Kinematics of Machines

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Module-13 Lecture-2

So far, we have discussed gears with reference to a pair of gears, but very often the input and the output shaft are connected by more number of gears and such a combination of gears to connect the input to the output shaft is called a gear train.

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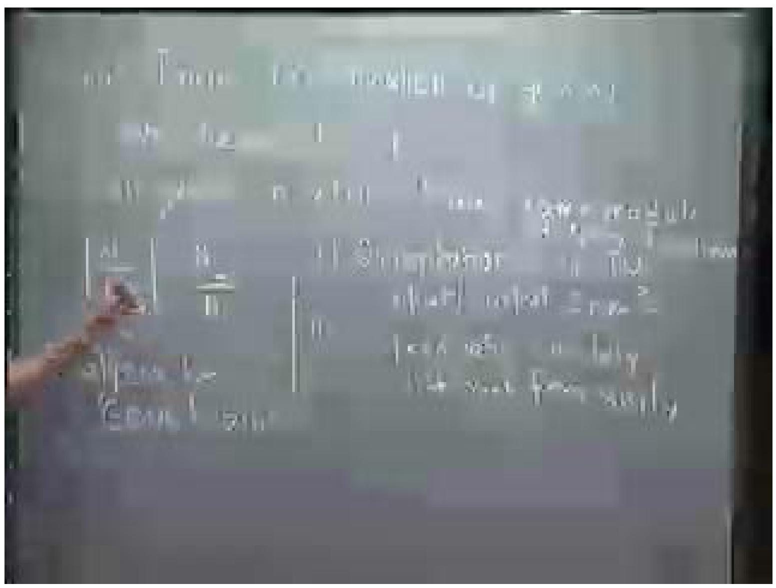

Gear train is nothing but a combination of gears. We can use to connect the input shaft to the output shaft. Now, a gear train may consist of all types of gears, like spur. The same gear train may consist of a few spur gears, few helical gears and bevel gears like that. So a combination of gears which is used to connect the input to the output shaft on the gear train and a gear train may consist of all types of gears or maybe only one type of gear. Now, unless other is specified, we will always assume that all gears in a train have same module. There are exceptional gear trains, where all gears do not have the same module but in general, all gears in a gear train will be assumed to have the same module, which means if two gears say 1 and 2 are in engagement, then their speed ratio will be given by the inverse ratio of the number of teeth. This is angular speed of gear 1 and gear 2 and these are the number of teeth on gear 2 and gear 1. It is an inverse ratio.

As we know, if this external gearing then $omega_1$ by $omega_2$ will have a negative sign and if it is internal gearing, then $omega_1$ by $omega_2$ will have a positive sign that we have to keep track of. Now the question is, why do we need more number of gears to connect a particular input shaft to the output shaft? There are various results for that. For example, the orientation of the input and output shaft may dictate that we need to have more than two gears orientation and spatial location. The input and output speed may vary by a wide margin, that is the ratio of the input and output speed is very much different from 1. It is much larger than 1 or much less than 1. In such cases also, we may have to use a gear train. The speed ratio of the input to the output speed is widely different from 1.

If the speed is say 60, then we cannot get by using only a pair of gear to connect the input and output shaft, to create a speed ratio 1 by 60 or 50. So under such situations, we need gear train. In this lecture, we will do this analysis of gear train. If a gear train is given and the numbers of teeth on the gears are specified, we should be able to find out, what is the speed ratio between the output and input shaft. Now, I will show a figure of a particular gear train which consists of all kinds of gear.

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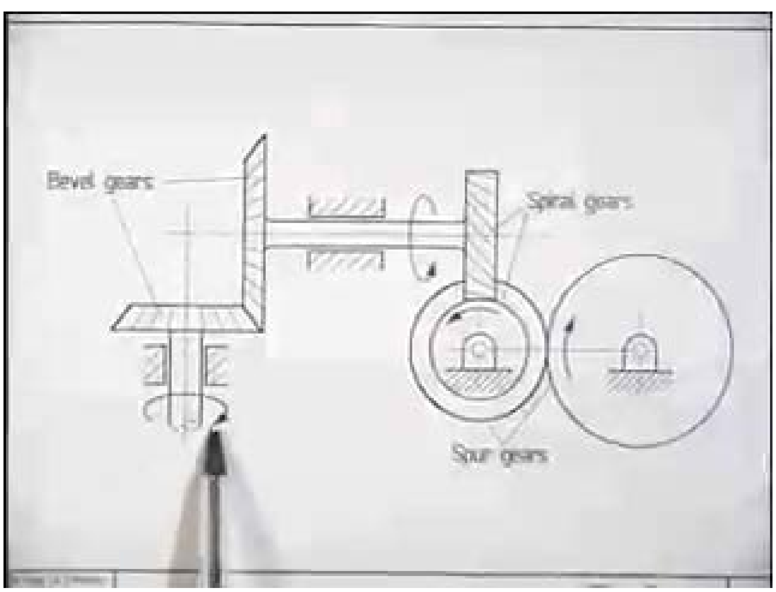

This figure shows a typical gear train. This is the input shaft which is rotating in this direction and the output shaft is located here, whose axis is perpendicular to the train of the board or train of the paper or train of the pin that is going like this. So to connect these two shafts, the input and output shaft and to get a particular speed ratio, we may need as we see, a pair of bevel gear, then a pair of spiral gear or cross helical gear, which are connecting these two skewed shafts and a pair of spur gears to connect this shaft to this shaft. So this pair of bevel gear connects these two shafts. This shaft and this shaft which is skewed are connected by a pair of cross helical gears and these two parallel shafts are connected by a pair of spur gear. So, this shows an example of a typical gear train.

In this lecture, we shall analyze gear train and we start our discussion with classification of gear train. Before analyzing gear train, let me first classify different types of gear train.

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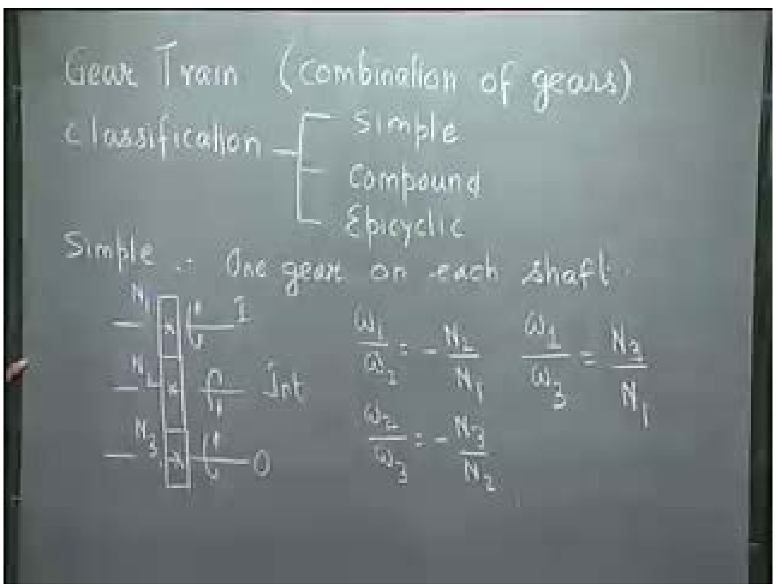

Gear train classification: gear trains are normally classified under three headings- namely, simple compound and epicyclic. These are the three normal forms of gear trains. Simple gear train is that gear train, where on each shaft there is only one gear. There is input shaft, there is output shaft and there maybe several or one intermediate shaft connecting the input to the output shaft, but on each shaft there is only one gear. So, simple gear train has one gear on each shaft.

For example, suppose this is one gear mounted on this shaft which is the input shaft. This is connected to another gear, which is on another shaft and that gear is down, mounted on another shaft and this maybe the output shaft. This is called the intermediate shaft. Input shaft, output shaft and there is an intermediate shaft, there is one gear on each shaft and the motion of this input is transmitted to the output. Now, analysis of such a simple gear train is almost trivial. As we see here, these are all external gearing and the number of teeth is say N_1 on this gear, N_2 on this gear and N_3 on this gear.

So, omega$_1$ by omega$_2$ as I said, will be minus N_2 by N_1, because this external gearing and this inversely proportional to the number of teeth. Similarly, angular speed of the intermediate shaft to the output shaft, which is omega$_2$ by omega$_3$ that will be minus N_3 by N_2. From here to there, again there is an eternal gearing so the direction of rotation changes and in the ratio of the number of teeth. If I multiply these two, we get the input speed which is omega$_1$ divided by omega$_3$ which is the output speed. If I multiply these two, omega$_2$ cancels and here N_2 cancels and minus into minus gives me plus, so it is N_3 by N_1. So, what we see is that the direction of rotation of this shaft is same as the direction of rotation of this shaft and this shaft rotates in the other direction. The input speed by output speed is N_3 by N_1. So we can get the reverse, if I want output speed by input.

The thing to note is that the number of teeth on this intermediate gear is of no relevant. The speed ratio is dictated only by the first gear and the last gear, the number of teeth on the gear mounted on input shafts and the number of teeth on the gear mounted on the output shaft. There may be several intermediate shafts, but the number of teeth on those gears will be immaterial, so far the speed ratio is concerned. Only the number of intermediate shafts will change this from plus to minus. Here, for example, this is only one intermediate shaft, omega$_1$ by omega$_3$ is positive and if I had another intermediate shaft then, omega$_1$ by omega$_4$ would have been N_4 by N_1 but we take negative side.

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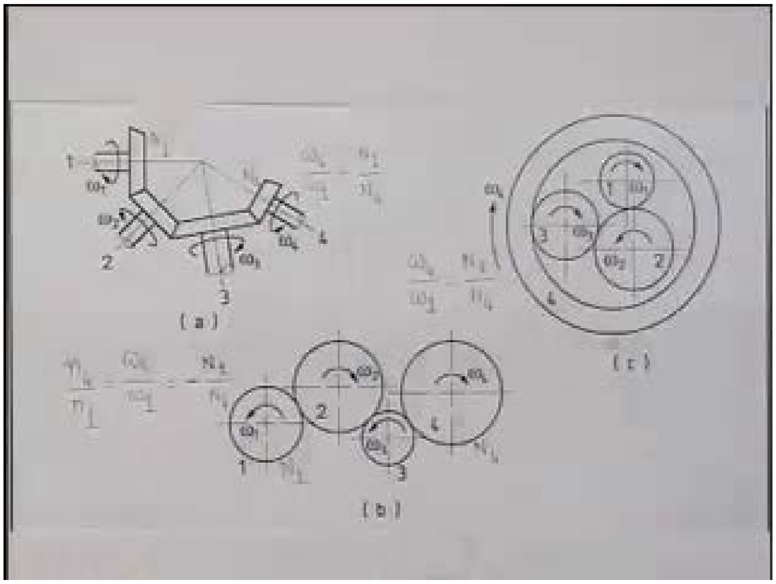

In this diagram, we show three different simple gear trains. We see there are four shafts in this figure, 1, 2, 3, and 4. Let this be the input shaft and this is the output shaft. These are all connected by bevel gear. Now, the speed ratio as we calculated just in the example we discussed before, the shafts are parallel. So, it was very easy to keep track of plus or minus. At every step, there is a change of sign but here as we see the shaft axes are all intersecting and not parallel, plus minus will be continuing. So there we have to follow, if this is the direction of rotation of the shaft 1 then, this is the direction of rotation of shaft 2 and this direction of rotation creates this direction of rotation for shaft 3 and that creates this direction of rotation for shaft 4. So if we want to write the output speed omega$_4$ by omega$_1$ that will be nothing but N_1 by N_4. The N_1 is the number of teeth on this gear and N_4 is the number of teeth on the final gear. The intermediate number of teeth is totally unimportant and the direction of rotation, which is as we have shown in this drawing.

Let me take the example of this train, where again it is a simple gear train. There are four shafts and on each shaft only one gear is mounted from input to the output. As we see, the counter-clockwise rotation rotates this gear clockwise, which makes this gear to rotate counter-clockwise which makes this gear to rotate clockwise. Here of course, the output speed omega$_4$ by omega$_1$ because there are two intermediate shafts, we get a negative sign N_1 by N_4. With one intermediate shaft, we have done earlier, they rotate in the same direction, but as soon as there are two input shafts, we get in the opposite direction that is the negative shaft. The N_1 is the number of teeth on this gear, N_4 is the number of teeth on this gear and number of teeth on the intermediate gear does not match. Very often this angular speed, we will write in terms of rpm and I will denote by small n. The small n refers to rpm and capital N will refer to the number of teeth on various gears, which are written in the circle.

Now, let us come to the example of this simple gear train. Here again, 1, 2, 3 and 4 which is an internal gear, which axis is somewhere here, at the centre of this gear 4. The thing to note is, this is a pair of external gearing, this is a pair of external gearing but at 3 and 4 there is a pair of internal gear. So, this omega$_1$ chases this omega$_2$, that chase this omega$_3$ and because of internal gearing omega$_3$ and omega$_4$ are in the same direction. So here again, we can write omega$_4$ by omega$_1$ is N_1 by N_4.

Though there are two intermediate shafts, but there is this plus sign, because there is a pair of internal gearing as oppose to this equation, where there are all external gears. Here 1 and 2 are external gearing, but between 3 and 4 it is internal gearing. So, there is no negative sign. So, analysis of such simple train as I said is pretty trivial.

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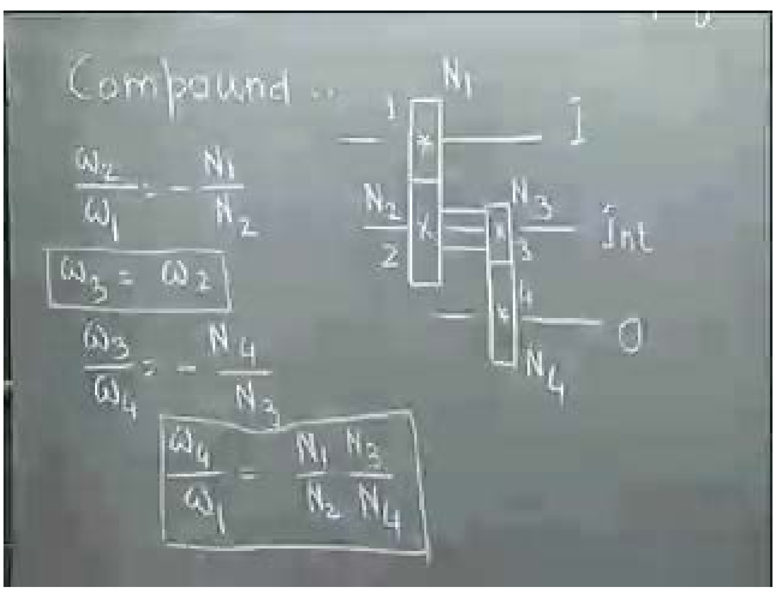

Now, let me discuss how to analyze compound gear train and what is a compound gear train? Let me now discuss what we mean by a compound gear train. In a compound gear train, there will be more than one gear on some shaft, either on the input shaft or output shaft or intermediate shaft, there must be one shaft in this transmission. In this gear train there will be more than one gear on one shaft.

For example, suppose this is the input shaft. This is the output shaft and this is one intermediate shaft. As we see on this intermediate shaft, there are two gears. So, this is a compound gear train from here to the output shaft. Let me find out, what is the speed ratio in such a compound gear train? Let the number of teeth be N_1, N_2, N_3, N_4, the gear number 1, 2, 3, 4, because gear 2 and 3 are both connected to the intermediate shaft, their speed will be same that is the speed of this shaft. So, we can find $omega_2$ by $omega_1$, because it is external gearing, it is minus N_1 by N_2 and $omega_3$ is same as $omega_2$, because they are mounted on the same shaft. So, $omega_3$ is $omega_2$ and again $omega_3$ by $omega_4$ will be minus N_4 by N_3, because this is again a pair of external gear.

Now, if I multiply these two, what do I find? $Omega_3$ by $omega_4$, N_4 by N_3 and this is $omega_2$ by $omega_1$, so I have to divide. If I divide this by this, $omega_2$ being same as $omega_3$, that will cancel. So we get $omega_4$ by $omega_1$, I am dividing this by this. So, $omega_4$ by $omega_3$ becomes this and N_1 by N_2 into N_3 by N_4, minus and minus cancels, I am dividing this by this, so N_1 by N_2 into N_3 by $N_{4.}$ So, this is the output speed to the input speed.

The thing to note is that, now the speed ratio from the output to the input or input to the output is decided by the number of teeth on all the gear, intermediate number of teeth becomes relevant. For a simple gear train, it was depending only on N_1 and N_4, but for such a compound gear train they are dependent on all the numbers of teeth on all the gears. Very often, such two gears are made out of one rigid body. The gear teeth of the machine on a same rigid body, the gear blank where [Refer Slide Time: 19:49] such that this becomes one rigid body which again ensures $omega_3$ equal to $omega_2$. So, this is how we analyze a compound gear train.

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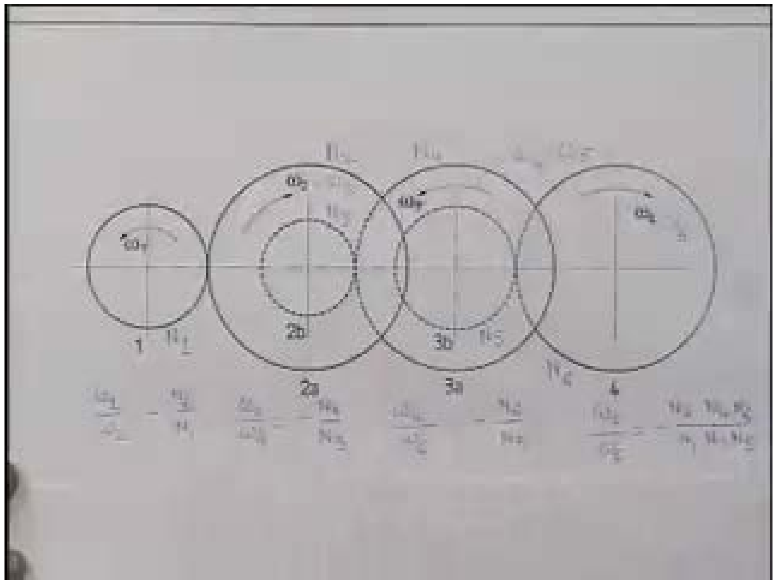

This figure shows again a compound gear train. As we see, on these two shafts there is more than one gear, there are two gears one here and one here. So this gear is connected to this gear, on the same shaft there is this gear which connects to this gear and on the same shaft there is another gear which is connected to this gear. So from here to here, the speed rotation is being transmitted, these two are intermediate shafts. So, this is a compound gear train.

If we say, the number of teeth on this gear is N_1, number of teeth on this gear is N_2, number of teeth on this gear is N_3, the number of teeth on this gear, which is connected to 3 is N_4, this gear is N_5 and on this final gear is N_6. So as we see omega$_4$ and omega$_1$ are in opposite direction that can be seen very clearly from here to there then this rotates clockwise, so this rotates anticlockwise, so this rotates clockwise.

But I can write omega$_1$ by omega$_2$ is minus N_2 by N_1. Omega$_3$ which is same as omega$_2$, I can write as omega$_2$ by omega$_4$ is minus N_4 by N_3. Omega$_2$ is same as omega$_3$. This omega$_3$, let us not be confused with this, I will remove this. This is 6, N_6 I will write, omega$_6$ this is the sixth gear. So, omega$_2$ is same as omega$_3$ then I get omega$_4$ and omega$_4$ is same as omega$_5$ that is the speed of this gear. We can write omega$_5$ which is same as omega$_4$ divided by omega$_6$ is same as, this I write omega$_6$. This I write omega$_4$ which is same as omega$_5$, this is omega$_2$ which is same as omega$_3$ and this is same as omega$_5$. So, omega$_4$ by omega$_6$ is minus N_6 by N_5.

This is actually omega$_5$ by omega$_6$ is minus N_6 by N_5 and omega$_5$ is same as omega$_4$. So I write this. If I multiply all this, as we see omega$_2$ and omega$_4$ cancel, I get the input by the output speed omega$_1$ by omega$_6$, minus into minus into minus gives me minus and here I have, $N_2N_4N_6$ divided by $N_1N_3N_5$. So input speed by output speed, negative sign implies they are in opposite direction and they are governed by the number of teeth on all of these gears in this compound gear train.

Next, we shall discuss the most difficult one where such visualization of rotation of various gears will not be possible and that is what we epicyclic gear train. In the simple and compound gear trains that we have discussed so far, we taught that all the gear axis on the shaft on which the gears are mounted were fixed in space. That means, the gear axis of all the gears were fixed in space. They are not moving in space, both in simple and compound gear train.

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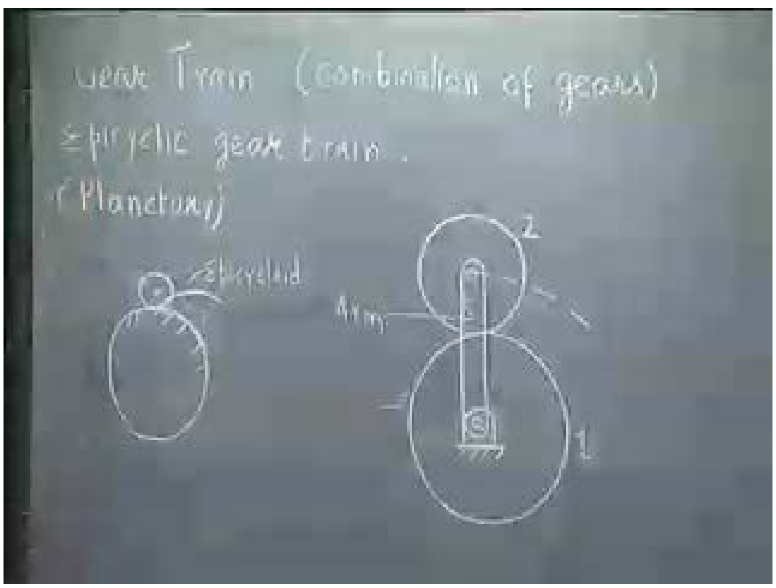

Whereas in epicyclic gear train, we will see that there will be one gear whose axis is not fixed in space, because the gear is mounted on an arm and that arm is rotating and carrying the shaft of that gear along with this. In such gear train, if there is one such gear, we call it an epicyclic gear train. Some books call it planetary gear train. To show, suppose I have one gear here and another gear there. This gear, let me call it 1 and this gear number 2, but gear number 2 is carried by this arm which is also hinged here. The axis of the gear 1 and this arm goes at hinged to the fixed link at this shaft.

When this gear train rotates as we see, this gear 2, the centre of the gear 2 moves on this circle, with this point as the centre. Consequently, the axis of this gear 2 does not remain fixed in space and this constitutes an epicyclic gear train or planetary gear train. In fact, this we call tongue gear around which the planet gear is moving that is why, the name planetary. Epicyclic, because we know if a circle rolls on another circle, then any point on this rolling circle generates a curve which is called epicycloid.

The curve that any point on this rolling wheel generates as if rolls around this circle is called epicycloid and from there, this name came epicyclic gear train, as a point on this gear will generate epicyclic gear. Let us not bother so much about the name epicyclic or planetary. We should note that this is a 4 linked mechanism.

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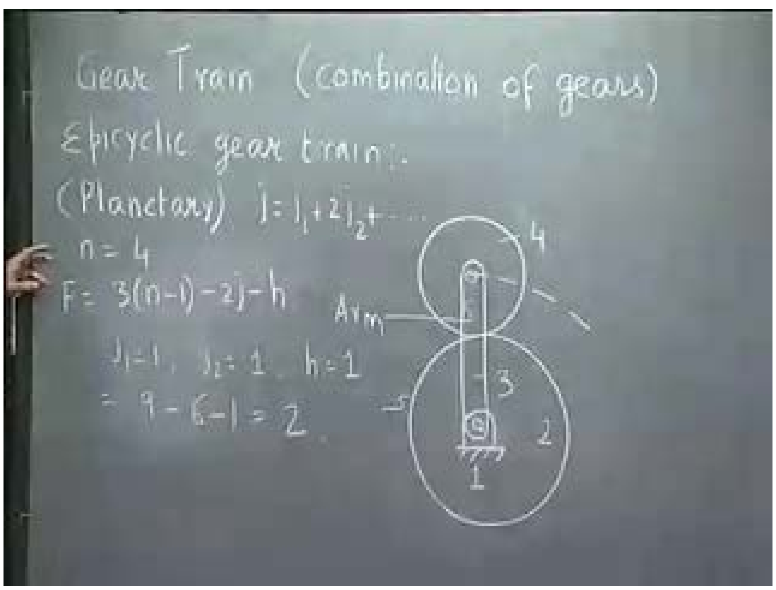

As usual we give them fixed link number 1. This gear let me call 2, this arm is 3 and this gear 4. So this is a 4 linked mechanism, n is 4. Let me first calculate, the degrees of freedom of this mechanism. Degree of freedom F was can be obtained from 3 (n minus 1) minus 2j minus h, where j is the equivalent number of simple hinges that is, j_1 plus $2j_2$ like that and h is the number of higher pair, n is the total number of links.

Now here as we see, there is a revolute pair between the arm and gear 4 here. So j_1 is 1. Here is a revolute pair but that connects three links namely, 1, 2 and 3. So this is a hinge of size j_2. So j_2 is also 1 and between the gear 3 and 4, these are the pitch circles, let me not draw, these are imaginary circles representing the gear, it is between 3 and 4, we have a gear tooth connection which is a higher pair. So h is also 1 between 2 and 4, between gear number 2 and gear number 4, they are having a higher pair between them. So if I now substitute n is equal to 4, I get 9 minus, j is j1 plus $2j_2$ that is 2 plus 1 is 3, j is 3, 2 into 3 is 6 minus 8 is 1, so it is 2. So the degree of freedom of an epicyclic gear train is 2. Further it implies that, we need two independent input teeth to fix the output teeth because it has a degree of freedom 2.

In other words, if we give only one input speed then we can get two output speeds, which will be coordinated but they will be non-unique. I can adjust both of them. They have to follow such a relationship but they can be otherwise non-unique. So I can generate two different speeds, two output speeds from the single input speed and that is where we use such epicyclic gear train.

Now, how do I analyze such an epicyclic gear train? Here as we know, it will be very difficult to visualize the direction of rotation of various gears because this gear is not fixed in space. So, let me try to make rather mechanical research, develop a analytical method to follow with no time to visualize which way which gear is rotating as it is possible incase of simple or compound gear train. So epicyclic gear train, we analyze rather mechanically for that, first of all let us derive what is the basic principle of analysis of an epicyclic gear train.

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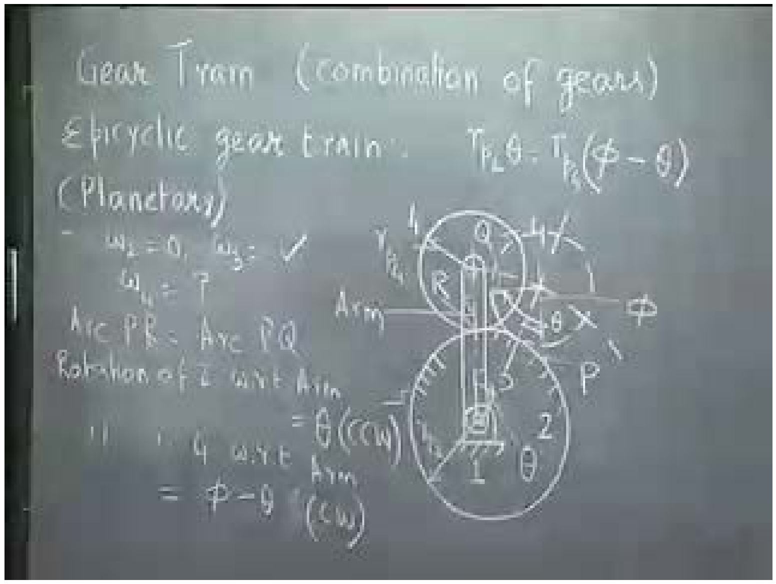

Towards that, we want to study the relative motion between 2, 3 and 4 and as I said, it has 2 degrees freedom, so two input speeds are necessary to fix the output speed. So what do we do? We give input to gear 2 and gear 3 and find what the speed of gear 4 is. We prescribe the angular velocity of body 2 and body 3 and we want to find that of body 4. We want to study this relative motion.

To study this relative motion without any loss of generality, I can say one body to be fixed and let me say this body 2 is fixed, that means omega$_2$ I am taking it to be 0 that is 1 input and then if I know omega$_3$, the other input speed, the question is what is omega$_4$? To do that, let us say I give the arm a rotation theta, link 3 has rotated by an angle theta. Centre of this gear has come here and this gear is taken as this position. This is the vertical line.

Now, this vertical radius after the gear has rotated say from here, this vertical radius has become this radius. So let me say this present contact point I call P, this point I call Q and this original contact point I call R. Theta is the rotation of the arm and what is the rotation of gear number 4? This vertical radius has become this. So the total rotation of this is so much. Phi is the rotation of body 4, theta is the rotation of body 3 and body 2 is held fixed because these are pitch circles that means, one rolls over the other without slip that means, these two arc lengths must be same. This arc length and the smaller gear, standard gear is same as this arc length on the sun gear or larger gear, because they are rolling on each other without slipping because this represents the pitch circles of the two gears.

Arc PR is same as arc PQ which means, if I say radius of this gear, pitch circle radius is r_{P2} and pitch circle radius of this gear, I call r_{P4}. So r_{P2} into theta is r_{P4} into phi. Phi minus theta, this angle is theta. If this angle is theta, the arm makes an angle theta with the vertical line, with this vertical line the arm makes an angle theta. So this angle is phi minus theta. Phi is the total rotation of gear 4 and theta is this angle. So this angle phi is minus theta. So this arc length PR is r_{P4} into phi minus theta.

We shall see that the relative rotation of the gears with respect to the arm still follows the teeth block that is, number of teeth is inversely proportional to the number of teeth with a appropriate sign. Let me say, what is the rotation of the sun gear with respect to the arm? Sun gear was this, arm gear rotated theta in the clockwise direction. So rotation of 2 with respect to arm is theta, but in the counter-clockwise direction. Rotation of arm with respect to 2 which was fixed is theta in the clockwise direction, so rotation of 2 with respect to arc is the opposite of that theta in the counter-clockwise direction. What is rotation of 4 with respect to arm? Gear 4 has rotated by an angle phi in the clockwise direction, arm has rotated by an angle theta in the clockwise direction, so rotation of 4 with respect to arm is phi minus theta in the clockwise direction.

Let me see again. Arm has rotated by an angle theta in the clockwise direction, gear 2 has not rotated. So rotation of 2 with respect to arm is theta but in the counter-clockwise direction. Rotation of 4 is phi, rotation of arm is theta, so rotation of 4 with respect to arm is phi minus theta in the clockwise direction and because of these two arc lengths being equal, this is the relationship between phi and theta.

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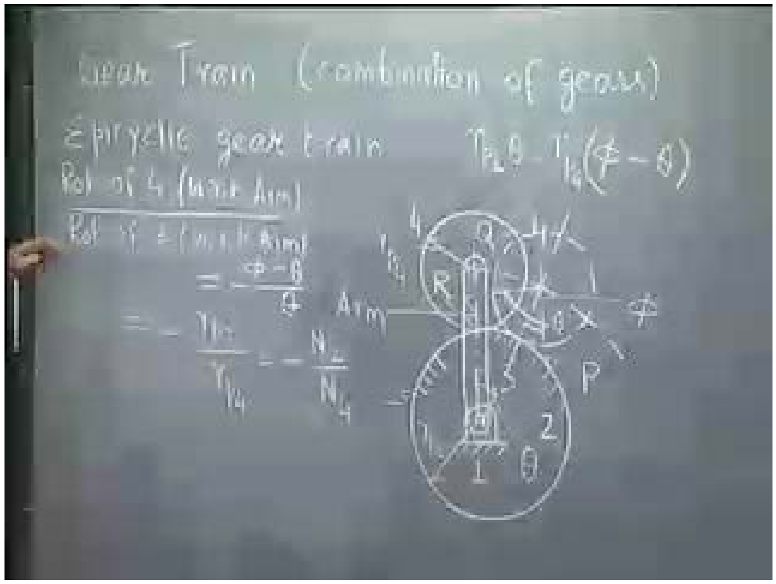

So if I divide this by this, I consider rotation of 4 but with respect to arm and rotation of 2 again with respect to arm. If I talk of this relative rotation and take the ratio, we are getting phi minus theta that is in the clockwise direction, divided by theta that is in the counter-clockwise direction. So I put a negative sign.

So we see in this epicyclic gear train, the relative rotation of two gears with respect to the arm, if I write, rotation of 4 with respect to arm, divided by rotation of 2 with respect to arm is phi minus theta divided by theta, where this is clockwise, this is counter-clockwise, so I put a negative sign. So from this equation, where I got by having these two arc lengths same, I can write this is equal to minus r_{P2} by r_{P4}. Phi minus theta by theta is r_{P2} by r_{P4} and it is a negative sign and this is same as minus number of tooth on gear 2 divided by number of tooth on gear 4, because these are the pitch circle radius which are proportional to the number of tooth, because they have the same module.

In case of an epicyclic gear train, because the arm is moving the absolute rotation of 4 and 2 are not governed by their number of teeth on them. Otherwise, this is exactly the same train value, only if we consider relative motion of 4 and 2 with respect to arm, not the absolute motion but the relative motion with respect to arm. That still follows the same value has minus because the external gearing and inverse ratio of number of teeth, rotation of 4 by rotation of 2 is minus N_2

by N_4. This forms the basic of analyzing epicyclic gear train in a mechanical way. We have to only remember that it is the relative rotation with respect to arm follows the same value given by the number of teeth.

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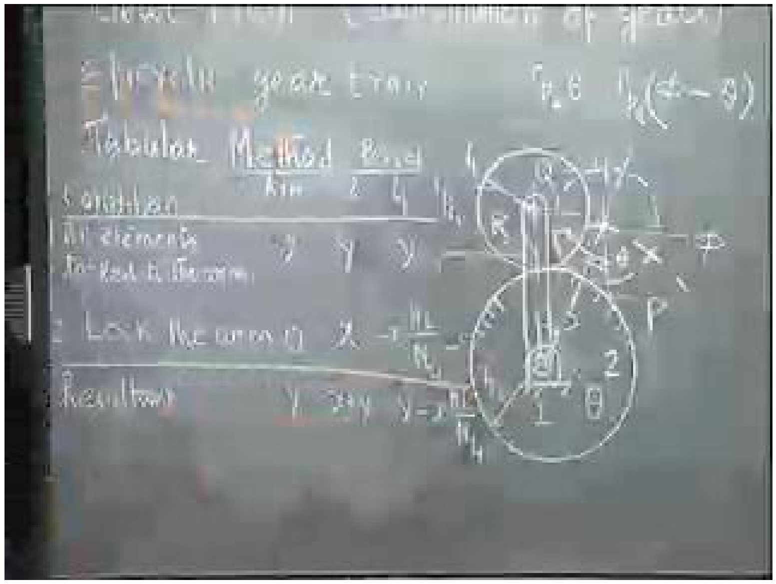

This we put in a some what mechanical tabular method. We first lock all the elements of a planetary gear train to the arm, all the elements are locked to the arm that means, the whole thing becomes one rigid body and we write condition, all elements, all the gears locked to the arm which means, the whole train becomes one rigid body and then gives arm some y revolution. Then all gears 2 and 4, because they are locked to the arm under this condition, if I give arm y revolution, I will get y revolution to all the gear and the second condition I consider is lock the arm. They do not allow the arm to move, these are revolutions of arm, gear number 2, gear number 4. Lock the arm that means, arm is not given any revolution and given any x revolution to any of the gear, say I give x revolution to gear 2. Then because the arm is locked it is not an epicyclic gear train anymore. It is just a simple gear train. So rotation of 4, I can easily get as minus x into N_2 by N_4. Minus is because it is external gearing and inverse ratio of the number of teeth. Add these two and that is what we get resultant motion. Here it is y, here it is x plus y, here it is y minus x into N_2 by N_4.

As I said, it is a 2 degree freedom mechanism, all the output speeds can be determined on this issue, give me two definite inputs that means two of these speeds must be noted then only I can determine the third. So I get two equations by equating the given speed and I can determine x and y and once I know x and y, I know the speeds of all the elements, because they are all existing in terms of x and y. Of course, the number of teeth is given. This is the method that we shall follow and I shall tell you few examples to show how to use this tabular method for analyzing epicyclic gear train.

Let me now conclude today's lecture, what we have learnt is simple, compound and epicyclic gear train. To analyze such gear trains, we found that analysis of simple and compound gear trains are pretty trivial. Only thing we need to be little careful about the direction of rotation if there is an external gearing or internal gearing and there are several numbers of intermediate shafts.

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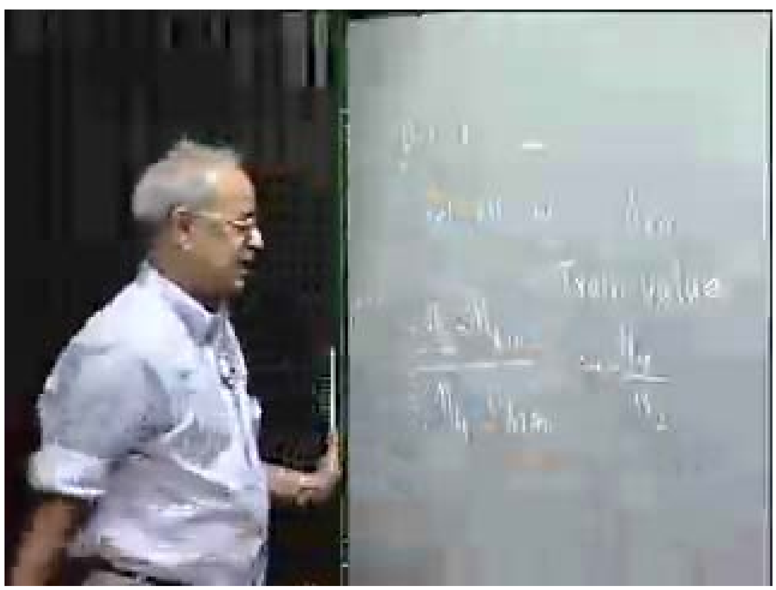

But when we come to epicyclic gear train what we have noted is that epicyclic gear train has 2 degrees of freedom. That means, two input speeds need to be specified to determine all other output speeds. Conversely, if there is one input then I can get two outputs which can be adjusted because, they have to follow one relation but we cannot determine exactly what will be the output speed that is the degree of freedom is 2 and which has lot of application as we will see later in our next lecture.

To analyze this epicyclic gear train the only thing to note is that, it is the relative rotation of the gear with respect to arm is equal to the train value. That is, as we get normally in a simple gear train or compound gear train using the number of teeth. So it is the relative rotation with respect to arm is given by the same value not the absolute rotation ratio of two gears. If I write to say, speed of gear 2 minus speed of arm, small n refers to the rpm by n_4 minus n_{arm}, where 2 was the sun gear, 4 was the standard gear and n_{arm} refers to the speed of the arm, then this is for external gearing was given by N_4 by N_2.

And we shall use this for a tabular method which I also explained, that we talk of two conditions- First, we lock all the elements to the arm then give arm a particular revolution then lock the arm and give one of the gears some other revolution and calculate what the revolutions of all the elements are. Then add these two, we get equations in terms of two unknowns and by two input speeds specified, we will be able to solve for those two unknowns and all other speed are expressed in terms of those unknown. So we should be able to determine those. This we shall see in our next lecture through an example.