Kinematics of Machines

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Module No. # 12

Lecture No. # 01

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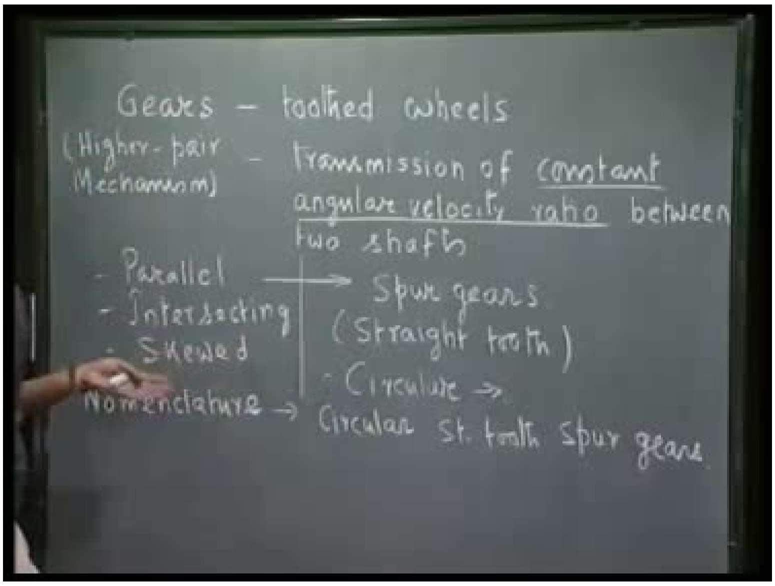

Today, we start discussion on a new type of higher pair mechanism, which are very commonly used and I am sure all of you must have seen this mechanism which are called gears; gears are nothing but as you know that toothed wheels; these are used all over machines to transmit constant angular velocity ratio between two shafts.

Let me emphasize for transmission of constant angular velocity ratio between two shafts. Actually, pair of gears use, one gear is mounted on one shafts, let say the input shafts and the other gear mounted on the other shafts that is the output shafts.

So, pair of gears is used to transmit a constant angular velocity ratio between two shafts. I want to emphasize this constant angular velocity ratio, this is very different from constant rpm; ratio rpm is not angular velocity, that is the average angular velocity and average angular speed, but gears are used, such that, at every instant, the instantaneous angular velocity of the two shafts maintain a constant ratio, that is why we used gears.

These two shafts can be parallel, between two parallel shafts or intersecting shafts, then is the shaft axis might be intersecting at a point or even skewed a neither parallel nor intersecting.

The special position of these two shafts can be either parallel or intersecting or skewed and they can be connected by a pair of gears, to transmit constant angular velocity ration. To start with a very simple type of gear to connect two parallel shafts are called spur gear and get to with straight tooth, that means, the tooth on these wheels are parallel to the axis of the shafts, which are these. Wheels are cylindrical and their teeth are surface of the cylinder and these teeth run parallel to the axis of the cylinder, for such part gears; as we see the cylinders in a projected view, if we view it from the front along the axis of the shaft we see just circle; so, these are called circular gears, see instead of cylinder I will use the one circular.

To start with let me define all the terms which are used for this description of gear, gear geometry and gears transmission. So, we start with nomenclature of gears, by doing that I will define all the relevant terms with reference to circular state tooth spur gears. So, now onwards, we may not emphasize, that we are talking of all that time are circular state tooth spur gears, that will be assume later on. We will talk about different types of gears, which are not circular state tooth spur gears. Then, now, show you it a figure, how these, a pair of two wheels which circular state tooth spur gears transmit motion from one shaft to another, maintaining a constant angular velocity ratio.

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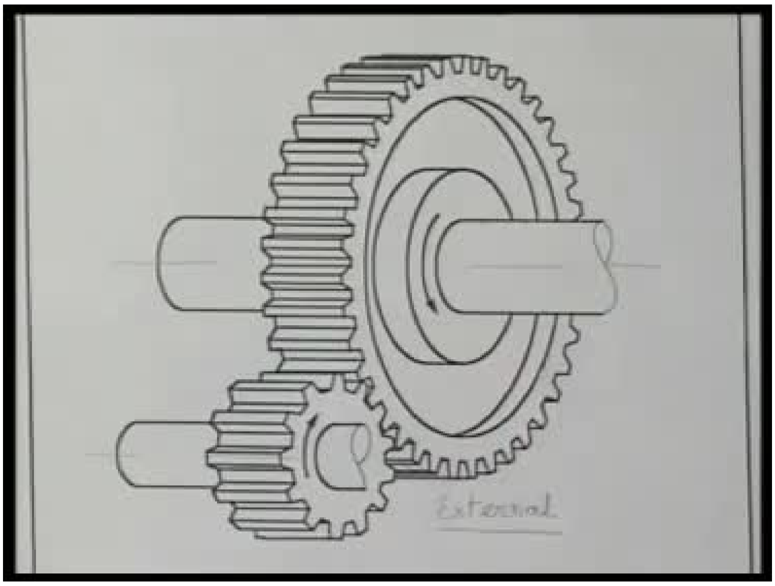

These figure shows a pair spur gears as you see the teeth on these (( )) are parallel to the axis of these shafts; these are the teeth, these are the gear blanks, on the surface of this gear blank the teeth (( )) and this is one shaft, on with these gear is mounted, which is another shaft where another gear is mounted; see, this is called external gearing extended of gear connection is called external gearing and as it gears, this shaft along with this gear rotating the clockwise direction, then they making here and the connected shaft will be in the opposite direction.

So, the direction of the angular rotation give us, due to this external gearing; similar gears can also be used external fashion, such that one gear drives the other, (( )) same direction maintaining the constant angular velocity ratio, which is the tooth of the gear that the blank or body of them.

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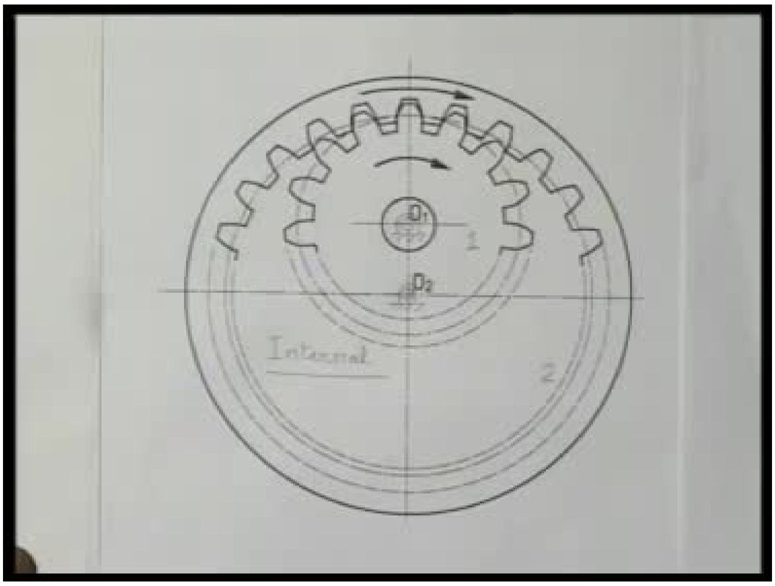

This figure shows a pair of internal gear; there are two shafts, there is a bearing between the (( )) of the machine and the shaft pertaining to this gear 1, there is revolute pair here this is gear number 2, which has a revolute pair with a fixed length; so, this constitutes a three c link mechanism. And as we see, if this I call gear number 1 and this I call gear number 2, the teeth are cut external surface of gear number 1, on this blank; gear number 2, the teeth are cut or machined, on external surface of gear number 2 and this teeth of gear number 1 gets into engagement with the teeth, in this case of teeth of gear number 2. And as it rotates, if gear number 1 rotates in the clockwise direction, it also rotates gear number 2 in the same direction.

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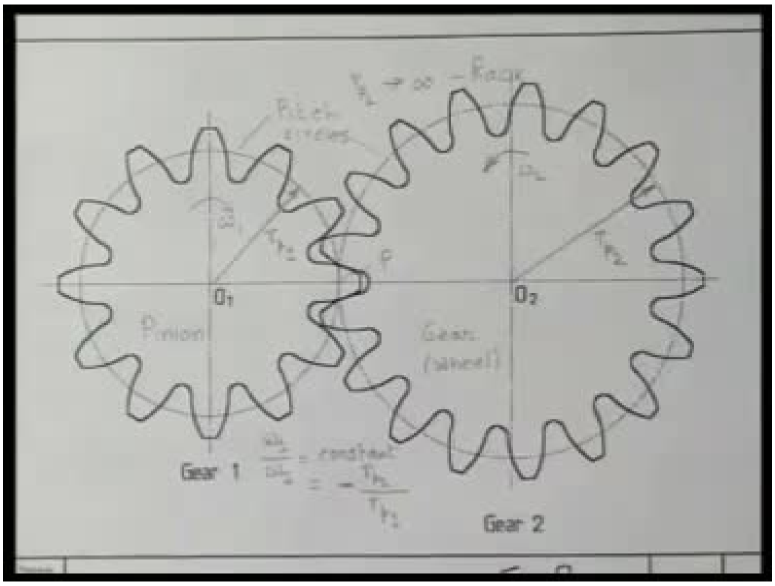

So, this is what we call internal gear. So, we have seen external gears and a pair of internal circular gears, now let me define different terms (( )) circular state to spur gears. This figure again shows a pair of external gearing connecting parallel shafts 1 at O 1 and the other at O 2. The smaller of these two gears, that is here; the number 1, the smaller gear is called pinion and the larger gear either wheel or gear. Now, let us observe these two circles, which are drawn on gear number 1 and gear number 2 touching each other, in the centers of these gears are at O 1 and O 2 respectively.

These two circles touch each other at this point, let me call this point P and we see due to this external gearing, clockwise rotation of this gear is converted to counter clockwise rotation of this gear and the angular velocity ratio omega 1 by omega 2 is constant. Exactly, the equivalent transmission is also possible by counting two friction disks, one disk of this radius and the other disk of this radius.

Imagine two friction disks with center here and radius this much and another center here and radius this much; now, if you imagine these two friction disk, then one can drive the other without slip, then the same angular velocity ratio is transmitted, that means, omega 1 by omega 2, if I call this radius say r p 1 and this radius r p 2, then this constant is nothing but minus r p 2 by r p 1, because there is no slip; so, at this contact points are velocities are same, so r p 1 into omega 1 must be r p 2 into omega 2, but omega 1 and omega 2 are in opposite direction, that is, taken care of by this negative sign. These two imaginary circle, these circle do not exist in gear, these two imaginary circles are called peek circle and had we see most of the gear tooth geometry will be refer to imaginary peek circles.

Now, let us imagine, that this r p 2 is made larger and larger and ultimately r p 2 tends to infinity, if r p 2 tends to infinity, then this peek circle is converted to a straight line and such a gear whose peek circle infinite is call a rack and this is the pinion which can also engage the rack, where this r p 2 is infinity and we have a straight line instead of this peek circle.

Such a rack and pinion arrangement can are very normally used, very frequently used to transmit constant speed from circular speed to linear speed or linear speed to circular speed, that means, a rack and pinion is very often use for conversion of uniform angular motion to uniforms rectilinear motion of the other way round; if I drive the pinion, then the rack will have uniform liner motion and if I drive the rack, then the pinion will have constant angular motion; so, for conversion of uniform rotary to linear or linear to rotary motion can be achieved by using a rack and pinion.

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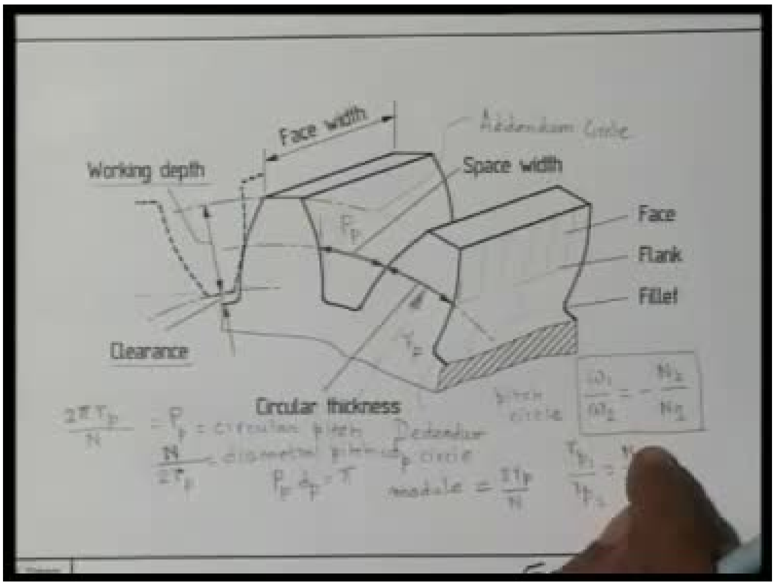

Let me now, define different terms of gear tooth geometry with reference to this figure; we have shown two adjacent tooth of a gear and one tooth of the making gear, this dotted lines shows the tooth of the making gear which is in contact with this gear, of which we have shown two consecutive teeth.

This circle is the pitch circle; this circle refers to the pitch circle of this gear. This width along the axis of the shaft, that is called face width of the gear and this circumferential distance measures along the pitch circle between two adjacencies - the right hand face and the left hand face, this is called space width.

This surface where the contact takes place, as it is taking place here; this surface where the contact takes place, that is called the face of the gear; this is face of the gear, this is face of the gear. On the face of the gear, the portion about the pitch circle, that is, this portion that is called flank face and this is the below this pitch circle, this is called the flank and as you see, there is a little fillet to connect to the body of the blank of the gear. The portion above the pitch circle level is called face, below this is called flank and the flank is connected to the body of the gear through fillet. This outer circle, if we imagine an outer circle passing through the top outer most surface of this gear, that circle is called addendum circle.

Similarly, this bottom circle which belongs to the body of the gear when the teeth ends this is called the duodenum circle. The thickness of this tooth, as you see varies from the bottom to the top, if this thickness is measured along the circumference of the thick circle, that is called circular thickness at the pitch circle The circular thickness give some varying, it is maximum here and here it is minimum. As you see we must prevent the robing of this top surface of the making a with the bottom surface of this gear; so, if this is the addendum circle of the top gear is the top surface from the center of that gear, if you draw a circle that would addendum circle of the making here and it is, this distance, if measured along the radius of this circle that is called clearance.

The radial clearance between the addendum of the making circle and the duodenum of the other gear; addendum of the one gear with the duodenum of the making gear this radial clearance is called clearance. And if we measure the radial distance between this addendum circle of gear 1 with the addendum circle of the making gear, that is, this distance; this circle refers to the top surface of this tooth, the making tooth and the radial distance between them is called working depth.

If we measure the distance along the identical points of two adjacent teeth of the same gear and measured along the pitch circle that is from here to there; this is one point on this tooth, I go the identical point of the next tooth that is here, then these distance measure along the pitch circle is called circular pitch.

If I call it P p, this distance measured along the pitch circle then P p is called circular pitch. The radius of this pitch circle, we have already denoted by r p, we also define something called diametral pitch; by diametral pitch, we mean the number of teeth unit length of the pitch circle diameter.

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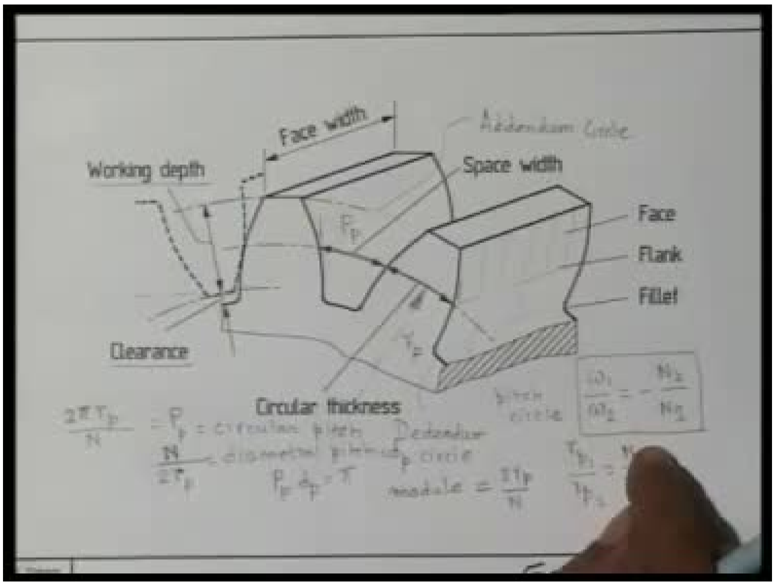

If the number of teeth is N, then N by 2 r p, that is number of teeth by the pitch circle diameter 2 r p d p the pitch circle diameter; this is called diametral pitch. And what is this circular pitch, it is easy to see, this total perimeter of this pitch circle will be 2 by r p and if there a N teeth on this gear, then P p will be 2 r p by N, where N is the number of teeth; so, 2 pi r p by N that is P p. From these two illusions, diametral pitch we write as a d p, so if we multiply by P p by d p, as we see N and 2 pi cancels, we get P p into d p is nothing but pi inverse of the diametral pitch, which is more commonly use these, rather than the diametral pitch is called module, module of the gear tooth and all making tooth of the same module; module is nothing but inverse of the diametral pitch that is twice r p by N, where r p is the pitch circle radius and N is the number of teeth.

Which clearly tells that this pitch circle radius are proportional to the number of teeth for a pair of making gears, because module is same, that means, r p 1 by r p 2 is r p by N is constant; so, N 1 N 2 r p 1 by p 2 is N 1 by N 2, we have already seen the angular velocity ratio omega 1 by omega 2 was minus r p 2 by r p 1, which means minus N 2 by N 1 for a pair of external gears.

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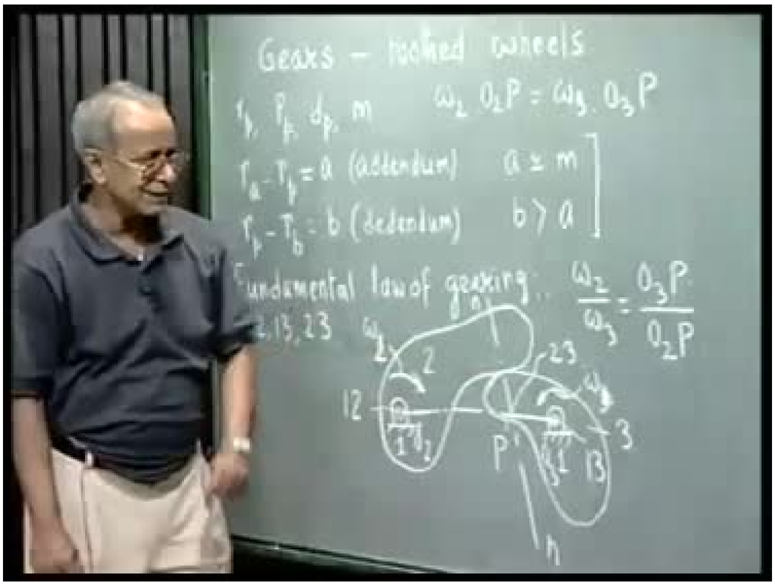

Angular velocity ratio is inversely proportional to the number of teeth in the gear, which are negative sign for external gearing and with the positive sign for internal. We have given so far various definitions, which define the tooth geometry of circular gear like pitch circle radius, then circular pitch, then diametral pitch, then module. We have also defined addendum circle and the difference between the radius of the addendum circle and the pitch circle is called addendum, we defined indicated by the symbol a; a is called addendum.

Similarly, we defined the duodenum as r p minus the duodenum circle radius, say r b this is what we called b and that is what we called duodenum this a and b everything is finally, standardized, because gears are used universally; so, they are standardized according to various standards and a is normally of the order of m and b is greater than a typical values of the standards will discuss much later. What we should know that to have that clearance b should be greater than a; and a is of the order of module. Next, we should discuss what should be the tooth profile, such that the constant angular velocity ratio is minted; so, we have a hard pair, there is a line contact between the, a pair of teeth of the making gears.

Now, what should be the tooth profile, such that constant angular velocity ratio at every instance is minted, this is what is comes from what we call fundamental law of gearing. To discuss the fundamental law of gearing, let say one gear as it is regulate pair with fix link here and the other gear is mounted there, that is the regulate pair second gear mounted to its gearing.

Now, the question that is asked, this is on gear a rigid body and this is another gear, but is the condition necessary, such that these two gears means tends a constant angular velocity ratio; so, let me say this is fix link 1, this is gear body 2, this is another gear body 3. So, this regulate pair obviously is the relative instantaneous center 1 2 and this is the relative instantaneous center 1 3. Now, the question is, what is, where is 2 3, 2 3 we do not know where, but definitely lies on this common normal, if I draw this common normal, then 2 3 lies on this line n n applying arrant hold kinetic theorem, we know that 1 2, 1 3 and 2 3 must be collinear.

So, if you draw the line 1 2, wherever it intersects this line n n, this must be the location of 2 3. Now, if these gear rotates in these direction, at this instant angular velocities omega 1 and this gear rotates angular velocity omega 2, let me call this point of intersection 2 3 as P 2 3, means, the velocity of this point consider to be point on body 2 must have the same velocity, if I consider this point P to be a point on body 3, Which means omega 2 into if this point I call O 2 and this point I call O 3, O 2 is 1 2, O 3 is 1 3; so, O 2 P into omega 2, this I called omega 1, let me call it omega 2 and this let me call omega 3, because body 1 is the fixed body, I am representing one of the gears as body number 2, so I denote it by omega 2 and the other gear is body number 3, so I call omega 3. So, omega 2 into O 2 P in this direction must be same as omega 3 into O 3 P; I consider the velocity of the point P to be a point on body 3, then is velocity due to these omega 3, is again in these direction which is perpendicular to this line O 2, O 3which is omega 3 into O 3.

Which tells me omega 2 by omega 3, where omega 2 are measured clockwise and omega 3 are measured counter clockwise; so, there is a negative sign in or here which is nothing but O 3 P divided by O 2 P. Now, if omega 2 by omega 3, I want to maintain constant, O 2 and O 3 are fixed point, then this point P also must rewind fixed, then only omega 2 the shape of the body should be search, then the common normal always passes to the same point P on the line O 2, O3; this line O 2 O 3 is call the line of centers and this line P is call the line of action, which is the common normal between the making surfaces.

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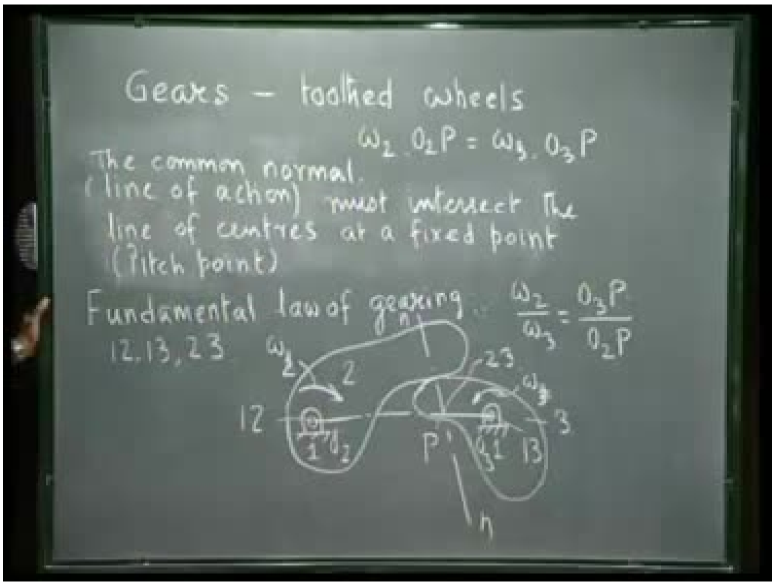

So, for omega 2 by omega 3 to even constant, this point P must not move, this point P must be fixed point on the line O 2, O 3 and this relationship is called fundamental law of gearing, which in words, I can express as the common normal which we call line of action must intersect the line of center, that is, O 2 and O 3 line of centers; O 2 is the center, O 3 is the other center of the gear. The line of the centers, the common normal must intersect the line of centers at a fixed point, then only the constant angular velocity ratio is minted and this is known as fundamental law of gearing.

This fixed point, I will see, will be called pitch point. Next, we shall discuss there are various profiles, which are possible to maintain this fundamental law of gearing and maintain this angular velocity ratio, but as we know most commonly, it is the invalid profile which is used for the gear tooth.

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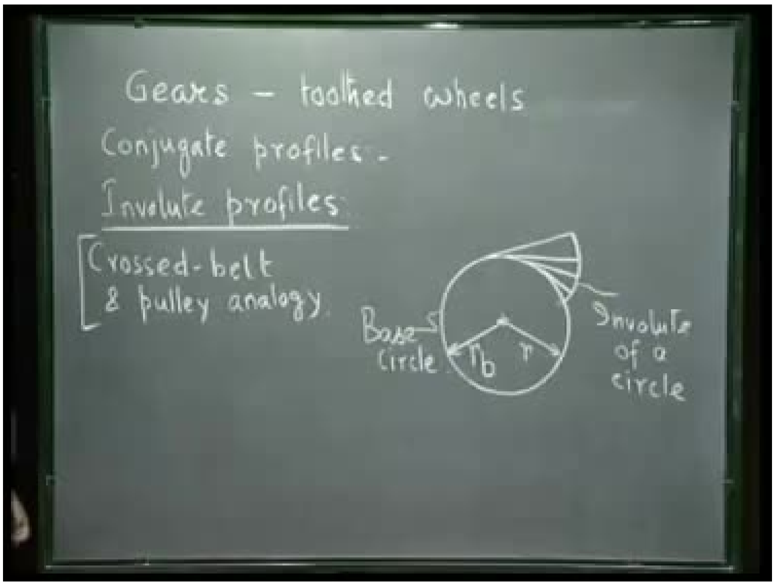

Now, let me see, what is a, what is an invalid, as I said just now gear to profile should be, such that they should satisfied this fundamental law of gearing. Actually, this profiles which satisfies the fundamental law of gearing are called conjugate profile. In fact, if one profile is given, whatever the arbitrary as long as it is continuous, we can always find another profile which is conjugate to the given profile, such that the fundamental law of gearing is satisfied.

But that is not, what is used in practice? In practice we use, what is known as in volute profile. Most common reacted are of in volute profile, as we discussed subsequently this in volute profiles, as lot of advantages including the manufacturing facility fabrication advantage convenience, we always normally use in volute profiles.

Now, let me discuss what is an in volute, in volute of a circle is defined as follows; let us talk of a circular cylinder say of radius r and think of is string, which is wound around this cylinder a string or tape which is wound around this cylinder; this is the end of the tape and now if I on wound this tape, from the cylinder keeping this tape or the string always taught, I am unwinding this tape and keeping the tape always taught, that means, this portion of the tape which has been unwound, which was originally on the surface of the cylinder and now it is being unwound.

So, this is the tape at one position, this is the tape at another instance, this is the tape at another instance. At every instance, the tape will be tangential to this original cylinder because it is kept taught, then the end of this tape, the card that is generate is called in volute of a circle, because initially the tape was wound on a circle, that is why you can define in volute of any card, that means, the originally the sting was wound on that particular cut, but we are not interested anything but in volute of a circle and this circle from which the string is being unwound is called base circle of this in volute and we will, shall denote it the radius of the base circle by r b and this is the in volute of this base circle of pool radius is r b.

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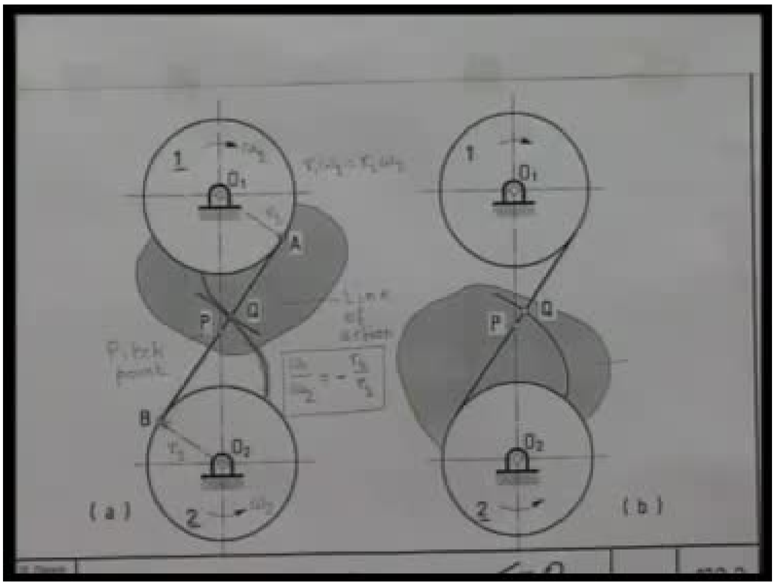

Then, let us show from a, what we call a crossed belt and pulley analogy, to explain why this in volute profile will satisfy the fundamental law of gearing or a pair of in volute will be conjugate profiles, that we shall discuss from what we call s crossed belt and pulley analogy. As I said just now, let me now extend how a pair of in volute tooth profiles can maintain conjugate action, that is, they can transmit constant angular velocity ratio satisfying, the fundamental law of gearing. This we shall do by an analogy with a pulley and crossed belt drive. Let say this is one pulley, pulley number 1 which can rotate about the point O 1; similarly, this is again another pulley, which is pulley number 2, which can rotate about this axis at O 2. This common tangent between these two circles representing a pair of pulleys represents the belt, one side of the crossed belt; you can imagine another common tangent on this side, which will complete the crossed belt; so, this is a crossed belt overlapping these pair of pulleys.

Suppose, the pulley number 1 rotates in the clockwise direction with angular speed omega 1, if we assume there is no slip between the and this pair of pulleys, then the speed of any point of the belt is given by r 1 omega 1 in this direction, where the r 1 is the radius of this pulley number 1, that is, this is r 1.

Because there is no slip between the pulley and the belt, the angular velocity of this pulley, omega 2 will be such that the speed of the belt, is also equal to r 2 omega 2, where r 2 is the radius of this pulley; so, this crossed belt and pair of pulley transmit angular velocity ration between these two bodies, namely 1 and 2, where omega 1 by omega 2, if we take care of the sign, this is clockwise, this is counter clockwise; so, from this relation we get minus r 2 by r 1. At this stage, let me imagine a pencil connected to a particular point on this belt, say at this point Q; if I attach a pencil here, as this belt moves, this particular pencil will draw this straight line A B, it will start from this point; this point Q, a sometime before at a and as the belt moves, the pencil moves along with the belt and draws this line A B, in this piece of paper or in the fixed phase.

Let us, now try to imagine, if I attach a body which is integral to pulley 1, this is an extension of pulley 1, that is pulley 1 and this is same rigid body. What is the curve gone by the same pencil, which are attached to this belt, on this body that is here. To find out determine that particular curve which is drawn by the pencil on body 1, what we do, we will hold the pulley 1 number 1 fixed; if we do not allow the pulley 1 2 rotate, then this string which is unwinding from this pulley unwinds, in the counter clockwise direction, because the pulley was rotating in clockwise direction and the belt was straight. Now, if we the pulley fixed, then the belt as it unwind rotate in the counter clockwise direction. So, as a result this point Q on this belt or string generates and in volute of which this pulley is the best circle, that is, this circle represent the best circle of this in volute.

This particular point when Q was here, this particular point of the pulley was there, when the Q as move down by the distance A Q, this point on the pulley, because pulley rotating in the clockwise direction as move there. So, the same pencil which is drawing this line A B in the space or in the piece of paper fixed space is drawing this particular in volute on this body 1. These circles represent the best circle of that pulley, best circle of this in volute and this is the string which is generating the in volute.

Exactly using similar logic, we can say the same pencil attached to the Q will generate this in volute on this body. This is an integral part of pulley 2, this is rotating in the counter clockwise direction; so, if we hold body to fixed, then is see the string which is winding on this particular cylinder should wind up in the clockwise direction.

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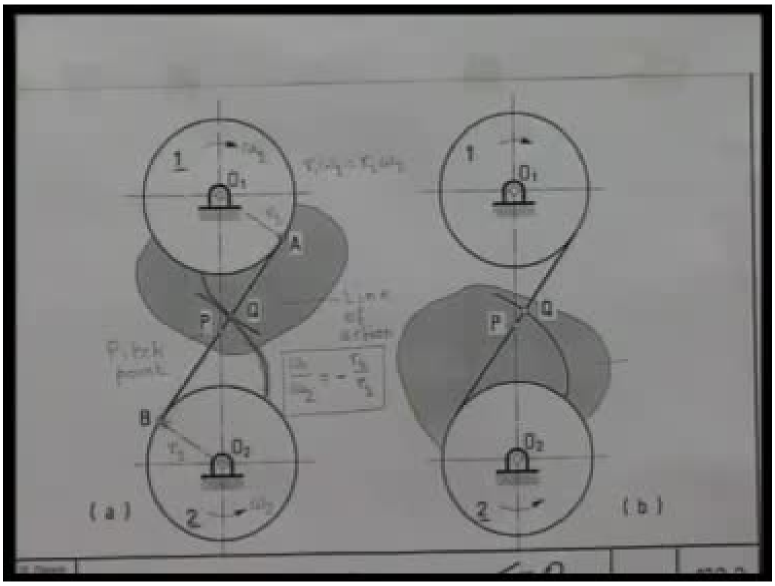

Because this was not moving, this was not having any rotation; this was rotating in the counter clockwise direction. So, if I hold this fixed this windup and also rotates in the clockwise direction and generates this in volute. Now, if we cut this body along this profile and this body along this profile, then this in volute I can have here. Now, the point of contact between these two in volutes is always on this line and if we remove this belt now and rotate this body 1 in the clockwise direction; as we see these in volute will push this in volute in this direction and this will rotate in the counter clockwise direction; maintaining, exactly the same relationship as it was doing, when they are given by this belt when they are connected by this belt.

So, exactly identical motion between this pair of pulleys with the crossed belt can be achieved by cutting an in volute profile on body 1 and an involute profile on body 2 and align one involute to drive the other in volute; again, omega 1 by omega 2 will be given by minus r 2 by r 1.

Since, this notice that as this body rotates the point of contact changes on both this profiles, but it is always lie on this line A B. A further clockwise rotation will may counter clockwise rotation, this point may come in contact with this, but both of this point will lie on this line somewhere So, this line A B which is normal to this in volute, because this is the string, this is the in volute; so, at every instant, the string is perpendicular to the in volute. So, this line A B which is common normal between these two in volute profiles defines what we call the line of action, this line A B what we call line of action and this line of action A B intersects the line of centers at this fixed point P.

So, let me repeat, these are the two in volute profiles, which maintenance the same motion as was been done by this cross belt; if you remove the cross belt and allow this in volute to drive this in volute, then this angular velocity relationship, constant angular velocity ratio is maintained. The point of contact moves along the line A B which we called the line of action and this line A B is the common normal at the point of contact to the pair of in volute. This is the line affection and this point P we called pitch point, P is what we called pitch point. Finally, let me complete this sentence, that using this analogy between pair of police given by a cross belt, we have shown that pair of in volute profile can maintain the same conjugate action, that is, constraint angle velocity issue, even if you remove the belt.

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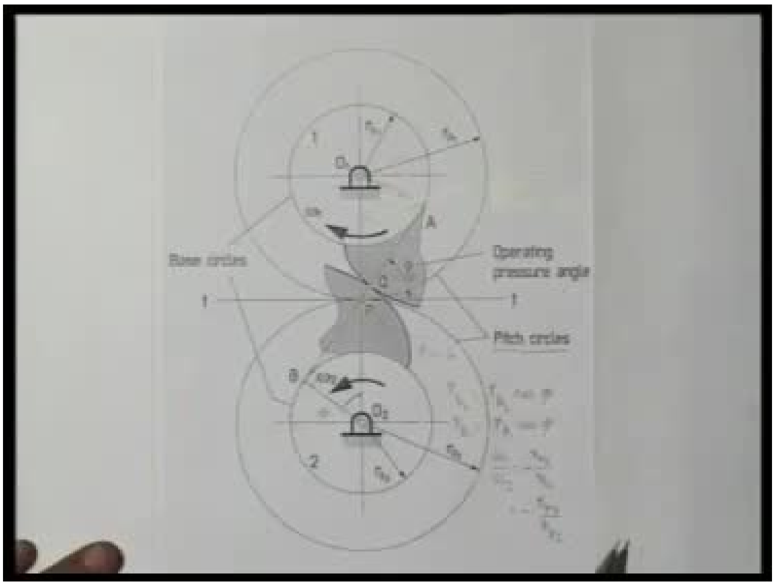

Because a point on the belt which is generating this line A B in space, a generating appear of in volute body 1 and body 2. Let me continue a discussion with belt fully analogy as we if seen just now, that this is pulley 1 of a circle radius are be 1; this is pulley 2 which is the best circle radius are be 2; this A B is a common tangent which represented the belt and these are the two in volute profiles, which as I said can be machine on body 1 and body 2 and remove the belt and rotation omega 2 to this in volute profile will called rotation angler velocity omega 2 of body 2. So, this contact between this two in volute profile, by the very contraction, if this is the sting which is always start and this point is generating this profile, it is obvious that this line A B is normal to both of this in volute profiles, that is, discommend tangent A B between this two base circle is the common known to this in volute profiles, which means, this is what we called line of action which is the common normal between the profiles.

And this common normal is intersecting the line of center O 1, O 2 at this point P and as you see because of this base circles, this point P never change is as we to belt rotate, because these always remains the line of action, the common normal when the contact is here and here, anywhere at this point, the contact is shown as the gee or rotate the contact changes from here to there, but this common normal, always remains the same.

And it is, this common normal which inch of takes the line of centers O 1, O 2 at this point P which is fix point, which is not changing; so, this point P, we called pitch point. So, this clearly establishes that these two in volute profiles maintains the conjugate action, because the pitch point is fixed, it does not change on this line O 1, O 2, it common normal A B intersects O 1, O 2 at the fix point P and if I draw these two circles with O 2 P and O 1 P as radius, these represent nothing but pitch circles; these and these of radius r p 1 and radius r p 2 are pitch circle.

The angle that this common normal A B makes is the common tangent to this two pitch circles t, t is the common tangent to these two pitch circles of radius r p 1 and r p 2.The angle that this common tangent A B to the base circles makes with this common tangent to the pitch circle; this angle phi is called operating pressure angle.

It will hope be as if this angle is phi and then this angle is also phi; similarly, this angle is also phi. O 1 a is the radius this is the point of common tangency A and B. The angle between O 1 A and the vertical line is same as the angle between A B and this horizontal line; so, this is phi, pressure angle; this is also phi, the pressure angle and this angle is also phi, the pressure angle; so, what we see that r b 2 is nothing but r p 2 cos phi, because this angle is 90 degree angle between the tangent and the radius is 90 degree.

Similarly, this angle is 90 degree, angle between the radius and the tangent and this is r p 1 and this is r b 1 and the angle between them is phi; so, what we get r b 2 is r p 2 cosine phi, where phi is called the operating pressure angle and r b 1 is r p 1 cosine phi; so, the angular velocity ratio omega 1 by omega 2, which we earlier have seen as minus r b 2 by r b 1 is also equal to minus r p 2 r p 1, because if you divide these by these, cos phi cancel, we get this.

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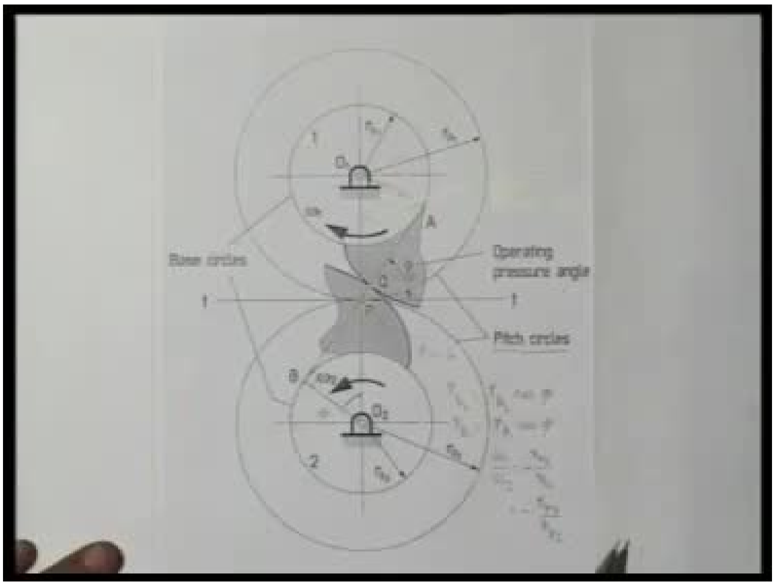

So, we have defined operating pressure angle, the pitch, point the line of action for these two in volute profiles maintaining conjugate action. In our next lecture, we shall discuss what the advantages of such in volute profile.