Kinematics of Machines

Prof. A. K. Mallik

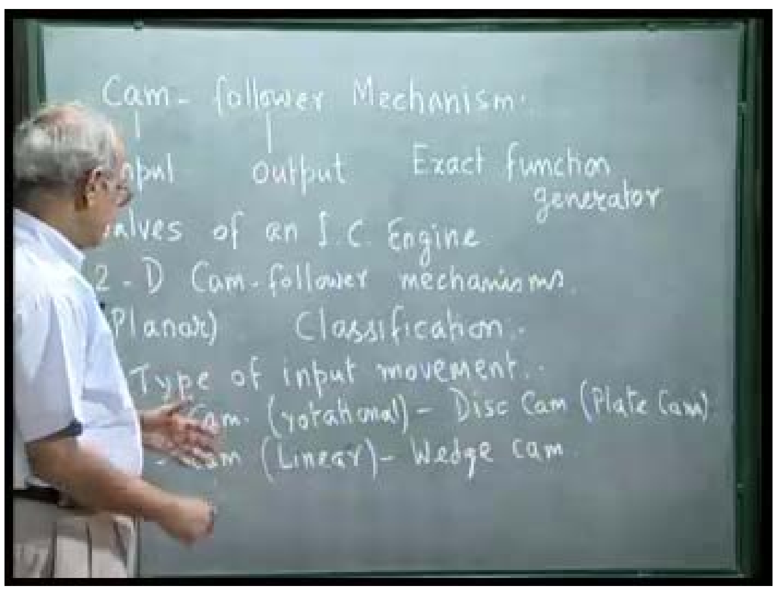
Department of Civil Engineering

Indian Institute of Technology, Kanpur

Module 10 Lecture 1

In this course, so far we have discussed planar linkages, which consisted entirely of lower pairs like revolute pair and prismatic pair. The rest of the course will now devote to higher pair mechanisms by which we mean, there must at least one higher pair in the entire mechanism. Higher pair means where the two links are connected either along a line or at a point. Two such higher pair mechanisms will be included in this course. We start our discussion with what we call cam-follower system.

(Refer Slide Time: 01:03)

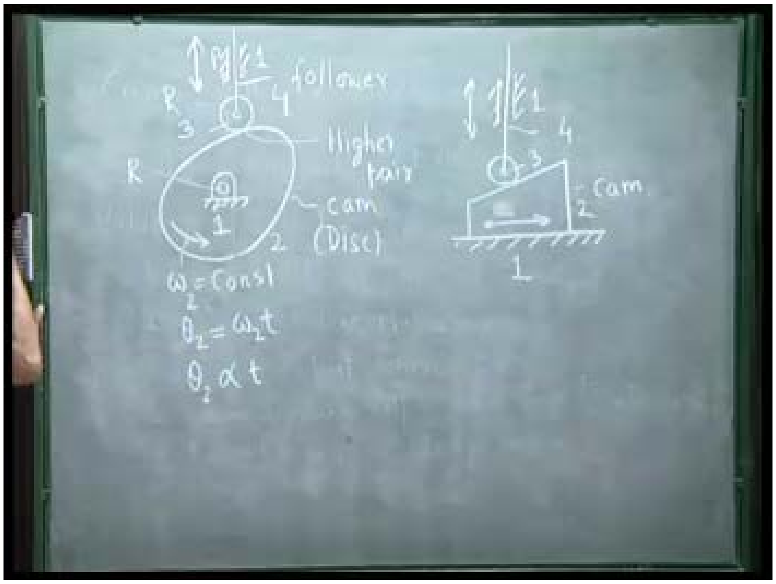


A higher pair mechanism is known as cam-follower. In this cam-follower mechanism, the input link is the cam and the follower is the output link. As we shall see, this cam-follower mechanism is nothing but an exact function generation. By linkages, we had approximate function generator but by using this cam-follower mechanism we can have exact function generator. Such complicated coordinated movement between the output and the input link is possible by using cam-follower mechanism which is widely used in various machines, particularly for timing purposes. As in the case of moving the valves of an automobile, as we know that in an IC engine the valves have to be kept open; first move to open it, then keep it open, then close it and keep it closed. All these timing operations can be easily incorporated by having cam-follower mechanisms.

In case of linkages we restricted our discussion only to planar linkages or two dimensional linkages. Similarly, for cam-follower mechanisms also we will restrict our discussion only to planar or two dimensional, that is 2-D, which means, all the points of this mechanism, will move in parallel planes. So a single view a projected view along the line perpendicular to the plane of movement can give the true movement of all the points of the mechanism. These 2-D cam-follower mechanisms, we start our discussion with classification.

Classification can be based on various criteria and the first we base our criteria on type of input movement. If the input link which we call the cam rotates as an angular motion, then we call it the cam has rotational or angular motion, then we call it a disc cam or plate cam. If the cam movement is translational that is linear oscillation, let us say linear to and fro motion, and then we will call it as a wedge cam. Before we get into the detail discussion of this disc cam, let me first draw a figure to tell you how a cam-follower mechanism looks like.

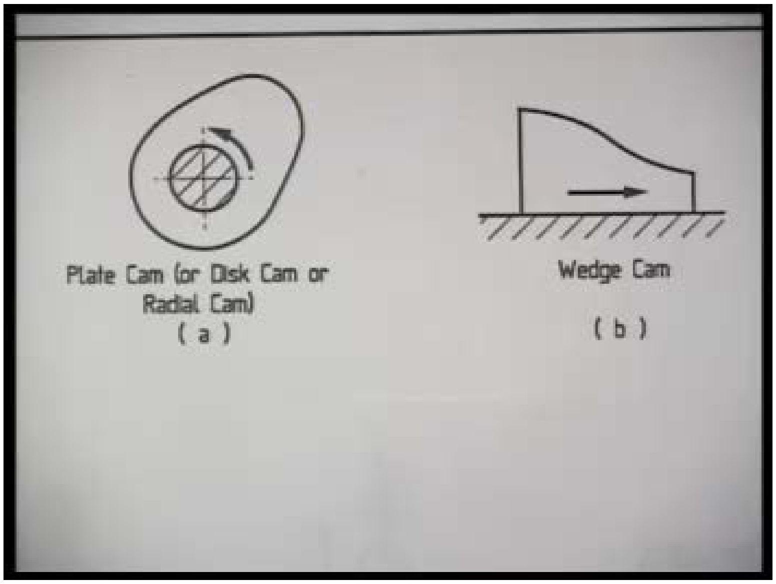
(Refer Slide Time: 05:35)



This is the cam-follower system. This profile body is called the cam. This has the revolute pair with the fixed length that is the foundation which I call link number 1. So cam is link number 2. There is a revolute pair between 1 and 2 and this is the output member which is the follower. If this cam rotates depending on the profile or shape of the cam, the follower will have translatory motion along this prismatic pair between 1 and the follower. This is again a 4 link mechanism fixed; link number 1, cam is 2, this roller is 3 and this roller is having a revolute pair with this output member of the follower which I call four. So uniform rotary motion of this cam will have rectilinear oscillation of the follower along the guide, this is a prismatic pair. There is a revolute pair and between 2 and 3 we have a line contact which is a higher pair. Similarly, we can have a wedge cam which will look like here, again, we have a four link mechanism, this is a fixed link 1, this is the cam which looks like a wedge, which I have given number 2; this is the wedge cam and this is what we called disc cam.

Depending on the profile of this wedge, as this cam oscillates in the horizontal direction, the follower will oscillate along the vertical direction along this prismatic pair or this guide. Here, again roller is 3 and the follower is 4. Our job is to generate a complicated output motion, that the given or desired output motion, what should be the cam profile? That is the objective of designing cam-follower system. This is one classification based on the type of movement that the input member or the cam has. Normally, we will see that this has constant angular velocity, $omega_2$ is constant. The rotation of this cam $theta_2$, which is $omega_2$ into t; t is the time, we can see this $theta_2$ is proportional to time and the movement of that output link will depend on the profile of this cam.

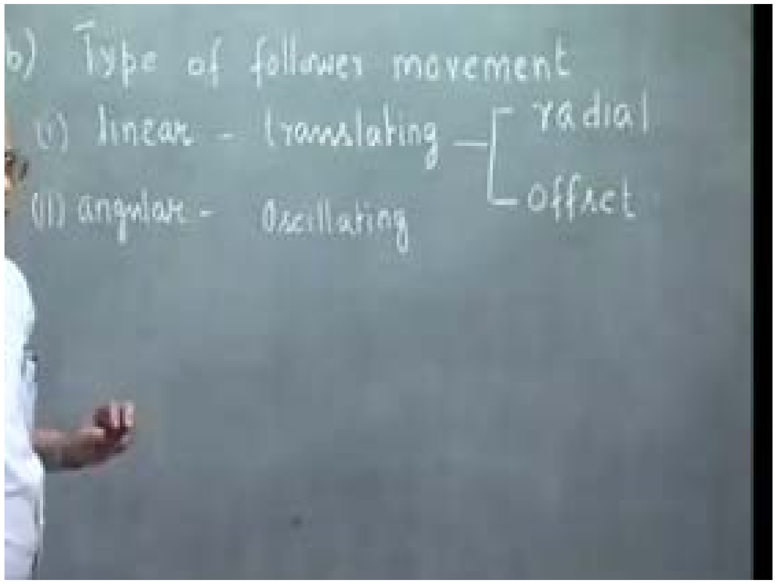
(Refer Slide Time: 09:51)



As we have just now said that, the planar cams can be classified in various wedge cams depending on various criteria. The first classification we made depending on the type of input motion is here; the input motion, that is the cam has an angular motion or rotational motion, then we call this cam as a plate cam or a disc cam or even radial cam whereas if the cam has a linear motion, then we call it as a wedge cam.

Our next classification will be based on the type of movement that the follower has. As I said just now, the cam can be classified also on the basis of the type of movement that the output member or the follower has.

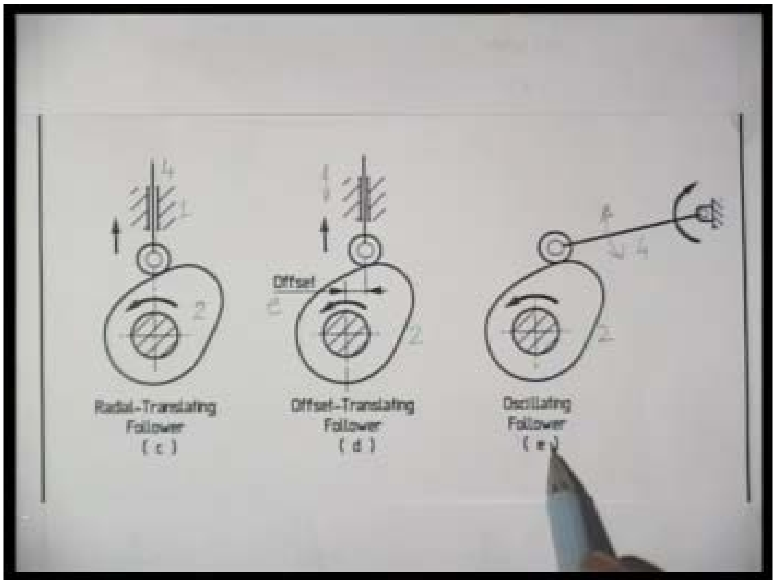
(Refer Slide Time: 10:42)



Accordingly, the second classification is based on type of follower movement. If the follower has linear motion as we have shown in the diagram, then we call it as translating follower and if the follower has angular motion, then we call it as an oscillating follower. The translating follower, the axis of translation that is the axis of that prismatic pair, if it passes through the cam centre, then we call it radially translating. We call it radial translating follower if the follower axis passes through the cam centre, centre of the cam shaft. If it has a little offset, that means the axis of the translation of the follower does not pass through the cam centre, it is little bit offset then we call it as an offset translating follower.

Later on we will discuss why this offset is necessary and the proper direction of the offset. Right now we are concentrating only on the different classification. Based on the type of input movement or cam movement, we had disc cam or wedge cam based on the type of follower movement, we call it. If the movement is linear; the follower movement, then we call it as a translating follower which can be either radial or offset. If the follower movement is angular, then we call it oscillating follower.

(Refer Slide Time: 12:54)

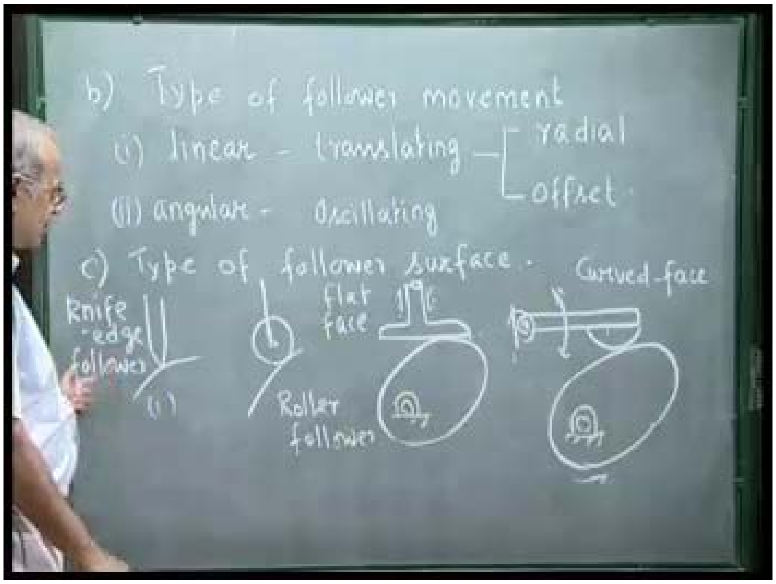


As we said, these are the diagrams to explain radially translating follower, offset translating follower and oscillating follower. As we see, this is the cam that was link number 2; this is the follower link number 4. As the cam rotates, this disc cam, the follower translates along this vertical direction. Here is the prismatic pair between this 1 and 4.

As we see, the axis of this prismatic pair passes though the cam centre, so I call it as radially translating follower. Similarly, here again the cam rotates, the disc cam; the follower translates, that the axis of this prismatic pair passes not through the cam centre but has an offset little away from this vertical line. This offset we will normally denote by the variable e by the symbol e. This is called offset translating follower. Whereas, in this case, as we see, the cam rotates as before but this is the follower due to the shape of the cam, the follower undergoes an oscillatory motion and the follower is hinged at this point. So, this is called oscillating follower.

The third classification of cam-follower system is based on the nature or shape of the follower surface. The contact surface between the follower and the cam, what kind of surface the follower has at the contact?

(Refer Slide Time: 14:45)

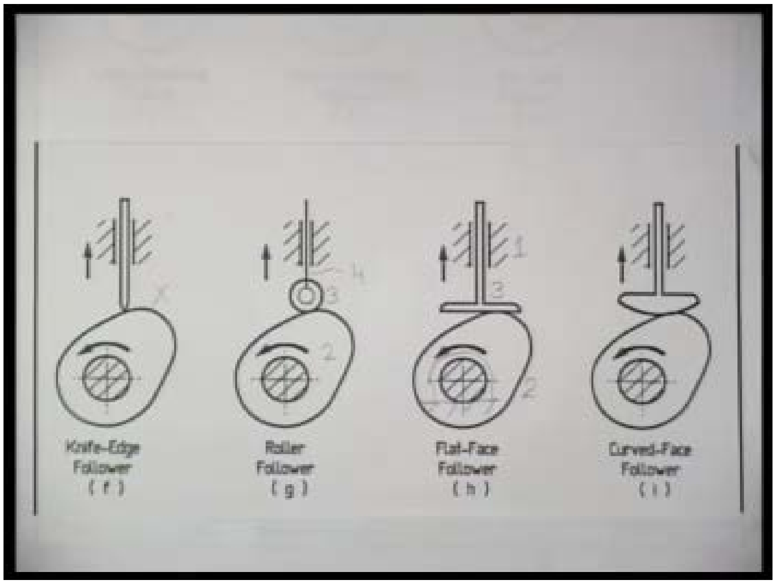


This is based on type of follower surface. If the follower has just a knife-edge with the cam then we call it as a knife-edge follower. It must be told that this knife-edge is only theoretical because knife-edge follower is never used because of very high [wire length…..15:29]. The follower as we have seen already, the follower surface can be that the follower is hinged to a roller and this roller is in contact with the cam as we have seen so far, this is called roller follower. The follower can also be in the form of a flat face.

As we see, the follower surface which is in contact with the cam is in the form of a flat surface, this is called flat face follower. The follower surface, instead of flat it can be also a curved surface. This is the cam which rotates and the follower which is hinged here oscillates, so this is called curved face.

These are the four common types of surfaces that we will be talking about; knife-edge, roller, flat face and curved face out of which as I said knife-edge is only theoretical but not used in real life, it is the roller or the flat face, or the curved face. The roller follower is used when a large force has to be transmitted like in stationary IC engines, to move the valves a large force has to be transmitted and we use a roller follower. If the space is constricted, if there is not enough space to use the large roller because this pin has to be sufficiently big to transmit the force between the cam and the follower and the roller has to be bigger than the pin at least twice as big as the pin, then the roller needs a lot of space. If the space is restricted then we can use flat face follower, if the force involved is not too large as we use in the case of automobiles.

(Refer Slide Time: 18:27)

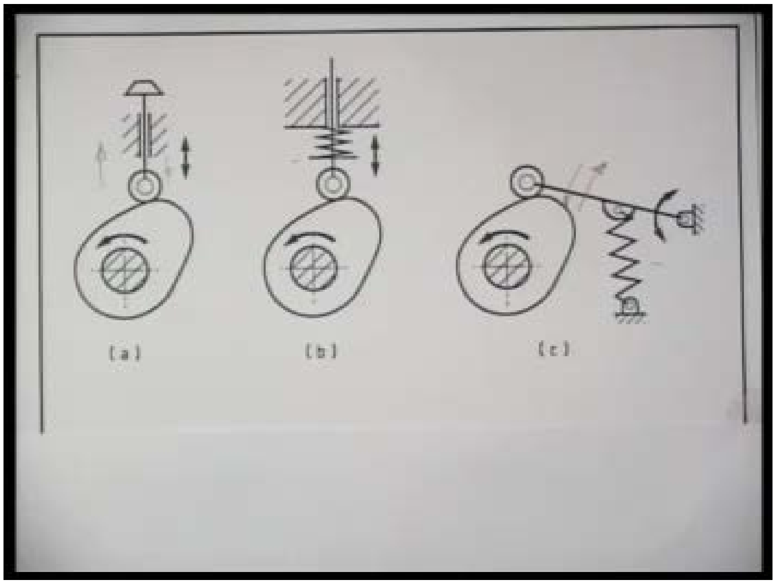


Let us just recapitulate the four different types of follower surfaces that we have discussed. This is the diagram of a knife-edge follower which as I said is really only for theoretical purpose. This is never used in practice because the knife-edge will wear out very fast. This is the follower is connected to a roller through a revolute pair and this roller is in contact with the cam surface. The follower is not directly connected to the cam surface, the follower is hinged to a roller and this roller is in contact with the cam surface. Here, we call it a roller follower.

We can use the flat face follower which is directly in contact with the cam surface so this becomes a three linked mechanism, the fixed link 1. Cam 2 has the revolute pair with the fixed link here, this is the cam shaft in its bearing and the follower is link number 3 whereas here we had 2; roller is 3 and follower is 4.

Instead of a flat face, we can also have a curved face as shown here then we call it as a curved face follower. We have just finished the classification of different kinds of cams and followers based on the type of input movement; based on the type of output movement and based on the type of surfaces that are in contact between the cam and the follower.

(Refer Slide Time: 20:04)

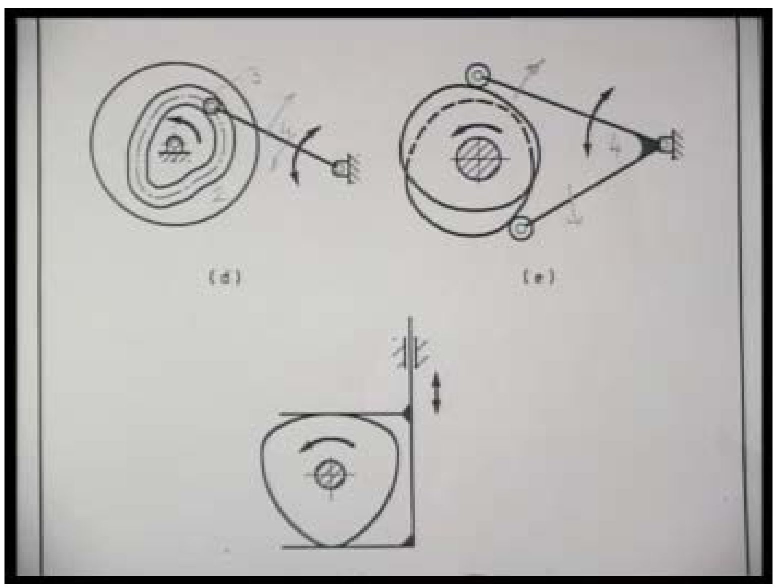


Before we get into the further discussion of various nomenclatures that will be used to describe, analyze or synthesize cam-follower system, I would like to bring out one important point in the cam-follower mechanism. For that let us look at this diagram. This is the cam which is the disc cam and this is a translating roller follower. We should note that as the cam rotates the cam can drive the follower only in the upward direction. Whereas, when the follower tries to come down there is no tensile force that can be applied by the cam on to this roller so it comes down by gravity. To ensure that the contact is always maintained during the upward motion and the downward motion of the follower normally we use springs. This spring is connected between this guide and the follower such that when the cam is rotated the follower goes up, the spring is compressed.

When the cam shape is such that the follower wants to come down, it is this spring which will try to retain it back ensuring that the contact between the cam and the roller is never lost. In the absence of the spring as we see if we leave it only to the gravity the maximum acceleration can be g in the downward direction. If we want the follower to have higher than g acceleration, then there will be loss of contact. The cam will rotate, but the follower will not follow the cam profile. To ensure the contact during the entire stroke of motion we use this spring.

Similarly, for this oscillating follower, roller follower, again we have a spring to ensure that the follower can be driven in this direction by the rotation of the cam. To ensure that the contact is always maintained even while the follower has to come in this direction, it is this spring which will help to maintain the contact between the roller and the cam surface, these are the springs. There are other ways of ensuring this contact during the entire stroke of motion even without the spring, which will be explained in the next diagram.

(Refer Slide Time: 22:32)



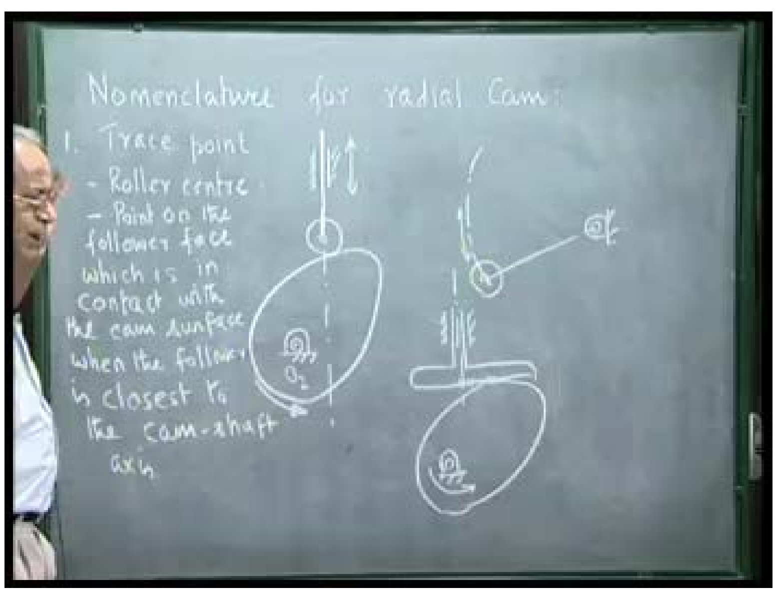
In this diagram, as we have shown, no spring is necessary because the roller follower is placed in groups, the shape of the groups is nothing but the shape of the cam which will dictate the type of angular motion that this follower will have. This is the cam and this is the follower and this roller 3 has been placed inside the groups in this body 2. As this body 2 rotates, depending on the nature or the profile of this centre line this roller will be pushed up or down and it will be always within the groups, there is no chance of losing contact either during this motion or during this motion.

As we see there is no need of spring, but this cam is more complicated than the previous one, where we used a spring because we have to part a group. The same thing is shown here. Again we do not need a spring, but we are using two rollers with this follower. This is the follower; it has two rollers and as this cam rotates either of these two rollers will be pushed. The cam can only push the roller and never pull it, so either this roller will be pushed during this motion or this roller will be pushed during this motion. Both clockwise and counter-clockwise motion of the follower is ensured without the use of a spring.

Similarly, here with a constant breadth cam the cam profile is such that, it will be in contact at both surfaces where both up and down motion can be ensured by the rotating cam so there is no need to use the spring, this is called constant breadth cam. We will restrict our discussion on the cams where we shall assume there is a spring to ensure contact during the entire stroke of motion.

We have discussed all types of cam-follower systems, we have classified different types of cam-follower systems. Now let us get into the details of the design of cam-follower systems. But for that first we need to establish the nomenclature that is, the terms which will be used very frequently.

(Refer Slide Time: 24:55)



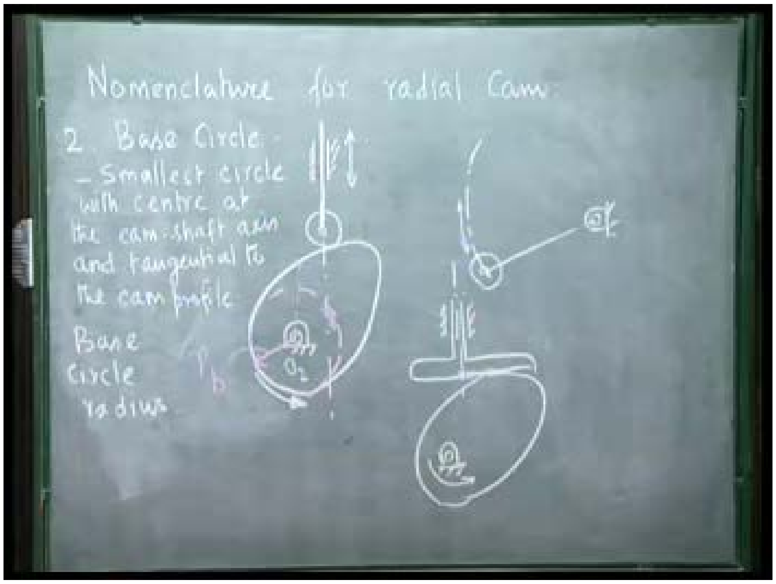
We will be discussing only radial cam, so we will look into the nomenclature for radial cam. We will introduce all the terminology with reference to an offset roller follower. Let us talk of a plate cam, which has an offset translating roller follower. That means the follower axis is not passing through the cam centre, it is offset. We assume that the cam is rotating in the counter-clockwise direction. This is the roller centre, this is the roller, this is the follower and this is the guide for the follower to move in the vertical direction, that is the prismatic pair.

First, we define a trace point. If we want to describe the motion of the follower for a translating follower or for an oscillating follower, we must say, by using which point of the follower we are going to describe the motion of the. The trace point is a theoretical point on the follower the movement of which describes the follower movement. For roller follower it is the centre of the roller. So the trace point is roller centre, which means the movement of the follower will be described in terms of the motion of this roller centre.

Of course in this case, it is same for every point of the follower. But an oscillating follower will still concentrate only on the circular arc along which the roller centre is moving. This is the movement of this roller centre that will be used to describe the motion of the follower. If it is a flat face follower, then the trace point we use is the point on the follower face which is in contact with the cam surface when the follower is at one of the extreme positions. We normally use that extreme position when the follower is closest to the cam centre. When the follower is closest to the cam shaft axis, this cam shaft axis is O_2.

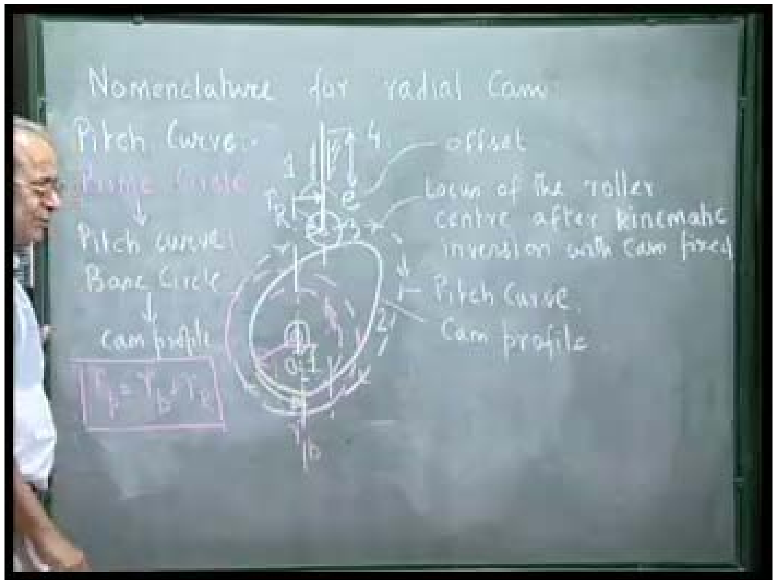
As we see, in the case of a flat face follower, at this instant it is this particular point of the follower face (Refer Slide Time: 29:30) is in contact with the cam surface. As the cam rotates and the follower moves up and down, this point of contact shifts on the follower face. From the follower to the lowest position, that is closest to the cam centre, whatever point on the follower face is in contact with the cam surface that configuration I will use as the test point. Or we can use this axis wherever it intersects this point as the trace point. Trace point is the theoretical point on the follower which is used to describe the movement of the follower, so that is why we call it as trace point.

(Refer Slide Time: 30:22)



Second thing we would like to define is the base circle. Base circle is the smallest circle. Let us think of the smallest circle that can be drawn with cam centre as the centre and touching the cam profile. This is the smallest circle that can be drawn with cam centre as the centre and touching the cam profile, this circle we call the base circle. It is the smallest circle with centre at cam shaft axis and tangential to the cam profile. This red circle will refer as the base circle. As we see, the base circle really defines the size of the cam; it is the difference of the distance of the cam surface from the cam centre and this base circle radius that defines the movement of the follower. This is the smallest circle when the roller is in contact with the base circle that means from here to there the follower is at its lowest position. This radius of the base circle we call r_b that is called base circle radius.

(Refer Slide Time: 33:00)



We define a pitch curve. To define the pitch curve we think of a kinematic inversion. If we remember the kinematic inversion, this is the four link mechanism, fixed link, cam, roller and follower. In this four link mechanism, this link number 1 which is fixed, but in this kinematic chain, if we make a kinematic inversion holding the cam fixed. That means we allow link 1 to move and hold the cam fixed instead of allowing the cam to rotate in the counter-clockwise direction with respect to link 1. We hold the cam fixed that means link 1 we now rotate in the opposite direction that is in the clockwise direction. If we do that, the locus of the centre of this roller that will generate a curve which is parallel to the cam profile. This is the locus of the trace point or roller centre after kinematic inversion with cam fixed.

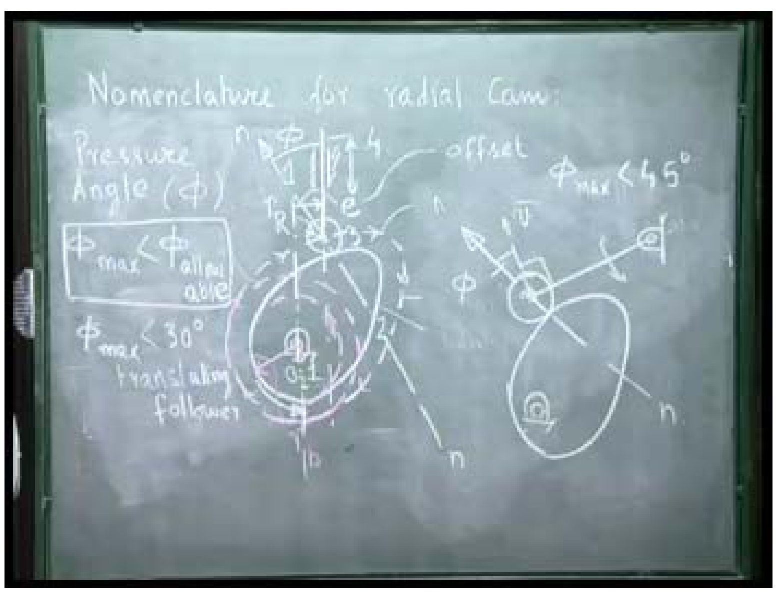
As we see, the cam rotates in the counter-clockwise direction. If we hold the cam fixed and allow the fixed link to move in the clockwise direction the roller centre will move in the clockwise direction, but it will lie on this curve which we call the pitch curve. If we now have a parallel definition like base circle to the cam profile, this pitch curve is a parallel curve to the cam profile. Now think of a circle, the smallest circle that can be drawn with cam centre as the centre and tangential to the pitch curve. This circle has centre at the cam surf axis and tangential to the pitch curve. This circle is called prime circle.

The base circle to the cam profile is nothing but prime circle to the pitch curve. This is pitch curve and this is cam profile. We have defined trace point, then, this is a cam profile, then we define base circle, then, we define the pitch curve and then, we defined the prime circle. So, prime circle to pitch curve is same as base circle to the cam profile.

If the base circle radius is r_b and the roller radius if we write r_R, is the radius of this roller and r_b is the base circle radius. Then, it is easy to see that prime circle radius r_p is nothing but r_b plus r_R. We have already defined that if we draw this vertical line through the cam shaft axis and this is the line of reciprocation of the follower then this distance we call offset denoted by the symbol e, e is what we call offset.

We must remember that this offset to the right is for counter-clockwise rotation of the cam, why? That will be explained little later. If the cam rotates in the clockwise direction then we must provide offset to the left. This we can call positive e for counter-clockwise rotation which I take as a positive direction of rotation of the cam. Now we define one more important parameter for this cam-follower system which we call pressure angle.

(Refer Slide Time: 38:03)

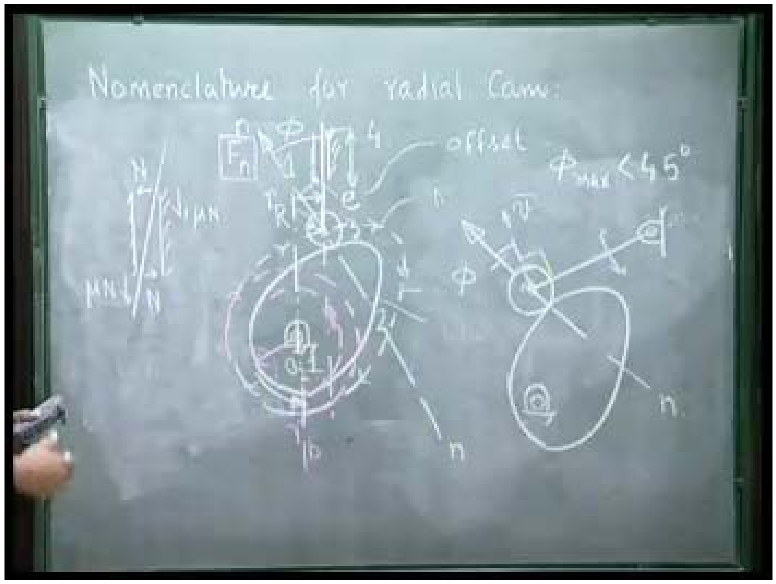


The importance of pressure angle can be clear if we look at this diagram. The common normal between the roller and the cam is passing through the roller centre and normal to the cam profile. This is the common normal between the roller and the cam profile. If we neglect the friction, the force that the cam exerts on the roller is along this common normal. This is the direction of the force between the cam and the roller in the absence of friction. I want the roller to move in the vertical direction. But I am pushing the roller along this direction, so this angle should not be too large. The direction of the follower movement is this direction of the force in the absence of friction is along the common normal. The angle between these two directions between the contact force normal contact force and the direction of the follower movement, that we call pressure angle. This angle phi is called pressure angle.

Obviously, for smooth movement of the follower this phi_{max} should be less than some maximum value $phi_{allowable}$. The phi will keep on changing depending on the cam profile and normally, for a roller follower we want phi_{max} for translating follower the phi_{max} should be less than 30 degree. Similarly, for an oscillating follower; the velocity of this point and the common normal they are not in the same direction and the angle between them will be called pressure angle and for oscillating follower; this is for translating follower, phi_{max} should be less than 30 degree, whereas, for oscillating follower suppose, we take the case of a roller follower but oscillating, this is hinge here, then the direction of the velocity of the trace point is perpendicular to this line, this is the direction of the velocity of the trace point and the common normal between the roller and the cam surface. This is the direction of the contact force in the absence of friction and this angle we will as call pressure angle. For such oscillating follower, the pressure angle is less important and phi_{max} should be less than 45 degree; whereas, for translating follower phi_{max} is less than 30 degree.

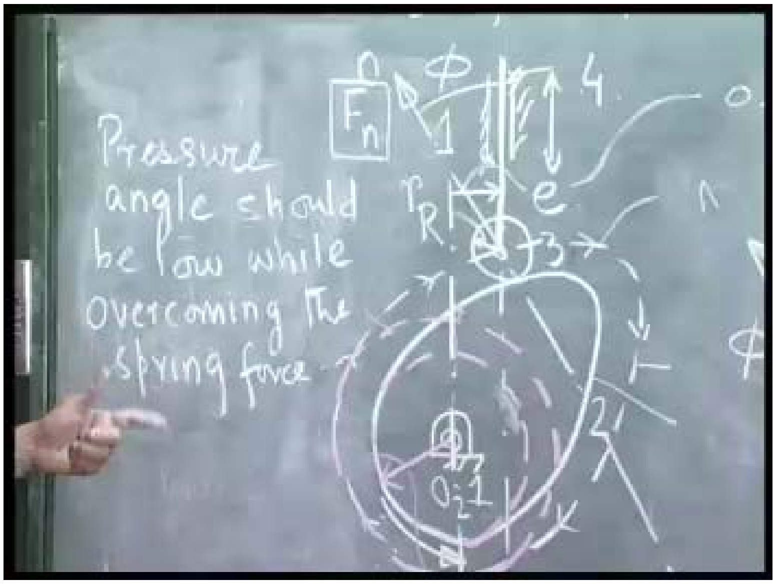
Now that we know the pressure angle we will be able to appreciate why this offset is given in a particular direction depending on the direction of the cam rotation? If we remember that there is a spring which tries to push it back, so pressure angle is more critical during the upward motion of the follower. As the follower is pushed out from the cam shaft, it is compressing against the spring and this contact force let me call it $F_{n,}$ this is the normal force that has the tendency to rotate the follower in its bearing in this guide. It has a tendency to rotate it clockwise, for this force the follower has a tendency to rotate clockwise and the contact will take place here and there. The guide which is supposed to guide this vertically, actually due to this force has a tendency to rotate and the contact will take place here and there.

(Refer Slide Time: 43:48)



If I exaggerate this corking movement it will be like this (Refer Slide Time: 44:00) then a normal force acts in this direction, a normal force acts in this direction and these two normal forces balances the corking moment, the moment due to this force F_n. The friction force which tries to oppose this vertical motion will be mu time based normal force. If I call it N then this will be mu N and this will be mu N. As we see during upward motion, the follower has to overcome not only these two friction forces, it also has to overcome the spring force. As a result, a large F_n is necessary to overcome this friction force and the vertical component of this F_n will overcome this two friction force and the spring force whereas during the downward movement the spring force is helping the follower to come down, so this contact force will be less. It is obvious, that this vertical component will be F_n cos phi, if the phi is very large then this vertical component will be reduced. As a result, during the upward movement, I want phi to be low and during the return movement, that is, the down movement phi can be large, so phi_{max} is more critical during the upward movement.

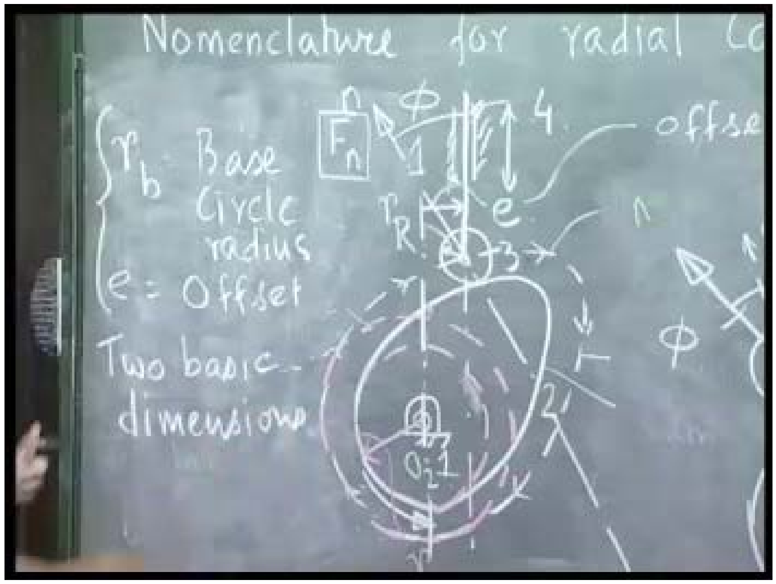
(Refer Slide Time: 45:40)



Pressure angle should be low while overcoming the spring force and this particular offset, this positive offset ensures that the pressure angle reduces during this motion when it is comprising this field and it will increase obviously while the follower is coming down but at that time spring force is there to make the follower move downward, so I do not need a large contact force.

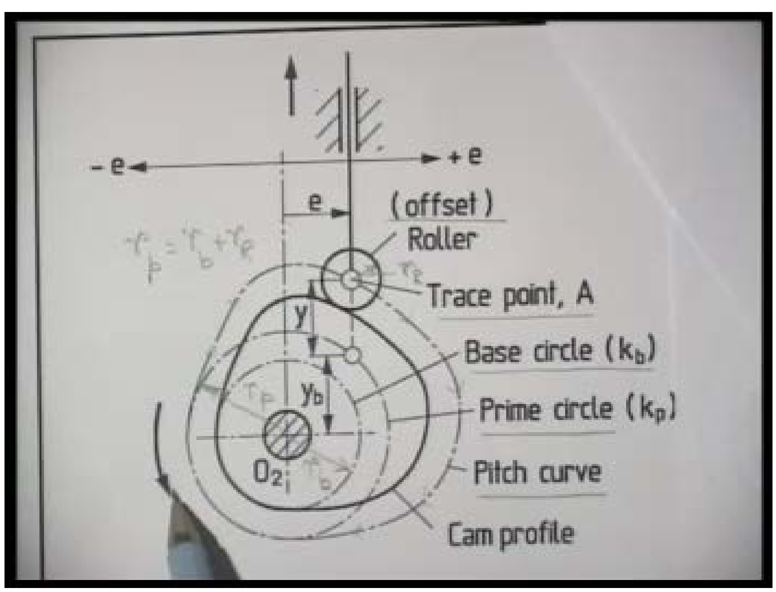
Pressure angle can be allowed to be larger during the downward movement but has to be smaller during the upward movement assuming the spring is resisting the upward movement. This is the need for offset. Of course, all this we will have analytical expressions later on that how pressure angle is related to all this other dimensions.

(Refer Slide Time: 47:06)



We have base circle radius r_b and e the offset. These are the two basic dimensions. r_b defines the cam size and e gives the relative position between the follower and the cam shaft. These are the two basic dimensions which we have to first determine before we can go for designing the cam. At this stage, let me discuss a little bit, what is the effect of this offset in the flat face follower case?

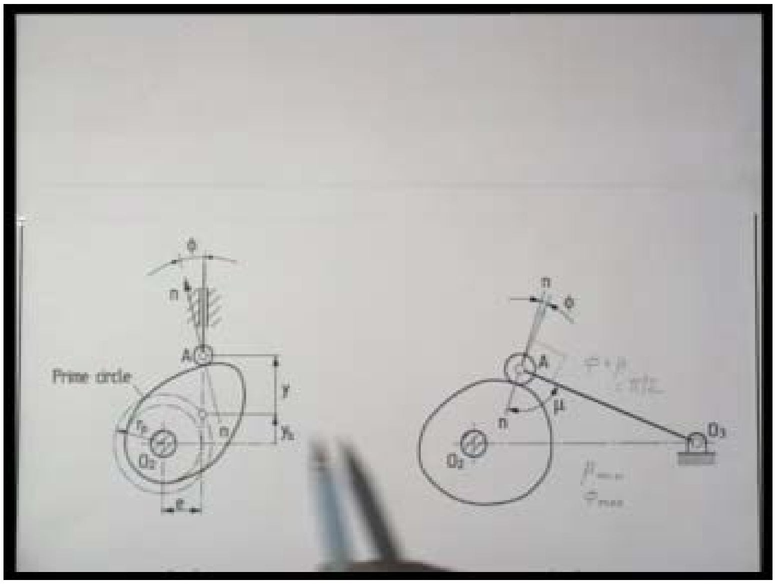
(Refer Slide Time: 48:01)



Before getting into the flat face follower place, let me recapitulate whatever we have discussed so far with reference to a roller follower. As we see; this is the cam profile, this is the roller and this is the follower axis. We have defined trace point that is the centre of the roller. The movement of this particular point describes the movement of the follower. This circle whose centre is at the cam shaft and this is the smallest circle that can be drawn that means it becomes tangential to the cam profile. Any bigger circle will intersect the cam profile.and the smallest circle can be drawn with cam centre as the centre and tangential to the cam profile which we call base circle.

The radius of the base circle is called base circle radius and we denoted it by $r_{b.}$ This curve which is parallel to the cam profile passing through the centre of this roller is called pitch curve. How do we generate the pitch curve? By kinematic inversion, holding the cam fixed and aligning the fixed link to rotate in the clockwise direction; the cam is rotating in the counter-clockwise direction, then this locus of this roller centre after this kinematic inversion, it will move along this curve in the clockwise direction. This curve we called pitch curve and the smallest circle that can be drawn with cam shaft centre as the centre and tangential to this pitch curve we call it as prime circle. This circle we call prime circle. If the radius of the prime circle is r_p, then we can easily see that r_p is nothing but r_b plus r_R where, r_R is the roller radius, this is r_R. This distance that is, the vertical line, so this cam shaft centre and this vertical line of the follower movement, this distance is called offset. This is the positive offset for counter-clockwise rotation.

(Refer Slide Time: 51:04)



In these two figures, we explain again the pressure angle, we have just now seen if this is the roller follower and this is the cam profile, let this be the common normal, which passes obviously though the roller centre. This is the direction of the follower movement but the contact force, if we neglect the friction force then acts along this direction and the angle between the direction of this force and the direction of the follower movement, this angle phi we called pressure angle.

Similarly, for the oscillating roller follower, let this be the common normal passing through the roller centre and the follower is hinged here, so the velocity of this point A is perpendicular to O_3A that is this angle is 90 degree. This is the direction of the movement or velocity of the point A and n, n is the direction of the contact force. Angle between these two we call the pressure angle phi.

We can see that, this angel which we called mu; phi plus mu is 90 degree. This is 90 degree, this total angle is 180 degree so phi plus mu is 90 degree and this angle mu which we call transmission angle incase of linkages. This pressure angle is complementary to the transmission angle incase of linkages. In case of linkages we ensure that mu should not fall below a particular value, we are concerned with mu_{min}. Whereas in case of cam, we are talking of the pressure angle phi and we are concerned about the maximum value of phi which must not be beyond a given allowable value. As we said phi_{max} for this is normally about 45 degree for the oscillating follower and for the translating follower phi_{max} is normally kept within 30 degree.

Let me now summarize what we have learnt today. We started our discussion with higher pair mechanism namely cam-follower mechanism. We will be discussing only 2-D or planar cam-follower mechanism. We defined the cam and the follower and we have also classified cam-follower systems based on three things namely: the type of cam movement, type of follower movement and the type of follower surface.

After classifying these different types of cams we have also seen, the need of a retaining spring between the follower and the fixed link such that the contact between the cam and the follower is ensured during the entire cycle of movement. We defined different terms which will be used while designing cam-follower systems namely: the base circle, prime circle, pitch curve, pressure angle and things like that.

In our next class, we shall start our discussion, first with a flat face follower and show what is the need of making this offset, as we have seen in case of roller follower.