Kinematics of Machines

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Module 8 Lecture 2

Today, we start our discussion with function generation by analytical method, with reference to 3R-1P that is a slider-crank mechanism.

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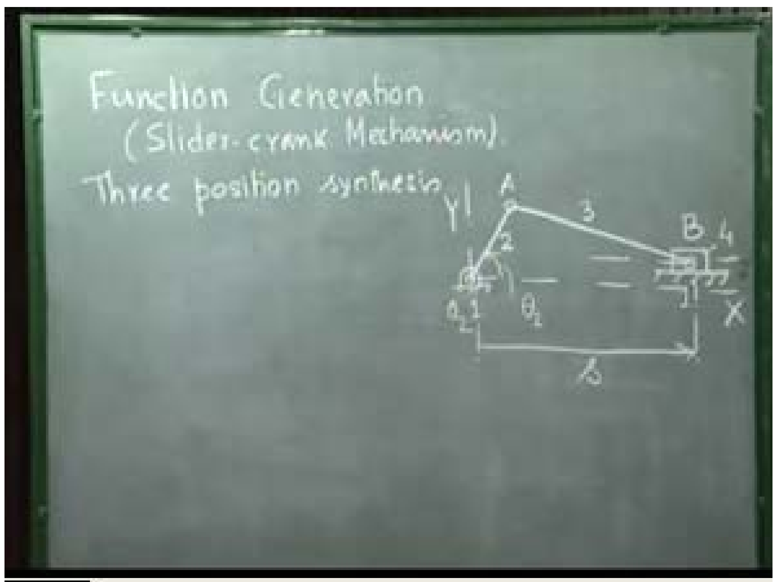

As in the case of a 4R-linkage, we shall restrict our discussion to only three point synthesis. That is, two pair of coordinated movement. Three position synthesis: The reason for this as usual is that, we can get away with linear equations. If you go for four positions or higher five position synthesis then, we have to handle non-linear algebraic equation, which is a little cumbersome and beyond the scope of this course. So, let me start with slider-crank, O_2AB is the slider-crank, where 1 is the fixed link; O_2A is link number 2, which is the crank, AB, which is the link number 3 is the connecting rod. At B I have link number 4, which is the slider. The direction of the sliding is horizontal and through O_2, I draw a line parallel to the direction of sliding of the slider and this we call our X-axis; perpendicular to that, so this origin O_2, we draw our Y-axis. The crank angle is measured from the X-axis and angle that, this crank O_2A makes with X-axis, we call it $theta_2$ and the position of the slider along the X-axis that is the X co-ordinate of the slider which we write s, as the slider position.

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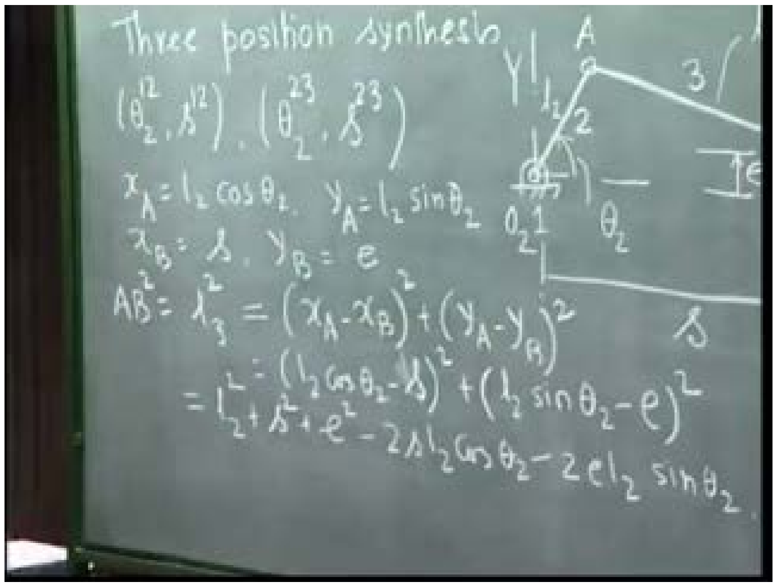

By three position synthesis, we mean that, we have to generate two pair of coordinated movements of the input link that is the crank and the output link that is the slider. 2 this crank is input link and link number 4 this slider is the output link. So, as the crank rotates from position 1 to 2 by an amount $theta_2 12$, the slider should be displaced by a given amount or prescribed amount s12. Similarly, as the crank goes from position 2 to position 3, the slider should move by s23. So, these two pairs are prescribed. We have to come out with the kinematics dimensions of this linkage namely the crank length that is l_2 that is O_2A, AB that is the connecting rod length l_3, the amount of offset that is the distance of the slider movement line from the X-axis along the vertical direction or Y- axis that is e. That is the amount of offset. So, it has three link length parameters in this linkage namely kinematic dimensions are l_2, l_3 and e. So, to generate these two pairs of prescribed movements, what must be these three dimensions that is what we mean by three position synthesis function generation by slider-crank mechanism. Towards this end, we first obtain the displacement equation as we did in case of 4R-linkage. So, we write: X coordinate of point A is l_2 cosine $theta_2$ and Y coordinate of the same point A is l_2 sin theta$_2$. The X coordinate of point B is s, the position of the slider and Y coordinate of the point B is e the amount of offset. So, you can write AB square, which is nothing but l_3 square, where l_3 is connecting rod length. I can right as X_A minus X_B whole square plus Y_A minus Y_B whole square. Substituting for this coordinate X_AY_A and X_BY_B in this equation, we get l_2 cosine theta$_2$ minus, X_B is s, whole square plus Y_A is l_2 sin theta$_2$ and Y_B is e. If we expand this, we get l_2 square plus s square plus e square minus twice s l_2 cosine theta$_2$ minus twice e l_2 sin theta$_2$.

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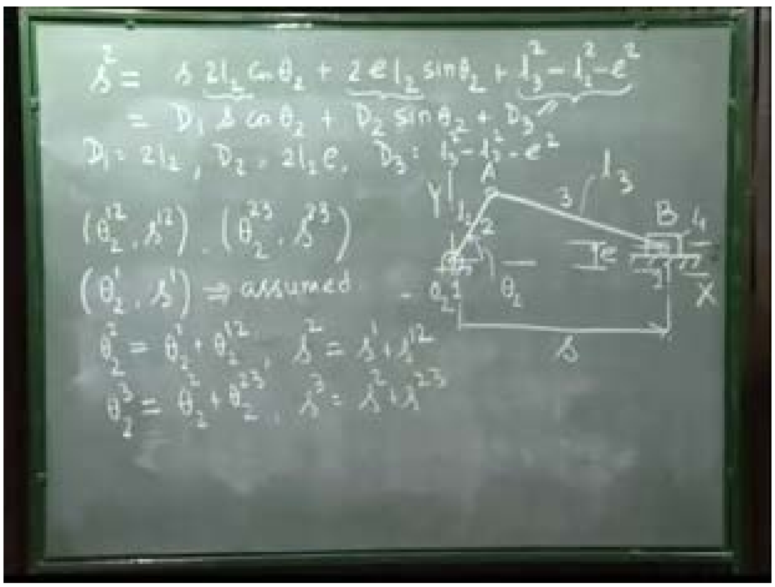

Let me now re-arrange the terms of these equations in the following manner: We write: s square is equal to s $2l_2$ cosine theta$_2$ plus 2e l_2 sine theta$_2$ plus l_3 square minus l_2 square minus e square. Let me now re-arrange the terms of this equation in the following manner: We write s square is equal to twice s l_2 cos theta$_2$ plus twice e l_2 sin theta$_2$ plus l_3 square minus l_2 square minus e square. So, here we write this as: D_1s cos theta$_2$ plus D_2 sin theta$_2$ plus D_3. This we replaced by D_3. D_2 is this and D_1 is $2l_2$, with D_1 is equal to $2l_2$, D_2 is equal to $2l_2$ into e and D_3 is equal to l_3 square minus l_2 square minus e square. For three position synthesis, we assume the values of theta$_2$ and s corresponding to the first accuracy position. So, these are assumed. From here I can find the values of theta$_2$ and s_2 corresponding to the second accuracy point as theta$_2$1 plus theta$_2$12, this we have assumed, this is given, so I can find theta$_2$2. Similarly, I can find value of s2 corresponding to the second configuration or second accuracy point as s1 plus s12. s1 we have assumed and s12 is prescribe so I can find s2. Exactly the same way, I can get the values of theta$_2$ and s corresponding to the third configuration theta$_2$3 as theta$_2$2 plus theta$_2$3 and s3 is equal to s2 plus s23. We already calculated s2 and we have given s23. So, I can find s3. Thus, the function generation with 3 accuracy points by the slider crank has boiled down to generation of three pairs of coordinated values of theta$_2$ and s namely theta$_2$1 and s1 which are assumed, theta$_2$2 s2 and theta$_2$3 s3. We can plug in three sets of values in this equation. So, we get three equations by plugging in the values of theta$_2$ and s2 in this displacement equation and we get s1 square is equal to D_1s1 cosine theta$_2$1 plus D_2 sin theta$_2$1 plus D_3.

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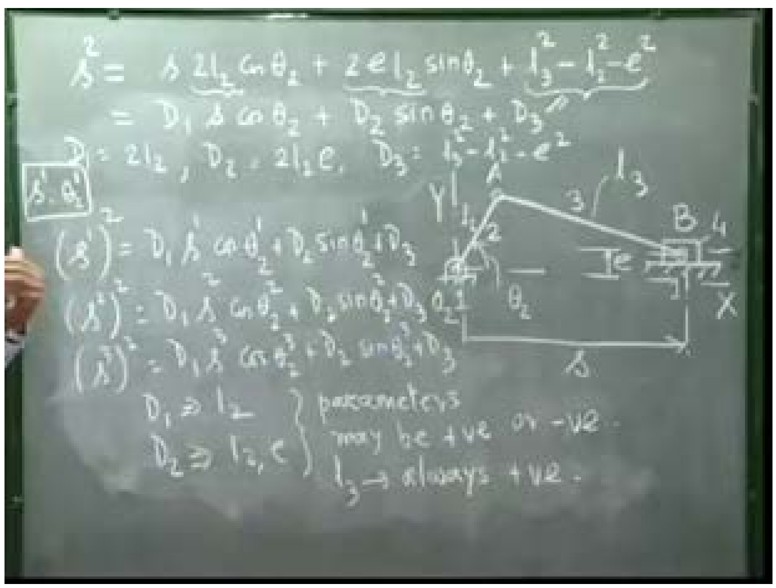

Same way, this equation will be valid for the second configuration s2 and theta$_2$ by writing D_1s2 cosine theta$_2$2 plus D_2 sin theta$_2$2 plus D_3 and for third position, s3 square is equal to D_1s3 cosine theta$_2$3 plus D_2 sin theta$_2$3 plus D_3. So, we get a set of three linear equations in three unknowns, namely $D_{1,}$ D_2 and $D_{3,}$ which can be easily solved because, we have already obtained theta$_2$1 is one, we plug it in, I get a linear equation in $D_1D_2D_3$. Same way, I get two more linear equations involving the same set of unknowns namely $D_{1,}$ D_2 and $D_{3.}$ So, we can solve for D_1 D_2 D_3 and once I get, D_1 I get the crank length l_2; once, I know D_2 and also the crank length l_2, I can find the offset e from D_2.

Once, I know both l_2 and e from $D_{3,}$ I can obtain l_3. As we have mentioned in-case of the 4R- linkage, we can see that both l_2 and e may turn out to be positive or negative. Then, if D_1 that is l_2, D_2 in combination of l_2 which means l_2 and e these parameters, these crank length and the offset may be positive or negative and they must be interpreted in the vector sense that means, if e is negative which means the offset will be below this horizontal line passing through O_2, the diagram shows e as positive and if e turns out to be negative, it will only imply that the line of sliding is below the x-axis. Similarly, if l_2 is negative then $theta_2$ must be replaced by $theta_2$ plus pi as we discussed in case of 4R-linkage. Whereas l_3, which I will get from this after you know l_2 and e will take the positive root of this l_3 square, we take the positive square root and l_3 is always positive. The thing to note is that, if you want to extend this to four positions synthesis that is to generate three pair of coordinated movements, we have to leave either s1 or $theta_2$1, which we assume one of these two parameters have to be left as an unknown to be determined and the resulting equation will no longer remain linear in the design variable. That adds to the difficulty of the solution to the set of non-linear algebraic equation.

Just as in case of four-bar, we can use this so called displacement equation to generate instantaneous kinematics relationship by taking derivatives. That means, if you have to coordinate to particular positions, the linear velocity of the slider and the angler velocity of the crank that can be done by differentiating this equation. Before finishing our discussion on kinematics synthesis by geometric and analytical method, I would like to point out two things.

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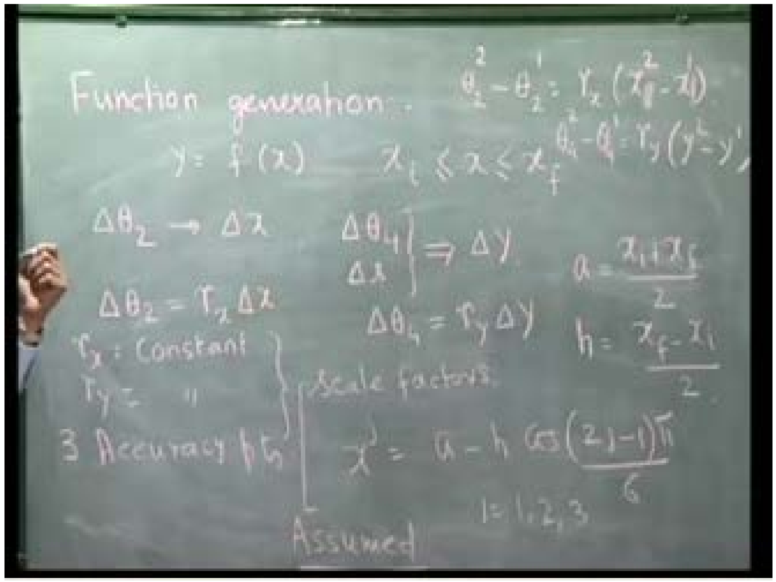

One is, what we have just now discussed, we call a function generation problem, which is nothing but position coordination. What you mean that, if y to a given function of x, where x is the independent variable and y is the dependent variable. If this function has to be generated in a range, say from x initial to x final. x_I is x initial and x_f if x final within this domain of the variable x, I have to generate the function y by a mechanism. What it basically means is that, the input movement, for example: The movement of the crank delta $theta_2$ represents the change in the independent variable x that is delta x. Similarly, the change in the output variable, for example: delta $theta_4$ in case of 4R- link or delta s in case of slider crank, these are change in the output variable.

This represents the change in the dependent variable y. That is they represent delta y and this may represent delta x. It is usually we take that they represent vector in a proposal manner that is we write: r_x into delta x, where r_x is the constant. Similarly, here we represent the output movement proportional to the change in the dependent variable and the proportionality constant is r_y, which is again a constant. So, r_x and $r_{y,}$ these are constants and these 2 constants are called scale factors. In this precession point approach, which we are discussing in the entire course, what you mean, that I know, I cannot generate this function in the entire range. So, we take some accuracy points and the first choice I said, we should take the Kirsch's accuracy point where suppose for three accuracy points, I should take $x_{1,}$ x_2, and x_3 in this range. I write: x j which will be a minus h cosine 2 j minus 1 into pi by six and j goes 1, 2, and 3. Three accuracy points in this range is given by this formula which we then call…. we are taking see heavy check accuracy points namely x_1, x_2, x_3 and correspondingly find what is the value of y which will be $y_{1,}$ y_2 and y_3.

In this expression, if we remember a means the midpoint of this range and h is the half of the range. So, if we have to generate this function by a mechanism in this range, I calculate a and h, then I calculate x_1, x_2, x_3 by putting j equal to 1, 2, and 3. Once I know x_1, x_2, x_3 from the given desired relationship I can find y_1, y_2, and y_3 and then we use this to get the input and output variables. How do I get that, that means we assume $theta_2$1 and $theta_2$2 minus $theta_2$1 will be r_x into x_2 minus $x_{1.}$ So, the scale factors are again assumed. This r_x, r_y constant, this scale factor these are assumed. Once the scale factor as are assumed, $theta_2$1 is assumed. I already got x_1, x_2. I should write others x_1, x_2 the second accuracy point x_2 j equal to 2 and first accuracy point j equal to 1 $x_{1.}$ So, x_2 one unknown r_x is assume $theta_2$1 we have assumed, so I can $theta_2$2. Similarly, for $theta_4$ I write $theta_4$2 minus $theta_4$1 is r_y into y_2 minus y_1.

This is what you mean by function generation by a mechanism that, the change in the independent variable is the represented by the input movement. Change in the dependent variable is the represented by the output movement, mechanism only takes care of the movement not their exact values. So, there we assume $theta_2$1 and $theta_4$1 and this movement is controlled by the mechanism. r_x and r_y, we assume suitably such that, the range of the movement is within the desired range. Now, we discuss very important point, so far as the kinematics synthesis of linkages are concerned, which will tell us that will must always verify after the design has been attained, why the linkage is performing satisfactory because, whether we solve by geometric method or by analytical method, we can never ensure that a mechanism performs satisfactorily because of two reasons.

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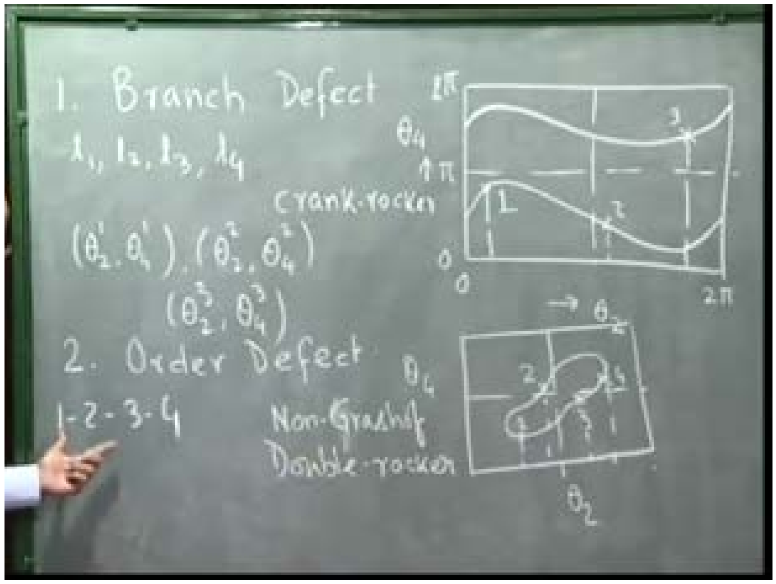

One is known as: branch defect. A mechanism synthesized by either geometric method or analytical method, may undergo what is known as branch defect. Let explain what is branch defect? If we remember that, if a 4R-linkage is Grashof type then there are two distinct branches. If this is the input angle $theta_2$ and this is the output angle say $theta_4$. Suppose, we are talking about a crank-rocker which obviously means it must be Grashof type. If it is a crank-rocker then, we know the input angle rotate completely that is from zero to 2 pi; whereas, the output link rocker only rocks between zero and pi or between pi and 2 pi and there are two distinct branches in the sense that one mode of assembly, may be this is the relationship between $theta_2$ and $theta_4$; whereas, other mode of assembly this will be relationship between $theta_4$ and $theta_2$. When we do three position synthesis like suppose that desired values are $theta_2$1 $theta_4$1 $theta_2$2 $theta_4$2 and $theta_2$3 and $theta_4$3.

Suppose, these three sets are like: one point here, this is $theta_2$1 this is $theta_4$1, this is the second point that means, this is $theta_2$2 and this is the $theta_4$2, and this is the third point this is $theta_2$3 and this is $theta_4$3. What we realize it is the same link length all this three points are satisfied, but all the three points are not satisfied by the same mode of assembly. So, our method by the geometric method and analytical method may give us the link length l_1, l_2, l_3, l_4, which is capable of attaining all these three positions, but not by the same mode of assembly. One mode of the assembly takes the position one and two, whereas the other mode of assembly takes the third configuration then, it will be total useless because, we cannot move from one assembly to the other without dismantling mechanism. This is called branch defect. There is another defect, which is also should checked for after the design is the complete, that is called order defect.

For example: If we remember that, suppose we have double rocket type mechanism and I draw the $theta_4$ verses $theta_2$ for a non-Grashof law; double rocker. If we remember that, non-Grashof linkage has single mode of assembly, so there cannot be any branch defect. Suppose, this is the relationship between $theta_4$ and $theta_2$ for a non-Grashof double rocker and this entire curve is obtained by the same mode of assembly. But, suppose to go for four positions synthesis and the points are like this; this is point 1, this is point 2, this is point 3 and this is point 4.

We want to attain these four combination of $theta_2$ and $theta_{4,}$ 1, 2, 3 and 4 but, as we see will get some link length which is satisfied all this four configuration but, if we drive the linkage, then there is an ordered defect, it does not go from positions 1 to 2 it goes from position 1 to 3, then 3 to 4, then comes to 2. If we drive the mechanism the other way then it will go from 1 to 2 then 2 to 4 then 4 to 3. So, this four positions 1, 2, 3, 4 are not taken up in the same order as be there. If, I rotate it this way I am getting 1 to 3 to 4 then back to 2 and if I drive the mechanism the other way, it is going from 1 to 2 to 4 then to 3. So, if we want these four positions to be taken up sequentially in this order then, this mechanism though theoretically satisfied the four configurations will not be of any use because, these four positions will not be taken up in the proper order. This is what we call ordered defect. An ordered defect can happen both in Grashof and non-Grashof type linkage whereas, branch defect can occur only in Grashof linkage because, Grashof linkage two branches are totally separate for the crank-rocker, double-rocker, or double-crank. In all see cases there are two distinct branches and you cannot move from the same mode of assembly from one branch to the other. I will show this with a typical example.

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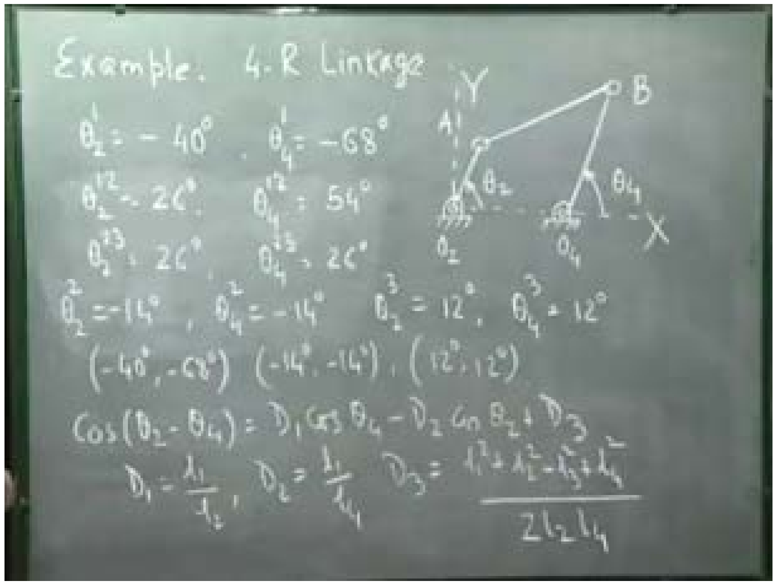

Let me now discuss an example to explain what we mean by branch defect? We talk of an example with respect to 4R-linkage. Let O_2ABO_4 is 4R linkage which we want to design such that, it satisfies three coordinate positions of theta$_2$ and theta$_4$ namely theta$_2$1 is equal to minus 40 degree. Let me now talk of synthesis of this 4R-linkage with the following specification: We assume theta$_2$1 is minus 40 degree and theta$_4$1 is minus 68 degree then, we assume theta$_2$12 is 26 degree and theta$_4$12 is 54 degree and theta$_2$23 is again 26 degree and theta$_4$23 is 26 degree. It is clear that, we want to solve this problem by analytical method using Freudenstein's equation because that is why, we are assume theta$_2$1 and theta$_4$1. For this two prescribed sets, I can find out value of theta$_2$ corresponding to the second configuration that is theta$_2$2 which is minus 40 plus 26 degree is gives me minus 14 degree and theta$_4$2 is minus 68 degree plus 54 degree that gives me again minus 14 degree. Similarly, theta$_2$3 is 26 degree minus 14 degree that gives me 12 degree and theta$_4$3 is 26 degree minus 14 degree that gives me 12 degree. So, the whole problem has boiled down to generation of these three pairs of coordinated value of theta$_2$ and theta$_4$, namely 40 degree minus 40 degree minus 68 degree then, minus 14 degree minus 14 degree and 12 degree and 12 degree. So, we write the Freudenstein's equation for this configuration of the 4R-linkage with the input link on the left. That is, cosine theta$_2$ minus theta$_4$ is D_1 cosine theta$_4$ minus D_2 cosine theta$_2$ plus D_3 where D_1, D_2 and D_3 at the c design parameter for D_1 is l_1 by l_2, D_2 is l_1 by l_4 and D_3

is l_1 square plus l_2 square minus l_3 square plus l_4 square divided by twice l_2l_4. Let me now plug in these three sets values of theta$_2$ and theta$_4$ in this equation.

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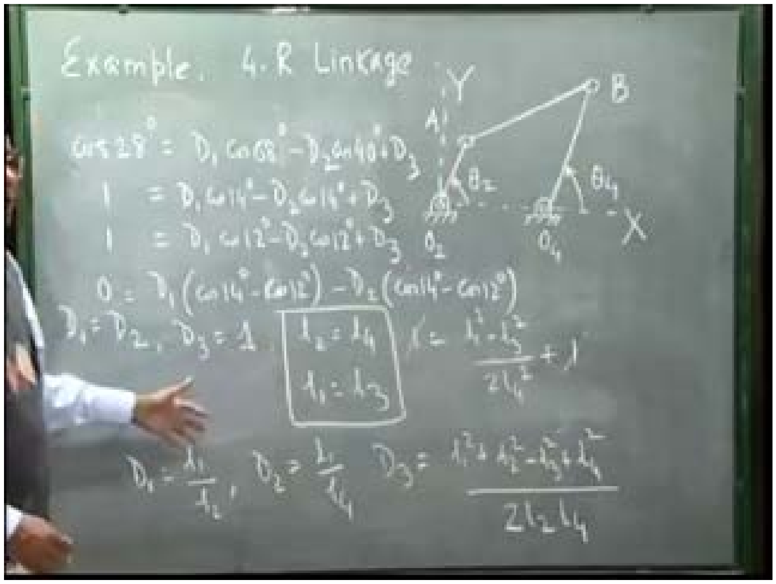

What we get? cosine minus 40 that is cosine 28 degree is D_1 cosine 68 degree minus D_2 cosine theta$_2$ which is 40 degree plus D_3. Now, if I use these set theta$_2$ and theta$_4$ are same. So, this is 1 and here we are getting D_1 cosine 14 degree minus D_2 cosine 14 degree plus D_3 and if I use the third set, again theta$_2$ and theta$_4$ as same so, cosine 0 is 1 and I am getting D_1 cosine 12 degree minus D_2 cosine 12 degree plus D_3. If I subtract these two equations, I get 0 is D_1 cosine 14 degree minus cosine 12 degree minus D_2 cosine 14 degree minus cosine 12 degree, which clearly tells me, D_1 is same as $D_{2.}$ Now, D_1 is same as D_2 then, from here I am getting D_3 any of these equations I am getting D_3 is equal to 1. What is the value of D_1? I can get by substituting D_3 equal to 1 and D_1 is equal to D_2 in this equation. The thing to notice, if D_1 is equal to D_2 that clearly tells me l_2 is same as $l_{4.}$ That means, the crank length and the follower length are equal and D_3 is 1 and l_2 equal to l_4, let me put it here. If, I put l_2 equal to l_4 and D_3 is 1 that is 1 equal to l_1 square minus l_3 square divided by $2l_4$ square plus l_2 square plus l_4 squared is $2l_4$ square and this is $2l_4$ square so, that is 1, which clearly tells me 1 cancels 1, l_1 is equal to $l_{3.}$ So, the final mechanism has come out to be that the fixed length is equal to the coupler length and the crank length is equal to the fall wall length. That means two opposite link lengths are equal which means it can be either the parallelogram or in anti-parallelogram. It is obvious note that 14 degree for both $theta_2$ and $theta_4$, can be taken up by a parallelogram configuration. Similarly, this minus 14 degree, plus 12 degree will be also taken up by the parallelogram configuration. Whereas, minus 68 degree and minus 40 degree cannot be taken up by the parallelogram configuration and we will see by the model that it will be taken up by the anti-parallelogram configuration. Which means, we have to go from the parallelogram to the anti-parallelogram bunch to obtain all the three configurations? This will not demonstrate with respect to a model.

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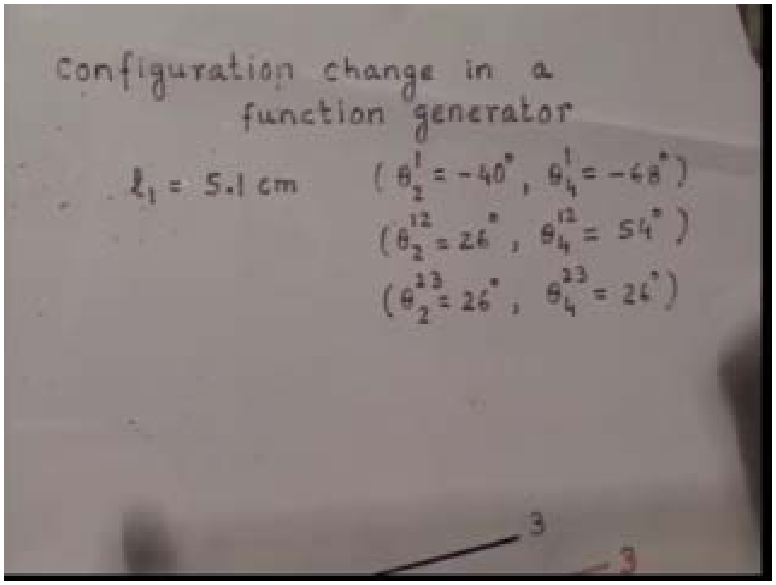

Let us now look at the model, which has been designed with the values that we have just obtained. We said $theta_2$1 is minus 40 degree, $theta_4$1 is minus 68 degree, $theta_2$12 is 26 degree, $theta_4$12 is 54 degree, $theta_2$23 is 26 degree and $theta_4$23 is again 26 degree. We have already seen l_2 comes out to be equal to l_4 and l_1 comes out to be equal to $l_{3.}$ We take l_1 equal to 5.1 centimeter in this model and we have seen that, D_1 and D_2 which we found to equal could have been solved by using the first equation and if we solve that equation it transfer to about point 3.

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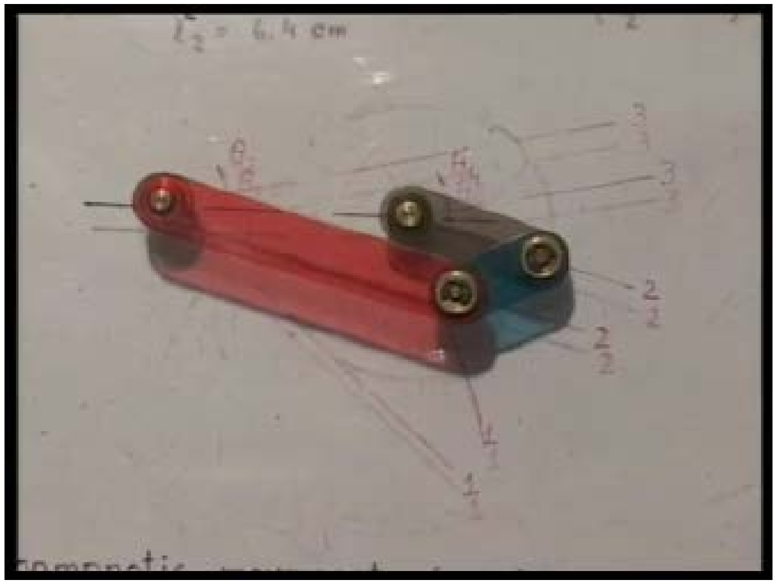

This is the model that we have got this l_1 is 5.1 centimeter and l_1 is equal to the coupler length l_3 and l_2 is also equal to the follower length l_4. This is the first configuration, where $\theta_2 1$ that is angle that this link mix with O_2O_4 that is minus 40 degree; whereas, the angle that this follower makes with the same line O_2O_4 is minus 68 degree. So this configuration is taken up this anti-parallelogram assembly; whereas, the second position, which is minus 14 degree for both, is these two configurations. This link should come here and this link should go here. These are the third configuration both at 12 degree. This is where this link should come and this where this link should come. If we draw this link, as we see, this second position is not been taken because, it is still in the anti-parallelogram assembly. If we not change it to the parallelogram assembly, then we see the second configuration is satisfied by the mechanism. Same way, if we go to third configuration, this only the parallelogram configuration that will take the third configuration but, if it gets into parallelogram configuration, then the first configuration is not satisfied, for obvious reasons because, θ_2 and θ_4 are not equal. In this case we have transition linkage because l_1 plus l_2 is l_3 plus l_4 that sum of these to link lengths is same as sum of these link lengths so, we can go from this uncertain configuration either to parallelogram configuration or to anti-parallelogram configuration. But as the mechanism this will be useless because this switch over cannot take place as soon as it becomes a non-transitory Grashof linkage and various configurations may be taken up by different modes of assembly that has to be checked after we get the design parameters either by graphical method or by analytical method.

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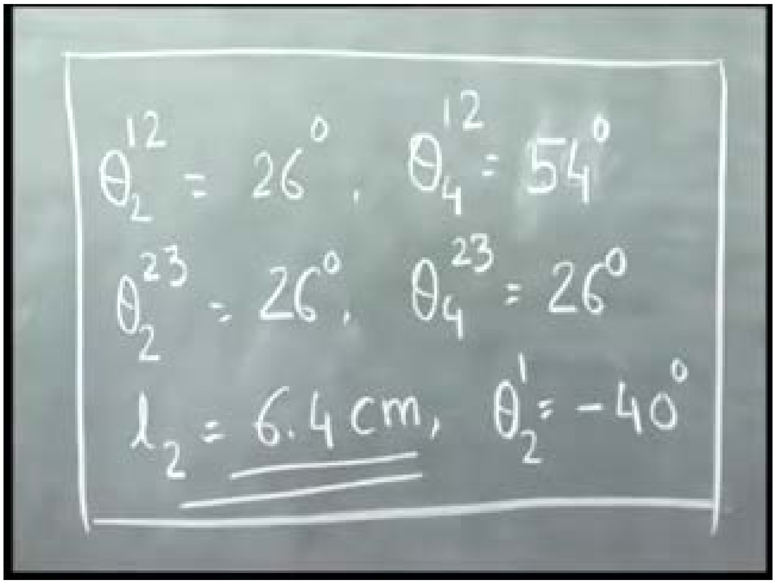

Let me now discuss with same problem, if we solve by graphical method, what kind of problem may arise? We talk of this theta$_2$12 26 degree as before, theta$_4$12 54 degree as we considered in the analytical method, same value to theta$_2$23 26 degree and theta$_4$23 is again 26 degree because, we are solving it by graphical method, we need to assume l_2 and theta$_2$1 or l_4 and theta$_4$1. We are solving it by assuming in this value of l_2 is 6.4 centimeter and theta$_2$1 is minus 40 degree as we assumed in the Freudenstein's method also. This is what we assumed in the graphical method and we do not assume the value of theta$_4$1. If we solve this problem by the graphical method, the solution that we get, we make a model out of it and I will show that model and see what kind of problem, we face in this graphical solution:

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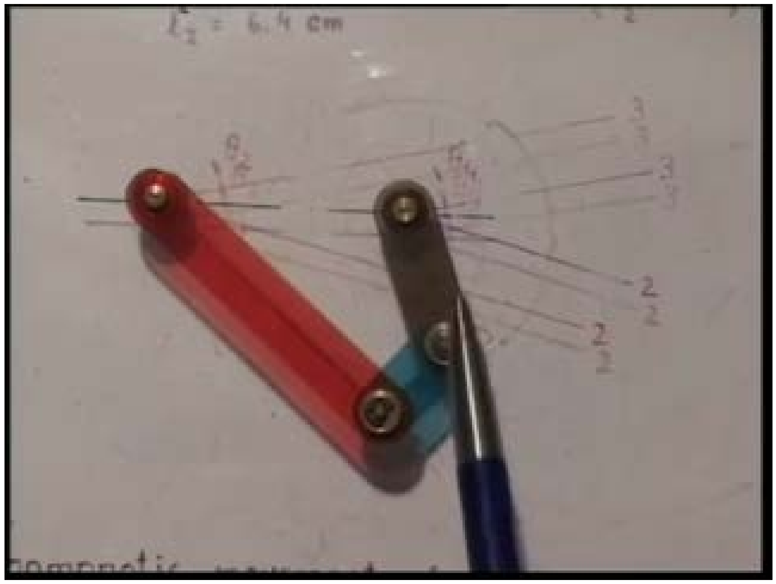

This is the model that we get for the solution to this problem by graphical method. This is O_2 and this is O_4. This is minus 48 degree and this is whatever O_4 makes. We have got l_2 as 6.4 centimeter and these are the other link lengths. This is the mechanism at the first configuration. If we move this linkage, these are the desired second configurations which are taken up by this length. These are the third desired configurations but as we see, if you drive this linkage when this link, this link is occupying the third configuration, the red link is not occupying the desired third configuration. But if you continue to drive while returning it is taken up the desired configuration. (Refer Slide Time: 43:20)This link is along this line and this link is along this line.

Now here, again we see that, when this link is along this line, the red link which is supposed to be along this line is not taking up the desired configuration. Same is to here for the first configuration. This link is along this line, but the red link is not along this line but if we drive then, during return it is taking up the desired configuration 1 and 2. What we see, there is non-monatomic movement, this positions is 1, 2 and 3 are not taking up the serial order, if we take up 1 and 2 then, 3 is taking up during the return stroke and if we take up the third configuration then, in the same stroke 2 and 1 are not been taken up. 1 and 2 are the taken up during the return stage. This is not a branch defect, but this is some kind of non-monatomic movement. That brings us to the end of the discussion so for a kinematics synthesis of planar linkages are concerned. We have discussed in great detail the graphical method which also gives you a feel of the movement of the mechanism. Then, we discussed in short, the analytical method only with respect to function generation problem. At the end, I have drawn your attention to the fact that, either we solved your problem by graphical method or analytical method the designed mechanism should always be checked for satisfactory performance, because there is always a chance of having either a branch defect or an ordered defect in the designed mechanism, which we have explained just now.