Kinematics of Machines
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Module No. # 09
Lecture No. # 04

In the last lecture, we have seen the steering requirement for the automobile when it takes a turn; that means, the front two wheels must be rotated accurately to prevent the tendency of side slip and we have also seen how the Davis steering gear has been fixed. We noted that the Davis steering gear is a fixed link mechanism which 4-Revolute and 3 systematic pairs. It is due to the systematic pairs the Davis steering gear is costly and also more difficult to maintain.

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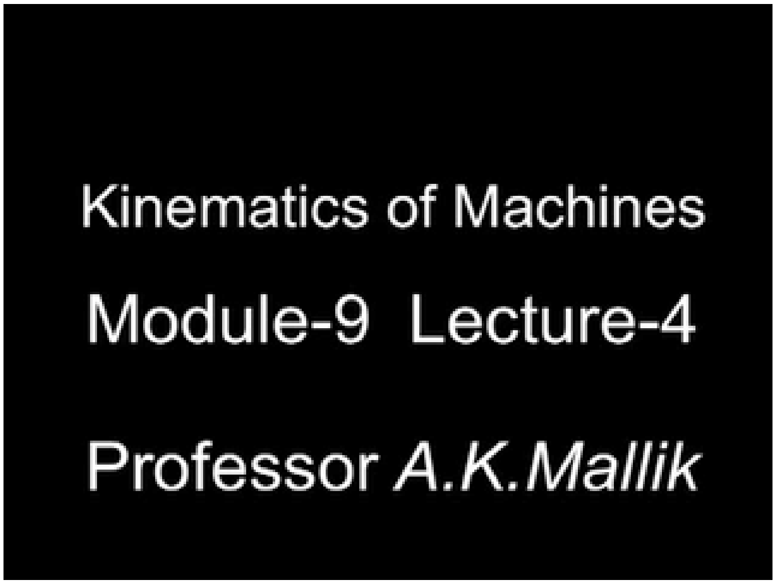

Today, we will discuss a simple version of steering mechanism known as Ackermann steering mechanism. We shall see the Ackermann steering mechanism in a simple 4-R linkage.

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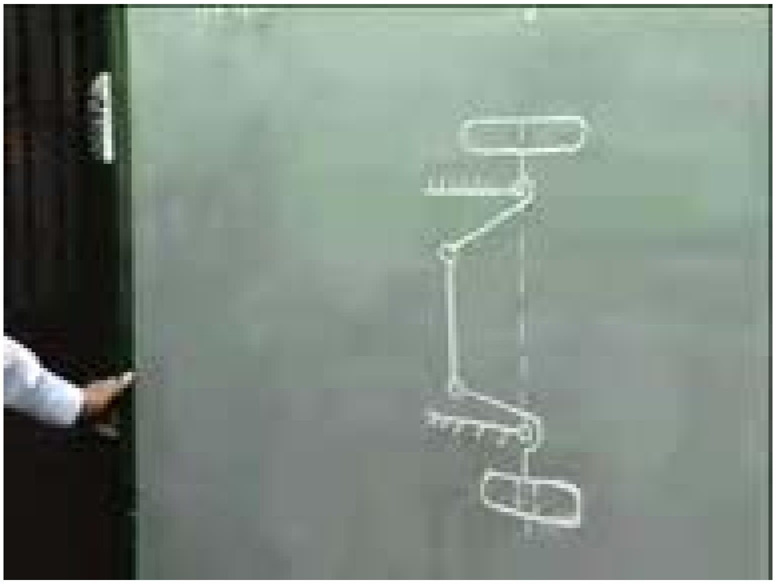

This is the Ackermann steering linkage which as we see, the top axle is mounted on the body of the automobile; this is the body (Refer Slide Time: 03:17). Similarly, this top axle is mounted on the body of the automobile and the 2 top axles are connected by a coupler. So, this is O 2, A, B, O 4; so O 2, A, B, O 4, is a 4-R planar linkage. This is the top view of the automobile (Refer Slide Time: 04:00). Here are the 2 front wheels and the back wheels will be here.

Now, this is also symmetric linkage in the since this link length is same as this link length and this angle is same and this angle (Refer Slide Time: 04:20). The car is going along a straight path; these 2 angles must be checked because, it has to be symmetric because the car may take a right hand turn or a left hand turn and these 2 curves must be identical. So, this linkage O 2, A, B, O 4, is the symmetrical 4-R link. Because it is simple, we have to also pay a price. In the sense that, the perfect steering conditions cannot be maintained for all radius of curvature of the turn; the perfect steering condition can be made only at certain points and at both, if we properly design that for other radius of curvature, that can give…….. so what you see that these are 4-R linkage has 4 link length; l 1 is the 6 link length which is given that will decide for the given location of the these two (Refer Slide Time: 05: 25) pivot O 2 and O 4 in the body of the automobile. Then, l 3 is the coupler length which we have to find out and l 2 is same as l 4 and we have to determine this link length for a proper steering action. We can simplify this 4-R mechanism and extend the synthesis procedure by an example and we can use what we have discussed earlier, as the Ferenstein method that is the analytical method of linked synthesis.

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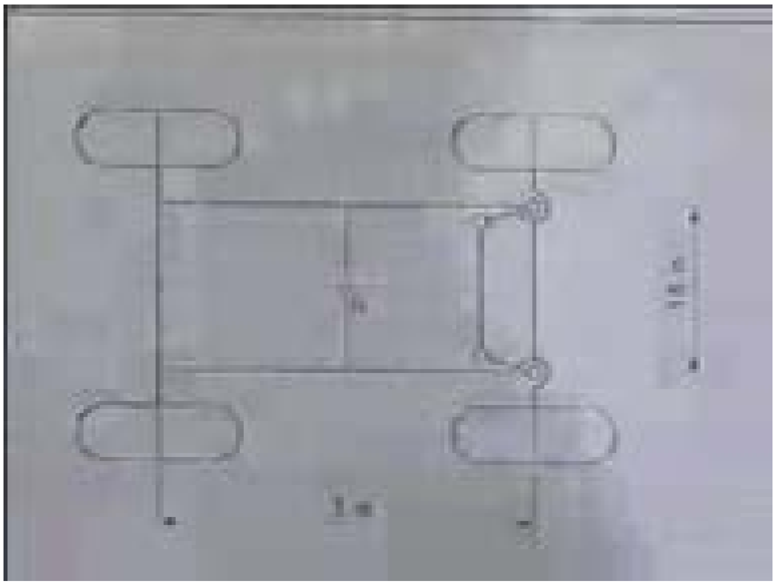

Let us discuss the design of the Ackermann steering mechanism with reference to this particular example. In this car, the wheel width that is, the distance from the rear of this to the front of this is 3 meter (Refer Slide Time: 06:25). If we remember, this wheel width is denoted by b and the distance between these 2 fixed pivots on the body of the automobile, is 1 point 8 meter which is call p. I is said that the radius, minimum radius curvature for this center point g of the body of the automobile it 10 meters. That means, the minimum value of r, if I call that r min for this center point g is desired to be 10 meters. The car has to obviously take right hand turn - that is clockwise or a left hand turn or counter-clockwise. As you see, the radius of curvature of this point g, can go from minus 10 meter to minus infinity or plus 10 meter to plus infinity. When the car goes along a straight path, the point g goes along a straight line that is this curvature is intact (infinite).

Now, as we noticed that the rotation of these 2 wheels must be correlated for perfect steering condition given by the following equation: say cot of theta 6 minus cot of theta 2 is to be p by b for theta 6 for the rotation of this outer wheel and theta 2 is the rotation of the inner wheel (Refer Slide Time: 08:33). This is what you derive when you discuss the (any) heavy steering gear. But, in this case, let us call the rotation of this outer wheel; and consider a turn in this direction, outer wheel so, I call it instead of theta 6 let me call it theta 4. So, the rotation of these two wheels - theta 2 for this wheel, theta 4 for this wheel, always will maintain this direction. But, such a function generation is obviously not possible for all values of radius; that means, for all values of theta 2 and theta 4. If we use a simple 4-R linkage, we can maintain this condition only for some specific values of theta 2 and theta 4 not for the entire link.

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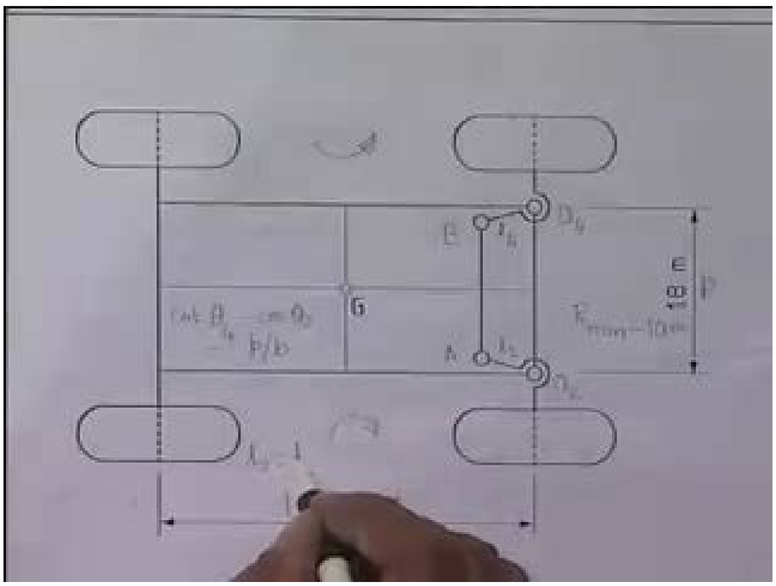

We call this fixed pivot O 2, this point a, this point b and this point O 4 (Refer Slide Time: 09:33). The rotation of O 2 a we represent by theta 2 and rotation of O 4 b is represented by theta 4. Now, we know if you have p accuracy point synthesis that these pairs of values of theta 2 and theta 4 can be satisfied by a 4-R linkage. However, in this set marker let seven set linkage that means, this O 4 be the link length l 4 and O 2 a then, the link length l 2 must be same to maintain the symmetry; that is identical turns for the clockwise or counter-clockwise.

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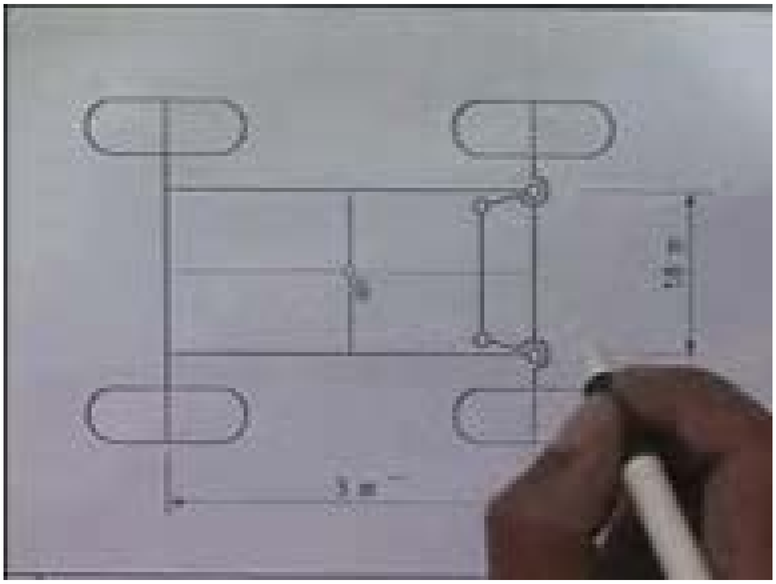

So, l 2 is same as l 4 because, this l 2 satisfied we do not require; obviously, we cannot even go for 3 point synthesis. We can go only for a two point synthesis. O 2, O4, this is the fix link length l 1; A B, that is the top level link length is l 3 (Refer Slide Time: 10:36). What we see here is, we have to determine, what is l 2 and l 4 and what is l 3. That l 1 is given to be 1 point 8 meter; the fixed point O 2, A, B and O 4. Before you can go for two point synthesis we decide that one of the accuracy points corresponds to the state configuration. And the state configuration, the linkage has to be symmetrical; that is this angle must be equal to this angle (Refer Slide Time: 11:18).

If we remember the Davis steering gear, this angle which we called alpha that time, found to be half of cot inverse p by b here p is to 8 meter and b is 3 meter. so this is half of crossing the point 6 which will be equal to approximately 73 degree. That is for the Davis steering gear, for c equal to 1 point 8 meter, b equal to 3 meter, this angle which is called alpha then found out to be half of cot inverse p by b approximately 73 degree; but, this problem we assume this angle to be 70 degree (Refer Slide Time: 12:16).

So, five as to be 73 degree I take this angle as 70 degree this angle is also 70 degree. This is l 1 this is l 6 and O 2 A and A B 10 which is say l 2; this length is l 2 this link also l 2. Now, the radius for curvature for point g as we denoted already goes from minus 10 meter to minus infinite and plus 10 meter to plus infinity and we have only one accuracy is 3 point 2 solve for because we already taken b then take and accuracy point and there are only two link length ratios namely l 1 by l 2 and l 1 by l 3. So, we can satisfy at the most only two accuracy points of the accuracy point corresponds to the state configuration. That leaves to only we one for that accuracy point. Of course, the symmetry of the linkage ensures that I need to consider only one kind of turn; say from 10 meters to the straight path; minimum ten meter radius for g and straight path for g. because, the negative part that is the other direction of the turn will be automatically satisfied now the question is which radius I decide to satisfied the steering requirement for that we go back to that our cheavyshev accuracy point.

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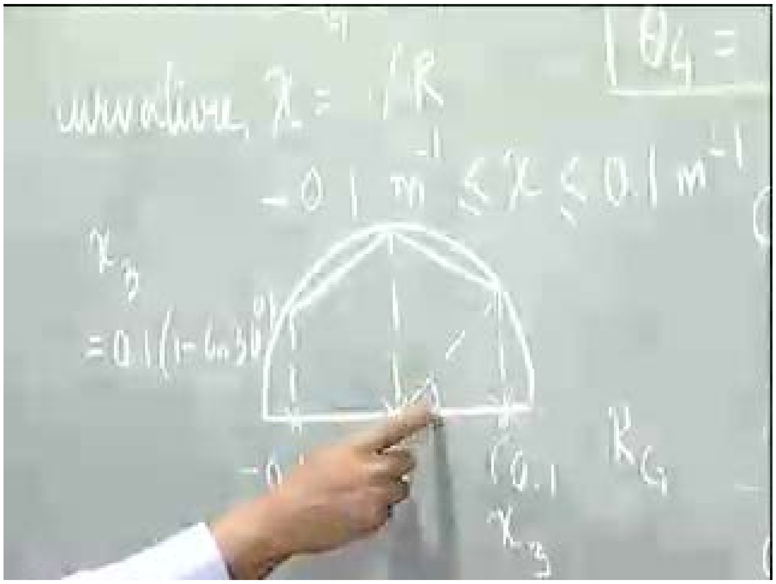

Let me now complete this problem of designing this Ackermann steering linkage represented by 4-R linkage O 2, A, B, O 4 to satisfy the steering requirement. The symmetric linkage ensures that the right hand and left hand turn are taken identically. So, one accuracy points I consider this straight path (Refer Slide Time: 14:29) and this angle obtained, this angle is also 70 degree. We have already chosen theta 2 equal to 70 degree and corresponding theta 4 which we measure from the line O 2 and O 4 is 100 c minus 70 degree is 110 degree. So, this is one pair of coordinated movement that is satisfied by the 4-R linkage.

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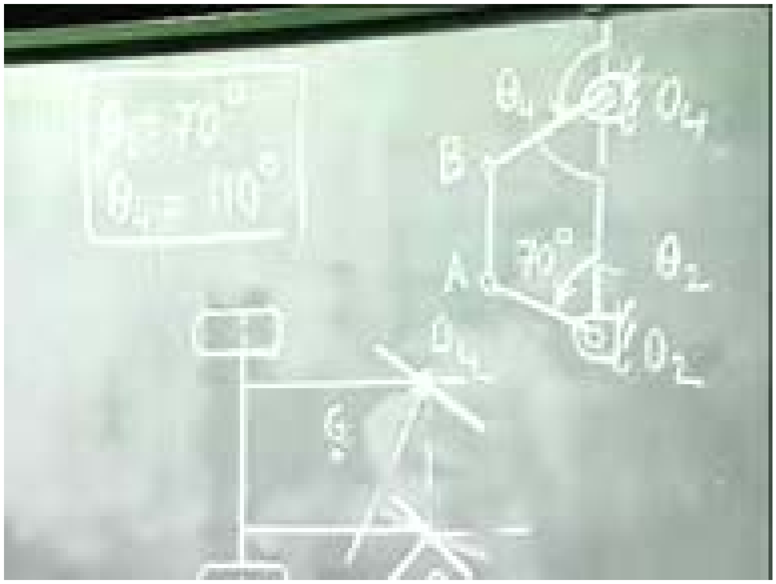

Symmetric linkage structure of the right hand and left hand turn identically; we are left with only one more accuracy point; let me check it as R G - for which value of the radius of curvature of this center point g I must satisfy the requirement. And, let as say we will consider the rotate for that value of R G; this I call R G. So, filling for value of R G, this requirement, this steering requirement that will be all perpendicular to the wheel axis, intersect at one point. This is the perfect steering condition. So, what is this rotation (Refer Slide Time: 15:54) and what is this rotation? We shall call say, delta theta 2 and this I will call delta theta 4. So how,… the accuracy point that is the value of R G for which the perfect steering action will be satisfied will proceed as follows:

We see that the curvature which is defined as 1 upon R goes from minus 0 point 1 meter inverse to 0 point 1 meter inverse; right. 10 meter radius of curvature means point 1 meter inverse curvature; this is call curvature (Refer Slide Time: 16: 45).

So, we can take three accuracy point in this line minus 1 to point 1 as follows: the line, this is 0, this is minus 0 point 1, this 0 point 1. Then, we draw regular hexagon; we draw this regular hexagon within this circle - semi circle which we have done earlier and this are the accuracy points.

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This is 0 point 1, this is minus 0 point 1 and this is 0. So, if I call it psi 3 which is same as minus psi 1; so psi c is, this will be 0 point 1 into 1 minus cosine of 30 degree. This angle is 30 degree. Radius is 0 point 1 this is 30 degree (Refer Slide Time: 18:23). So, this is point 1 into cosine 30 degree which is 0 point 0866 this is (Refer Slide Time: 18:49). The corresponding R G is 1 upon psi 3 which is 11 point 55. This is one accuracy point and other accuracy point will give you R G is minus 11 point 55 meter and other is R G is psi is 0 so R G is infinity; that is the straight path. Now, straight path we have already satisfied by this pair of coordinated movements; these two pairs of value theta 2 and theta 4. Similarly, for 11 point 55 which is to minus 11 point 55 will automatically satisfy because the linkage is symmetric.

So, now you can apply for Prete method to call for the link length pairs. This R G will take the 11 point 55. So, this was given to us as 1 point 8 meter (Refer Slide Time: 20:26). So, if I call this point I and this point C, this point D and this point E which is we are taking I G, the radius of the curvature of the path of the center point is 11 point 55 meter and this will get, for given to us as since. So, this is 1 point 5; the wheel rotates by delta theta 2, then its vertical, this line is horizontal; so, that rotation is written as delta theta 2. And, the rotation of that wheel is this much - which is delta theta 4. So, we can see tan of delta theta 4 is 3 meter. This opposite 3 meter divided by I G which is ic plus cg is point 9 meter and ic I can get from I G and cg; cg is 1 point 5 meter. So, this c divided by square root of 11 point 55 square I G is 11 point 55 square minus cg square; that is, 1 point 5 whole square that is ic and cg is 0 point 9. So, if we calculate this, we will get delta theta 4 will be 13 point 65 c.

Exactly the same way I can calculate delta theta 2 because, tan delta theta 2 that is the rotation of the inner wheel is O 2 E by IE O 2 E is 3 meter by IE that is Ic minus cE and Ic already got here so it is c divided by square root of 11 point 55 square minus 1 point 5 square minus 0 point 9. If we calculate this we will get delta theta 2 is 15 point 87 D.

So, if you see that, if you want to satisfy the perfect steering condition corresponding to the radius curvature of this point G for 11 point 55 meter that is little higher than the minimum value of 10 meter, this is our accuracy point for the steering condition will be perfectly satisfied that these two wheels from this neutral position that the straight position will rotate by delta theta 2 and delta theta 4 which is given by 13 point 65 and 15 point 87 degree.

So, we get a second set of coordinator values for theta 2 and theta 4. theta 2, let me call it second value which is 70 degree minus this is 70 degree (Refer Slide Time: 25:22) and it is rotated g prime for 13 point 65, 15 point 87 degree. So, 87 degree - so that gives me 66 point 1 degree and theta 4 too similarly we get 110 minus 13 point 65 degree which is me point 35 and 14 that is 96 point 35 this is not 56 this is 54, sorry. This is 54 degree 70 degree minus 15 point 87 degree is 54 point 13 degree and the corresponding value of theta 4 this 96 point 35.

So, we get two pairs of coordinator movement as the two pairs of values of theta 2 and theta 4 which must be satisfied by this four bounded value - O 2, A, B, O 4. So, we write the Cordister equation: cosine theta 2 minus theta 48 here 1 cosine theta 4 minus a 2 cosine theta 2 plus kc for k 1, k 2, k 3 are the k design parameters; where k 1 is l 1 by l 2, k 2 is l 1 by l 4 and k 3 is l 1 square plus l 2 square minus l 3 square plus l 4 square divide by twice l 2 l 4 (Refer Slide Time: 27:33). Now, you know you want same as l 2 because, l 2 is same as l 4. So, this I can write k 1 for theta 4 minus k 1 cos theta 2 plus k 3. Now, this equation cosine of theta 2 minus theta 4 k 1 – if I take common, I can write theta as k 1 cosine theta 4 minus cosine theta 2 plus k and these two pairs of movements have to be satisfied this equation. So, if you substitute we get two equations to solve for k 1 and k 3. If we write this equation if you substitute here we are getting cosine 40 degree is k 1 cosine 110 degree minus cosine 70 degree plus kc and if use this set of values then I am getting cosine point 22 and O 2, 954, 42, 42 point 22 degree is k 1 cosine theta 4 is 96 25 minus 54 point 16 plus theta.

So, from these two equations I can be able to solve for two unknowns, namely, k 1 and k 3. If solve for k 1 and k 3 by solving we get k 1 equal to 1 point 953 and k 3 is 2 point 102. So, l 1 is given as l 1 is O 2 O 4, which is 1 point 8 meter. So, from here I get l 2 which is same as l 4 is equal to point 9217 degree (Refer Slide Time: 30:34). ac I can use to solve for l 3 but I can also use this configuration are I can see that l 1 is l 3 plus 2 l 2 cosine 70 degree. l 2 cosine 70 degree plus l 4 which is same as l 2 this is also 70 degree. So, again l 2 cosine 70 degree plus l 3 is there. I can either solve it from this value of kc because, I know all the l 1 and l 2; l 4, I can find l 3 or I can directly use that and use l 1 and l 2 which are obtained to find value of l 3 which will turn out to be 1 point 17.

So, l 1 is given to us as 1 point 8 meter. So, you can use the prudentecis method to solve for this, to synthesize this Ackermann steering mechanism to satisfied the requirement that at a particular value of the standing radius for this center point g which we took as 2 point 11 point 55 meter that steering condition will be exactly satisfied. It will be exactly satisfied for identical left hand turn; this is to be a right hand turn and obviously if we right for the straight part but, all other values of the radius of curvature of the part this steering condition will not be exactly satisfied. So, this is a much simpler mechanism than Davis gear but, it is obviously not satisfying the steering requirement for all horizontal values of the steering radius. It is satisfying only for a particular value of this steering radius and for all of the radius we almost ensure that the deviation is not much this intersection of these 3 line will not be too far off such that, the turning of the side steering is not true.

That is, we finished this section on steering mechanism and next, we shall discuss another special class of mechanism which we call universal joint Hooke’s joint. Here, now we shall discuss another type of special mechanism which is also used in automobile which is known as universal joint or Hooke’s joint. If you notice we will find that the output from the gear box of an automobile is taken by a propeller shaft which connects to the differential here and this propeller shaft and the differential input shaft are connected by a joint which is known as universal joint are Hooke’s joint.

Actually, this is use to connect two intersecting shafts which transfer between two intersecting shafts. The axes of two shafts, if there is intersection at a point then the power can be transmitted from one shaft to another by what is known as the Hooke's joint.

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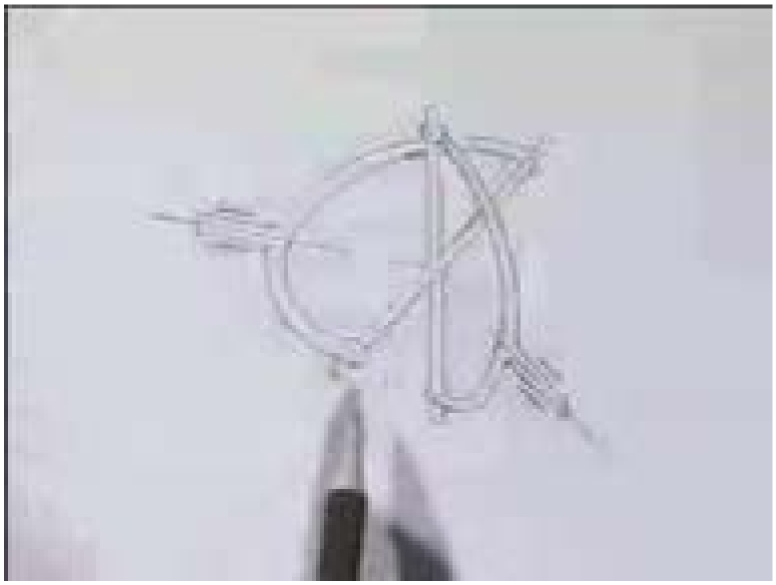

First let us have a look at the physical construction of the Hooke's joint. Here, this is how the physical construction of a Hooke's joint looks like. This is one shaft - the input shaft and this is the output shaft. Here, these 2 shafts are intersecting at this point (Refer Slide Time: 34:30). Then, this input shaft can be connected by this mechanism which is called an universal joint or Hooke's joint. We see, this part - this is called a fork is incident with this input path. Similarly, this fork is also incident to this output path and this is a fork arc and the angle between these 2 arcs of this shaft is 90 degree. This is vertical; this is horizontal at this configuration. The horizontal path of this fork bar is mounted in 2 bearing in this fork. Similarly, the vertical path of the this fork arm is mounted in these 2 bearing carried by this fork.

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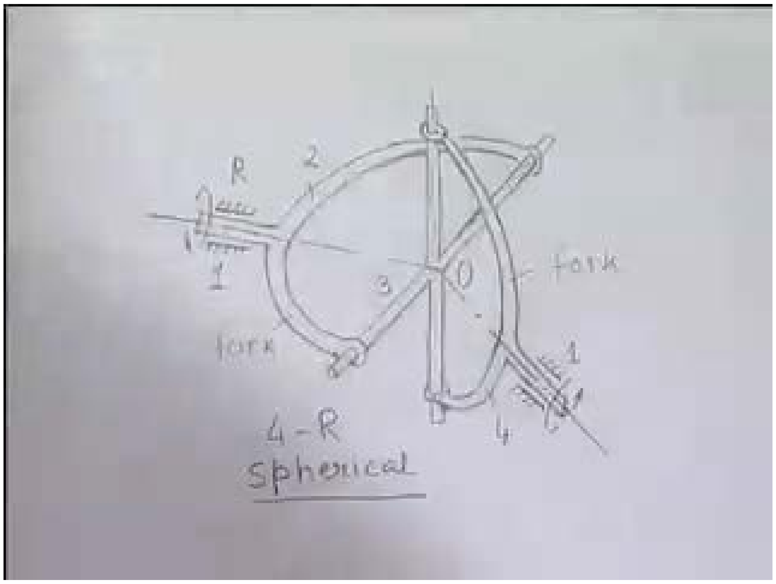

This is how a, and now, this shaft if this shaft rotates, this fork arm also rotates and it able to transmit power to this shaft. This is what is known as universal joint or Hooke's joint (Refer Slide Time: 35:42). In the next figure, we will show what is the kinematics representation of this mechanism. We should know that there is a revolute here; this shaft is rotating with respect to the pair or foundation, this is link number 1 this is link number 2 and there is revolute and the axis of revolute is this (Refer Slide Time: 36:04).

Similarly, there is revolute pair here; this is link number 1 as this is four-coupled is link number 4 and there is revolute and the axis of the revolute is along axis of the path and these 2 revolute pairs' axis too are intersecting at this point. There are 2 revolutes here; that is, there is two bearings and kinematically they are one revolute here because, the axis of these 2 revolutes is same and they are connecting the same 2 members - this shaft arm which I can give link number 6. Between 2 and 6 there is two bearings that kinametically there is only one element here because, the axis of both the bearings are same. Similarly, there is revolute here (Refer Slide Time: 36:15); this link 3 and 4 this fork of shaft is link number 4 and this is link number 3 and there is a revolute here and axis of this revolute is also passing through this point.

So, all the revolute axes are intersecting at this point O (Refer Slide Time: 37:09). So, the 4-Revolutes kinematically, one here, one there and one here, one there and all the 4 are intersecting at one point. These 2 revolutes kinematically are equivalent; I can take any one of them.

Similarly, these 2 revolutes are kinematical equivalent; they are connecting to same to link and axes is same and they are kinametically equivalent to one revolute. So, this is also a 4-R mechanism but not a planar mechanism because, the axes of these 4 revolutes are not parallel and all the points of this mechanism and are moving in parallel; same so, this is what is called a 4-R spherical mechanism. All these 4 arms of the axes of the revolute are intersecting at the point O. Consequently, the distance of any point of this mechanism cannot change from O as the mechanism moves. So, all the points lies on the surface of the sphere here. This point always lies on the surface on the sphere; that is why it is called a 4-R spherical mechanism.

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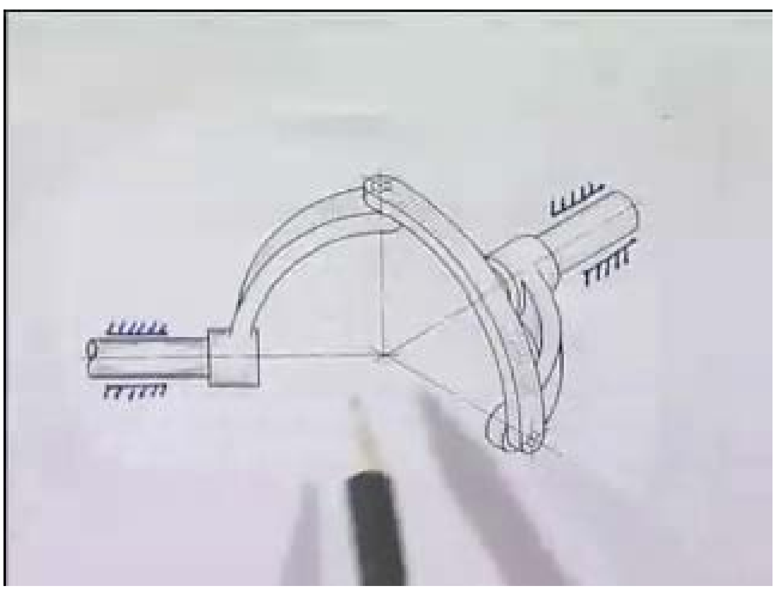

Now, we represent the same 4-R spherical mechanism or Hooke's joint are in this diagram as we see this is link number 1, are in revolute here whose axis is this line (Refer Slide Time: 38:35); this is the shaft and the fork which is the same digit body this is the link number 2 and this fork arms which connect the 2 forks is link number 3. Link number 2 and 3 of revolute here and axes of this revolute sphere also passes through this point O. This fork on c is connected to this part which is an integral part of this shaft to this revolute and axis of this revolute sphere between 3 and 4 also passes to the point O;

this is link number 1 (Refer Slide Time: 39:18). So, this is a Hooke's line where this angle is 90 degree and this angle is also 90 degree and this angle is also 90 degree.

If we consider the 2 revolutes to belonging to same link, the link number 2 has been linked to revolutes here and here then, its actual angle between the axes of this 2 revolutes too is 90 degree. Link number 3 between 2 revolutes here and 1 here, the angle between the axes of these 2 revolutes is also 90 degree. Link number 4 has this to revolute 1 here and 1 here and the angle between the axes of this 2 revolutes here, is also 90 degree.

So, this Hooke's joint is the safer state of a spherical mechanism or spherical linkage. All these angles are 90 degree and such a linkage as I said is used to transmit power from the coupling shaft that comes out of the gear box to the input of the differential. And, these axes are intersecting; as we see, the input shaft and the output – this one is the input and this one is the output (Refer Slide Time: 40:25), does not matter, these 2 shaft axes are intersecting at point O.

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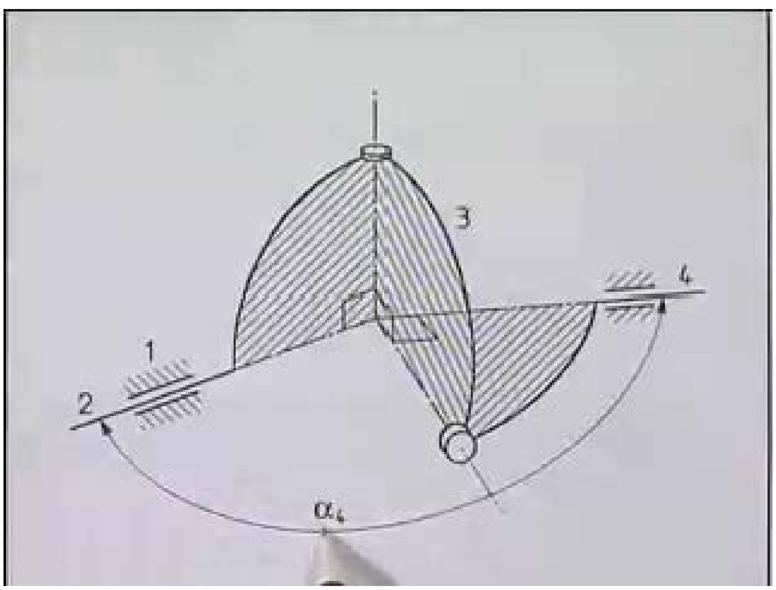

In this diagram, the Hooke's joint is represented by the kinematics diagram. These are the two revolutes at fixed like this. This is the output shaft and this is the input shaft and 1 represents the body of the automobile of the same of the foundation.

But, you should know that the angle between shaft axes is represented by alpha 4. These 2 shaft axes which are intersecting at the point O, this to line lie in a plane and the angle between these 2 lines that is, input shaft and output shaft is alpha 4 is called the shaft angle. Alpha 4 is called shaft angle. If link number 2 would first rotates along this axis, then link number 4 will rotate along this axis. So, I represent the rotation like this; the rotation of these 2, I represent by angle theta 2. That this plane goes down in this direction in theta 2 and this plane goes down in this direction theta 4 (Refer Slide Time: 42:00). It confirms that the relationship between theta 4 and theta 2 can be expressed by the equation tan theta 4 will be cosine of the first angle alpha 4 into cosine in cot of theta 2.

So, tan theta 4 is cos alpha 4 into cos theta 2; the 2 is measured this way and theta 4 is measured this way (Refer Slide Time: 42:47). As we see here the alpha 4 is more then 90 degree; so, cosine alpha 4 is negative; so cos theta 2 is positive; then tan theta 4 will be negative. That means, this will go in the other direction when the shaft rotates in this configuration, in this direction, this shaft rotates this way. not the way it is found here but, this relationship will hold good only if you measure, this is the axis of rotation for theta 4 and this is the axis of rotation for theta 2. It is very easy to see by differenctial expression that the angular velocity of these 2 shafts will not be same. If I differentiate this we get, x square theta 4 into theta 4 bar with is I call omega 4 will be cosine alpha 4 and differentiation of these 2 will be me 4 square theta 2 into theta 2 bar which is omega 2.

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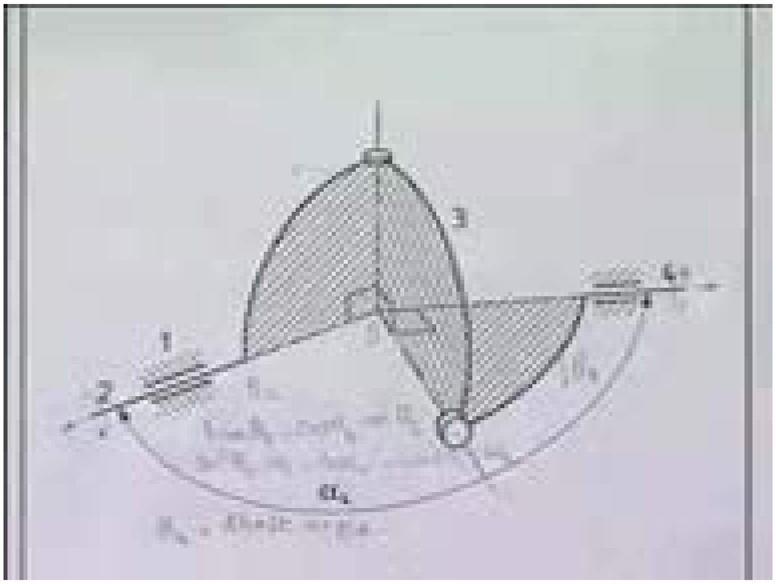

So, omega 2 and omega 4 are different depending of the values of theta 4 and theta 2 and obviously, the shaft angle alpha 4. If omega 2 is constant, omega 4 will keep on gaining. So, even if the input rotates at constant angular velocity, the output will not rotate at constant angler velocity. We can differentiate velocity once more assuming omega 2 to be constant then we can find what will be omega 4 bar. We can differentiate this once more this expression and get what is the alpha 4, this is when alpha 4 is there, when alpha 2 is 0. So, it does not transmit a constant angular velocity ratio; also, it does not transmit uniform angular velocity ratio. However, we can use 2 such Hooke's joints in series to transmit a constant uniform angler velocity ratio. That is called constant setting or self-constant setting act. Hooke's joint which has two Hooke's joints in series, we can now show you the shape.

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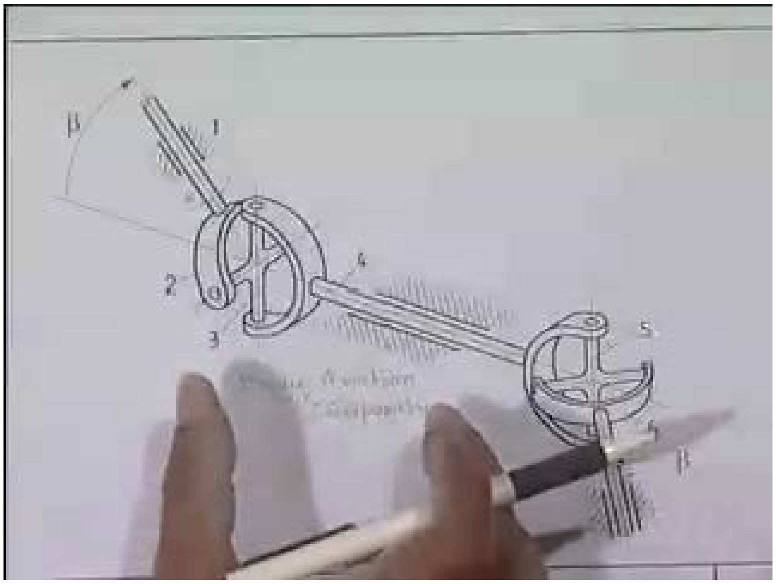

This link pole a self-constant setting Hooke's joint which is nothing but 2 identical Hooke's joints in series having the same shaft angle; for this is the input shaft which is link number 2 continues along this fork; this is the fork arm and this is the output shaft of this Hooke's joint here (Refer Slide Time: 45:35). One is the same. 2, 3, 4 and shaft angle is beta and this link pole acts as the input shaft for the rest of the mechanism which again has its shaft and it's fork connected identically and the shaft angle between this input shaft and this output shaft is same beta. It is same as this shaft angle. In that case, the angular velocity of this shaft will be same as the angular velocity of this shaft. So, this is called 2 Hooke's joint in series - will act in a self-compensating manner. However, as we see, this mechanism is cumbersome and takes quite a lot of space. In fact, there is patent of a compact, compensating universal joint which is patented by Bentinck Aviation Corporation. This is a different type of mechanism but which is patented by Bentinck Aviation Corporation which is a very compact self compensating Hooke's joint. That means, you can transmit power between 2 intersecting shafts using a very compact mechanism.

Let me now summarize what we havediscussed today. We started our discussion with a simple steering mechanism called Ackermann steering mechanism which is nothing but a 4-R simple planar linkage; symmetrical linkage such that, identical turns can take place for the right hand turn and left hand turn. What we found is that because it is a simple mechanism, it cannot satisfy the ideal steering requirement for all radius of turning. In fact, we can satisfy the condition of change only for a particular value of the radius and we have also shown an example - how to see the particular value of the radius so that, either for all other radius is somewhat less. After that, we discussed what you call a Hooke's joint or universal joint which is used again in a automobile to connect 2 intersecting shafts. This is nothing but 4-R mechanism but it is in a spherical linkage because, all the revolute axes are not parallel rather they are intersected one point. Then we showed that in this kind of connection, we can not transmit uniform constant angular velocity ratio. That means if the input are rotate are constant speed the output can rotator varies to compensate this. We can use 2 such joint, identical joints in series and make a self-compensating universal joint which can transmit uniform angular velocity ratio within 2 intersecting shafts.

However, that mechanism is not very compact. It becomes quite cumbersome and big it becomes a fixed link mechanism. However, this compact mechanism to do this satisfactorily, that is to connect 2 intersecting shafts without having any change of feed from one end to the other which has been patented by Bentinck Aviation Corporation.