Kinematics of Machines

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Module – 13 Lecture - 1

So far in this course, we have discussed straight tooth spur gears which are used to connect two parallel shafts.

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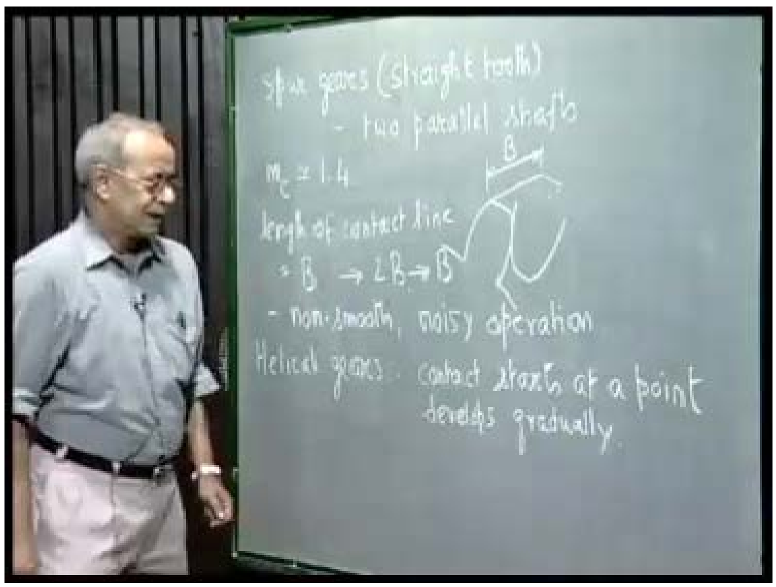

We have discussed spur gears with straight tooth. These are used for connecting two parallel shafts and the tooth on the gear surface runs parallel to the axis of these shafts. There are other types of gears which are used to connect either parallel or intersecting or stud shafts. In today's lecture, we will briefly discuss all these other types of gears. In case of spur gears, we discussed something called contact ratio and contact ratio is always more than 1 and the recommended desirable value is of the order of 1.4. What does it mean? For some part of the cycle either one pair of gear teeth are in contact and for rest of the cycle two pairs of gear teeth are in contact. So, on an average 1.4 pair of gear teeth are in contact.

If we have straight tooth spur gears: suppose these are the two teeth which are engaged at this point because these teeth are running straight parallel to the axis of the shaft, the contact takes place at the same instant along the entire face width. The width of the gear face is face width, if I denote it by B, then the length of contact line between a pair of gear teeth is B if one pair of gear teeth in contact. When two pairs are in contact, this B changes to 2B. Again when one pair of gear teeth comes in contact, because contact ratio is not 2, it is 1.4 for some part of the cycle, it will again come back to one pair of teeth in contact. So, the contact line length will change from 2B to B. This way, the length of the contact line suddenly changes from B to 2B and again suddenly from 2B to B because the contact along the entire length of B takes place at the same instant. This results in non-smooth, noisy operation. To get rid of this noisy operation, we can go for a little costly gears, which are known as helical gears. These are again spur gears that means, used to connect two parallel shafts, but the teeth of the gear are not straight along the gear surface, outer surface of the gear. They do not run straight parallel to the axis of the shaft, rather they are in the form of a helix and in that case we call it helical gears.

Now I will show a diagram of a pair of helical gears and in this case, that contact between a pair of teeth starts at a point between two helix and then develops gradually. The contact does not take place simultaneously at the same instant over the entire width of the gear rather, it starts at a point and then gradually develops. As a result the length of the contact length does not change suddenly and it gives rise to smoother less noisy operation. These are called helical gears.

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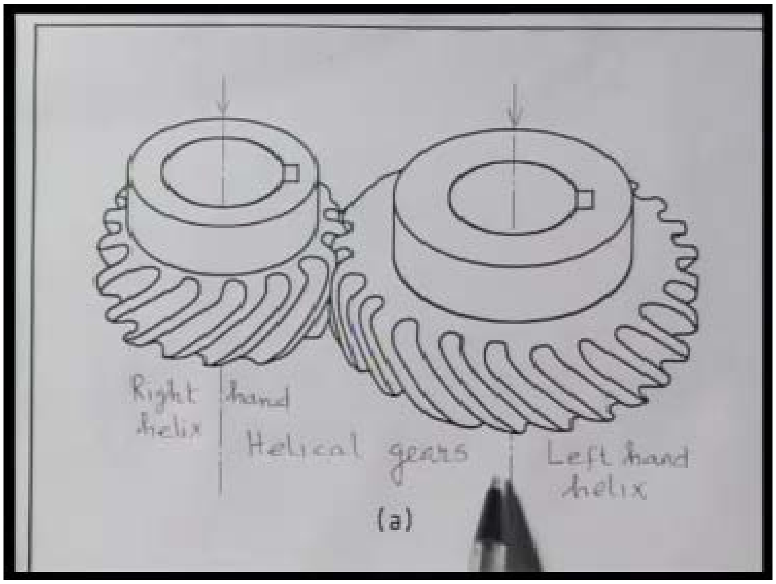

This diagram shows a pair of helical gears. As we see, this is one shaft. This is another shaft. Both are vertical that is, both are parallel shafts. These two shafts are parallel and on this shaft this helical gear is mounted and on this shaft this helical gear is mounted. Here at this contact point, as I said the contact between the pair of teeth, between these two helical gears starts at a point and gradually develops that gives rise to smoother operation. So, these are called helical gears.

The thing to note, when two parallel shafts are connected by a helical gears, there helix are in opposite direction. In this gear, the helix are in this direction and the meeting gear, the helix are in opposite direction. Accordingly, we call them either left hand helix or right hand helix. To decide whether it is left hand or right hand, we follow this convention. Look at this gear along the axis and as we see the teeth are inclined, if I view it from this side, the teeth are inclined to the left. The tooth starts from here and gets inclined to the left. So, this is called left hand helix.

Similarly, if I look at this gear along this axis of the gear, then the teeth is getting inclined to the right. Starting from here, all the teeth are getting inclined to the right. So, this will be called a right hand helix. The two parallel shafts are connected by a pair of helical gears, one of which is left hand and the other is right hand.

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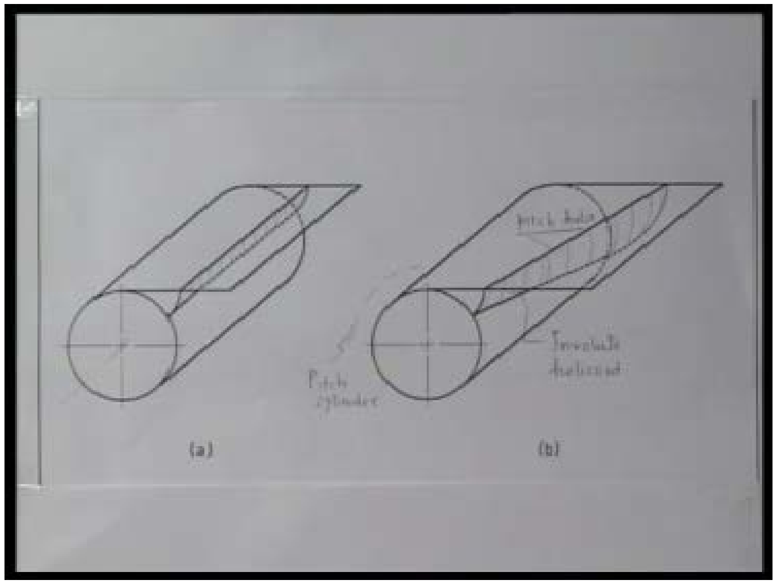

While discussing straight tooth spur gears, we said that one of the most common profiles of the gear tooth was an involute. This is the involute of a circle. Actually, this represents the base circle from which, this string is unwinding to generate this involute profile. If we consider the width of the gear that is this base circle, I can think of a base cylinder, then this strip of tape or paper which was wound on this base cylinder. If it is unwound, then the surface that is generated by any line of this paper like this particular line which was here is generating this involute surface and the gear tooth of a straight tooth spur gear was this involute profile.

Let us see what happens incase of a helical gear. In the case of a helical gear, this generating line which was parallel to the axis of the base cylinder. If this is the axis of the base cylinder then this generating line which was generating the involute profile was parallel to the axis. Here, we take this line on the base cylinder which is inclined to the axis, this line. This is the base cylinder. This is the tape or strip which is being unwound from this base cylinder, but the generating line is not parallel. It is on this base cylinder which is inclined and as this tape unwinds, this particular straight line generates this surface and this surface is the helical tooth surface of helical gears. This surface is called involute helicoid.

We shall discuss a little bit of the geometry of this helical tooth surface. To discuss the geometry of this helical gears tooth, let us consider the pitch cylinder. This is the base cylinder, so a little larger than will be that imaginary pitch cylinder rather than the pitch circle. We can think of a cylinder of radius r_P which we will call pitch cylinder. The intersection of that pitch cylinder with the tooth surface is this line which we call pitch helix. Pitch helix is the intersection of the pitch cylinder with the tooth surface. If we think of a pitch cylinder, then that the tooth surface, if I draw wherever it intersects that pitch cylinder that is called pitch helix.

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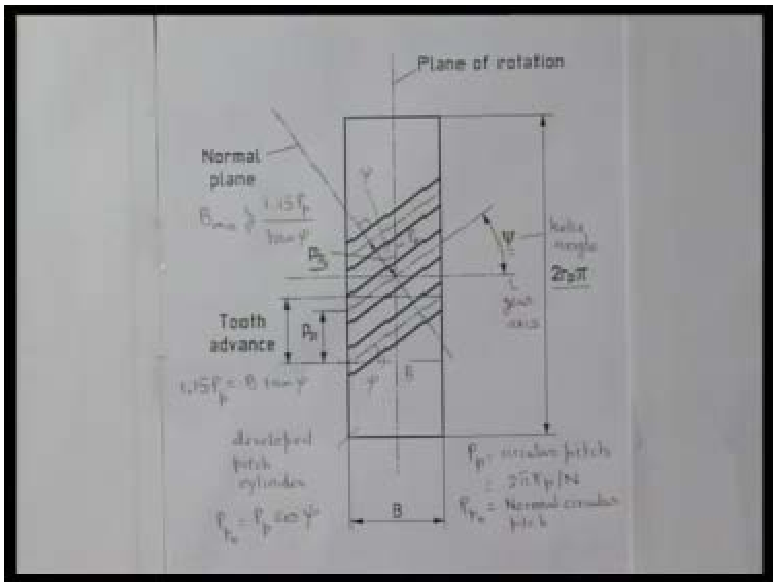

Let me now define the typical gear tooth geometry for a helical gear with reference to this figure. This rectangle of this side B represents the width of the gear and this length of psi 2 r_P into pi that is if the pitch cylinder is developed, then in the form of a rectangle, if a cylinder is developed, it becomes a rectangle. This is the length of the cylinder and this is the circumference of the cylinder, the pitch cylinder that is why I write the radius as r_P. So, this distance is twice pi r_P. This is developed pitch cylinder. B is the width of the gear and r_P is the radius of the pitch circle. This line represents the gear axis that is the axis of the shaft. These are the pitch helixes. These are the three consecutive teeth which have been shown when we wrap this rectangle on to a cylinder of radius r_P. This can get converted into helix. These lines become helix. The helix angle is defined as this angle psi. The angle of this pitch helix makes with the gear axis. This is the axis of the gear and these are the pitch helix. So, the angle of the pitch helix makes with the gear axis, this psi is called helix angle. This line represents the plane of rotation.

If we measure the distance between the identical points between two adjacent teeth suppose, I consider this point on this tooth and the identical point on the adjacent tooth that is here, the distance measured along this pitch circle is called circular pitch. This we have done earlier, even for spur gears. P_P is circular pitch which is 2 pi r_P divided by the number of teeth, where r_P is the pitch circle radius. If I consider a plane which is normal to this pitch helix, then that I defined as normal plane. This angle is 90 degree. If I measure the distance between identical points of two adjacent teeth like this point and that point along this normal plane, then that I call normal circular pitch. P_PN is nothing but normal circular pitch.

If we measure the distance along this pitch circle or plane of rotation between two identical points of two adjacent teeth, then this distance I call P_P as it is here and if I measure along the normal plane, then I call it P_PN. As is clearly seen, if this is P_P, this distance and this angle is psi, the helix angle. So, these two are related. P_PN is nothing but P_P cosine of psi. So I can write P_PN is P_P cosine of psi where psi is the helix angle.

Next, we define what is known as tooth advance? As we see in this particular tooth, if the contact starts here, then the contact ends there. The distance that is covered from the start to the end along the pitch circle is called tooth advance. That is this distance. From here to there, measured along the pitch circle. This is the tooth advance. We consider the same tooth from the start to the finish. The movement along the pitch circle is called tooth advance. To ensure that the contact starts with the next tooth before it ends with the previous tooth then this tooth advanced must be more than P_P because this tooth starts contact here and this tooth ends contact there and this end is coming later than this. It is ensured. Continuous rotation is transmitted. One pair of teeth comes in contact before the previous pair of teeth loses its engagement. So, tooth advance should be more than P_P. In fact, to ensure this that the tooth advance is more than P_P, I must have some minimum value of B. If this is B and this angle is psi, then tan psi is tooth advance divided by B. Tooth advance divided by B is tan psi. So this is B tan psi. Tooth advance should be more than P_P. Normally, it is recommended the tooth advance should be 15% more than the circular pitch that is tooth advance should be 1.15 into P_P which means minimum value of B that we need we call B minimum should be greater than or equal to 1.15 P_P divided by tan psi where psi is the helix angle. So that finishes our discussion with the very rudimentary or fundamentals of the teeth geometry of helical gears.

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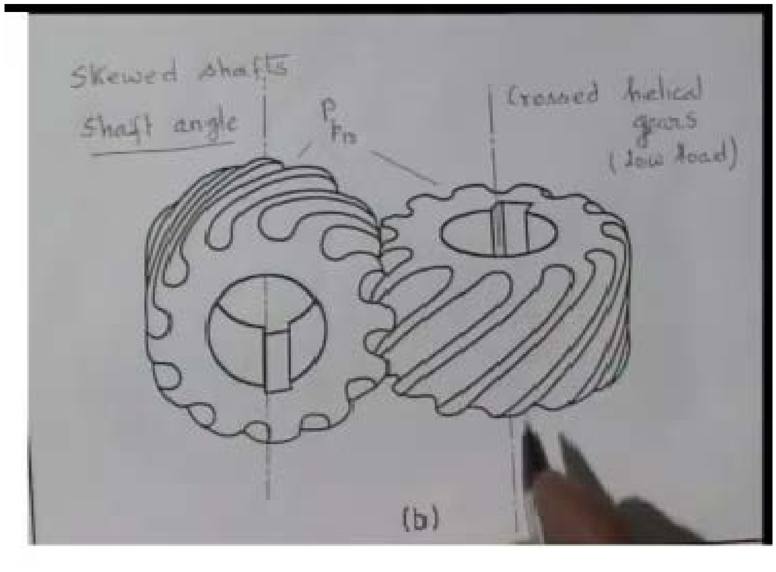

So far we have discussed spur helical gears. That is two helical gears of opposite handedness like right hand helix and left hand helix to connect a pair of parallel shafts, but helical gears can also be used to connect two non-parallel, non-intersecting shafts. A pair of helical gears can be used to connect to skewed shafts. As shown in this diagram, this shaft axis is vertical, whereas this shaft axis is horizontal and they can be connected by a pair of helical gears and such gears are called crossed helical gears. In a pair of crossed helical gear, the contact between a pair of teeth is always at a point. So, it cannot transmit too much of load. They can be used only for very low load application. They are incapable of transmitting very high value of torque. So they are used only for very low load application like in instrumentation. The thing to note is that these are nothing but helical gears as soon as they are mounted on two skewed shafts, we call them crossed helical gears. These are nothing but ordinary helical gears. To act as a pair of crossed helical gears the only condition necessary is that their normal pitch which we just now defined as P_PN of these two gears must be same. They can be of same handedness both can be right hand, both can be left hand or one of them right hand and other left hand, like that. Handedness is not important of this helix. Only thing the normal pitch of these two helices P_PN must be same. Now we define the shaft angle. Between these skewed shafts, how do I prescribe the angle? To define the angle between these two skewed lines, we drew it along the common perpendicular. We drop a common perpendicular between these two skewed lines and view these two lines along that line. The next diagram we will make it clear.

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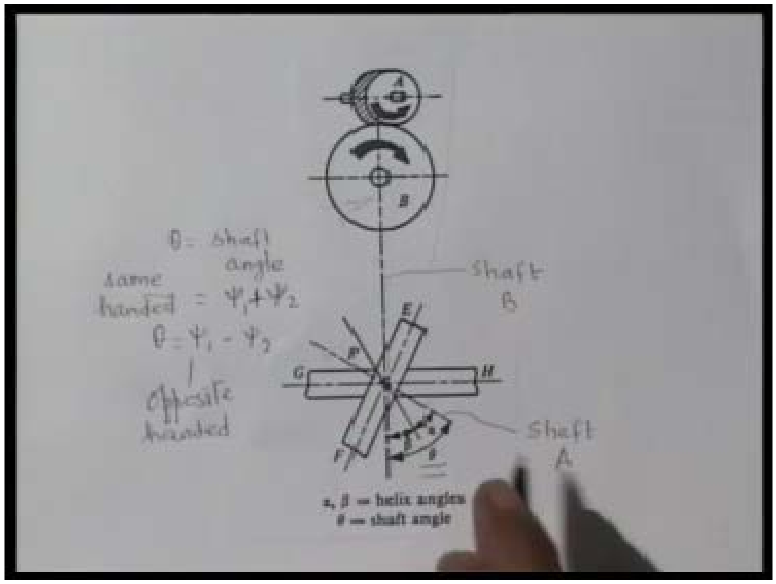

This diagram shows a pair of crossed helical gears, gear A and gear B. This is the shaft of gear A and this is the shaft of gear B. If I view it along the common perpendicular between these lines and this line which is plane perpendicular to the plane of the paper, then this is shaft of gear B and this is gear A and this line is shaft A. The angle between these two shafts is measured by this angle theta. This vertical line represent the shaft of B which is perpendicular to the plane of the paper in the top view, I am seeing it here. This line represents shaft of B and this line represents shaft of A and the angle between them, we call shaft angle. This theta is called shaft angle.

If both this helices are of the same handed that is both are right hand or both are left hand then I call it right handed. If these two helical gears are of right handed helix, then theta is given by psi_1 plus psi_2, where psi_1 is the helix angle of one gear and psi_2 is the helix angle of the other gear. This is plus when they are of same handedness and if they are of opposite handedness then the shaft angle theta will be psi_1 - psi_2. This is for same handed helix and this is for opposite handed. This is for the addition of these two helix angle, if those two have the same handedness, both right hand or both left hand and the shaft angle is psi_1 - psi_2, if these two are of opposite hand, one of them is right hand other is left hand, subtract the smaller from the bigger helix angle and we get the shaft angle.

The shaft angle is that if I rotate one of the shafts such that these two axis become parallel then what is the amount of rotation necessary to make these two shafts axis parallel and these two gears rotate in opposite direction? As shown here, this is rotating in this direction. If I rotate this and make it parallel to here, then that gear will rotate in opposite direction. The amount of rotation that is necessary is what we defined as the shaft angle theta.

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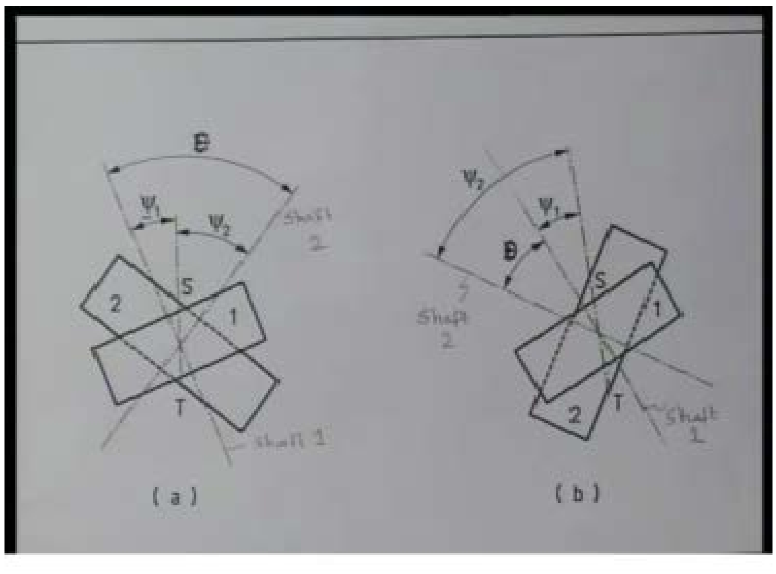

Just now we have explained the shaft angle for a pair of crossed helical gears which are used to connect two skewed shafts. Say this is gear 1 and this is the axis of gear 1. This is shaft 1. This represents gear 2 and this line represent shaft 2. This ST represents the tooth which is just now engaged between gear 1 and gear 2. This line ST represents the tooth. So, the angle between this ST and the shaft 1 that is this line is the helix angle psi_1. Similarly, this is the axis or shaft 2 and this is the tooth. So the angle between them is the helix angle for gear 2 which is psi_2 and they are of same handed. So the shaft angle theta is clearly seen to be psi_1 plus psi_2. The diagram changes as follows if the two gears are of opposite handed helix. This is gear 1, this line is shaft 1, axis of gear 1 and this is gear 2 and this line is shaft 2. So, the angle between shaft 1 and shaft 2 is this which is theta and here the gears are of opposite handed. So this is psi_2 and this is psi_1 where ST is the tooth. This line represents the tooth in contact. This is one helix angle for a shaft 1 gear 1. This is psi_1 and angle between ST and this shaft 2 is psi_2 which is the helix angle of gear 2 and the shaft angle theta as this clearly seen is psi_2 - psi_1.

So far, we have seen how to connect 2 parallel shafts either by straight tooth spur gear or by helical spur gears. Helical gears can also be used as crossed helical gears to connect 2 skewed shafts. Now we discuss another very common type of gears which are called bevel gears

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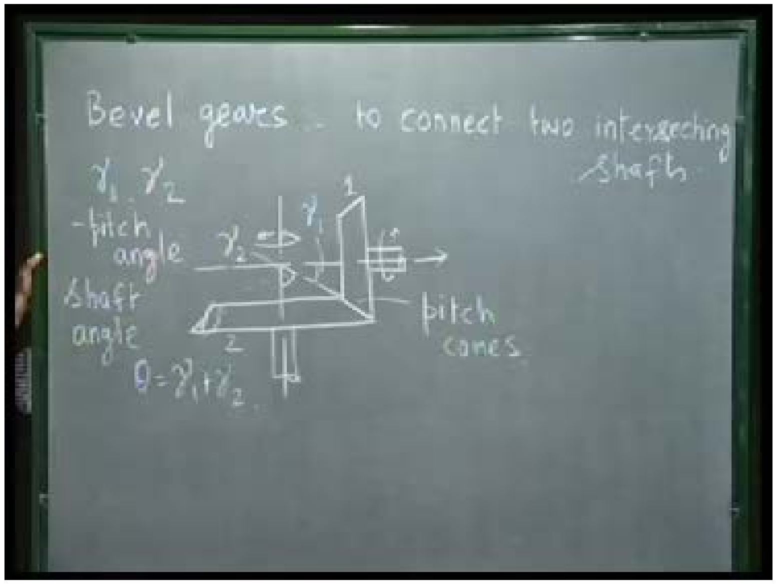

These bevel gears are used to connect two intersecting shafts. Suppose, this is one shaft axis and the other intersecting shaft axis is like this. I am taking the angle as 90 degree. It need not be 90 degree, just as an example, I am talking of two shafts intersecting at 90 degree. Then they can be connected by a pair of bevel gears. If this gear is rotating in this direction, the teeth contact is here and that rotates this gear in that direction. If I see it from here, this is the direction so that gear will rotate this gear in about the vertical axis but in which direction, this is getting into this. So it is like this. So these are called a pair of bevel gears.

The thing to note that for spur gears, we have this pitch surface which are cylinders. Gears teeth were cut on the surface of a cylinder. Here, the gear teeth is cut on the surface of a truncated cone. These cones represent, instead of pitch cylinder, we have pitch cones and the semi vertex angle of this cone, say I call it this is gear 1 and this is gear 2. The semi vertex angles of these cones are called pitch angles. Instead of pitch cylinder, we are having pitch cones and instead of pitch circle radius, because there is no constant radius here, we are defining this pitch cones by their semi vertex angle. So gamma_1 and gamma_2 are called pitch angle or cone angle, instead of pitch circle radius. The shaft angle that angle between these two shafts which are connected by these two gears are clearly seen to be gamma$_1$ plus gamma$_2$. I will show these things again with a better diagram.

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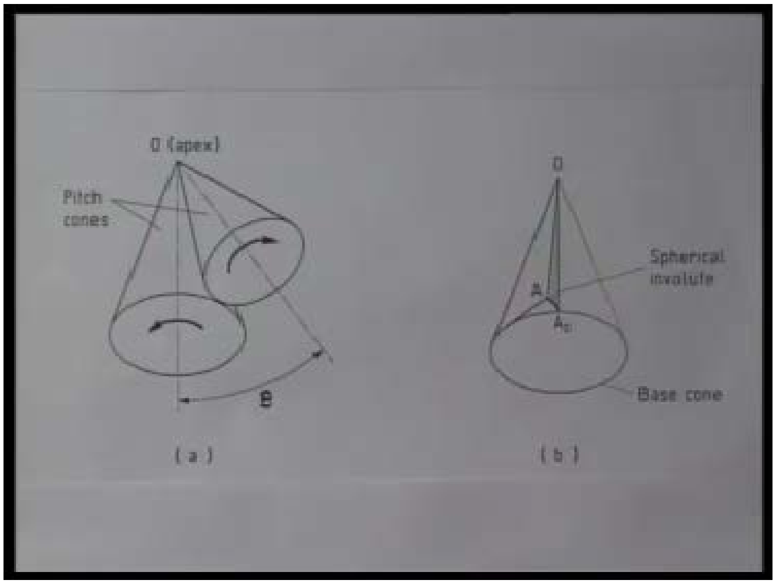

In case of spur gears, we have seen the equivalent motion that is transmitted by a pair of spur gears can be represented by rolling of one cylinder over the other without friction. Those are called pitch cylinders or in case of two dimensional representations pitch circles. It was this pitch cylinders rolling over each other without slip transmitted the same motion as by a pair of involute teeth. In case of bevel gears, those pitch cylinders, I replace by pitch cones. These are two pitch cones and if one rolls over the other without slip then this rotation of this, assume these are friction cones, there is no slip along the line of contact. Then this rotation of this cone will generate this rotation of the other cone and this is the equivalent motion that is transmitted by a pair of bevel gears.

Instead of pitch cylinders, in case of spur gears, we have pitch cones for the bevel gears. The angle between the axes of these two cones is called the shaft angle theta and this axis represents the shaft axis which are intersecting and meeting at the vertex of this two pitch cones. For spur gears, we have seen from pitch cylinders by unwinding a strip, we generated the tooth surfaces which were involute.

In case of bevel gears, we have to unwind from this base cone rather than base cylinder or base circle, incase of spur gears, now we unwind the strip which was wound on to this base cone. By unwinding, we are generating this surface as the strip unwinds and this surface will represent the tooth of a bevel gear and this surface is called spherical involute. Instead of involute of a circle as we got from the spur gears, here the radius of the base circle keeps on reducing as the result this surface changes from involute to a spherical involute.

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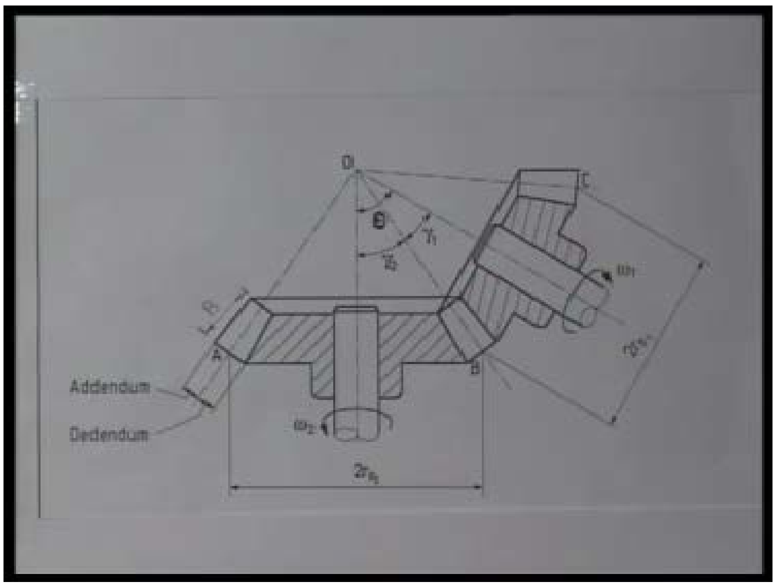

This figure represents a pair of bevel gears along with its teeth. As we have said, this is one pitch cone. The diameter at the base of this is 2 r_{P1}. This is gear 1 which is rotating with angular speed omega$_1$ and consequently transferring the motion to this gear of diameter at the pitch cone is 2 r_{P2} and this rotates at an angular speed omega$_2$. It is r_{P1} into omega$_1$ will be same as r_{P2} into omega$_2$ because at the pitch cone, there is no slit. The semi vertex angle of this pitch cones as we see is gamma$_1$ and gamma$_2$ and the shaft angle is gamma$_1$ plus gamma$_2$ as I explained earlier. So for the tooth is concerned, we measure above and below the pitch cone level and this is the addendum of the tooth and this is the dedendum of the tooth. Everything is measured at the level of the base of the pitch cone. BC is the base of the pitch cone of gear 1. AB is the base of the pitch cone of gear 2.

In the above diagram, we have just seen the tooth. Only one thing was left out that is this width of the tooth which we call the face width, is measured along this length. At the addendum, we measure this which we call or here, this length we call the face width which we represented by B.

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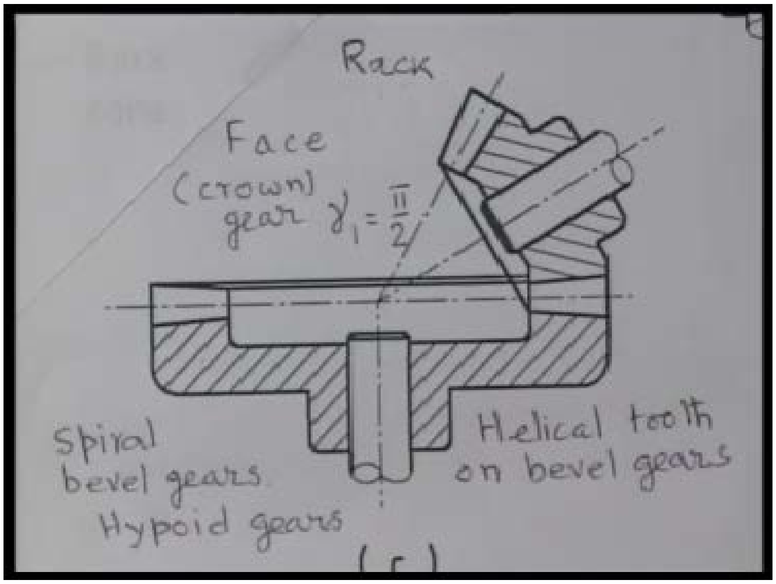

While we discuss spur gears, we talked of rack. Rack was a gear of apparently infinite pitch circler radius then it becomes a straight. A straight rack is nothing but a gear of pitch circle radius infinity. The analogous is in case of bevel gear, is called a face gear or a crown gear. Face or crown gear which is analogous to rack, in case of spur gears. These are a pair of bevel gears but if this is what we call face gear or crown gear. The thing to note is here the pitch circle radius was infinity and in case of crown gear, the pitch cone angle is 90 degree. The teeth are cut on this circular track rather than on the surface of a cone, because this cone has a semi vertex angle pi by 2. So all these teeth of the crown gear or face gear lies on a circular track and this is analogous to rack, in case of a spur gear.

The bevel gears which we have shown so far all have straight teeth, but we can have bevel gears also with helical teeth. We can have helical tooth on bevel gear. If we have helical tooth on bevel gears, then those are called spiral bevel gears. As I said earlier, the gear is a vast subject. We are not going to discuss all the details of these spiral bevel gears, but there is one type of spiral way bevel gears which are very commonly used in automobiles and which are called hypoid gears. These are used in automobiles and I will show the diagram of a pair of hypoid gears.

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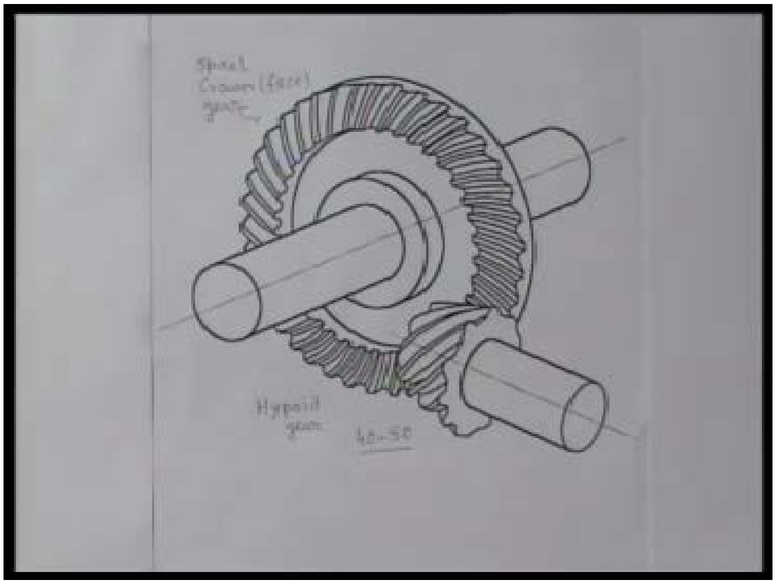

This diagram shows what I called a hypoid gear. The thing to note is that they look like bevel gears and the gear teeth are in the form of helix rather than straight. So, this is a crown or face gear because the semi vertex angle, if I think of these as a cone is 90 degree and the gear teeth are lying on the circular track. This is a crown gear but with spiral tooth. We can call it a spiral crown or face gear. This gear is in contact with this gear. As I said, these are used in automobiles with this as the input gear and this as the output gear and this is called a hypoid gear. The thing to note is that the axis of this shaft and the axis of this shaft that is these gears are not intersecting. Rather, they are offset. So these hypoid gears are very similar to bevel gears but not exactly bevel gears because bevel gears are used to connect two intersecting shafts, but here there is an offset. This line does not intersect this line. So there is an offset.

This is very similar to a pair of bevel gears and this gear is very similar to a crown gear or face gear but with spiral teeth and this type of gears are called hypoid gears, which are used in automobile. The advantage of this is that, by a single pair of gears a very large reduction is possible of the order of 40 to 50 speed reduction by this value. Ratio of the speeds by 40 to 50 can be achieved by this hypoid gear.

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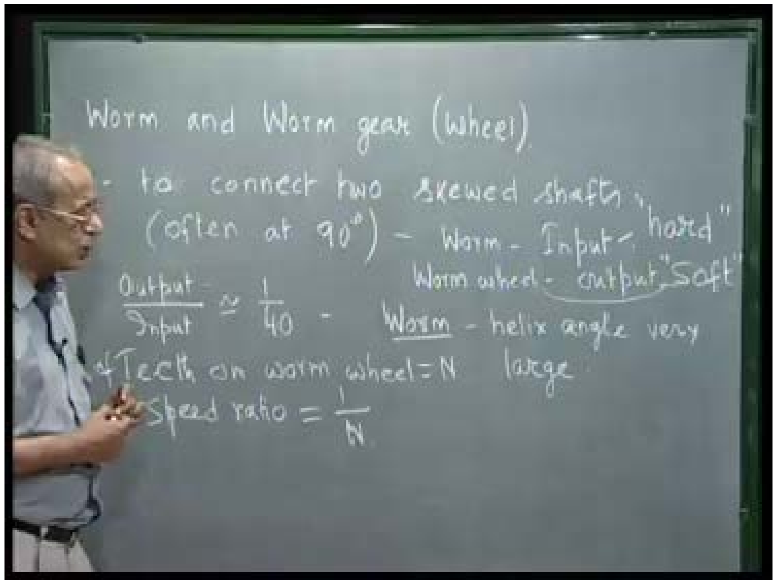

Today we have discussed different types of gears like helical gears, bevel gears, spiral bevel gears, hypoid gears and we finish this lecture by mentioning a very commonly used another type of special gear which are known as worm and worm wheel or worm gear.

These are normally used to connect two skewed shafts vary often at 90 degree perpendicular to each other but non-intersecting, they are one above the other but the angle between them is 90 degree. These are used for a very high speed reduction. Worm is the input shaft and worm wheel or worm gear is connected to the output shaft and the speed ratio, output speed by input speed in one stage can be as high as 1 by 40.

As we will see that if we start from a helical gear, but if the helix angle is so large, that one tooth makes a complete revolution on the pitch cylinder, then it becomes a worm. Worm is a helical gear or very large helix angle. Helix angle is so large that it makes a complete revolution on the pitch cylinder and then it gets converted into a worm and it looks like a screw and this screw drives the worm wheel. I will show the complete picture a little later. The thing to discuss is that as we will see the ratio of the speed reduction can be very high at single stage by using a worm and worm gear and it is 1/3rd of the worm which comes in connection with a large number of teeth of the worm wheel. If number of teeth on worm wheel is N then speed ratio output by input speed will be 1 upon N because it is the same side of the worm which comes in engagement with

N number of teeth on the worm wheel. It is always necessary to make this worm of hard material like steel whereas the worm wheel is made of softer material like brass such that, vary out almost at equal rate and the motion transmission remains uniform. If the varying rate is different on the worm and worm wheel then the engagement will be non-smooth. To maintain equal wear rate on the worm and worm wheel, worm wheel is made of softer material like brass, whereas worm is made of a harder material like steel. Now I will show the diagram of a worm and worm wheel.

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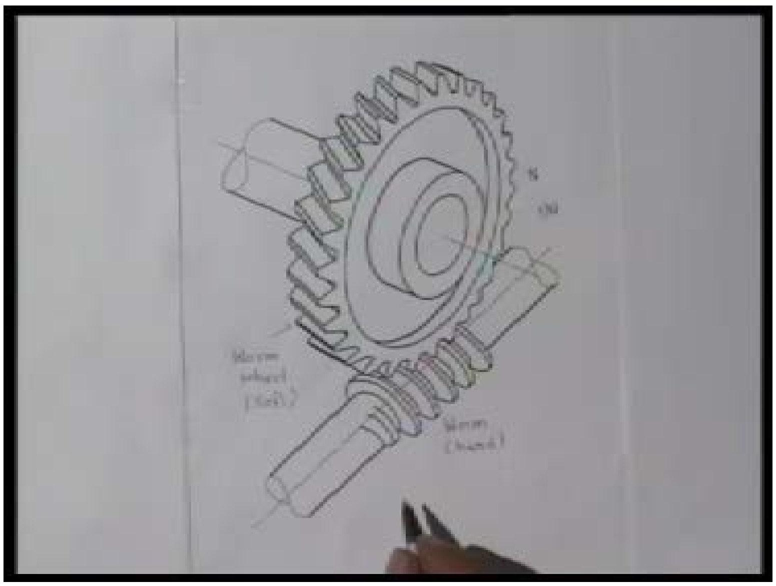

This is the diagram of a worm and worm wheel. This screw figure like is worm which is always the input speed. The worm wheel can never drive the worm. It is the worm which drives the worm wheel and this is called worm gear or worm wheel. If the number of teeth on this worm wheel is N, then the speed ratio is 1 upon N. The output speed by input speed is 1 upon N, the speed ratio. So if we have 40 teeth on the worm wheel, in single stage, I can get a speed direction of 40 and that is not very uncovered. This worm as I said is made of harder material of the two and worm wheel is made of softer material, because it is the same worm thread is getting coming into engagement without all the worm teeth. So the wear rate is lower here and moves here. So this to make uniform were rate on both this is made of harder material and this is made of softer material. As we see here the angles between these two shafts are 90 degree, though they are skewed.

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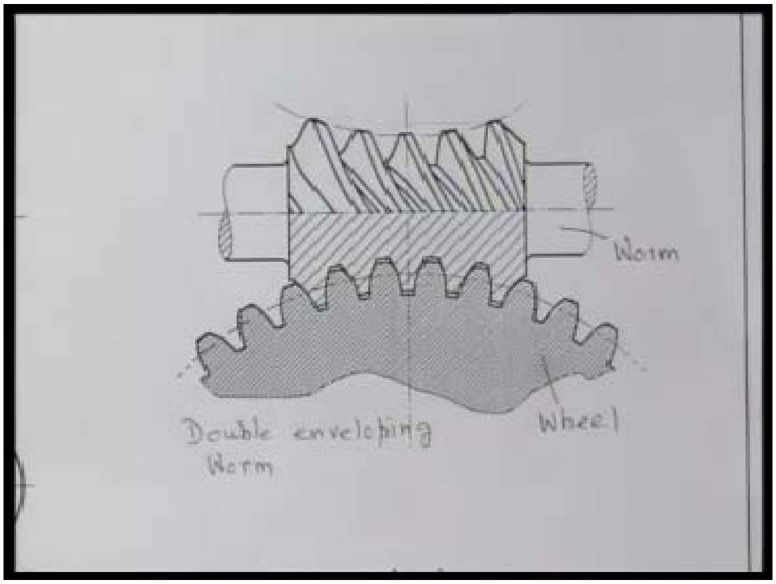

We conclude this lecture by showing this picture of a better design of a worm and worm wheel which is known as double enveloping. This is the worm and this is the worm wheel. The thing to note is that the threads of the worms are not straight, rather there is a curvature and this curvature is exactly same as the curvature of this worm wheel, such that there is much better contact between the thread of the worm and the teeth of the worm wheel. So it is doubly curved and that is why it is called double enveloping worm. This is nothing but a worm and worm wheel with a better design. We have, of course single enveloping worm, where this is the worm threads are straight. This curvature is not provided. This is obviously more costly. So there we conclude only the glimpse of different types of gears. We have discussed in detail the spur gears and in our next lecture, we will discuss the analysis of gear trends.