Kinematics of Machines

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Module – 11 Lecture – 3

Synthesis of Cam Profile (Flat-Face Oscillating Follower)

Today, we continue our discussion on synthesis of cam profile for a flat-face oscillating follower.

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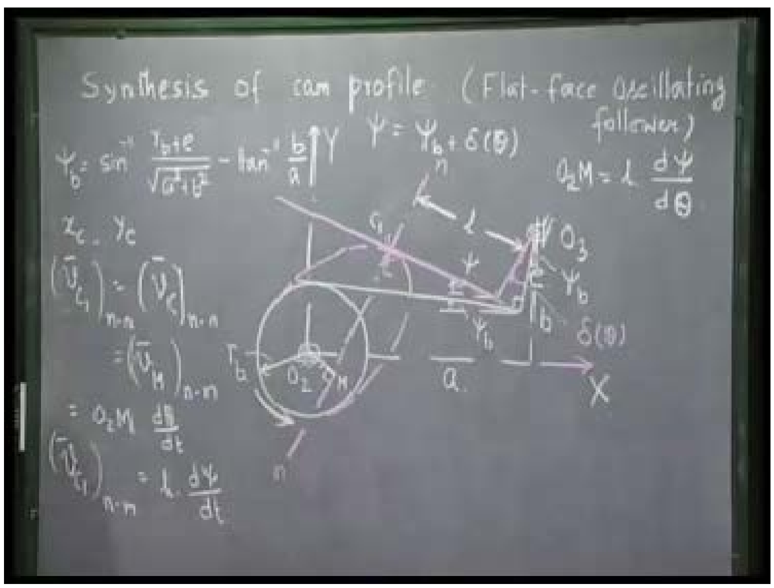

Let the base circle of the cam be given and when the flat-face follower is in contact with the base circle, that is the lowest position of the follower and the oscillating follower is hinged at this location. We call this dimension as ‘e’; the location of the follower hinge from the cam shaft is given by these two dimensions, ‘a’ and ‘b’. The angle that the follower face makes with the horizontal at this lowest position, we denote it by psi_b that is the angle e makes with the vertical or the follower face makes with the horizontal. In our last lecture, we got an expression for this psi_b which was sin inverse r_b plus e divided by the distance O_2O_3, which is nothing but a square plus b square minus tan inverse b by a -base circle radius is r_b. When the cam rotates from this position by an angle theta, let the follower rotates by an angle, delta theta. The follower has rotated by an angle delta theta and this is the follower. This is the cam profile. We denote the distance of this contact point on the cam, let me call it C. This is the common normal within the cam profile and the follower surface. Let us say this distance is l, which keeps on changing as the cam rotates, because the contact point shifts from this cam surface.

As we have done for the translating follower situation, we take our X and Y axis passing through O_2, this is the Y-axis, this is the X-axis. Our objective is to find the co-ordinates of the contact point, namely x_c and y_c. To find this x_c and y_c, I will need this quantity l. So, as the first step, I try to find what this quantity l is. From O_2, I drop a perpendicular to this common normal and let me call this point as 'M'. The follower position is at this instant, again measured from the horizontal is given by psi. As we noted in our last class, psi is nothing but psi_b plus delta theta. We have already got psi_b; delta theta is prescribed for us so I can find psi. To find this distance l, we consider a point C_1 on the follower surface, but at this instant coincident with C. C belongs to the point on the cam and C_1 denotes the point on the follower surface which is in contact. To maintain the contact, velocity of this point C_1 and this point C along the direction of n-n must be same. Otherwise, the contact will be either lost or one point will get into the other point, that means, because they are rigid body and that cannot happen.

To maintain the contact, velocity of the point C_1 along the direction n-n must be velocity of the point C along the same direction. Because M and C are two points on the cam which is a rigid body, the velocity of C and M along the line CM that is along n-n must be same. $[V_C]_{n-n}$ is nothing but the velocity of this point M in the n-n direction because the two points are from the same rigid body. So, this distance CM does not change, that means, the velocity along this direction must be same. Velocity of C and C_1 along n-n must be same; otherwise the contact will be lost.

To find the velocity of the point M along n-n is nothing but O_2M into the angular velocity of the cam. I can write this as O_2M into angular velocity of the cam d theta by dt and that is velocity in this direction. If omega d theta by dt is counter-clockwise, the velocity of M is along n-n, and O_2M into d theta by dt. Velocity of C_1 along n-n is nothing but l times psi dot. $V_{C1,}$ the velocity of the point C_1 along n-n, where n-n belongs to the follower surface is nothing but d psi by dt, where, psi is measured clockwise.

Because these two are same, I can immediately write O_2M is nothing but l into d psi by d theta. Once I get this O_2M equal to l into d psi by d theta, then I can write, as I see if this angle from the horizontally psi and this line is perpendicular to this line, this line is also perpendicular to this line. So, these two lines are parallel, this angle is also psi.

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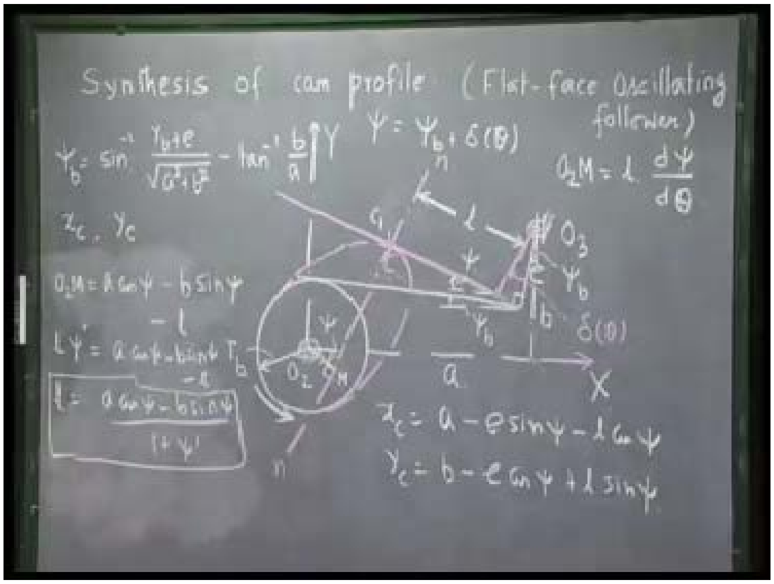

I can write O_2M is a cos psi minus b sin psi; e is perpendicular to n, there is no component and then minus l. I am going from O_2 to M in this way: O_2 to this point, to this point, to this point, to this point and this point. O_2M, if I add up these vectors, the components if I take, a will give me cos psi, b will give me minus b sin psi, e will not give anything, a will again give l cos psi, but with a negative sign, l will give again l, but in this direction, so minus l and O_2M is l psi prime. This is a cos psi minus b sin psi minus l. That gives me l equal to a cos psi minus b sin psi divided by one plus psi. I get this distance l, which changes with theta, because psi is a function of theta.

Now, we are in a position to write the coordinate of x_c, which is a minus e sin psi minus l cos psi. x co-ordinates of this point C is a; there is no projection of b on the horizontal direction. Horizontal position of e is e sine psi, but in this direction, projection of l is l cos psi, and in this direction, that is the co-ordinate x_c. Similarly, we can write y_c is b minus e cos psi plus l sin psi;

out of which I get l from this expression. I have got x_c and $y_{c,}$ everything in terms of theta, once these parameters a, b, e are given to us.

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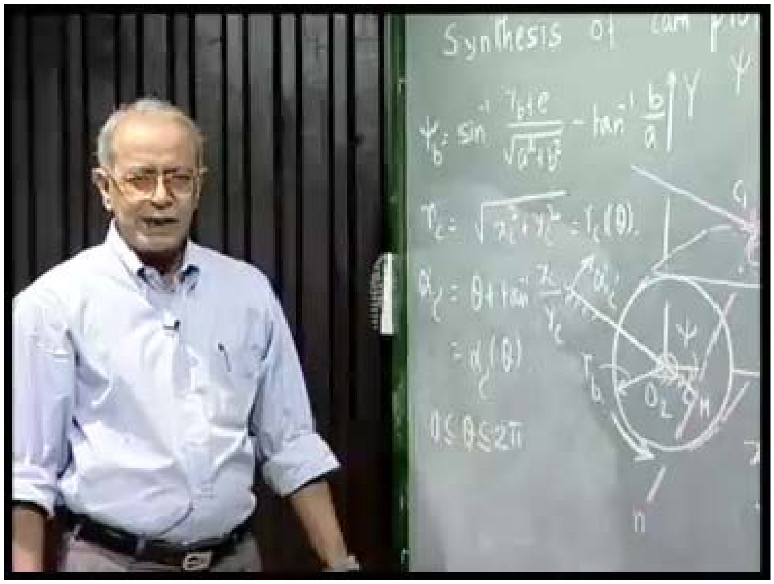

Rest is as before. Once I get the contact point, I can write r_c is square root of x_c square plus y_c square, as we see this is a function of theta. From any line fixed on the cam, if I measure the polar angle from here, if I call it $alpha_{Cprime}$, then $alpha_{Cprime}$ is theta plus tan inverse x_c by y_c and this is also a function of theta. This is the parametric equation of the cam profile in polar co-ordinates with the origin here. r_C which we can get for various values of theta, $alpha_{Cprime}$ which I can again get for various values of theta, where theta represents the cam rotation. Varying theta from 0 to 2 pi, I can get r_C and $alpha_C$ for various values of theta. Then, plotting those values of r_C and $alpha_{Cprime}$, I get the cam profile as we have done in case of translating follower. This portion is exactly as before.

I would like to emphasize one point at this stage that once we get the r_C that the distance from the cam shaft axis and the polar angle for the contact point for all types of follower, translating or oscillating does not matter.

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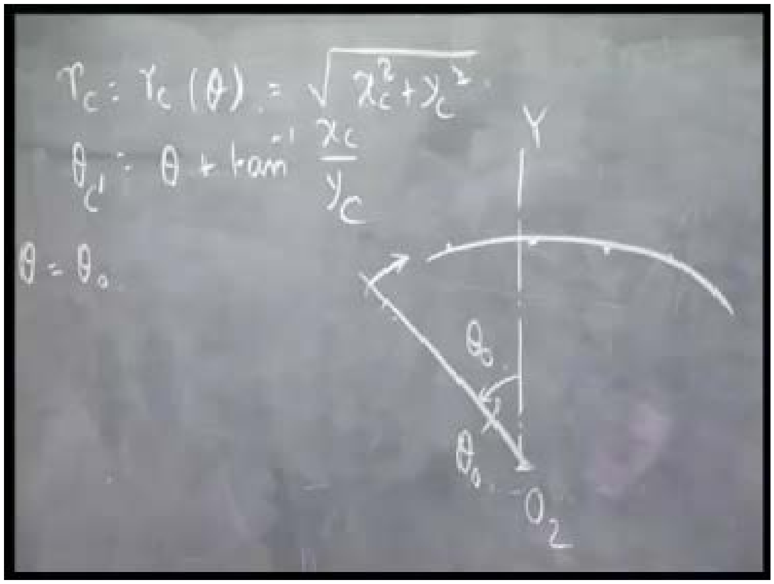

If I summarize this method of analytical method for synthesis of cam profile, we are getting r_C as a function of cam rotation and a polar angle which I denoted by phi_{Cprime} or psi_{Cprime} or $alpha_{Cprime}$ in the various cases. Let me write one of them. I just call theta C prime, which is theta plus tan inverse x_c by $y_{c.}$ This r_c is nothing but square root of x_c square plus y_c square. I can calculate these two quantities for all the values of theta. To draw the cam profile, after getting this series of values what I can decide that I will draw the cam profile, when the cam has rotated by an angle $theta_0$. I take an origin which is O_2, draw a line which is vertical, and then draw a line at an angle $theta_0$. Then, take these values of r_C and $theta_{Cprime}$, but I measure $theta_{Cprime}$ in the clockwise direction from this line. For some values of theta, I get r_C and $theta_{Cprime}$ this point, then this point, then this point like that, and joining these points, I will get the cam profile when the cam has rotated by an angle $theta_0$. This is where we end our discussion on analytical method of cam profile synthesis.

Next, we will do how to do it, the same thing synthesis of cam profile for various types of follower by graphical method. Now that we have finished discussion on the analytical method of cam profile synthesis, we will do the same problem through graphical method.

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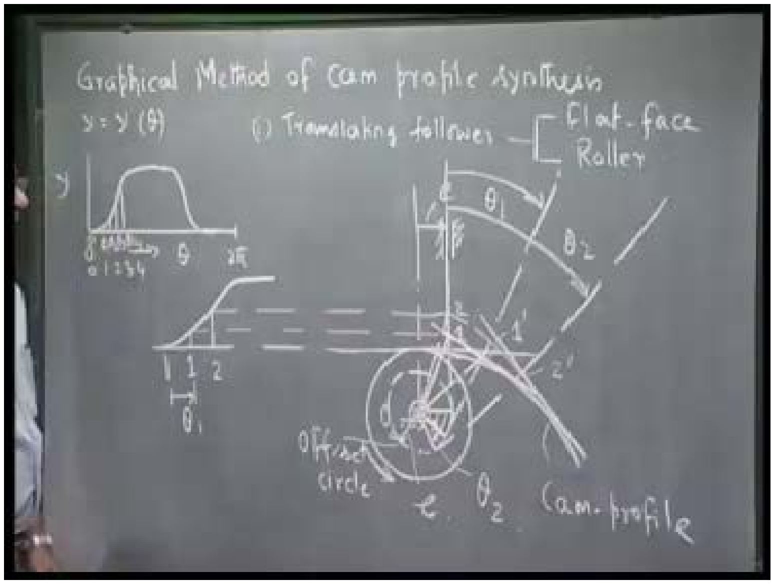

We assume all the geometrical parameters like base circle, radius, offsets, and all these quantities are known. Only thing is, instead of y the follower movement being given in the form of an analytical expression, we have it in the form of a displacement diagram. Given the displacement diagram, how can we obtain the cam profile through a graphical method? As before, we start with translating follower. First, we discuss if the follower is flat-face type, then we will discuss if the follower is a roller follower.

What we do? We first draw the base circle, this is the cam shaft. The offset is also given; let this be the follower axis, this is the follower face at the lowest position and this is the offset. In the graphical method, this total rotation of theta from 0 to 2 pi, we divide in certain number of station points. Let us say, this is 0. Then we take various values of theta like theta$_1$, theta$_2$, theta$_3$, theta$_4$, these are called station points 0, 1, 2, 3, 4. How many divisions we take is the question, but obviously more number of points we take, more accurate will be our drawing. Obviously, we cannot take too many points, because then it will be very time-consuming.

I will just illustrate by taking 2, 3 station points and rest will be same; it can be easily followed. This line represents the follower axis. This corresponds to station point 0, when the follower is at its lowest position. We draw the displacement diagram at the same level. Whatever is the displacement diagram, I draw it at the same level. This is the station point 1 and this is station point 2. We project these points onto this line on the follower axis. Let us say this is 1, this is 2. Here again I mark this as 1, this as 2. To obtain the follower cam profile, as we have done in case of analytical method, we hold the cam fixed and make a kinematic inversion, such that this fixed link rotate in the opposite direction.

Suppose this is the rotation of the cam for station point 1, the cam rotation is $theta_1$ in the counter-clockwise direction. The follower axis has to rotate, for the same station point the kinematic inversion is that the follower axis must rotate by same value of $theta_{1,}$ but in the clockwise direction. If I draw a circle with radius e, then I make this $theta_1$ and draw a tangent. This follower axis has rotated by $theta_1$ in the clockwise direction, corresponding to this station point. That is the kinematic inversion that I hold the cam fixed and allow the follower axis of the fixed link to rotate by the same amount, but in the opposite direction. The distance of this corresponding station point 1 from O_2 is here, this is the distance. This line represents the follower axis at this distance; I rotate this radius along this circular arc and intersect here. This is what I call 1 prime and this is the follower axis, the follower face is perpendicular to that, but at this distance. I draw a line perpendicular to the follower axis. This line represents the follower face after kinematic inversion corresponding to station point 1.

Similarly, corresponding to station point 2, I rotate it by $theta_{2.}$ This particular point of the follower axis should be at this location, that is, so much distance from O_2, I rotate O_2 and this is what I call 2 prime. Then, the follower face is perpendicular to this line; this is the follower face corresponding to station point 2. In this way, I can draw the inverted position of the follower face corresponding to all the station points 1, 2, 3, and so on. The cam profile is nothing but a curve, which is an envelope to this family of straight line representing the follower face.

I draw an envelope to this straight line, which represents the follower face after kinematic inversion. Then, draw an envelope to this family of straight lines and that is nothing but the cam profile. This is the cam profile which is always tangent to the follower face and these lines represent the follower faces corresponding to station point 1 and 2. The same construction can be continued for all the station points and we have to draw a curve, which is tangent to this family of straight line representing the follower faces. So, the basic thing is the kinematic inversion. The kinematic inversion is simple, if we draw these lines tangent to the circle. In some books it is mentioned as offset circle, which is a circle of radius e with center at O_2. But the simplest thing is, to rotate this line O_2a_1 by an angle $theta_1$, then O_22 by an angle $theta_2$. It is very easy to show from the triangle law that this distance is same as this distance, because both of them are e. Then, if I consider station point 1, this distance is same as this distance. This particular right angled triangle is same as this right angled triangle. This whole right angled triangle has been rotated by an angle $theta_1$ in the clockwise direction. I can just as well as say O_2a_1 rotated by $theta_{1,}$ I get this point, O_22 rotated by $theta_{2,}$ I get this point. Once I get this point, then I can draw the follower face perpendicular to the follower axis. Next, I will take up a roller follower, but translating. Let me now explain the graphical method for a translating roller follower.

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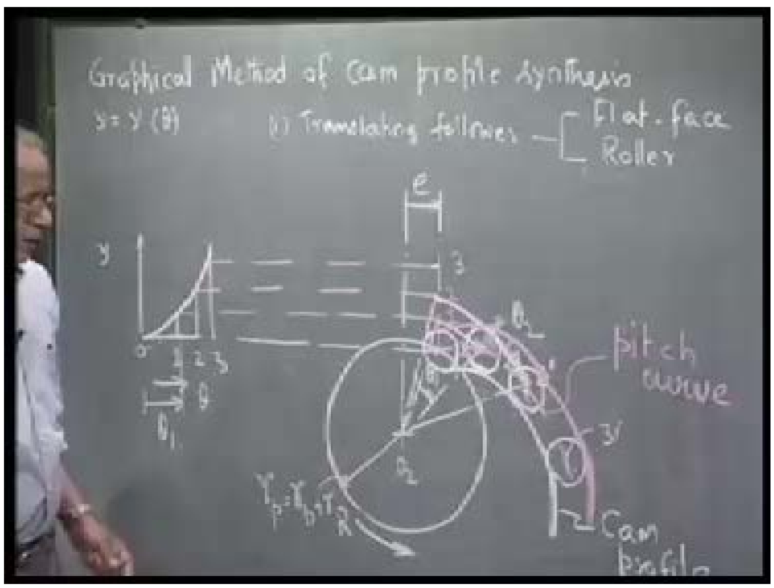

For the roller follower, first, let me draw the prime circle. This is the cam center O_2 and this is the prime circle. So, the radius is $r_{p,}$ which is r_b plus r_R and let us say the offset is e, this is the offset. As we have discussed in case of analytical method, this is the lowest point of the roller center, that is, the trace point. The displacement of the follower will be measured from this level. I draw a horizontal line and draw whatever is the displacement diagram here. As before, I consider few station points namely 1, 2, 3; this is the plot of y versus theta and this is zero. Now, I know that corresponding to the station point 1, the trace point should be here. Similarly, for the station point three, the trace point should be here and for two, the station point is here. To get the inverted position of the station points, we know this offset means, the cam is rotating in the counter-clockwise direction.

Corresponding to station point 1, the cam rotation is known which is $theta_1$. I get the inverted positions of this trace point corresponding to one, by rotating O_2a_1 to an angle $theta_{1,}$ but in the clockwise direction. If I rotate this by $theta_{1,}$ this I call inverted position of the station point 1, which is 1 prime. Similarly, for this 2, I rotate about $O_{2,}$ this line O_22 to an angle $theta_2$ in the clockwise direction. I get the inverted position of this trace point as two pi. Similarly, I do for all the station points namely 1, 2, 3, and 4 like that. These are the inverted positions of the trace point. What is the trace point? Trace point is the roller center and the roller radius is given. Corresponding to 0, this is 0; so I draw the roller whose radius is given.

Here again, with this point as center, I draw the roller; with this point as center, I draw the same roller. In this way, I keep on drawing roller. This is three prime, the inverted position of this trace point corresponding to third station point. What is the cam profile? Cam profile is nothing but the curve, which is envelope to this family of circles representing the roller after kinematic inversion. I have to draw a curve which is tangent to the series of circles. If I draw a curve which is tangent to all these circles, that will give me the cam profile. The methodology is just the same as the analytical method; I make a kinematic inversion with the cam fixed, get the inverted position corresponding to that trace point, and draw the family of straight lines or circles representing the flat-face of the roller follower. Then, the cam profile is nothing but the envelope to this family of circles, in case of roller followers or in case of flat-face follower to the family of straight lines, representing the follower face at the inverted positions. Principle of kinematic inversion is very important that we hold the cam fixed and allow the fixed link to rotate in the opposite direction. The cam is rotating in the counter-clockwise direction; this follower axis for translating roller follower is rotated in the clockwise direction. That rotation can be done very easily geometrically, by rotating this line O_2a_1, O_22, O_23 by $theta_1$, $theta_2$, $theta_3$ in the clockwise direction and that gives me the location of this trace point on the roller centers.

Once I get the roller centers, I can draw the rollers and cam profile is enveloped to this family of circles representing the roller fixed. In fact, if we can imagine, I can draw another envelope touching the circles on the outer side. This was touching the circles in the inner side. Then, I can use this as the groove in the cam face where I do not need a spring. As I told you earlier that without the spring for returning the follower, I can use a groove in the face of the cam and put the roller in that groove. Then, cam will always push it up and pull it down and there is no need to have a spring. Actually, this curve is nothing but what we call pitch curve. This is the cam profile, if we have a spring and this groove is the groove where I have to put the roller, when I do not need to use the spring using a roller follower. Next, we will show how to use the same principle to obtain the cam profile for an oscillating follower either roller or flat-face.

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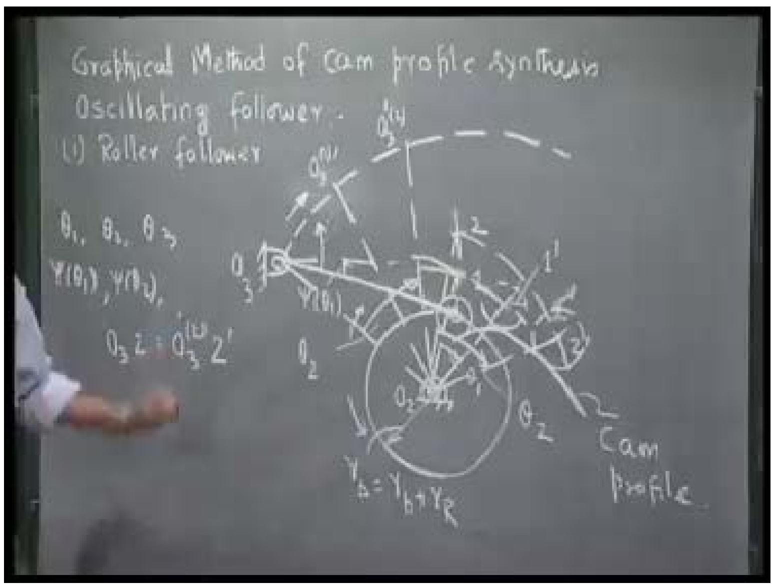

Let me now explain the graphical method of cam profile synthesis with reference to an oscillating follower. Here again, we shall discuss both roller follower and flat-face follower. First let us take a roller follower. We assume all the required geometric dimensions like r_b, r_R -all these things are already known and the displacement function or displacement diagram is given to us. For the roller follower, let me first draw the prime circle. This is the prime circle radius which is as we have noted earlier, base circle radius plus roller radius. When the roller center or the trace point is in contact with the prime circle, that is, the extreme position or the lowest position of the follower, from where we measure the movement of the follower. This point is called O_2.

Suppose this is the roller center at the lowest position and this roller follower is hinged here to the fixed link and this point we call O_3. When the cam rotates in the counter-clockwise direction, let us say this follower also goes in the counter-clockwise direction and this rotation psi as a function of theta is known. The trace point will obviously go on this circle with center at O_3 and this as radius. This will move on this circle. If we take station points corresponding to $theta_1$, $theta_2$, $theta_3$, then we know the corresponding values of psi $theta_1$, psi $theta_2$, and so on. Corresponding to station point 1, let us say the trace point is here, that means, this rotation of the follower is psi $theta_1$. Similarly, I can get all the trace points corresponding to station point 1, station point two, and so on.

As before, we make a kinematic inversion. To draw the cam profile, we hold the cam fixed, which means this fixed link O_2O_3 has to rotate in the clockwise direction. Corresponding to station point 1, the inverted position of O_3, this is the circle with O_2 as center and O_2 O_3 as radius. After the kinematic inversion with the cam fixed, this O_3 will go on this circle. So, corresponding to station point 1, when the cam has rotated $theta_1$ in the counter-clockwise direction, this fixed link will rotate in the clockwise direction by an angle $theta_1$. This I can say O_3 corresponding to one and the inverted position. The distance of the trace point from this hinge does not change. Distance of $O_3$1 should be same as O_3 prime 1 prime and the distance of one from O_2 must be such that this required or desired follower movement is reproduced.

To get the inverted position of one, I draw a circular arc with O_2 as center and $O_1$1 as radius. From $O_3$1 prime, I draw a circle with $O_3$1 as radius and where these two circles intersect; that is nothing but the inverted position corresponding to station point 1 and that is the roller center. At this point, I can again draw the roller whose radius is given to us. Let me again illustrate it with corresponding to station point two, we draw a circle with O_2 as center and $O_2$2 as radius. 2 prime will lie on this circle and $O_3$2 prime, the inverted position of O_3 corresponding to station point two on this circle such that this angle is $theta_2$. From $O_3$2 prime, I take the same radius as $O_3$2 and draw a circular arc. $O_3$2, this distance is same as $O_3$2 prime 2 prime. This distance is same as this distance and $O_2$2 is same as $O_2$2 prime. This is the inverted position of the trace point, that is, the roller center. Here again, I draw the roller of same radius. If I draw a curve which is tangent to this family of circles, then that gives me the cam profile.

In fact, very simple geometrical consideration can show that this point we call 1 prime, that 1 prime I can easily get if I rotate O_2a_1 $O_2$1 in the clockwise direction by theta$_1$, I get to 1 prime. Similarly, if I rotate $O_2$2 through an angle theta$_2$ in the clockwise direction, I get to 2 prime. This can be very easily proved from congruence of triangles. We consider a triangle O_2, $O_3$1, and $O_3$1 1 prime, O_2; we can show that very simply. These I leave for you to do. Instead of going through this by drawing two circular arcs, I can as well rotate $O_2$1 by theta$_1$ in the clockwise direction to get 1 prime; $O_2$2 by theta$_2$ in the clockwise direction to get 2 prime. Then, draw the rollers and then, draw an envelope to this family of circles.

This I did to explain the kinematic inversion. It is actually the fixed link which is rotating, that means O_3 is going along this circle in the clockwise direction, when I hold the cam fixed. But, from congruence of triangle, geometric construction can be much simplified, by just not doing all these; only rotating $O_2$1 through this theta$_1$ and $O_2$2 through theta$_2$. Next, we will take up the case of a flat-face follower.

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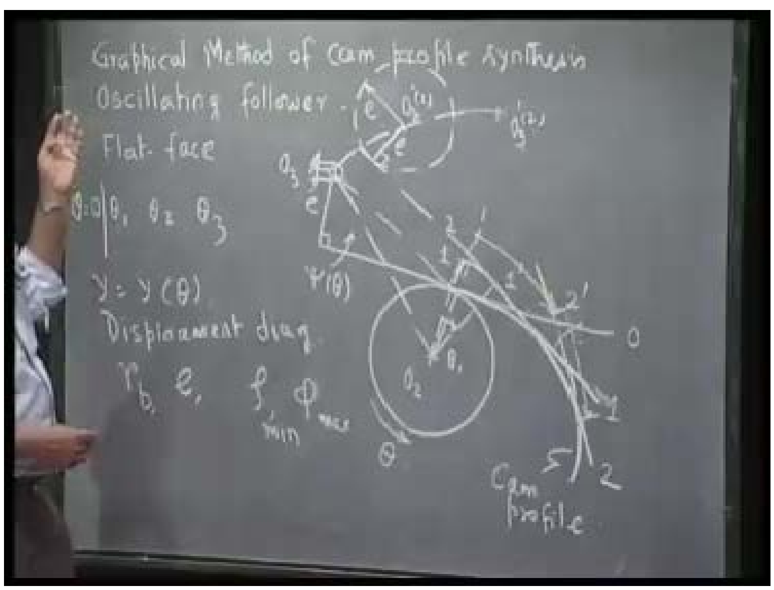

Let me now explain this graphical method for an oscillating follower, which is flat-face type. If it is a flat-face follower, let us say this is the base circle radius and the flat-face is tangential to this (43:57) (Not Audible) circle radius at the lowest point. This is where the follower is hinged. In the analytical method, this dimension is what we call e. As we see, as this thing rotates this angle, this is one rigid body and this angle always remains 90 degrees, which means the flat-face is always perpendicular to this line.

Let me use this particular contact point as the trace point. As the cam rotates in the counter-clockwise direction and as the follower oscillates in the counter-clockwise direction, this particular trace point of the follower moves on a circle with O_3 as center, and this as radius. This trace point moves on this side and where will it be? It will be decided by the rotation of this and this is we call O_2. Once this is prescribed, the movement of the follower again I consider station point $theta_{1,}$ $theta_{2,}$ $theta_{3,}$ and so on. This situation is shown for theta equal to zero. This is the situation when theta is zero. When it is $theta_{1,}$ the cam has rotated by $theta_1$ and this trace point goes here. So, I call it as before one, similarly for two, it comes here at two.

To draw the cam profile, I make use of the same principle of kinematic inversion holding the cam fixed. As I explained earlier for the roller follower, the inverted position of one can be easily obtained by rotating $O_2$1 through an angle $theta_1$ in the clockwise direction, because this O_2O_3 moves in the clockwise direction. This is what we call 1 prime. The next question is how I draw the flat-face passing through this point 1 prime, this flat-face in the inverted position.

For that, O_2O_3 I rotate as before the O_3 inverted position is here, which I call $O_3$1 prime. Here, I draw a circle of radius e with this point as center. This is of radius e and the trace point is here. Also, I know the follower face will be always perpendicular to this offset e, which means, this will be tangential to this circle, offset e is same and from 1 prime, I draw a tangent and this is e. This represents the follower face after kinematic inversion. Similarly, I can rotate $O_2$2 through an angle $theta_2$ to get the inverted position 2 prime. Then, I go to $O_3$2 wherever that is by rotating O_2O_3 through $theta_2$ in the clockwise direction.

Then, again I draw this circle of radius e and from 2 prime, I draw a tangent to that circle. Then, that tangent, I am not drawing everything, since it becomes cumbersome. Procedure is same; I go from 2 to 2 prime by rotating $O_2$2 through an angle $theta_2$ in the clockwise direction; that gives me this point. I rotate O_2O_3 through $theta_2$ in the clockwise direction and that gives me $O_3$2 prime. Then, with this point as center and of e radius, I draw a circle. From this point, I draw a tangent to that circle and that tangent let it be this, which represents the inverted follower face. A curve which is tangent to all these inverted positions of the follower face, an envelope to this inverted positions of the follower face, this is corresponding to one, this is corresponding two, this is corresponding to zero.

Once I get the inverted positions of the follower faces which is the family of straight lines, then draw an envelope to this family of straight lines, that is nothing but the cam profile. What we should note that here, the contact point is one, but here the contact point is not 1 prime, the contact point has shifted on the follower face as we have seen while we discussed the analytical method. Similarly, this is the 2 prime inverted positions, but the contact point is here. If we measure the maximum distance of the contact point from this trace point on this side, that will give me the required follower which is on the right hand direction. To the right of this contact point, how much the contact point moves, so that I must have the required follower face.

Similarly, if we continue, we will see that it will be to the left of this; right now, it is to the right. For these three positions, 0 it is alright; 1 it is here, but the contact point is here; 2 the point is here, but the contact point is here. This is the shift of the contact point on the follower face in position two. If we complete it, we will see that this contact point shifts in one direction, then returns, and then shifts in the other direction. The maximum distance that is needed for the contact point to move on the follower face, that decides the minimum width of the follower face.

That brings us to the end of cam follower mechanisms and let me summarize what we did. We first explained the different terms which are needed to describe the cam follower mechanism. After that, we described what the typical follower basic follower movements are. We had both the analytical expressions and the graphical construction for the displacement diagram. Follower motion, either y expressed as a function of theta or through displacement diagram.

Then, we showed how to get the basic dimensions like r_b, e, and so on, so that, certain constants like minimum radius of curvature or maximum value of the pressure angle, all these conditions are satisfied. We would always obviously like to go for the smallest value of r_b that we can get away with, so that the cam size is small. After getting these basic dimensions, we discussed both the analytical and graphical methods to obtain the cam curve or the cam profile and that is the synthesis problem. Of course, given the cam curve, we can also find what should be the follower movement; that is what we call the analysis of the follower movement. But, that is purely a geometrical problem, which is quiet simple; even we can do analytically or graphically. We are not going to discuss that in this course.