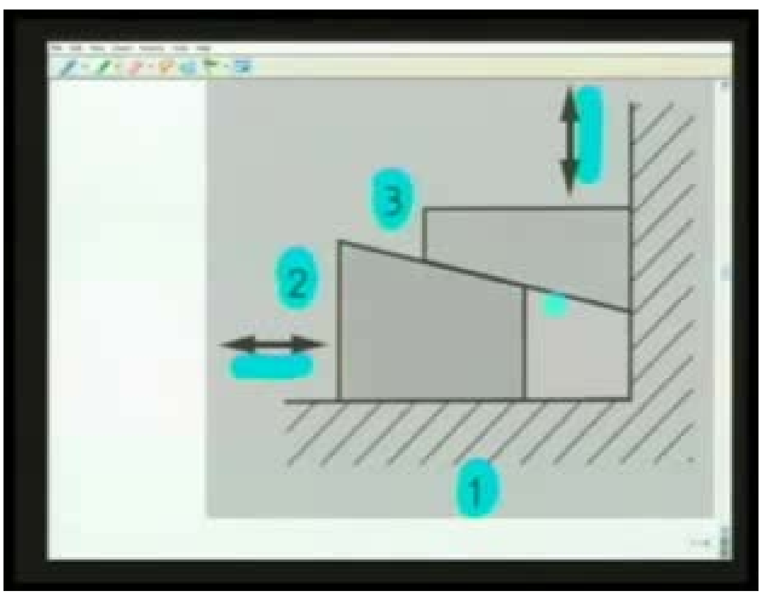

The kinematic representation of this is as follows: There are a three links: links 1, 2 and 3 having 3 prismatic pairs in different directions. It is obvious that here relative motion between various links is possible; it is not a structure, the degree of freedom of this loop is not zero.

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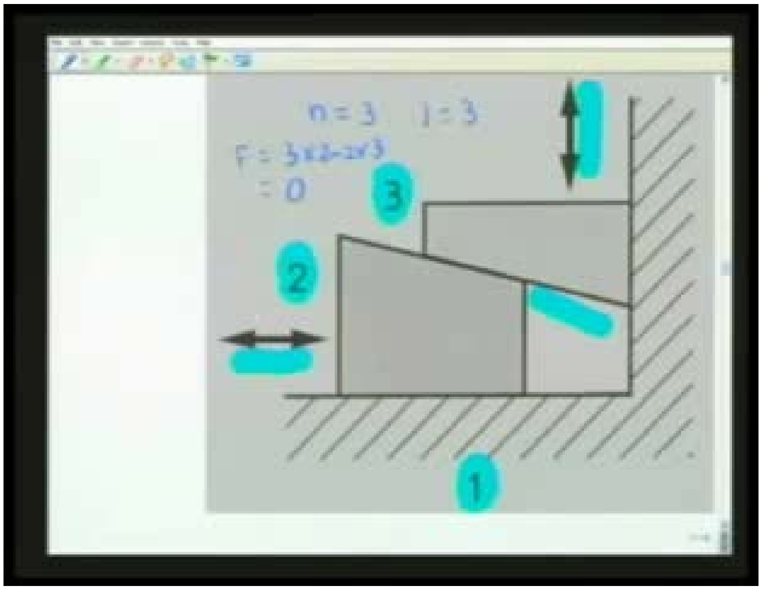

As we can see link 2 can be moved in the horizontal direction to produce a unique vertical movement for link 3. Thus, for this particular closed loop mechanism, n is 3, j is also 3. So according to the formula, we should have had 3 into n minus 1, that is 2 minus 2j that is 2 into 3 is 0, which is true for the revolute pairs, but not true for the prismatic pair. In light of this difference between revolute and prismatic pair, let us modify our formula for calculating the degrees of freedom. In view of this single degree of freedom, closed loop which is possible by 3 prismatic pairs connecting 3 links, let us modify the formula for calculating the effective degrees of freedom.

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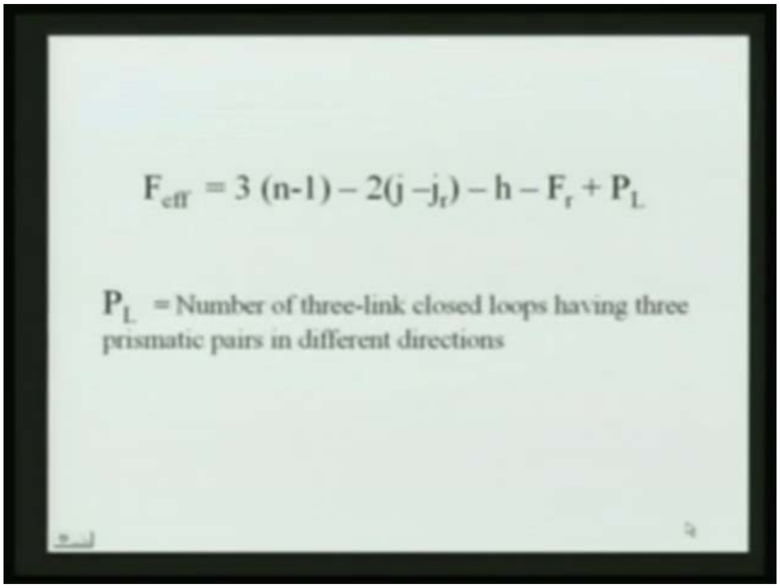

I would like to say F_{eff} is equal to 3 times n minus 1 into minus 2 times j minus j_r minus h minus F_r plus P_L, where P_L is the number of 3 link closed loops having 3 prismatic pairs in different directions. While deriving this formula, we have not bothered with the kinematic dimensions of the mechanism. So, this formula may have some exceptions for some very special kinematic dimensions, as we shall see shortly through a number of examples. We have already said that due to some special kinematic dimensions the formula that we derived may give wrong result.

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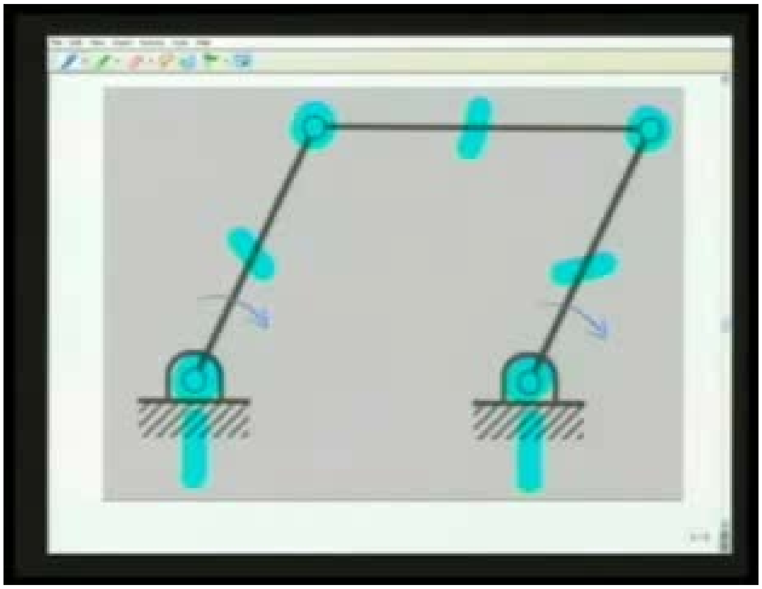

As an example, let us talk of this parallelogram linkage. This is a 4-link mechanism with 4 revolute pairs, but the opposite sides have equal lengths. These 2 links are of same lengths and this coupler length is equal to the frame length, that is, the distance between these 2 fixed pivots. Obviously, this is a 4R mechanism, which is degree of freedom 1 and it can transmit motion from this link to that link. During this movement, the opposite sides always remain of same length; so a parallelogram remains a parallelogram.

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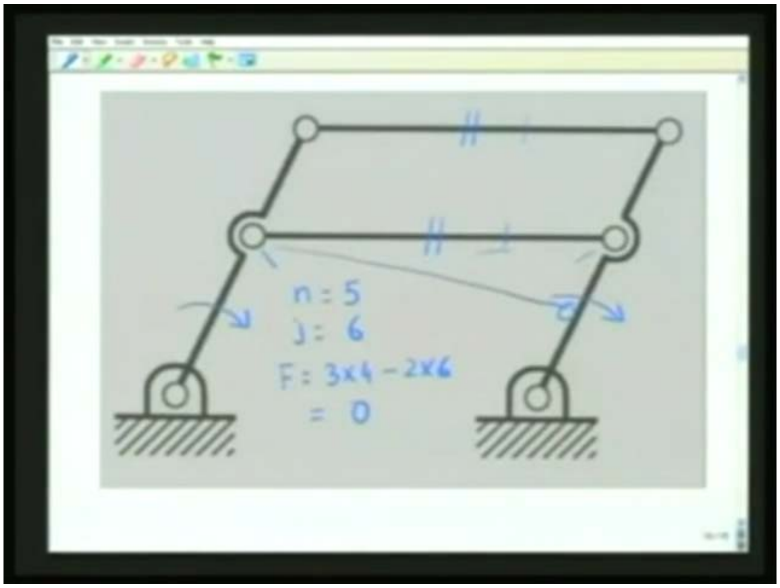

In this parallelogram linkage, if we add an extra coupler which is parallel to the original coupler then what happens? As we see now n has become 5 and due to this extra coupler, we have introduced 2 revolute pairs at its 2 ends: one there and one there. So, j has become 6. Consequently, from the formula, we get F equal to 3 times 5 minus 1, that is 4 minus 2 times j, that is 2 times 6, which is 0.

So the formula tells us that, this is structure, but intuitively we can realize that this extra coupler has not imposed any extra constant and the mechanism still retains its single degree of freedom and this moves like a parallelogram as before. Of course, this failure of the formula is only because these 2 couplers are parallel and the original diagram was a parallelogram. If this coupler, extra coupler, I introduced in an inclined fashion, say starting from this point to this point (Refer Slide Time: 26:15 min), then the formula will be correct and the assembly will become a structure. In fact, such an extra coupler is normally used to drive a parallelogram mechanism.

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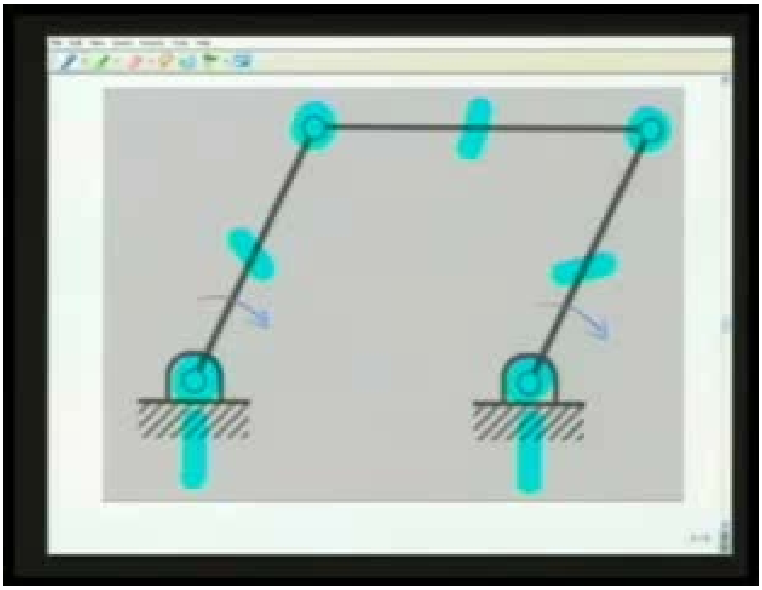

As we shall see in a model that, when the parallelogram moves, there is a configuration when all the links become collinear and that mechanism loses its transmission quality. In fact, it can go into a non-parallelogram or anti-parallelogram configuration. To ensure that a parallelogram always remains a parallelogram such an extra coupler is necessary.

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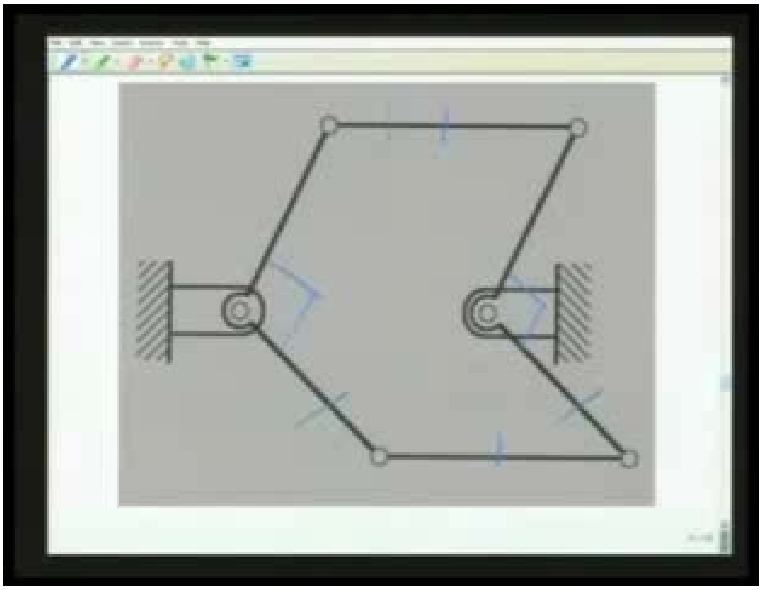

In fact, to maintain the good transmission quality at all configurations, these 2 extra couplers are connected to the input and output link by making a 90 degree angle between the extensions of this input link and the output link, such that, when this particular coupler is collinear with the line of frame, the other coupler is parallel to the line of frame, this portion of the links become perpendicular to the line of frame. This point will be much clearer when you demonstrate it through a model.

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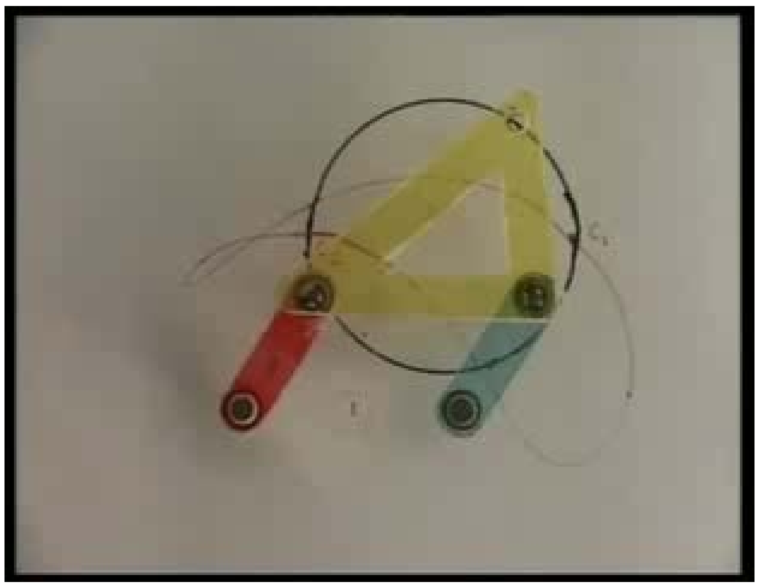

Let us now look at the model of this parallelogram linkage. Here this red link and this blue link are of equal link length. This coupler which is the yellow link has the same length as the fixed link or the distance between the 2 fixed pivots. As we see this parallelogram linkage when it moves always remains a parallelogram. However, when all the 4 links become collinear, there is a possibility that it flips into anti-parallelogram configuration and it does not move as a parallelogram linkage. Again, here, if sufficient care is taken, one may transfer it to a parallelogram linkage. To get rid of this uncertainty configuration, it is better to have an extra coupler as explained earlier and we shall demonstrate it through our next model.

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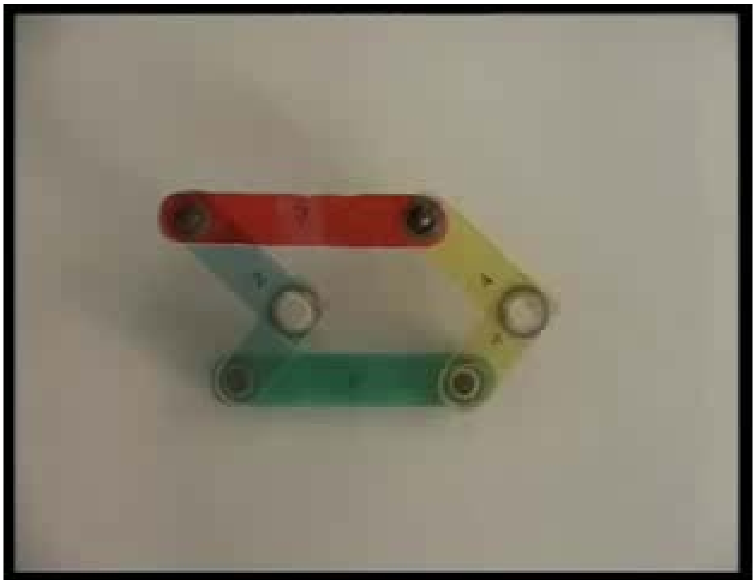

Let us now look at the model of this parallelogram linkage with a redundant coupler. As we see these 2 links are extended at 90 degrees and there are 2 parallel couplers. Consequently, here we shall be able to maintain the parallelogram configuration throughout the cycle of motion. It can never flip back into anti-parallelogram configuration.

As we have just seen that for very special kinematic dimensions, the formula for calculating the degrees of freedom may fail. In fact, when the formula was telling that the degree of freedom is 0, we are getting single degree of freedom mechanism. For special kinematic dimensions, when the degree of freedom calculation fails, according to the formula, such linkages are called over closed linkages.

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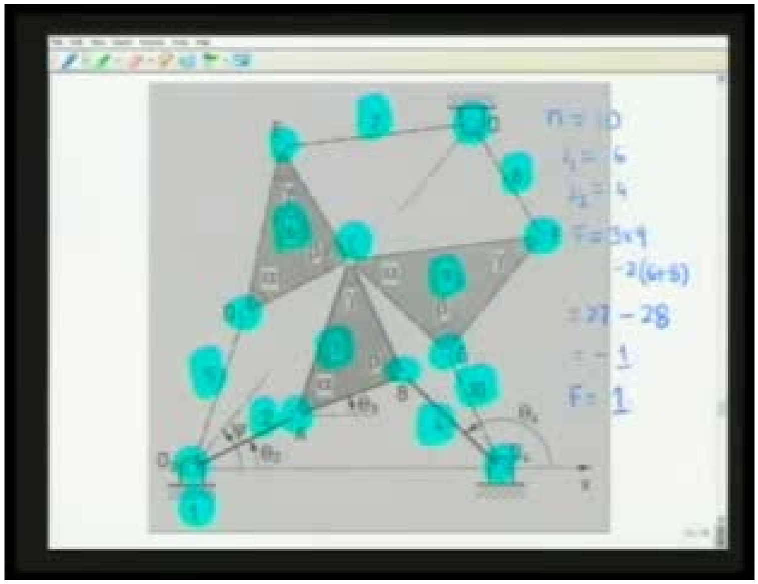

As a further example of over close linkages, let us look at this 10-link mechanism. Here, we have link number 1, which is the fixed link; link 2 connected to link 3 connected to link 4 which in turn is again connected to link 1. That means, we get a simple 4-bar mechanism. There is another 4-bar mechanism: link 8, link 9, link 10 and link 1. There is a third 4-bar mechanism consisting of: link 7, link 6, link 5 and link 1. All these 4-bar mechanisms are connected at this revolute pair C.

So, in all we have 10 linked mechanisms and let me also see, what typical revolute pairs are there. There is a revolute pair at O_2 which connects 3 links namely 1, 2 and 5. There is a revolute pair at O_4 which again connects 3 links namely 1, 4 and 10. There is a revolute pair at O which again connects 3 links namely 7, 8 and 1. There is a revolute pair at C which connects 3 links namely 3, 6 and 9. These hinges are of j_2 category and thus we have 4 such hinges of j_2 category. There are simple hinges at A, at B, at G, at F, at E and at D.

Let us try to calculate the degrees of freedom of this particular mechanism. We have already seen n, which is the total number of links are 10. j_1 - that is the number of simple hinges which are at A, B, G, F, E and D that is j_1 is 6; number of compound hinges each one of which connects 3 links, that is j_2 is at O_2, O_4, O and C that is j_2 is equal to 4. Degree of freedom of this mechanism according to the formula is F equal to 3 times n minus 1 that is 10 minus 1, 9, minus

2 times, that is j_1, that is 6 plus 2 times j_2 that is 2 into 4 is equal to 8, that is 27 minus 14 into 2 equals to 28, which is minus 1. So, without any special dimensions this assembly is a structure with degree of freedom minus 1. However, if we look at this figure what we see that O_2 A C D is a parallelogram; O_4 G C B that is another parallelogram and O F C E is another parallelogram. Not only that this ternary links that is number 3, number 9 and number 6, all these 3 ternary links are similar triangles as indicated by the angles alpha, beta and gamma. Due to these special dimensions, we will find that the degree of freedom of this assembly will become equal to 1. This is another example of an over closed linkage, where some of the constants may be redundant, but this will not be highlighted in this lecture. We will just show you the model of this particular mechanism.

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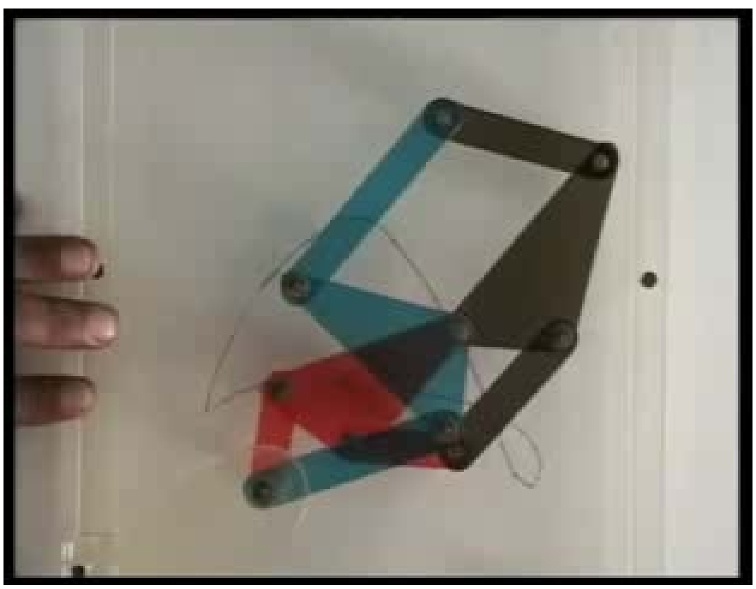

Let us now look at the model of this 10-link mechanism which has just been discussed. As we have seen according to the calculation the degree of freedom should have been minus 1, but notice that these 4 hinges constitutes a parallelogram; so does these 4. These 4 hinges also constitute another parallelogram. These 3 triangles, the ternary links are similar to each other. Consequently, this constitutes a single degree freedom mechanism, which is an over closed linkage which has mobility; it is not a structure.

As the further example of an over closed linkage, let us consider this 8-link mechanism which is known as Kempe Burmeister focal mechanism.

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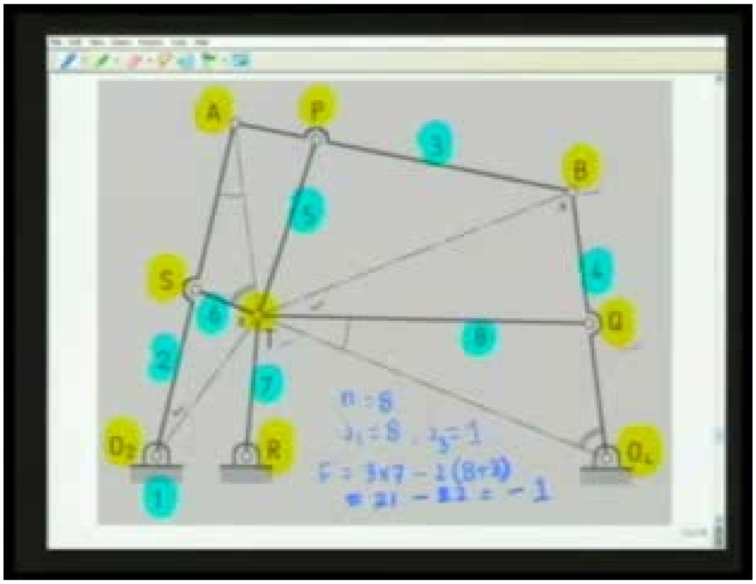

As we see, there are 8 links: link 1, link 2, link 3, 4, 5, 6, 7 and 8. These 8 links are connected by revolute pairs one at O_2, at S, A, P, B, Q, O_4 and R. There are 8 simple hinges and there is a higher order hinge at this point T where 4 links namely 5, 6, 7 and 8 are connected. So, if we calculate the degree of freedom, we see n is 8, j_1 is 8, j_2 is 0, but there is a j_3 at T, where 4 links are connected so j_3 is 1. The degree of freedom F is 3 into n minus 1, that is 7 minus twice j_1 which is 8 plus 3 times j_3 which is 3 into 1 is equal to 3. We get 21 minus 2 into 11 that is 22 which is minus 1. According to the formula, this should be a structure. However, for very special dimensions, as indicated by these similar triangles BTQ with O_2TS, this angle is equal to this angle and this angle is equal to this angle (Refer Slide Time: 36:34 min). Similarly, there are other similar triangles in this figure. For such special dimensions as we see in our model the degree of freedom will turn out to be 1. F will be 1that means it will be a constant mechanism with single degree of freedom.

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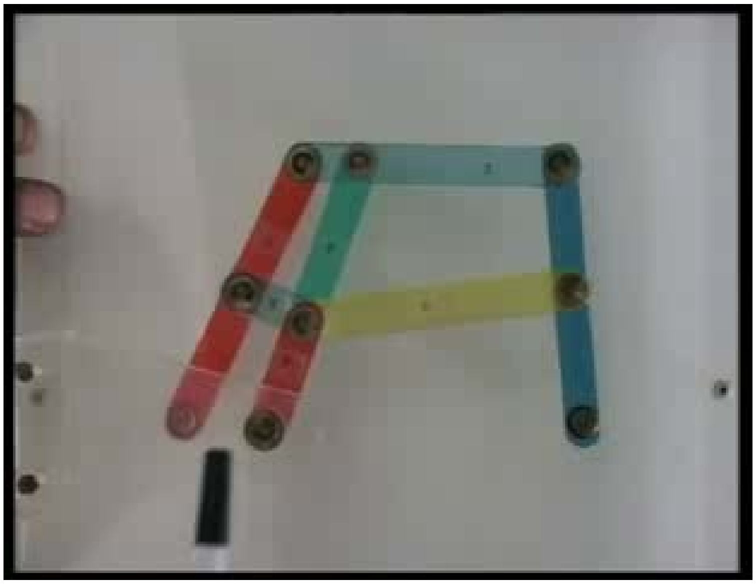

Let us now consider the model of this Kempe Burmeister focal mechanism, which we have just discussed. As we see including this fixed link, we have 8 links: 2, 3, 4, 5, 6, 7, 8 and this is a hinge where 4 links are connected and all other hinges are simple hinges. Accordingly, the formula said the degree of freedom should be minus 1. But however, as we see this mechanism can be moved very easily and there is a unique input-output relationship. That means, the effective degree of freedom of this mechanism is 1; that is only because of the special dimension. If we change any of these points a little bit this will really become a structure and no relative movement would be possible.

As a last example of an over closed linkage, let us look at this 5-link mechanism which is known as cross slider trammel.

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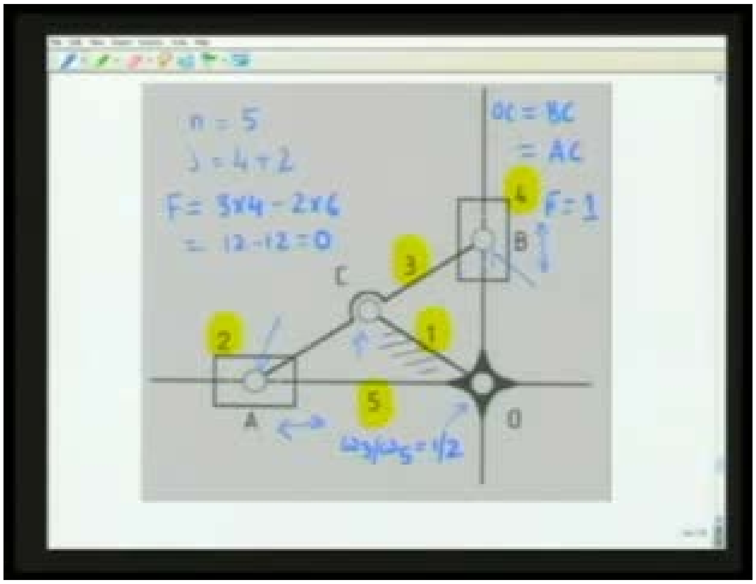

Here we have link 1 which is the fixed link. Link 3 which is connected to link 1 and link 3 is connected to link 4 and link 2. Link number 4 and 2 are having prismatic pairs with link number 5. Link 2 has a prismatic pair in the horizontal direction with link number 5. Link 4 has a prismatic pair in the vertical direction with link number 5. Thus, we have n equal to 5; we have 4 revolute pairs here and here and here and here (Refer Slide Time: 38:46 min). So, j is 4 revolute pairs plus 2 prismatic pairs. Thus, F turns out to be according to the formula 3 times 5 minus 1 which is 4, minus 2 times j that is 6.

So, the effective degree of freedom of this mechanism according to the formula: F is 3 times n minus 1 which is 4 minus 2 times j which is 6; that is F equal to 12 minus 12 which is 0. Without any special dimension, this will be a structure; there should not be any mobility any relative movement. However, for these special dimensions when I make OC is same as BC is same as AC then, we will see that there will be an effective degree of freedom of this mechanism will turn out to be just 1. In fact, as we see the angular velocity of link number 3 to that of link number 5 which are both rotating with respect to fixed link 1 will be exactly half for these special dimensions. This is known as cross slider trammel and I would like to encourage the student to show that why it moves by starting from the elliptic trammel that we have discussed in an earlier lecture.

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Now, I shall demonstrate this cross slider trammel through a model. Let us now look at the model of this cross slider trammel. This is that link number 3 (Refer Slide Time: 40:52 min), which has a revolute pair with fixed link here. This is that link number 5 (Refer Slide Time: 40:58 min), which has a revolute pair with fixed link here. There are 2 sliders - 2 and 4 which are hinged to link number 3 here and here (Refer Slide Time: 41:08 min) and these 2 sliders move in these 2 perpendicular slots. For the special dimensions, as we see this has degree of freedom 1 and rotation of link 3 produces unique rotation of link 5. In fact, we can see that 2 revolutions of link 3 produces 1 revolution of link 5. That is, one can show that omega_3 by omega_5 at all instance remain half.

Let me now summarize, what has been covered in today's lecture. What we have seen how we can calculate the degrees of freedom of a planar mechanism by counting the number of links and different times of kinematic pairs. Attention has been also drawn to the fact that there is a possibility of some redundant degrees of freedom that has to be accounted for.

We have also seen there may be some kinematic pairs which are redundant in the sense they do not serve any purpose so far kinematics is concerned, but they must be there due to some other practical considerations.

At the end we have seen, that these formulas which are derived only from the count without any consideration of any kinematic dimensions may fail when there are some special kinematic dimensions. We have also seen some such over closed linkages through the models, how they move, though the formula says they should be structures.