Kinematics of Machines

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Module-01 Part-03

In today's lecture, we shall discuss folding planar mechanisms. As we shall see such mechanisms have versatile applications. In fact, a major portion of this course will be devoted to the study of folding planar mechanisms.

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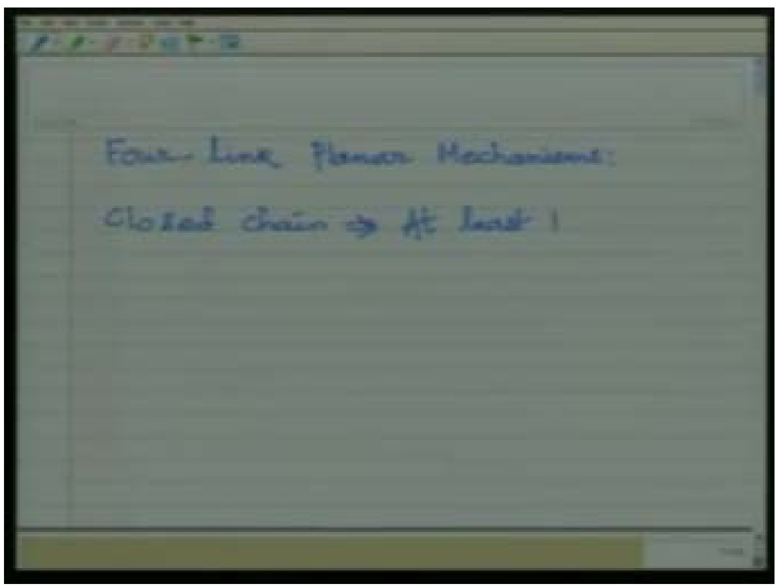

Before getting into the folding planar mechanisms, let us recall the technical, that is, kinematic definition of a mechanism. We have already defined mechanism as a closed kinematic chain with one of its links fixed. It is obvious, that to get a closed chain, we need at least three links. In the next figure, we shall see a three-linked closed chain with three revolute pairs.

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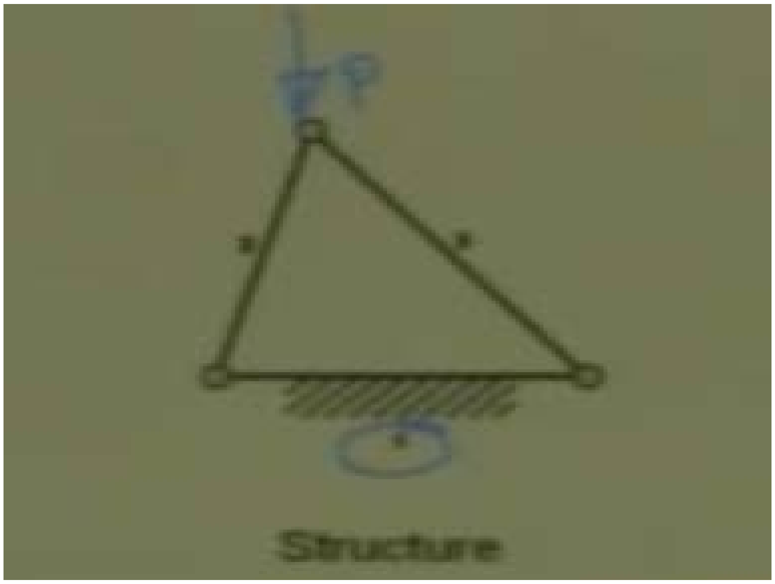

In such a three-linked closed chain, it is obvious that if one of the links, say, number 1 is held fixed, then there cannot be any relative movement between these three bodies. In fact, if we apply an external load there is no relative movement between various links, this load can be supported by this assembly. Such an assembly with 0 degree of freedom is called a structure. Hence, we see the folding planar mechanism is the simplest mechanism that we can think of. To start with, we consider a folding planar mechanism with four revolute pairs.

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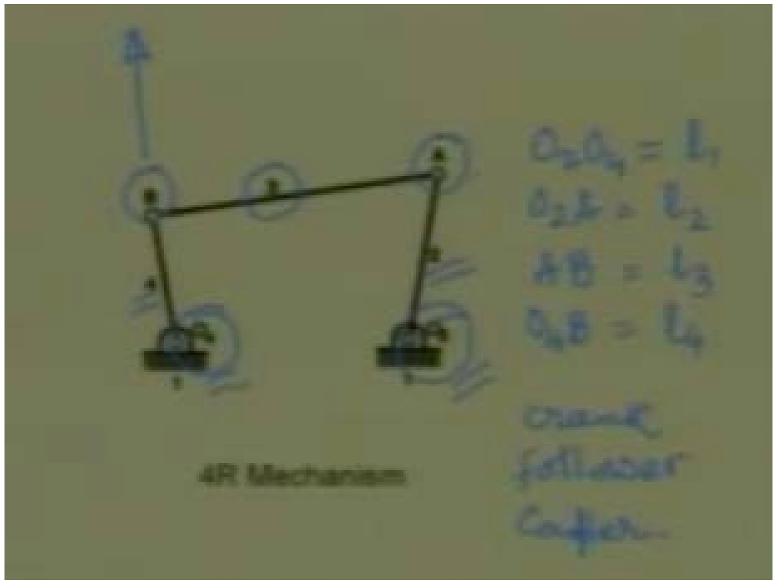

As we see in this folding planar mechanism, there are four revolute pairs: one at O_2, one at O_4, one at A and the other at B. The folding is numbered, 1 as the fixed link and 2, 3, 4 as the moving links. These two hinges at O_2 and O_4 which are connected to the fixed link are called fixed pivots and the revolute pairs at A and B are referred to as moving hinges. Link number 2 and link number 4, which are connected to the fixed link are normally used as the input and the output link. Link number 3, which connects the links 2 and 4, is called the coupler, which is the motion transfer link between 2 and 4. Such a 4R mechanism has four kinematic dimensions namely, O_2 O_4 which we represent by l_1 and O_2A which represent by l_2. Similarly, AB is equal to l_3 and O_4B as l_4.

If one of the input links which is connected to the fixed link makes a complete rotation, as is normally in the case, when mechanism is driven by an electric motor, then that link is referred to as a crank. Link number 4, which is the output link, is called follower and the intermediate connecting link is called the coupler.

Let us consider a 4R planar mechanism where one of these kinematic pairs at B goes to infinity along a vertical direction. Consequently, that kinematic pair is converted to a prismatic pair and what we get is a three revolute one prismatic pair, referred to as the 3R-1P mechanism.

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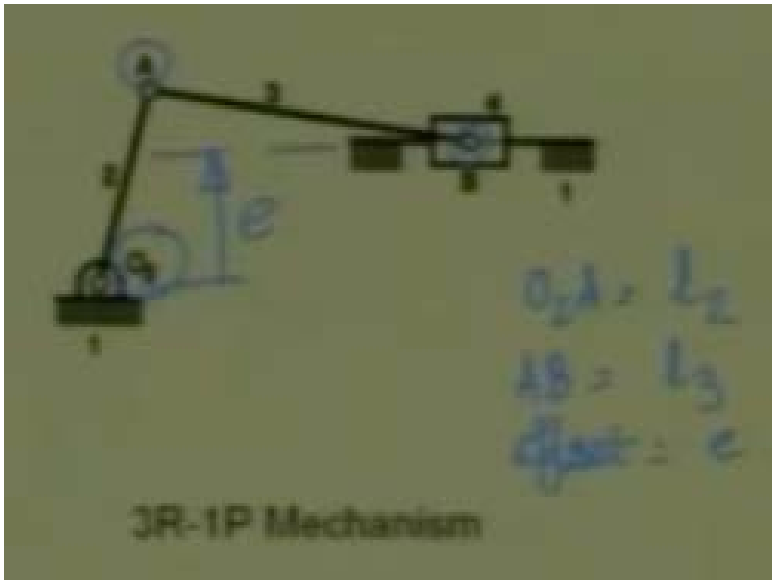

So we get a 3R-1P mechanism where there are three revolute pairs at O_2, A and B and the kinematic pair between link 4 and link 1 is a prismatic pair. Such a mechanism has three kinematic dimensions namely O_2A which is called the crank length is represented by L_2, AB which is called the connecting rod whose link is represented by L_3 and the line of reciprocation of B is at a distance from O_2 and this perpendicular distance is called the offset. The other kinematic dimension is the offset which is equal to e. This is also known as an offset slider crank mechanism which is used to convert uniform rotation of link 2 in to rectilinear to and fro oscillation of the slider 4.

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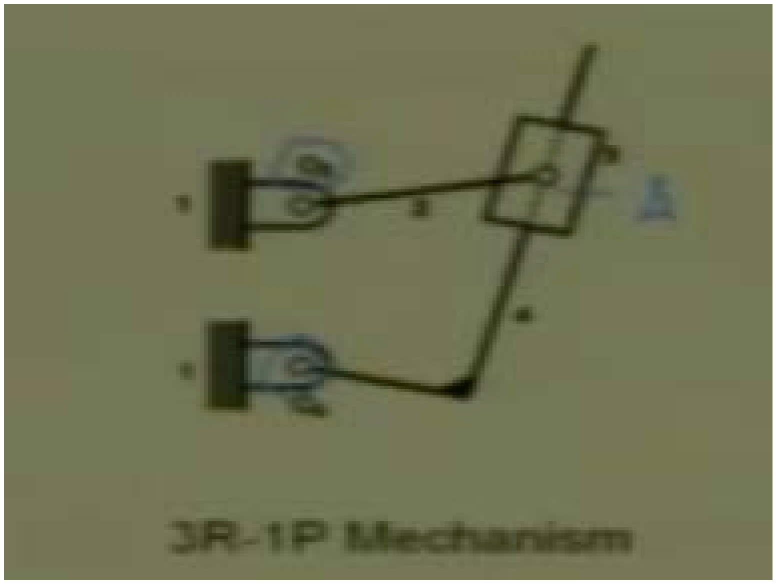

Let us consider different kinematic inversions which can arise out of such a 3R-1P chain as shown in this figure. Here, as we see there are three revolute pairs namely, O_2, O_4 and at A. Comparing to the previous 3R-1P chain, here the link 1 which is fixed has revolute pairs at both ends, that is at O_2 and O_4.

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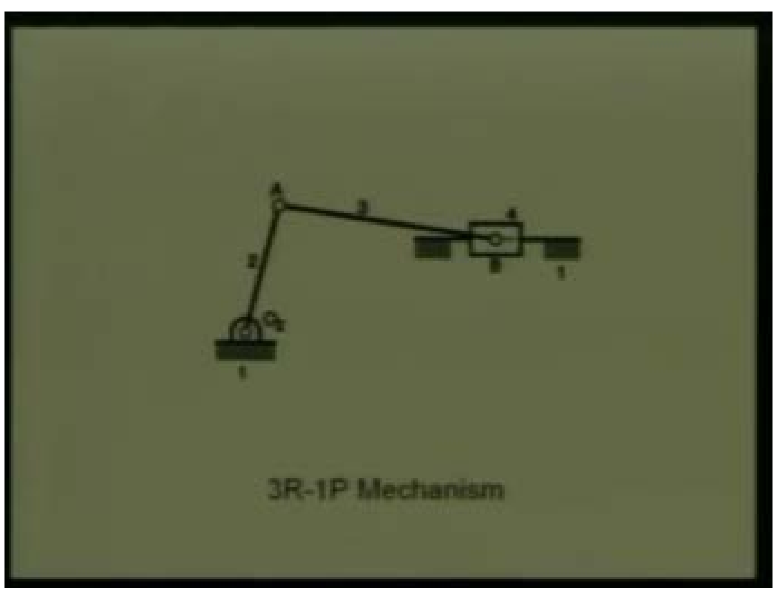

In the previous mechanism, the fixed link had a revolute pair at one end and a prismatic pair at the other end.

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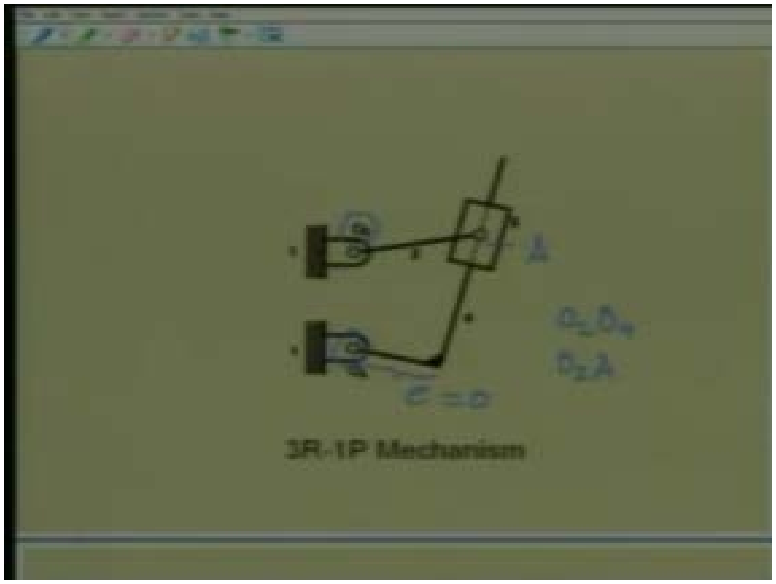

Here again, we have three kinematic dimensions namely O_2 O_4 - that is the fixed link length, O_2 A that may be the crank length and the distance of this line of reciprocation from these fixed changes. Such a mechanism with this offset e equal to 0 is used in a quick written mechanism that is used in a shaper.

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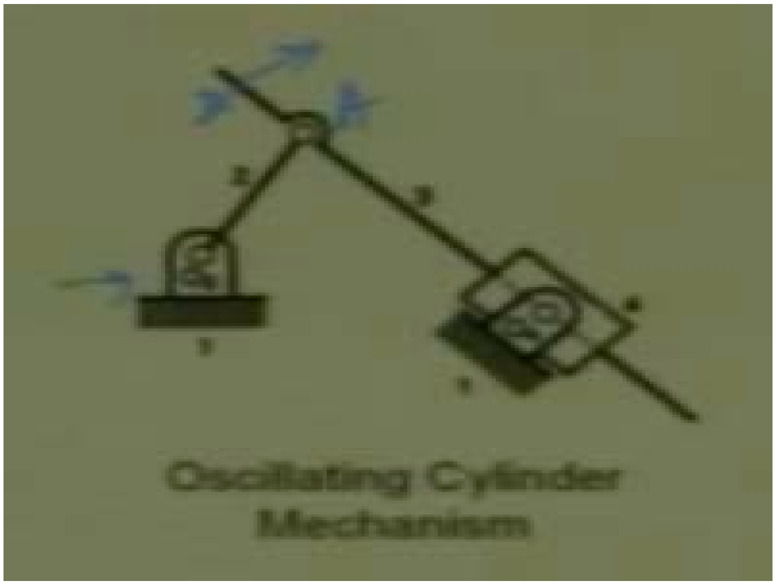

We consider again yet another kinematic inversion of the same 3R-1P chain. Here, we have three revolute pairs namely, at O_2, A and O_4. Here again the fixed link has revolute pairs at both ends namely O_2 and O_4. Such a mechanism is called an oscillating cylinder mechanism which is used, for example in a bicycle foot pump. The thing to note is that here the input member is link number 3 which is given an oscillatory motion. This piston link 3 moves within this oscillating cylinder which has the same angular velocity as link number 3.

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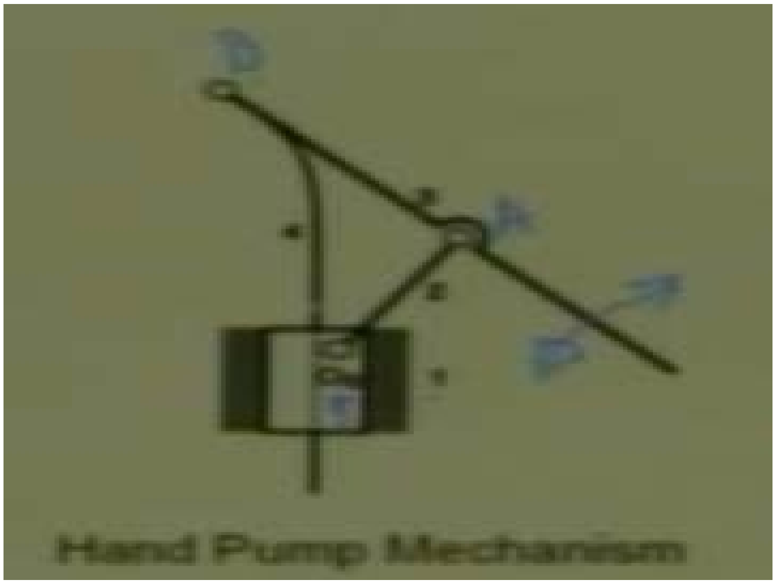

We see another kinematic inversion from the same 3R-1P chain where we have kinematic pair at O_2, A and B. The link number 4 which has a revolute pair at B has a prismatic pair in the vertical direction with respect to the fixed link 1. Such a mechanism, as we know is used in a hand pump mechanism and here again it is link number 3 which is given the input motion such that the piston which is link number 4 moves vertically up and down within this fixed link. So far, we have considered 3R-1P chain. Let us have another revolute pair converted into a prismatic pair and consequently we get what we call a 2R-2P chain.

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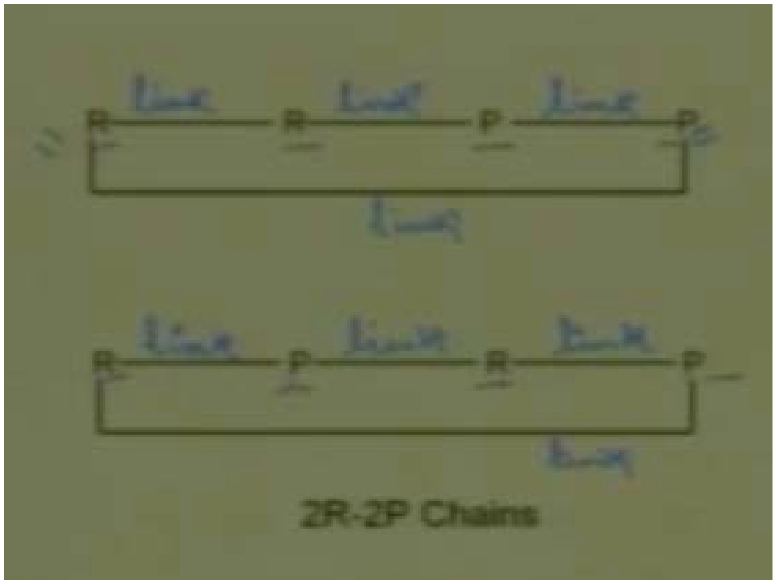

However, unlike in a 3R-1P chain, there can be two varieties that is the sequential order of this kinematic pairs. For example, we can have RRPP or RPRP. Here, the two revolute pairs are connected by one link, another link has a revolute pair at one end and a prismatic pair at the other end, another link has prismatic pair at both ends and this link has a PPR at one end and a revolute pair at other end. This is what we call RRPP chain, whereas here these kinematic pairs appear alternately as RPRP. All the four links have a revolute pair at one end and a prismatic pair at the other end that is same for all these four links. It may be emphasized that in a 3R-1P chain whichever way we go, we start from one prismatic pair then it is followed by three sequential R pairs. So, PRRR is the only possibility. Let me now consider some inversions from this 2R-2P chain.

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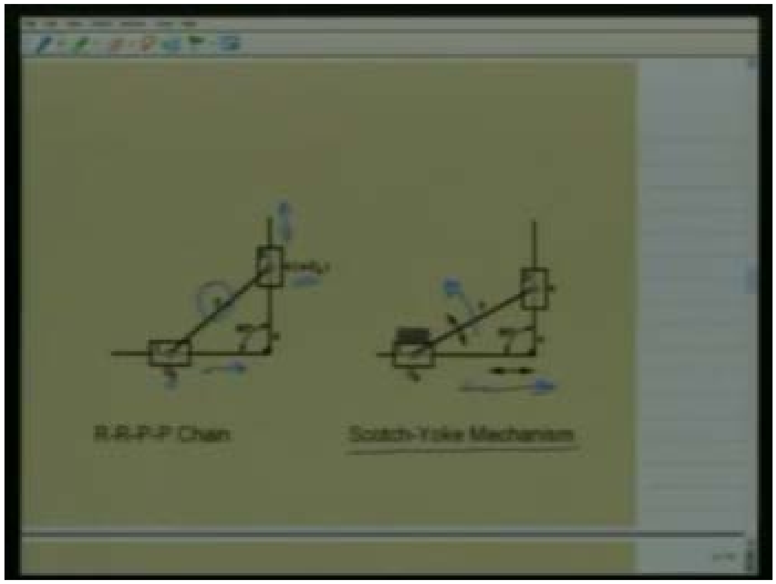

Here, we consider a folding planar mechanism with a revolute pair at O_2, another revolute pair at O, which I called as O_4, that is link number 2 has two revolute pairs at its either ends, and link number 3 has a revolute pair at O and a prismatic pair with 4 along the vertical direction. Similarly, link number 1 has a revolute pair with 2 and a prismatic pair with 4 in the horizontal direction. We consider a special case when these two prismatic pairs have an angle of 90 degree between them.

From such a 2R-2P, that is RRPP chain, we can have different mechanism by the process of kinematic inversion. For example, let us hold this link number 1 fixed. So we can see that link 4, the output link can have horizontal movement with respect to the fixed link as link 2 undergoes rotary motion with respect to link 1, that is the fixed link. This is the kinematic sketch. Let me see the physical construction of such a mechanism known as the scotch yoke mechanism. As we see, such a mechanism will convert uniform rotary motion of link 2 into simple harmonic translatory motion in the horizontal direction for link 4.

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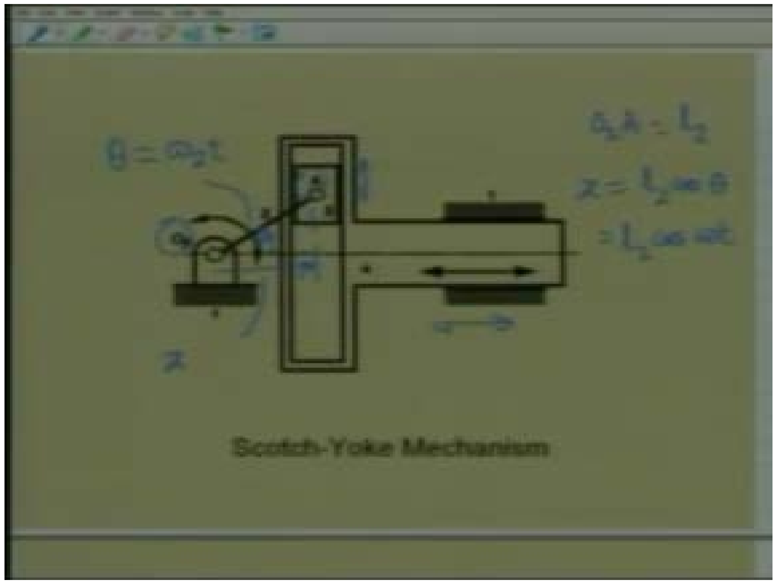

Again we have RRPP chain. There is one revolute pair at O_2, one revolute pair at A, there is a vertical prismatic pair between 3 and 4 and a horizontal prismatic pair between 4 and 1. If we consider the kinematic dimension O_2 A as L_2 and this angle if I call theta, assuming at t equal to 0 is theta is equal to 0, then I can represent theta is equal to omega$_2$ into t, where omega$_2$ is the constant angular speed of link number 2. Then the position of this link number 4 which can be represented by this point is given by x. It is easy to see that x is nothing but L_2 cosine theta, that is L_2 cosine omega$_t$. So, we have produced a simple harmonic motion out of continuous rotation. It may be emphasized that continuous uniform rotary motion would have been translated into to and fro rectilinear oscillation by a slider crank mechanism that we have seen earlier.

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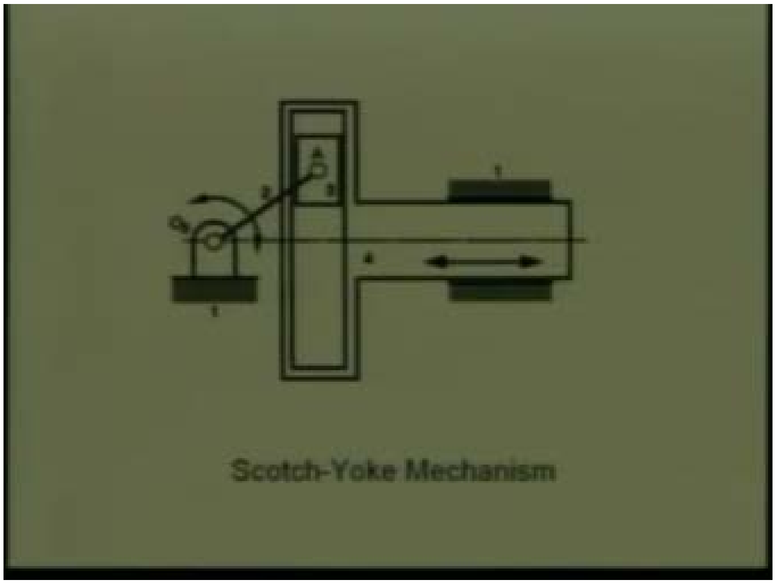

However, the to and fro rectilinear oscillation of the piston of a slider in a slider crank mechanism is periodic, but not purely harmonic. This periodic motion tends to be harmonic as the ratio of the connecting rod length to the crank radius keeps on increasing.

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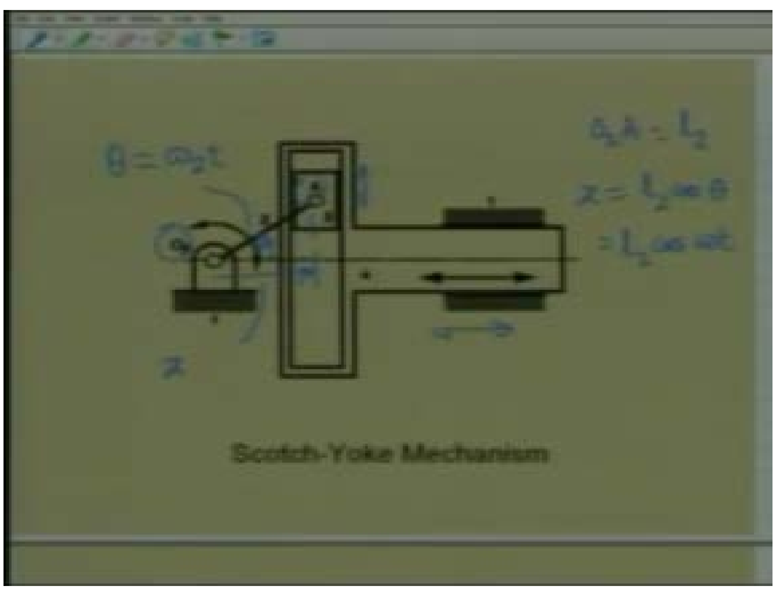

In fact, if the connecting rod length in a slider crank mechanism tends to infinity, then the slider motion becomes purely harmonic. As soon as the connecting rod length becomes infinity, one of the kinematic pair, which was the revolute pair for a connecting rod, gets converted into this prismatic pair, because we have already seen a prismatic pair is nothing but a revolute pair at infinity. I will now show this through a model.

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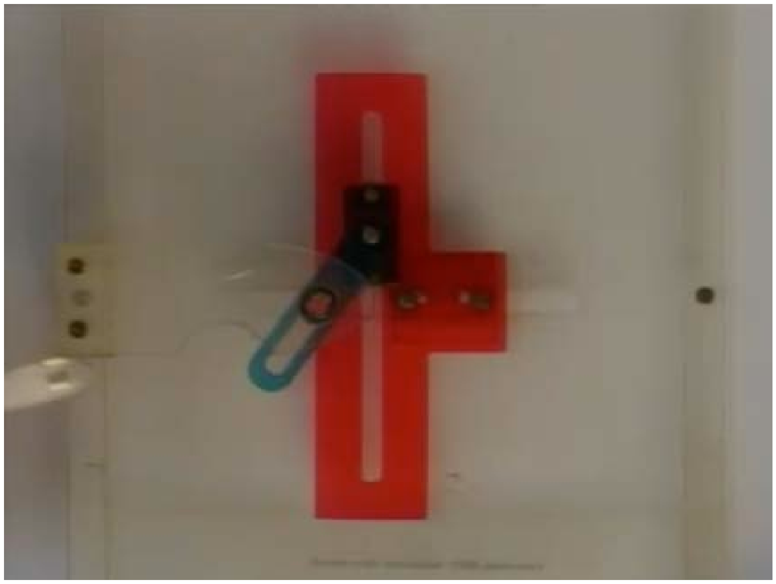

This is the model of that scotch yoke mechanism. This is the link number 2 that is the crank, which has a revolute pair with the fixed link at this point. Another revolute pairs between link 2, that is the crank and this block, is here. This block link number 3 has a prismatic pair in the vertical direction with link number 4 and link number 4 has a horizontal prismatic pair with link number 1. If we rotate link number 2 at a constant angular speed, as we see, the red link that is link number 4 is performing simple harmonic oscillation in the horizontal direction. Let us now consider another kinematic inversion of the same RRPP chain.

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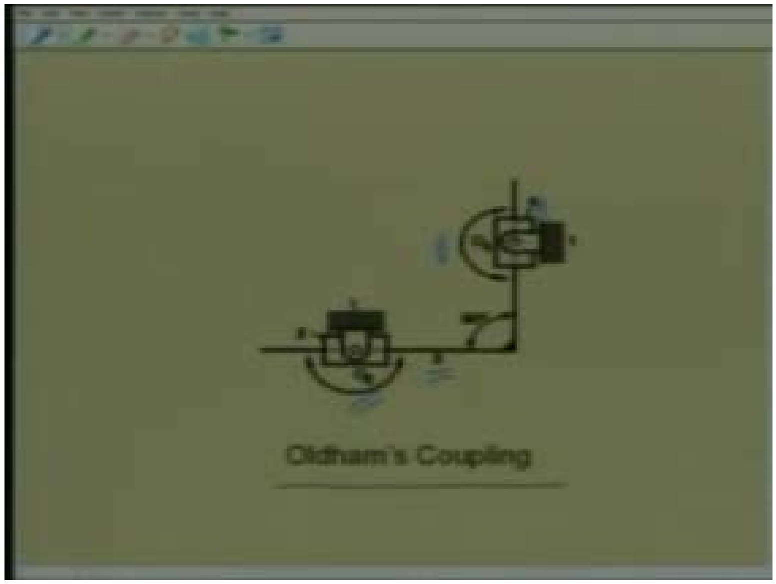

As shown in this mechanism, the link which has revolute pair at its both ends at O_2 and O_4 is held fixed. Link 3 has a prismatic pair in the horizontal direction with link 2 and a prismatic pair in a vertical direction with link 4. So, if we rotate link number 2 that is in translation with link number 3 which is again in translation with link number 4, thus link 2 and link 4 has only relative translatory motion. In other words, they have the same angular motion. Thus this mechanism known as Oldham's coupling can be used to connect two parallel shafts: one at O_2 and the other at O_4 and transmitting uniform angular velocity ratio.

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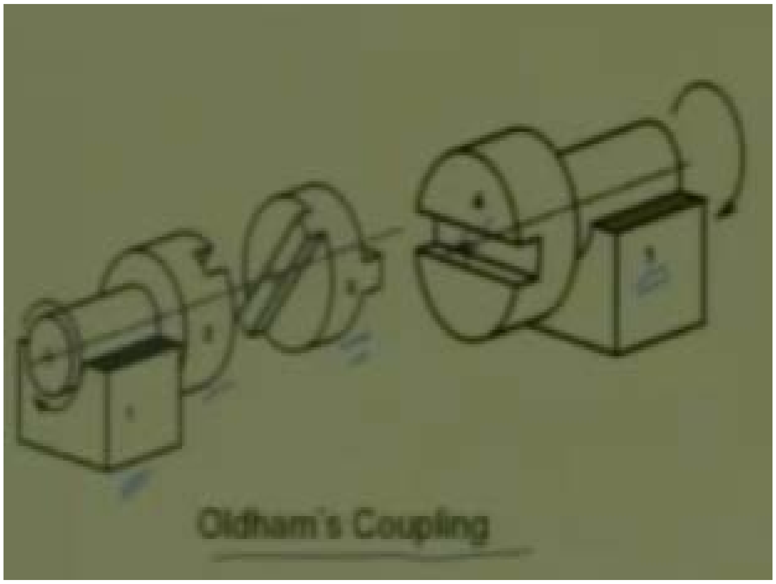

This is the kinematic diagram. The physical construction of this Oldham's coupling is shown in the slide. As we see, there is a revolute pair between link 1 and link 2, there is a prismatic pair between link 2 and link 3, another prismatic pair between link 3 and link 4. The direction of these two prismatic pairs are at right angles to each other. Link 4 again has a revolute pair with the fixed link 1. Thus link 4, that is, this shaft and link 2 that is, the other shaft which are parallel can be connected by such a coupling and it will transmit uniform angular velocity ratio.

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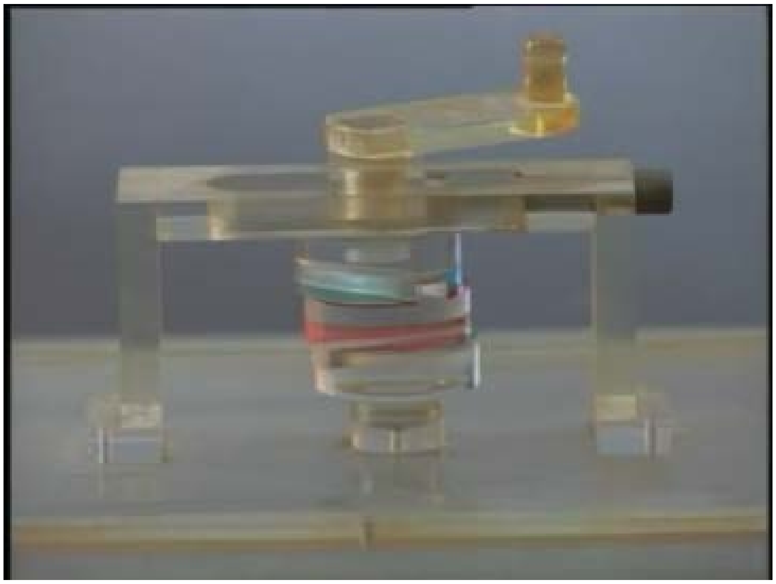

I will now show this through a model. This is the model of Oldham’s coupling. As we see, the rotation of this shaft is converted into the rotation of that shaft at the same speed through the intermediate member which has a prismatic pair here and a prismatic pair at the top. So, there are two prismatic pairs at 90 degrees to each other. This intermediate member moves in and out in this prismatic pair and also at this prismatic pair. Of course, this coupling is good enough to transmit power, only when the offset between these two shafts is not very large, because a lot of power is wasted in fiction at this two prismatic pairs. Let us consider another kinematic inversion from the same RRPP chain.

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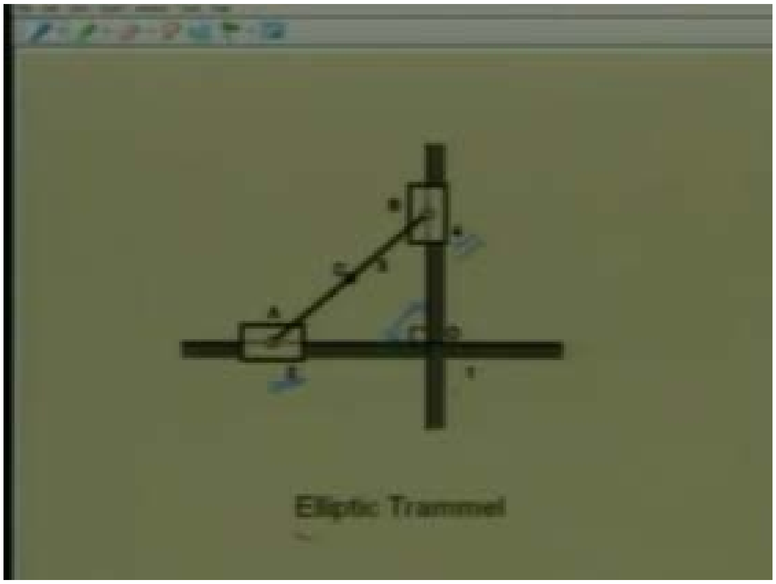

In this mechanism, we consider the link which has prismatic pair at both ends fixed. For example, this link 1 has a prismatic pair with link 4 and a prismatic pair with link 2 at right angles to each other. This mechanism is known as elliptic trammel. It will be obvious now why this name elliptic trammel? Let us consider the physical construction of this mechanism rather than this kinematic representation.

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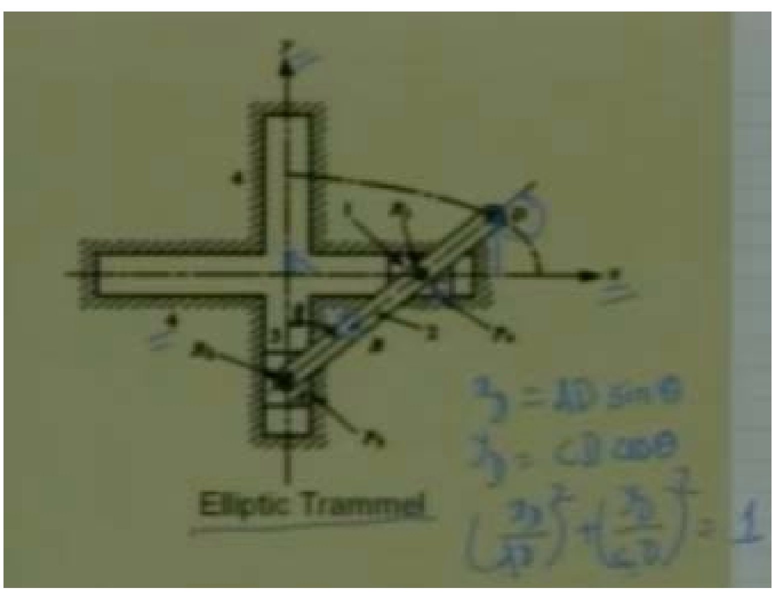

As we see, this link has two prismatic pairs, one in the horizontal direction and the other in the vertical direction. This rod A B C D has revolute pair with this block at C and another revolute pair with this block at A. As this rod moves, let us look at the coordinates of any point D on this moving rod. This is x axis and this is y axis. So the x coordinate of this point D, when the rod is making an angle theta with the vertical is easily seen to be AD sine theta. Similarly the y coordinate of this moving point D is CD and this angle is theta. So it is CD cosine theta. So, y_D is CD cosine theta.

If we eliminate theta from these two coordinates, we can easily see that x_D by AD squared plus y_D by CD squared is equal to 1. That is, as theta changes, this point D moves on an ellipse with semi major axis given by AD and CD. That is why it is called an elliptic trammel. It must be pointed out that there are three points on this rod AB which are exceptions namely, this point A which generates a vertical straight line because of this prismatic pair. Similarly, this point C which generates a horizontal straight line because of this horizontal prismatic pair and for the mid point B which is the midpoint of the distance AC, that is, AB is equal to BC. If AB and BC are equal, then the point B generates a circle of radius. If I call this point O_2, then the radius is O_2B. We can say that these are nothing but the degenerated cases of the same ellipse. Now, we show you a model to generate this ellipse through an elliptic trammel.

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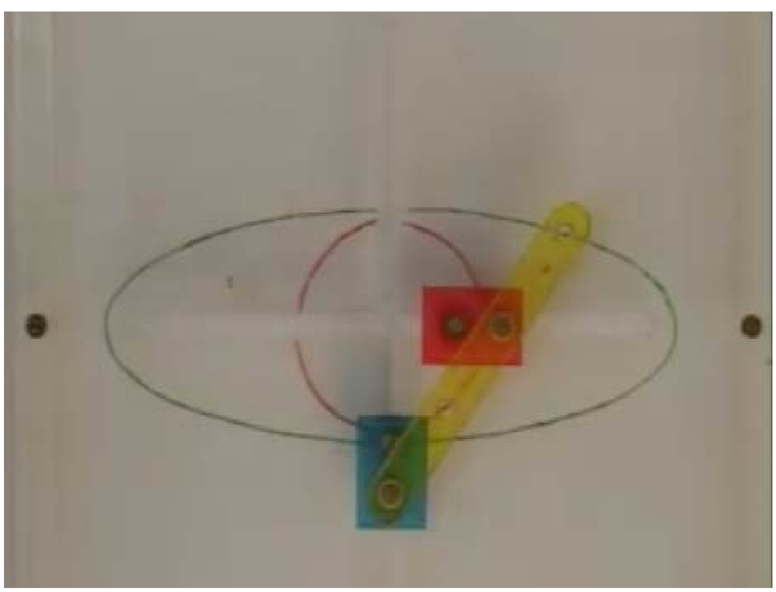

This is the model of the elliptic trammel. As we see, there are two perpendicular slots in this fixed link. This is that rod AB, which has a revolute pair with this block here and at this block here and these two blocks move along these two prismatic pairs. If we move this rod, as we can see, this particular point of this rod generates the ellipse which has been drawn with the green line. If we consider the mid point of these two revolute pairs, then as we see, as this rod moves this particular mid-point generates this red circle, as mentioned earlier. We have discussed different inversions from a RRPP chain. Let us go back to another 2R-2P chain namely, RPRP chain.

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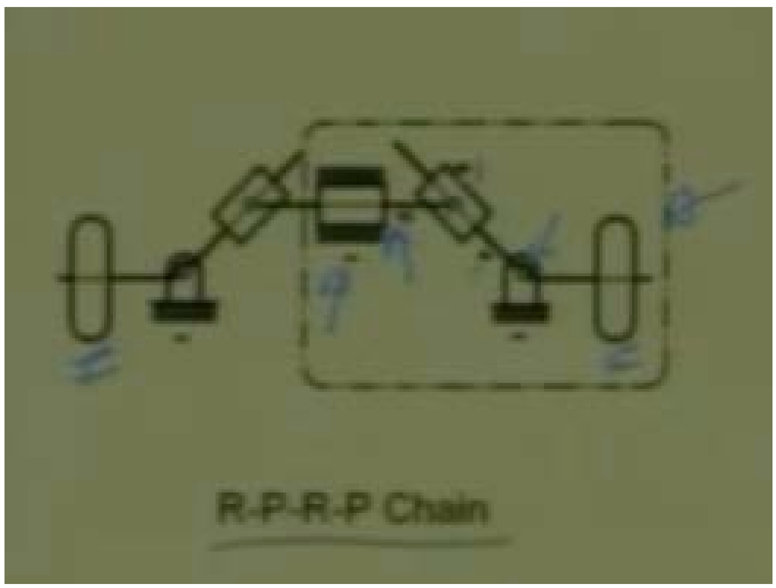

We consider here only the portion covered by this dashed lines. This is the part of an automobile steering gear known as Davis steering gear. Here, as we see there is a revolute pair between link 4 and link 1, a prismatic pair between link 4 and link 3, a revolute pair between link 3 and link 2 and then there is a prismatic pair between link 1 and link 2. These are the two wheels of the automobile. By moving the steering wheel, we move this link 2 in this prismatic pair between 1 and 2. Consequently, these two wheels will rotate. So, here is an RPRP chain. Now that we have discussed 4 linked planar mechanisms which is two revolute and two prismatic pairs, let me now increase one more prismatic pair instead of a revolute pair, that is, can we talk of 3P-1Rchain?

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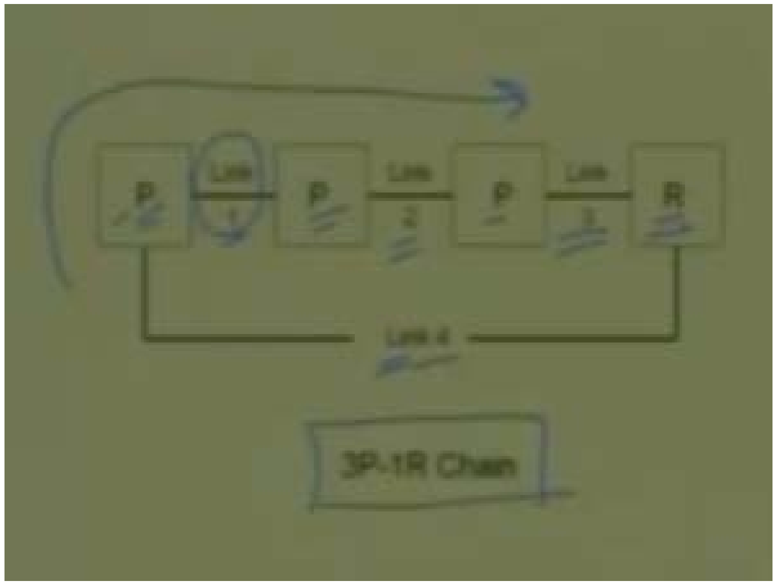

As we see, there is a P pair and there is a link 1 connecting to these two prismatic pairs. Similarly, link 2 connects to these two prismatic pairs and link 3 connects a prismatic pair and a revolute pair and a link 4 connects a revolute pair and a prismatic pair. It will be easy to show that we cannot have a mechanism with such 3P-1Rchain. For example, because there is a prismatic pair between link 1 and 2, there cannot be any relative rotation. Same goes between link 2 and 3 because they are connected between a prismatic pair. There cannot be any relative rotation between link 2 and link 3. There is a prismatic pair between link 1 and link 4 so there cannot be any relative rotation between link 1 and link 4.

Thus, if we follow these three prismatic pairs, we conclude there cannot be any relative rotation between link 4 and link 3. Thus, this revolute pair which allows only relative rotation between link 3 and link 4 cannot permit any relative motion. Thus, we can conclude that such a 3P-1Rchain cannot give rise to any mechanism. Now, can we have a mechanism with 4 prismatic pairs? A four linked planar mechanism with all pairs prismatic. We will see later that such a 4P mechanism is not a constant mechanism.

In fact, three links connected by three prismatic pairs having the relative translation at various angles itself constitutes a planar mechanism which is constant, but that we shall see later. So far we have discussed mechanisms only with revolute and prismatic pairs. Let me now change the topic a little bit. Can we consider mechanisms involving higher pairs as well? As usual, we show that a mechanism with higher pair can be replaced equivalently by a mechanism having only lower pairs that is revolute pair and prismatic pairs.

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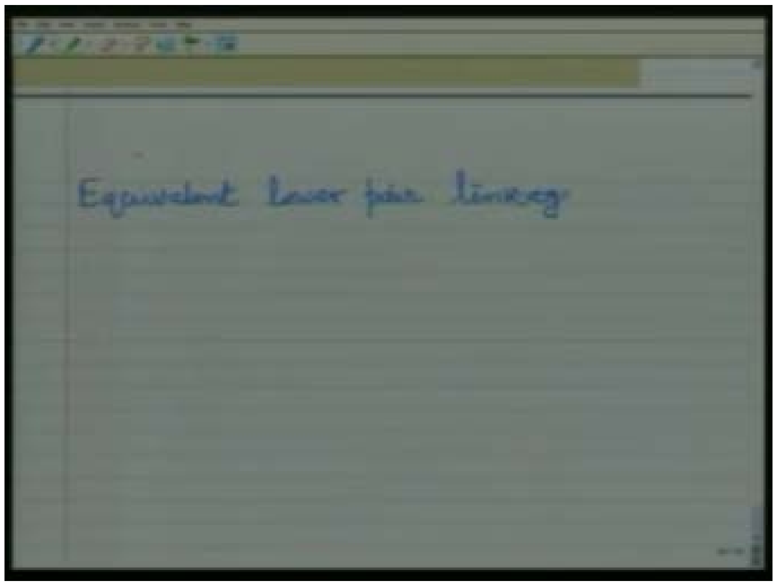

Of course, this equivalence is only instantaneous and holds good for velocity and acceleration analysis at a particular configuration. We are talking of an equivalent lower pair linkage for a higher pair mechanism, how a higher pair can be replaced equivalently by lower pairs.

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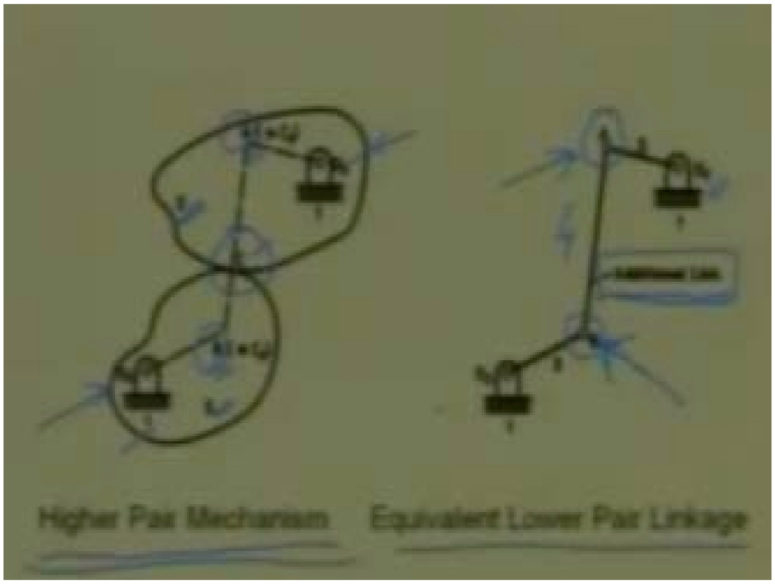

As an example, let us look at this higher pair mechanism which consists of 3 links namely, one the fixed link and 2 and 3. There is a revolute pair between link number 1 and 2 at O_2 and a revolute pair between link 1 and 3 at O_3 but, there is a higher pair at the point C between link number 2 and link number 3. Our objective is to replace this higher pair mechanism by a kinematically equivalent lower linkage consisting only of lower pairs. I repeat again that this equivalence is only instantaneous, that means, only for this particular configuration.

Towards this end, let us consider that the centre of curvature of body 3 at the point C at is at B. Similarly, the centre of curvature of the body 2 at this contact point C is at A. Due to the property of centre of curvature, that is circle of curvature or osculating circle, we can consider that for three infinitesimally separated time instance, these points A and B do not change. We can replace this higher pair by having a lower pair at the point A which is a revolute pair and a lower pair at the point B, which is again a revolute pair and because the distance between A and B are not changing for three infinitesimally separated time intervals, I can join these two points A and B by a rigid additional link.

Thus, this higher pair mechanism has been replaced by a 4R-linkage which has all 4 revolute pairs at O_2, O_3, A and B. As I said earlier, because as these two bodies 2 and 3 moves the centre of curvatures also move. So for every instance, we have a different equivalent lower pair linkage. Later on, we will explain this equivalence through a model but we should see that a higher pair at C has been replaced by an additional link that is link number 4 and two additional lower pairs, one at A and the other at B, where A and B are respectively the centre of curvatures of the contacting surfaces between 2 and 3 at the contact point C. Of course, this equivalence can be permanent if the centers of curvature do not change. That is, one of these contacting surfaces, say circular or cylindrical or coinstantaneous of curvature or flat that is again of infinite radius of curvature.

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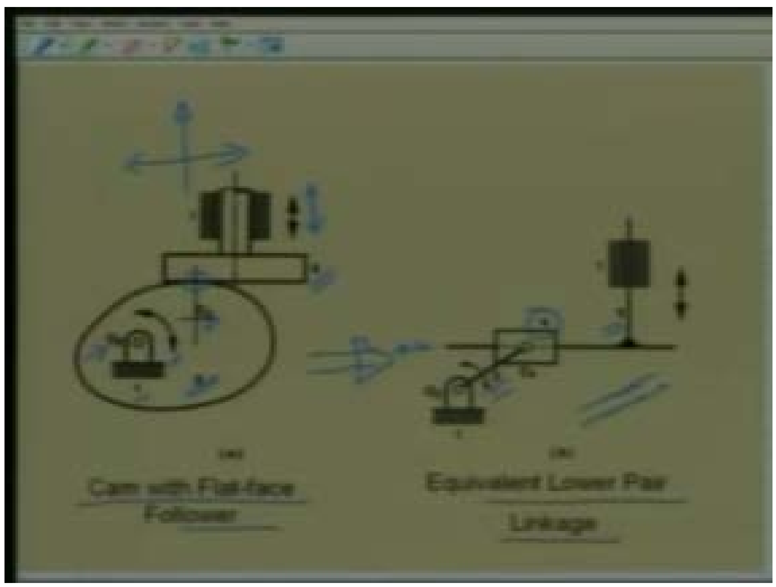

For example, let us look at this higher pair mechanism which is a cam with a flat face following. The cam that is this rigid link 2 has revolute pair with the fixed link at O_2 and this follower 3 has a prismatic pair with fixed link number 1 and there is a higher pair at this contact point. So, the rotary motion of this cam 2 is converted into rectilinear motion of this follower 3. Let the centre of curvature of the cam surface at this contact point is at C_2. The centre of curvature of the follower surface at the contact point is at infinity, that is, the revolute pair at the centre of curvature is converted to an equivalent prismatic pair in the horizontal direction. If we recall that a prismatic pair is nothing but a revolute pair at infinity. Consequently, this cam follower mechanism with a flat face follower is replaced by this equivalent lower pair linkage. This link 2 represents the cam, link 3 represents the follower and there is an additional link 4 which has a revolute pair with link 2 at C_2 and a prismatic pair with link 3 in the horizontal direction. This is the equivalent lower pair linkage of this cam follower mechanism. That means, the input output characteristics of this cam follower mechanism can be carried on by analyzing this lower pair linkage because C_2 keeps on changing if the cam surface is not circular. That is why, for every instance we have to have separate equivalent lower pair linkage because this link length C_2 will keep on changing. This equivalence is valid only up to acceleration analysis, velocity and acceleration because the centre of curvature or osculating circle is in contact with the surface only for three infinitesimally separated positions. Higher order derivatives do not match. Consequently, higher order time derivatives cannot be calculated. I will show these two equivalence lower pair linkage by models.

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Let us consider this model where we have a higher pair mechanism and this link 2 and link 3 is having a higher pair at this point. The centre of curvature for this body 2 at this point of contact is here at C_2. Similarly, the centre of curvature of this surface of body 3 is at this point C_3. As we said earlier, we can have an equivalent lower pair linkage by having a revolute pair at C_2 and another at C_3 and connected by an additional rigid link. Thus, we have a 4R mechanism which is instantaneously equivalent to the original higher pair mechanism. I have connected these two bodies, that is, this red link and the original link number 2, rigidly.

As we shall see that I can move this mechanism, the same motion is transmitted at least around this contact region between body 2 and body 3, whether, it is to the higher pair mechanism or through this equivalent lower pair linkage. To distinguish the movement of this body 3 in the lower pair linkage and this body 3 in the higher pair mechanism, let us notice at these two lines, there is a red line on this body and there is a blue line on this body. Around this point, as we see these two lines move almost the same way because velocity and acceleration relationship between the original higher pair mechanism and the equivalent lower pair linkage is just the same.

However, when there is a lot of movement these two lines separate out. This blue line and this red line are not same any more. Around this point, it is only here that these two lines separate out because the centers of curvature are very different from what it was at this configuration. This is what we mean by instantaneously equivalent lower pair linkage. We can show another model with one surface flat. That is, radius of curvature with infinity and the other curvature is a circle, that is the radius of curvature is constant. Under such a situation the equivalence will hold good for the entire cycle of motion

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A higher pair mechanism between link number 1, 2 and 3 and the centre of curvature of body 2 at the contact point is here, whereas, body 3 has a flat or a straight contact in surface. Consequently, the extra link that is having here gets a prismatic pair between the body 3 and that extra link and a revolute pair between body 2 and that extra link. If we connect these lower pair linkage, link number 2 with the link number 2 of the original higher pair mechanism rigidly then we can see that same motion is transmitted by the both lower pair linkage and the higher pair mechanism.

Let me now summarize, what we have talk so far in this lecture. What we have seen is 4R planar mechanisms of different varieties consisting of four revolute pairs or three revolute and one prismatic pair or two revolute and two prismatic pairs. When we have two revolute and two prismatic pairs, the order of the sequence of the pair becomes important. We have talked of two varieties namely, RRPP or RPRP. At the end, we have also seen we have a three link mechanism with a higher pair in that also can be equivalently represented by a four linked planar linkages having only R pairs or R and P pairs.