Kinematics of Machines

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Module – 10 Lecture - 2

In the last lecture, we have discussed cam follower systems and we have discussed two types of followers, namely, translating follower and oscillating follower. For smooth functioning of a cam follower mechanism, it is imperative that the follower should move smoothly without requiring too much of input power, that means, the follower should not jam, during its movement. Of course, the chance of jamming is more, in case of a translating follower.

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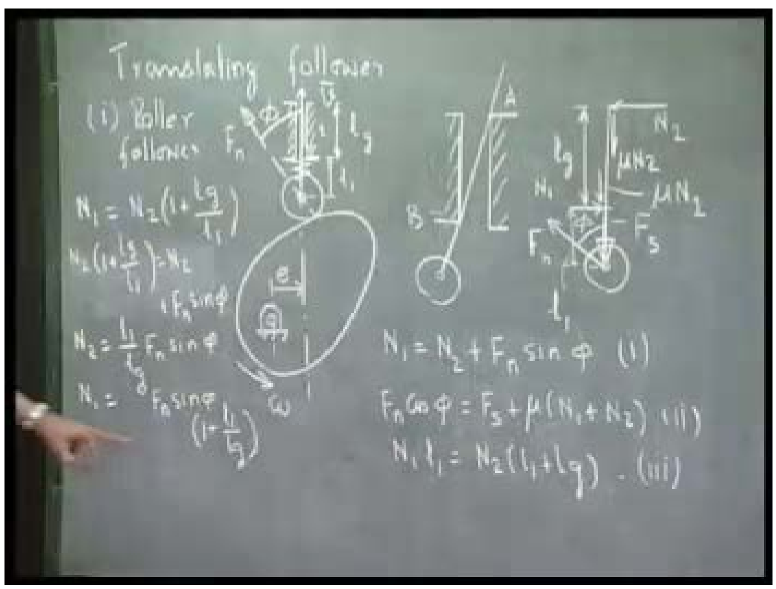

While designing a cam follower system, care must be taken such that a translating follower does not jam in its prismatic guide. We have already seen that, the restriction to the movement of the oscillating follower is much less and we did not worry so much, when the follower is oscillating. Now, a complete dynamic analysis of this translating follower is beyond the scope of this course because this course is restricted only to kinematics. However, a simplified static analysis can highlight the basic features and that is what we are going to start right now.

First, we discuss the case of a roller follower. Suppose the cam is rotating in the counter-clockwise direction at constant speed and the roller follower is mounted in an offset manner as shown in this figure. This distance as we know, is called the amount of the positive offset because it is rotating in the counter-clockwise direction. We are doing a static analysis that means we will not talk of the acceleration of the follower.

If we neglect the friction between the roller and the cam surface, then the driving force on to the follower is along this common normal which we call F_n and the direction of the velocity of this point of the roller centre that is, the point of the follower or the trace point, is in the vertical direction. So, this is the direction of the velocity. This angle between the direction of the velocity and direction of the normal force, we define as pressure angle phi.

As we know, due to this force $F_{n,}$ there is a tendency of the follower to cock or rotate clockwise within its guide. Let the guide length is l_g, which as we you see, will be an important parameter and let us say this distance from the roller centre to the lower end of the guide is $l_{1.}$ We know there is a spring which connects this fixed link 1 and the follower. There is a spring here. So, as the follower is pushed away from the cam centre, this spring is compressed. If I take larger view of the guide, this is the prismatic pair and due to this force, the follower gets tilted and this will be the situation. So there is contact here, at this end of the guide and this end of the guide. At these two points, contact forces are generated.

Of course, this gap has been shown much wider. This gap is very small, that angle of tilt is not so much. Now what we do is, we take the free body diagram of this roller follower. Let us say, this point is B and this point is A, we neglect the tilting of the roller, I still draw it vertically. The forces that are acting are: this driving force F_n; there will be a spring force downward as the spring is compressed, let us say, this is a spring force which I write, F_s; and at this point A, there will be a normal force say N_2; and at this end, there is a normal force say $N_{1.}$ This distance, we have taken to be length of the guide as l_g and the distance of the point B from this point, this we have called l_1 and this angle is the pressure angle, phi.

As we said, we are neglecting the acceleration of the follower, so we do a static analysis and see the relationship between these forces. If there is no acceleration of the follower then the system must be in equilibrium. If I take the horizontal forces, we get N_1 is equal to N_2 plus F_n that is this driving force and sine phi.

If we take the vertical forces, at this point we assign it has just started moving so the frictional forces will be downward as I can show here because the follower is trying to go upward mu N_2 and at this point, there is a force downward mu N_1. So at the point B, I have N_1 and muN_1 and at point A, I have N_2 and muN_2. The vertical forces gives me F_n cosine phi is equal to F_s plus mu times N_1 plus N_2 and if we take the moments of all the forces about the roller centre. Then F_s, F_n, mu N_2, mu N_1, these forces are not producing any moment, the moment due to this forces are negligible because this angle is very small. It is almost vertical. So the all new moments are due to N_1 and N_2 and that gives me $N_1 l_1$ is equal to N_2 into l_1 plus l_g.

If I substitute, we can get N_1 and N_2 from here solving these two equations, what we get is, N_1 is equal to N_2 into (1 + l_g by l_1) and substitute it here we write, N_1 equal to this, I write, N_2 (1 + l_g by l_1) is equal to N_2 + F_n sine phi and I substitute for N_1 here from this equation. So I get N_2 is, cancels, so l_1 by l_g into F_n sine phi. This is equal to N_2 and N_1 I can get from here, N_1 is N_2 l_g by l_1, so it gets cancelled, so F_n sine phi into 1 + N_1 is N_2 into 1 + l_1 by l_g here. This l_g by l_1 gives me 1 and this one gives me l_1 by l_g.

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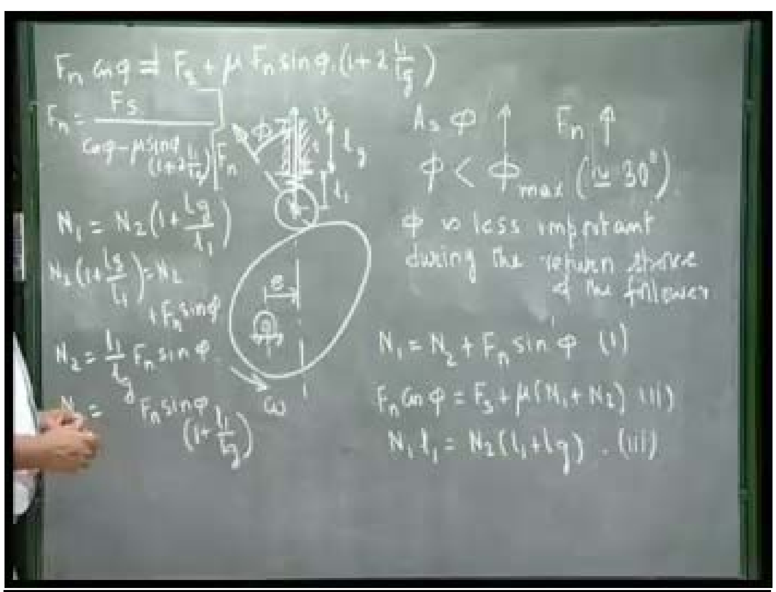

Using the second equation, now I can write F_n cos phi equal to F_s + mu times $N_1 + N_2$ which is F_n sine phi into 1 + 2 l_1 by l_g. So we get the driving force F_n equal to F_s divided by cosine phi – mu sine phi into 1 + 2 l_1 by l_g. So this is the driving force F_n which ultimately this input member has to provide.

From this expression, as phi increases cos phi decreases and sine phi increases, so the denominator becomes small and F_n becomes too large. So as phi increases, F_n increases. Consequently, to have F_n within reasonable limit for smooth operation, I must not have phi too high, that is phi should be less than some phi max, which we said is normally around 30 degree.

We also see that, this l_g appears in this expression. So a proper guideline is necessary to control the value of F_n. The length of the guide is important. Of course, while follower is coming down, it is the spring force which pushes the roller down and keeps in contact with the cam and the pressure angle is not important. Phi is less important during the period, what we call the return stroke, this is called the rising stroke and that is the return stroke during the return stroke of the follower. Later on we shall see that this positive offset will decrease the value of phi as compared to when the follower offset is 0. If the offset is 0 for the same movement of the follower and same cam the maximum pressure angle will be more and the maximum pressure angle will reduced during the rising portion, during the movement that is going up.

During this movement when the follower is being pushed by the cam that is when the driving effort is necessary, the maximum value of phi will be reduced by having this positive offset that is to the right when the rotation is counter-clockwise. During the return as I said, the spring pushes the follower down and the value of the pressure angle is less important. We have just now had a simple static analysis of a roller follower which is translating in its guides.

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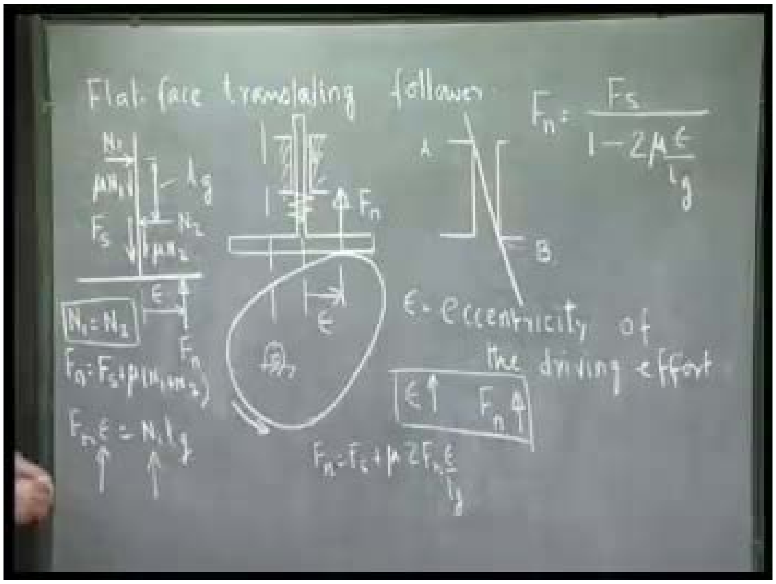

Next, let us continue with a similar analysis, simplified static analysis for a flat face translating follower. We have a flat face translating follower. This is the diagram of a cam and a translating flat face follower. This is the prismatic pair between the follower and the fixed length.

Here we see, if we neglect the friction force between the follower and the cam surface, the driving force is along this common normal. Due to this force, the follower has the tendency to rotate in the counter-clockwise direction and contact will take place here and there. So this contact point let me say A and this contact point let me call B.

As I said, the tilting has been shown exaggeratedly, actually it is little bit away form the vertical, I will still draw it vertically. So this is the representation of this follower and at the point A, there is a normal force say N_1 and at the point B, there is a normal force N_2, here, there is a normal driving force which we called F_n because the follower is trying to go upward, the friction forces will be downward will be mu N_1 where mu is the coefficient of friction and here, it will be mu N_2 and there is a spring that applies a force downward which we called F_s.

If we consider that under these forces, the follower is just going to lose its equilibrium, that means it is going to move upward then horizontal force equilibrium gives me N_1 is equal to N_2 and vertical force gives equilibrium gives me F_n equal to F_s plus+ mu times N_1 + N_2. This distance from the guide axis to the driving force we call eccentricity, let me denote it by epsilon. This epsilon is called eccentricity of the driving effort.

In ideal condition, this eccentricity epsilon should have been 0. If I push it here, obviously it will go more smoothly, but due to this eccentricity e, because the contact point is here and the contact point will travel over this follower face at this particular instant, the contact point is here and epsilon denotes the eccentricity of the driving effort which is producing this tilting tendency.

So if I take moments about this point and if I call this distance l_g, the length of the guide as before, this moment is balanced by this moment of F_n, that is F_n into epsilon is N_1 into l_g, because N_1 is same as N_2. These two forces are same so they produce a pure couple of magnitude N_1 into l_g. This clearly tells me, if eccentricity is more, then N_1 will be more and if N_1 is more, N_2 is equal to N_1, so F_n will be large. So that tells me, if epsilon increases, F_n also increases.

We can write the expression of F_n, if we want, we can write, F_n is F_s plus mu times $2N_1$ which is $2F_n$ epsilon by l_g. So, F_n is F_s by 1 minus 2mu epsilon by l_g. If epsilon increases, then the denominator decreases, so F_n increases. So, more is the eccentricity that is this distance, more driving force is necessary and this is more critical while going up because we have to overcome the spring force and this offset obviously decreases the eccentricity. Had this axis been here, the eccentricity would have been so much and now the eccentricity has been reduced by this amount of offset. So this explains the importance of the free running of the follower in a cam follower system.

We have discussed the importance of the pressure angle and the eccentricity, in case of roller follower and flat face follower respectively.

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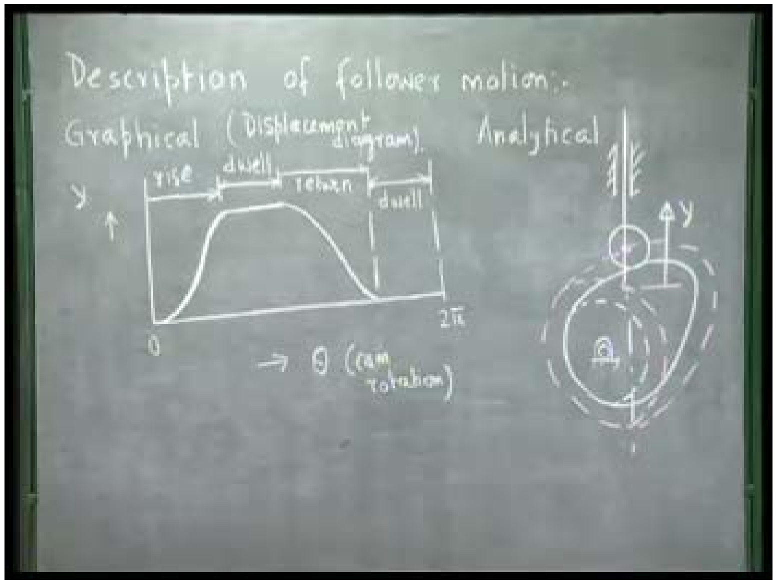

Our next task is, how to design the cam profile that is, how to synthesize the cam profile, if the desired motion of the follower has to be generated. Before getting into that let us see how we describe the follower motion that is, description of the follower motion. How do I describe the desired follower motion?

As in the case of linkage synthesis, we can use either graphical method or analytical method for cam profile synthesis and accordingly, this follower motion can be described in terms of a diagram, if we want to apply a graphical method or in terms of an analytic expression, if we want to have an analytical method.

We have graphical description and analytical expression. In the graphical description, we call it the displacement diagram. We plot the displacement of the trace point which we denote by y, y is the displacement of the trace point that is a roller centre or the in case of flat face, it is the axis of the follower where it is intersecting the flat face, a particular point of the follower face that is called the trace point. We plot the displacement of the trace point versus rotation of the cam, say represent it by theta.

Obviously, this y versus theta is a periodic function as cam rotates from 0 to 2 pi, everything gets repeated. This is one cycle of the cam rotation. Normally, in the cam, this y is measured from the lowest point, that is when it is nearest to the cam centre, if we go back to the diagram. This is the cam and the corresponding base circle (Refer Slide Time: 27:18) let us say this is a pitch curve and the corresponding prime circle which we defined in our last lecture.

As we know, this is the roller centre which is the trace point. This lowest point of the roller centre is here. It can never get into the prime circle because the roller surface cannot get inside the base circle. When the roller is touching the cam along this base circle, this is the lowest point of the roller centre, when it intersects the prime circle. So this is the lowest level that the stress point can come and we measure y from here. So this follower displacement is measured with y equal to 0 at this point (Refer Slide Time: 28:51).

Normally, the follower as it goes up, y is positive, y always remains positive, it goes away from this cam centre. After that the follower does not move So there is a dwell period. This is called dwell period. After some dwell period the follower will return to the same level and again has a dwell period. This is the typical diagram for the automobile valve. The valve opens, is kept open, returns, gets closed and remains closed and then again open. So, this is again a dwell period. When the cam goes away from the lowest position, this duration is called the rise and this is called return (Refer Slide Time: 30:10).

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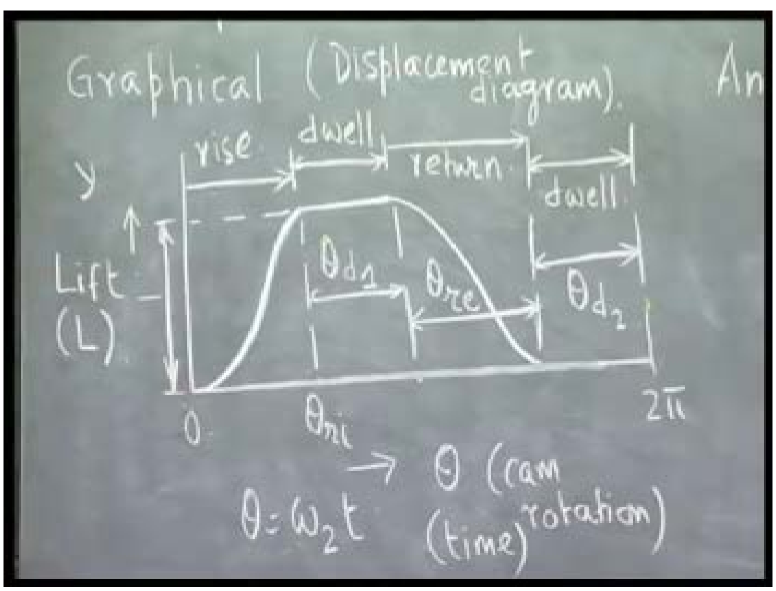

So normally, at least in automobile cams for operating the valves we have a rise when the valve is open, then the valve is kept open. There is a dwell, the follower does not move, then the valve gets closed, the follower comes down and then the valve is kept closed and the follower does not move, it follows a dwell. So we have a rise, dwell, return, dwell and it is this maximum displacement of the follower, this is called lift. We will denote lift by the symbol L.

The value of theta during which the rise takes place, we call it $theta_{ri}$. Then this dwell period we call $theta_{d1}$ because it is the first dwell period and this angular interval, we call $theta_{re}$ followed by again another dwell $theta_{d2}$. We should remember that, normally the disc cam rotates at constant speed. So theta is $omega_2$ t where $omega_2$ is constant, so this theta is nothing but representing time. So this is the graphical description of the follower motion and this diagram is called displacement diagram.

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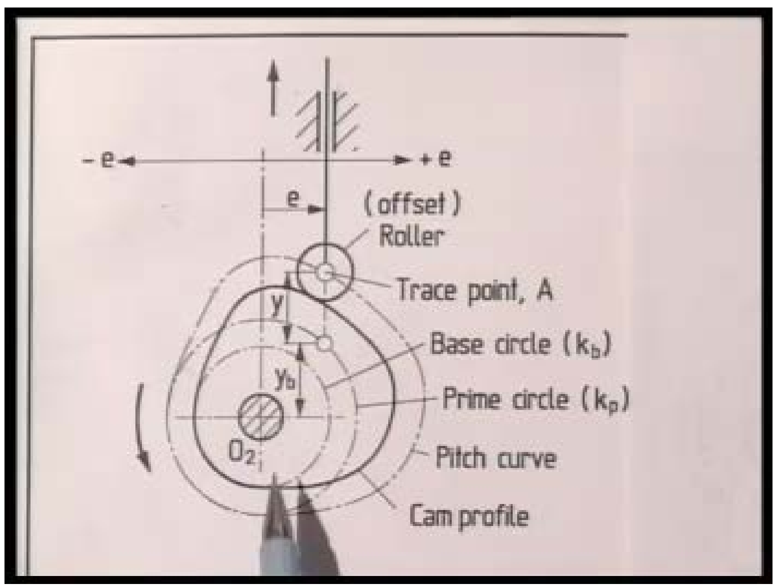

Let me now repeat, what we have explained just now. We are talking of a roller follower which is translated. This is the cam profile; this is the pitch curve; this is the base circle; and this is the prime circle. As we know that the lowest point that the roller centre can come is here. It can never get into the prime circle because then the roller will get into the base circle. The movement of the follower is measured from this lowest level. So this distance y is called the displacement of the follower, y is always positive it starts from here goes up and comes down, y always remains positive.

The velocity is positive that is dy dt is positive, during the rise period, dydt and all derivatives of y, y remains constant, so all derivatives of y remains 0 during the dwell period then velocity is negative while it is the return period and again y remains 0 during the second dwell period.

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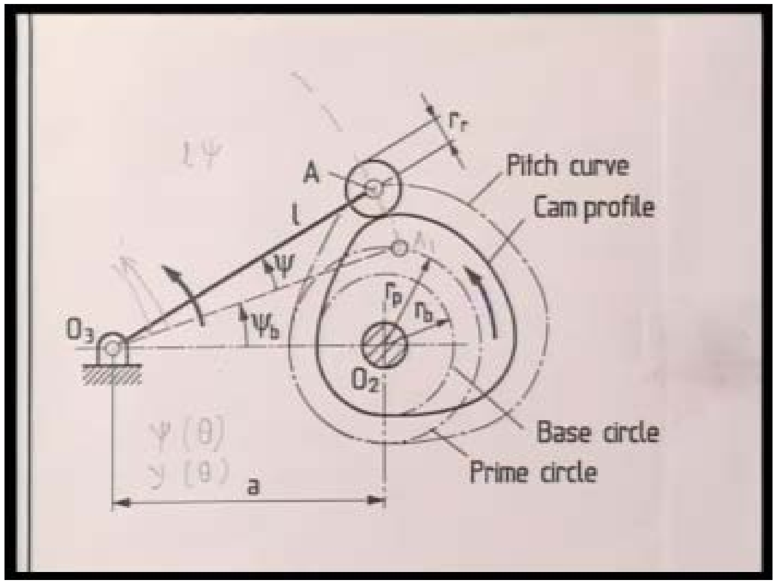

Just now we have seen, how we describe the displacement of a translating follower. Now, let us look at this picture, which shows a roller follower but which is oscillating. The roller follower is hinged at O_3 and this is oscillating. Again, this is the cam surface or cam profile; this is the base circle; this curve is the pitch curve; and this circle is the prime circle.

As the roller follower oscillates, the lowest point that this trace point can come, the trace point moves in a circle with O_3 as centre, is moving along this circle. So this is the lowest point of the trace point, the roller centre and this is the follower O_3A. This is the lowest position of A. Let me call it A_1. Here, there are two more dimensions, that is, how this O_3 is placed with respect to the cam centre $O_{2.}$ Here, we have shown only A, but there can be a vertical distance also which could have been B which has been taken 0 in this diagram.

This is the lowest position of the follower and it oscillates from here and comes down up to this point. It can never go below this level. So this is how we measure the displacement of the oscillating follower by this angle psi measured from this O_3A_1. This angle is psi and this psi B will be decided by the other dimensions. The angle of this lowest position from the line O_2O_3

and the movement of this will be l times psi along this circular arc,where l is the length O_3A. So this is how we describe the movement of the follower. The psi has a function of cam rotation theta, whereas previously for the translating follower we had y the movement of the follower as a function of cam rotation theta. Next, we shall discuss some typical basic motion of the follower which is used.

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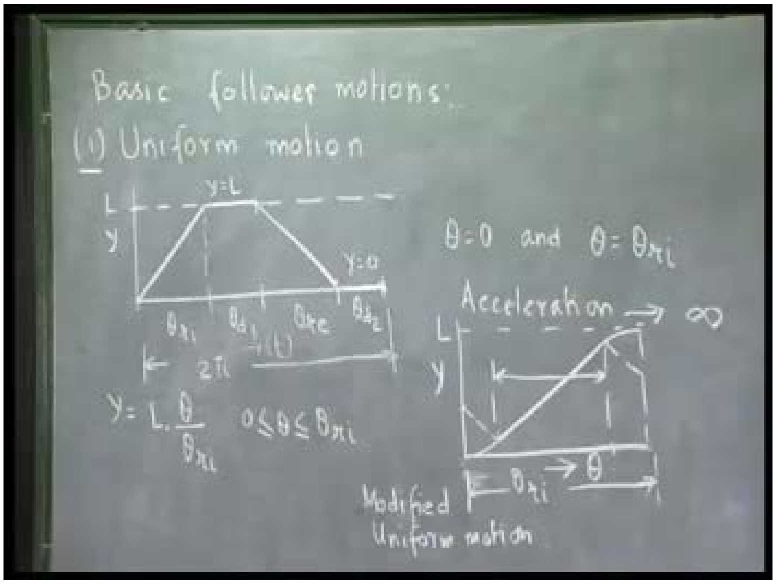

The next topic of our discussion is the basic follower motions. Those are the motions of the follower which are normally desired and we group it as basic follower motion. The first one we discuss is known as uniform motion. By uniform motion, we mean that both the rise and return of the follower takes place with constant velocity. If we look at the displacement diagram, say this is $theta_{ri}$; this is $theta_{d1}$; this is $theta_{re}$; this is $theta_{d2}$; and this whole thing is 2 pi.

The lift of the follower is say L. The displacement diagram is very simple in this case, that during this rotation from 0 to $theta_{ri}$, it attends a distance L and moves it to a constant velocity because theta is proportional to t. So, this we can think of as a time scale, the theta scale is same as a time scale, so uniform velocity means it is a linear motion. Then here, it remains at L.

During this rotation of the follower, during $theta_{re}$, it comes back with constant velocity and then again remain at y is equal to 0. So, during this y is L, during this y is 0, during the two dwell periods. During the return period, we can write as simply y equal to L into theta by $theta_{ri}$. This is valid because 0 is less than theta, less than theta_{ri}. This is the analytical expression for the same follower movement. As we have seen, during this rise period, the follower displacement y as a function of theta can be easily expressed by this linear function, y equal to L into theta by theta_{ri}. When theta is 0, y is 0 and when theta is theta_{ri}, y is L. This is what we call uniform motion.

Mathematically, this motion is very simple but we can see what are the undesirable features of such a uniform motion are. If we look at this point, that is, this point velocity was 0 and suddenly, the velocity is constant, a constant velocity is being picked up, so acceleration here is infinity. So at theta equal to 0 and at theta equal to theta_{ri}, velocity changes suddenly. Here it changes from 0 to a positive velocity and here it changes from positive velocity to zero suddenly, which causes acceleration of the follower tending to infinity. That means, a very large force is necessary to produce such acceleration of the follower.

That is why we would like to modify this, such that it takes off smoothly and also ends smoothly at the beginning of the rise and at the end of the rise. Let us concentrate only during the rise period exactly similar treatment can be done for the return period. So now onwards we will consider only this rise period, theta going from 0 to theta_{ri}.

If we want to have uniform motion then bulk of the motion will be take place at constant velocity, but at the beginning and at the end, we draw a circular arc with some point as centre and this as circular arc, such that the tangent here is horizontal and tangent here is again horizontal and these two points we can join with it steadily.

During this rise period the velocity is uniform but it is modified near the beginning and at the end. This is called modified uniform motion. This is the lift (Refer Slide Time: 41:55). So this is what we mean by description of the follower motion and this is the basic follower motions. Only one we have discussed today, that is the simplest which is known as the uniform motion. but we have already seen what the undesirable feature of this uniform motion is and it is this to avoid infinite acceleration or infinite jerk that is sudden change in acceleration we have to think of a more complicated motion than such a simple uniform motion. But that is what we will discuss in our next lecture.