Kinematics of Machines

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Module 8, Lecture 1

Kinematic Synthesis

So far, we have discussed graphical method for kinematic synthesis.

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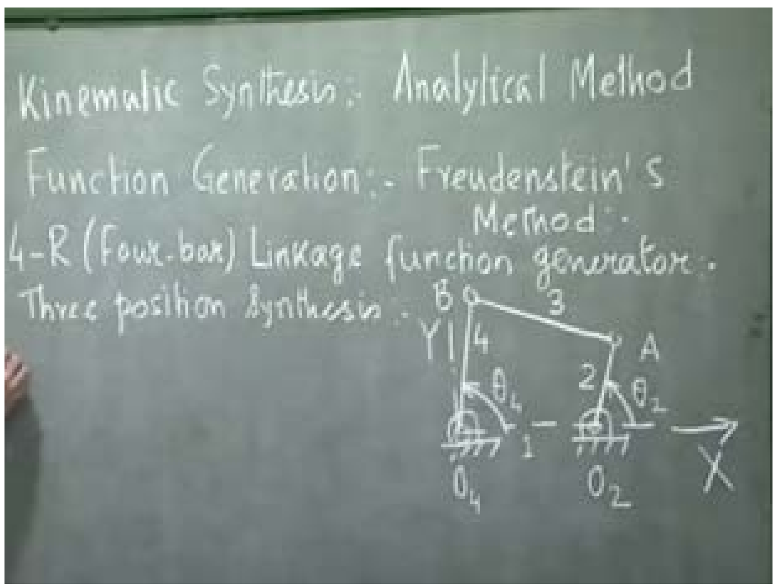

Today, we shall start discussing on kinematic synthesis of planar linkages, but by analytical methods. If we recall, we discussed four different types of problems when we had, the graphical methods namely: motion generation, function generation, fast generation and generation of prescribed dead-centre configurations.

However, when we discuss analytical method, we shall restrict ourselves only to one type of problem namely function generation. There are various methods like analytical method for function generation. But we shall discuss only one such method which is known as Freudenstein's method.

As an example, we take a 4R-linkage function generator and show how to get the link lengths for generating a prescribed function. If you remember, when we discussed graphical methods, we had function generator for three position synthesis and four position synthesis. However, we shall restrict ourselves only to three positions synthesis so far as 4R-linkage function generator is concerned. As we shall see later, for three position synthesis we can get away by considering only linear algebraic equation which is very easy to solve.

Towards this end, let me first draw a 4R-linkage. These are the two fixed changes O_4O_2 which defines the fixed link and we take our X-axis joining O_4 and O_2. O_2, A, B, O_4 is the 4R-linkage where O_4O_2 is link number 1 that is, fixed link; O_2A-the input link is link number 2; AB is the coupler, which is link number 3; O_4B- the output link is link number 4. The orientation of the input and the output link is defined by the angle that they make with X-axis namely, theta$_2$ and theta$_4$. We complete the coordinate axis by drawing the Y-axis with O_4 as the origin.

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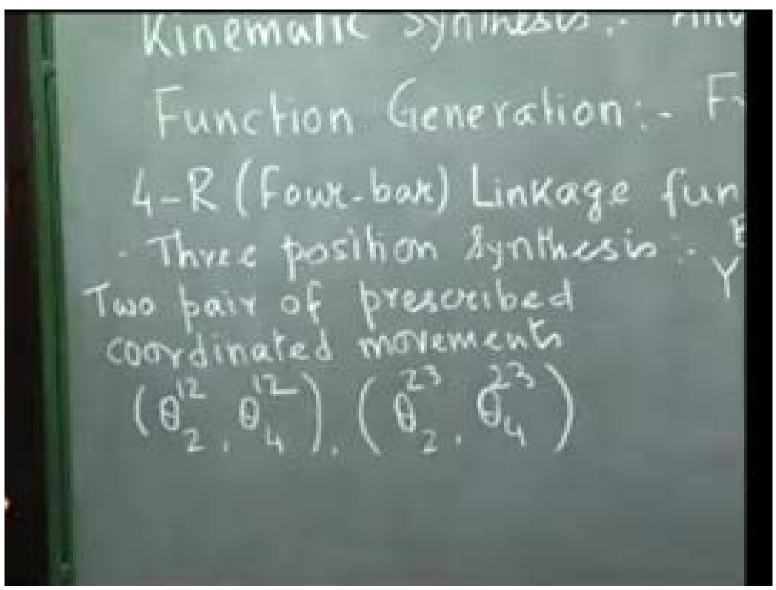

For three position synthesis, what we mean is to generate two pair of prescribed coordinated movements of the input and output link, that is, as the input link goes from position one to two by changing theta$_2$ by an amount theta$_2$ 1 2. The output link should move from position 1 to 2 by a prescribed movement theta$_4$ 1 2. Similarly, another pair of prescribed movement is theta$_2$ 2 3 and theta$_4$ 2 3. So, from position 1 to 2, change in angle theta$_2$ is given by theta$_2$ 1 2 and the corresponding change in the angle theta$_4$ should be theta$_4$ 1 2. This is a pair of prescribed movement.

This is another pair of prescribed movement. As theta$_2$ changes from second to third configuration by an amount theta$_2$ 2 3, the corresponding change in theta$_4$ should be theta$_4$ 2 3.

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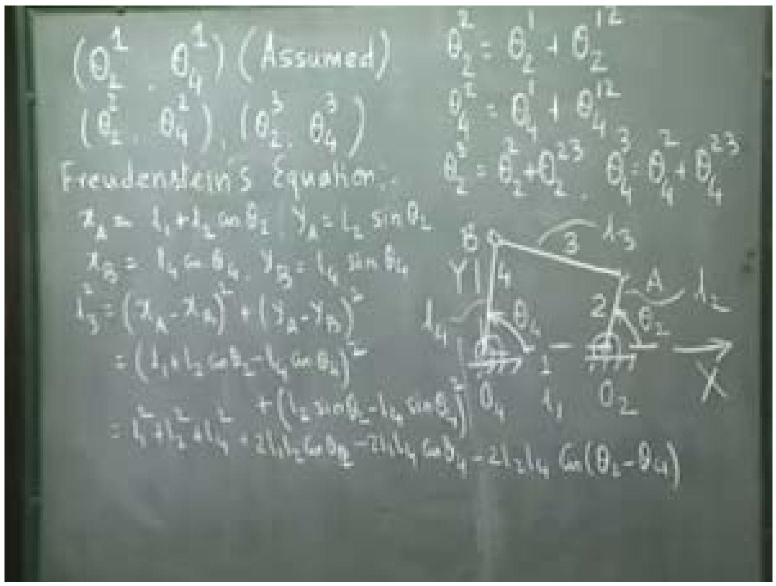

In Freudenstein's method for the three positions synthesis, we assume the values of theta$_2$ corresponding to the first configuration and value of theta$_4$ corresponding to the first configuration. I would like to emphasize that we should note the difference in the assumed parameters as far as the analytical method and the previous graphical method is concerned.

In the graphical method, what we assume is the location of the point A that is in the corresponding to the first configuration, that is, the link length l_2 and theta$_2$ 1 or we could have assumed the point B corresponding to the first configuration, which means assume l_4 the fall wall length and theta$_4$ 1. Whereas here we are leaving the link lengths as unknown parameters and we are assuming the value of the theta$_2$ and theta$_4$

corresponding to the first configuration. This gives me θ_2 2 which is θ_2 1 plus θ_2 1 2 and θ_4 2 is θ_4 1 plus θ_4 1 2. These are already prescribed and we have assumed. So, we get these two values namely, θ_2 2 and θ_4 two. Exactly the same way, I can get θ_2 three, which is θ_2 2 plus θ_2 2 3 and θ_4 three is θ_4 2 plus θ_4 2 3.

Once, we know θ_2 2 and θ_4 2 and we have given θ_2 2 3 and θ_4 2 3, I can get the values of θ_2 and θ_4 corresponding to the third configuration namely, θ_2 3 and θ_4 3. So, the three position function generation has while down to determine the link length such that these three pairs of values of θ_2 and θ_4 can take place in this linkage. As a first step towards Freudenstein's methodwe obtain the displacement equation in a particular form which is known as Freudenstein's equation.

First, we derive Freudenstein's equation, which is nothing but the displacement equation for this 4R-linkage in a particular format. Towards this end, we note that the x-coordinate of the point A, x_A is given by l_1 plus l_2 cosine θ_2, where l_1 is the length of the fixed link O_4O_2. This is the l_1, O_2A is l_2, AB is the coupler length l_3 and O_4B is the fall wall length l_4; y-coordinate of the point A, y_A is easily saying to be l_2 sine θ_2 and x and y coordinates of the point B namely, x_B is l_4 cosine θ_4 and y_B is l_4 sine θ_4. So, the distance between these two points A and B, I can write, which is l_3 as we see the square of the distance $l_3$square is equal to x_A minus x_B whole squared plus y_A minus y_B whole squared. Substituting for $x_{A,}$ x_B and $y_{A,}$ y_B here, we get l_1 plus l_2 cos θ_2 minus l_4 cosine θ_4 whole squared plus l_2 sine θ_2 minus l_4 sine θ_4 whole squared. Expanding this, we can easily get l_1 squared plus l_2 squared plus l_4 squared plus twice l_1l_2 cosine θ_2 minus twice l_1l_4 cosine θ_4 minus twice l_2l_4 cosine θ_2 minus $\theta_{4.}$

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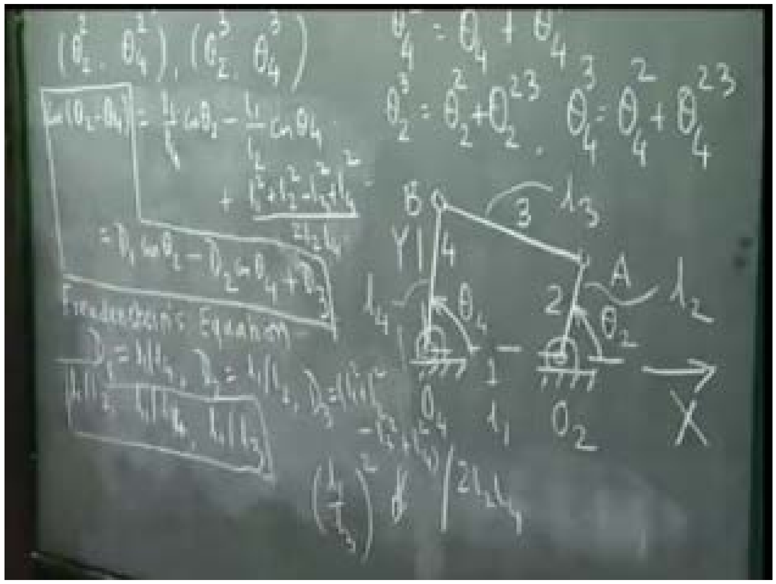

We rewrite this equation: Cosine theta$_2$ minus theta$_4$ is divided by twice $l_2 l_4$, l_2 and l_2 cancels, We get l_1 by l_4 cosine theta$_2$ minus l_1 by l_2 cosine theta$_4$ plus l_1 squared plus l_2 squared minus l_3 squared plus l_4 squared divided by twice $l_2 l_4$. We write l_1 by l_4 as a design parameter, D_1 cosine theta$_2$; $l_1$by l_2 as another design parameter, D_2 cosine theta$_4$, and we write this quantity as another design parameter, which is D_3. This equation is called the Freudenstein’s equation, where three design parameters are D_1 is equal to l_1 by l_4, another design parameter D_2 is equal to $l_1$by l_2 and the third design parameter D_3 is l_1 squared plus l_2 squared minus l_3 squared plus l_4 squared whole divided by twice $l_2 l_4$. I can substitute.

This equation has to be satisfied by these three states of theta$_2$ and theta$_4$ values. I can substitute theta$_2$ 1 theta$_4$ one, then this equation will be satisfied; If I substitute theta$_2$ 2 and theta$_4$ 2 for theta$_2$ and theta$_4$, then also this equation will be satisfied; I can also substitute theta$_2$ 3 and theta$_4$ 3 in these equations. So this way, I get three equations which are linear in three design parameters D_1, D_2 and D_3. The values of theta$_2$ and theta$_4$ are known, the only unknown in these three equations are D_1, D_2 and D_3 and this equation is linear. So, I can easily solve for three unknowns, namely, D_1, D_2 and D_3 by substituting these three pairs of values of theta$_2$ and theta$_4$ in this equation to generate three equations. This is known as Freudenstein's method of function generation for three position synthesis by a 4R-linkage.

Things to notice: From these three design parameters once we have solved I can get the link length ratios, namely l_1 by l_4 from D_1, l_1 by l_2 from D_2 and substituting for l_2 and l_4 in this equation from D_3, I can get l_1 by l_3. So, the three link length ratios, namely l_1 by l_2, l_1 by l_4 and l_1 by l_3 These three link length ratios can be obtained by solving a set of three linear algebraic equations involving D_1, D_2 and D_3. At this recall that after solving this state of linear equations,D_1 turns out to be negative, so stage, it may be worthwhile to is D_2. D_2 may be positive or negative; D_1 may be positive or negative.

What does it imply?

The negative value of D_1 and D_2 imply that this l_2 and l_4 has to be interpreted in the vector sense. That means if l_2 transferred to be negative, this $theta_2$ is $theta_2$ plus 5 such that l_2 becomes negative.

Similarly, l_4 has to be interpreted in the vector sense that D_1 is negative then l_4 is negative which means $theta_4$ will be changed by $theta_4$ plus 5. l_1 is always taken as positive and l_3 also as positive by solving l_1 by l_3 whole squared from this equation and taking the positive root of this l_1 by l_3.

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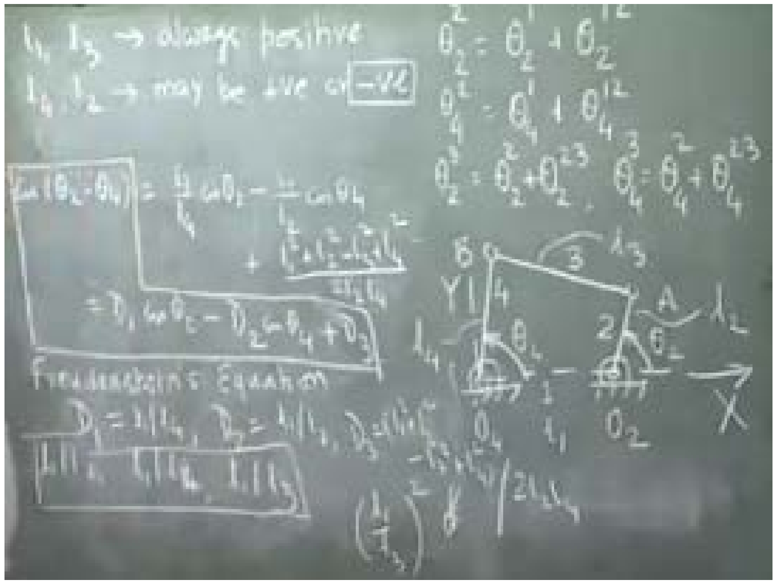

So to summarize, I can say that six-link length and the coupler length is always positive. If D_1 turns out to be negative, that means, l_4 can be negative and if D_2 turns out to be negative, then l_2 is negative. So, l_4 and l_2, that is, the input link length and the output link length may be positive or negative. The negative link length has to be interpreted in the vector sense, that is, we have to think of this O_2A in this direction, that is, theta$_2$ changes by theta$_2$ plus 5. Exactly the same is for the output length. If l_4 is negative, then O_4B will be in this direction, that is, theta$_4$ has to be replaced by theta$_4$ plus 5.

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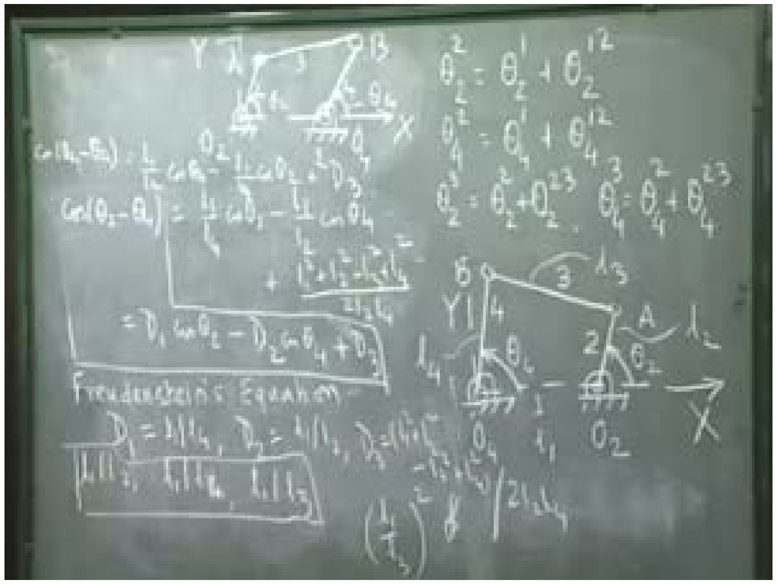

I would also like to say that sometimes, may be, the input link is on the left-hand side and the output length is on the right-hand side. That means, the 4R-linkage may be in this configuration: this is input link; this is coupler; this is the output link; this is the fixed link. It is easy to see that in such a situation, I take X-axis again along the fixed that is along O_2O_4 and Y-axis perpendicular to that. This is theta$_2$ and this angle is theta$_4$. So, we get Freudenstein's equation, corresponding to this 4R-linkage exactly from this equation, only by replacing theta$_2$ by theta$_4$ and l_4 by l_2 and theta$_4$ by theta$_2$ and l_2 by l_4.

Here, of course, l_2, l_4 interchanging is not changing this at all; only D_1 and D_2 will be changed; D_3 remains the same. So, this equation, cosine theta$_2$ minus theta$_4$ will be l_1 by l_2 cosine theta$_4$ minus l_1 by l_4 cosine theta$_2$ plus D_3. So we define, l_1 by l_2 is one design parameters and l_1 by l_4 as another design parameter like the situation we faced here. So, that is the Freudenstein's equation and this is the Freudenstein's method.

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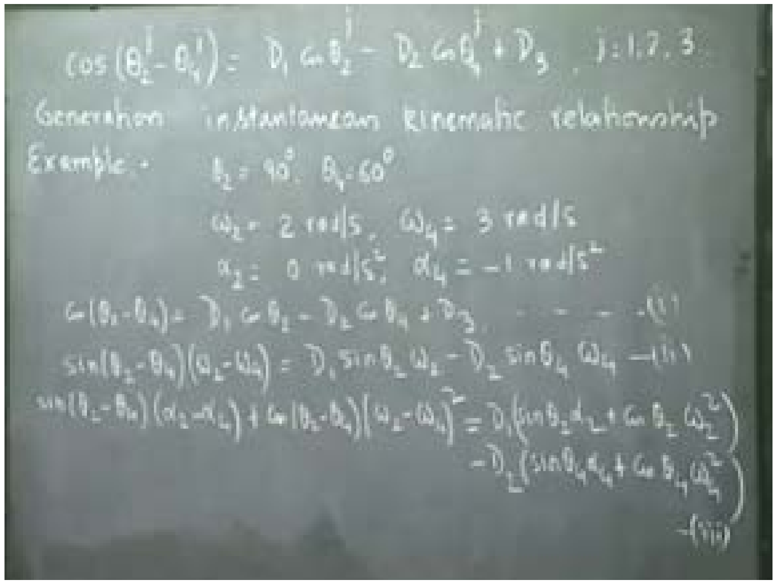

As explained just now, for three position synthesis, we write Freudenstein's equation in this form: cos θ_2 minus θ_4 is D_1 cos θ_2 minus D_2 cos θ_4 plus D_3; θ_2 and θ_4 takes three pair of coordinated values, namely, θ_2 1, θ_4 1, θ_2 1, θ_4 1, or θ_2 2, θ_4 2, θ_2 2, θ_4 2 and θ_2 3, θ_4 3, θ_2 3 and θ_4 3 that gives three sets of linear equations in D_1, D_2 and D_3 to solve for. Once we solve for D_1, D_2 and D_3, we can get the three link length ratios. It may point it out; it is the link length ratios that matter because, if the whole figure is killed up, we get the same relative movement between various links. That is the linkage remains same, it is only of the scale of the drawing.

So l_1, I can choose arbitrarily or conveniently and then we can get the other three link lengths l_2, l_3 and l_4. This equation can also be used for generation of instantaneous kinematic relationship, not just position, but velocity and acceleration as well. Instead of generating just position relationship between the input and output link, we can use this equation for generation of instantaneous other kind of kinematic relationship between the input and output links namely, the angular velocity of these two links or angular acceleration of these two links.

As an example, let us use this Freudenstein's equation to design a 4R-linkage, such that: theta$_2$ is 90 degree; theta$_4$ is 60 degree; at this configuration, omega$_2$ is 2 radians per second and at this configuration, omega$_4$ is 3 radians per second. At the same configuration, that is theta$_2$ equal to 90 degree and theta$_4$ equal to 60 degree angular acceleration of link 2 is 0, but angular acceleration of the output link is, say, minus1 radian per second square.

How to determine the link lengths of a 4R-linkage to satisfy these three conditions when theta$_2$ is 90 degree, theta$_4$ should be 60 degree and this same configuration omega$_2$ and omega$_4$ are prescribed, alpha$_2$ and alpha$_4$ are also prescribed?

We have to determine the link length. Towards this end, I start from the Freudenstein's equation for j is equal to 1, that is, cosine theta$_2$ minus theta$_4$ is D_1 cosine theta$_2$ minus D_2 cosine theta$_4$ plus D_3. Since, this equation is valid for all instances what we can do, we can differentiate this equation with respect to time and that gives me sine theta$_2$ minus theta$_4$ into omega$_2$ minus omega$_{4,}$ where omega$_2$ is theta$_2$ dot, theta$_2$ to D_2 is the velocity of the input link, the omega$_4$ is the angular velocity of the output link and that is equal to D_1 sine theta$_2$ omega$_2$ minus D_2 sine theta$_4$ omega$_4$.

Differentiating this equation once more with respect to time we get sine theta$_2$ minus theta$_4$ into alpha$_2$ minus alpha$_4$ plus cosine theta$_2$ minus theta$_4$ into omega$_2$ minus omega$_4$ whole squared is equal to, if I differentiate the right hand side with respect to time, we get D_1 sine theta$_2$ alpha$_2$ plus cosine theta$_2$ omega$_2$ squared minus D_2 sine theta$_4$ alpha$_4$ plus cosine theta$_4$ omega$_4$ squared. So I get three equations, namely, 1, 2 and 3. In these three equations, I substitute the desired conditions which are given here.

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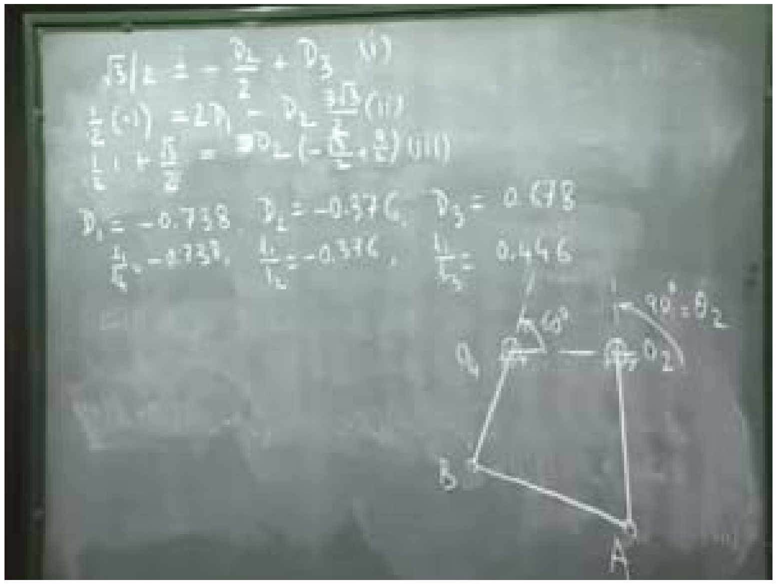

If I substitute in the first equation, $theta_2$ equal to 90 degree and $theta_4$ equal to 60 degree, we get 90 minus 60 that is cos 30 degree that is root 3 divided by 2 is equal to D_1 cos $theta_2$ is zero minus D_2 cos 60 degree is half minus D_2 by 2 plus D_3. This is what I get from the first equation, after substituting $theta_2$ and $theta_4$. If I substitute $omega_2$, $omega_4$, $theta_2$, $theta_4$ in the second equation, we get sine 30 degree that is 1/2 into $omega_2$ minus $omega_4$ that is minus1 should be equal to D_1, sine $theta_2$ is 1 and $omega_2$ is 2, so $2D_1$ minus D_2 (sine $theta_4$ is root 3 by 2 and $omega_4$ is 3) so, 3 root 3 divided by 2. This is equation one. This is equation two.

In the third equation, I substitute all these values to get sine 30 degree is half, $alpha_2$ minus $alpha_4$ is 1 plus cos 30 degree is root 3 by 2 into $omega_2$ minus $omega_4$ is minus 1 squared is 1 that is, 1 is equal to alpha2 is 0, so that term is zero; cos $theta_2$ is 0, so this term is also 0; $alpha_2$ is 0 and cos $theta_2$ two is 0,so this term is 0. So, we get minus D_2 sin $theta_4$ is root 3 divided by 2 and $alpha_4$ is minus 1 and that gives me minus root 3 divided by 2; cos $theta_4$ is 1 by 2 and $omega_4$ square is 9 so 9 by 2. This is equation three. From the third equation, I can solve for D_2. After I solve for D_2, from the second equation, I can solve for D_1 and if D_2 is known, I can solve for D_3 from the first equation.

If we do the algebra, we will get for this particular set, D_1 is minus 0.738, D_2 is minus 0.376 and D_3 is 0.678. As we said earlier, that $D_{1,}$ D_2 can come out to be negative and this particular problem D_1 and D_2 are turning out to be negative, which means l_1 by l_4 is minus 0.738, l_1 by l_2 is minus 0.376 and from D_3 if I substitute these values of l_4 and l_2 in terms of l_1, we can find out l_1 by $l_{3,}$ which turns out to be equal to 0.446. So, we get all three link lengths ratio and as we said, l_1l_3 are always positive. For this particular problem, l_2 and l_4 are turning out to be negative. So, if we choose the length l_1, we can solve for $l_{2,}$ l_4 and l_3.

If we draw this mechanism at this particular configuration we see that this is O_4 and this is O_2 theta$_4$ was 60 degree, but l_4 has turned out to be negative. So, I have to draw it at 240 degree. This is l_4 which is negative; l_2 is longer than $l_{4,}$ but l_2 is also negative and theta$_2$ is 90 degree that is this way; so, I draw it this way. All the prescribed instance relationship will be valued for this 4R-linkage where, this is $l_{2;}$ this is $l_{4;}$ this is $l_{3;}$ for a given value of l_1, which I can solve once I know these three values. This is what we mean by Freudenstein's method as applied to a 4R-linkage.

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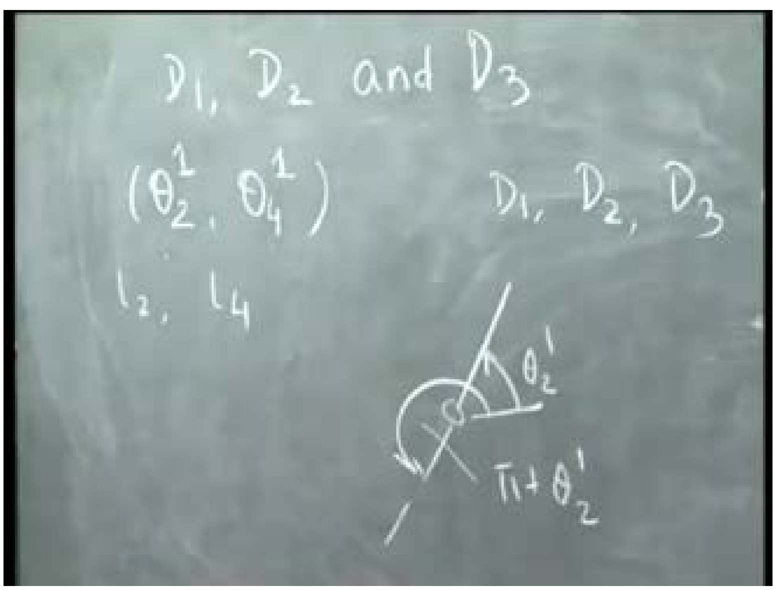

We discussed the function generation problem with reference to 4R-linkage function generator for three position synthesis but we emphasize that we got linear equations in the design variables namely, $D_{1,}$ D_2 and $D_{3,}$ only because we assumed $theta_2$ one and $theta_4$ one. If we have to extend this method to four position syntheses, then beyond $D_{1,}$ D_2 and $D_{3,}$ we have to leave one of these parameters also as an unknown to be decided or to be determined. That means $D_{1,}$ $D_{2,}$ D_3 and either $theta_2$ one or $theta_4$ one have to be left undetermined and then we can plug-in the four sets values of $theta_2$ and $theta_4$ to solve for these four unknowns $D_{1,}$ $D_{2,}$ D_3 and whichever is left unknown, but the trouble with that is that the equations will not be linear because of this angular term coming into the equation. We have cosine and things like that non-linear functions of these angles.

Consequently, the algebraic difficulty increases. Though it can be done, but we are keeping it outside the scope of this course. So, we have discussed Freudenstein's method, for function generation with reference to three position synthesis or two pair of coordinated moments. Things to remember is that, after solution input or the output length l_2 and l_4 may turn out to be negative. They have to be interpreted in the vector sense because we are assuming this $theta_2$ one and $theta_4$ 4 one arbitrarily. There cannot be any solution with those values if l_2 and l_4 turn out to be negative, but the solution will be available if we had pi to these terms. That means l_2 instead of O_2. This is what I call $theta_2$ one; l_2 negative means $theta_2$ one, I have to take this angle, which is pi plus $theta_2$ one.

In our next lecture, we shall discuss how the same methodology can be used for function generation by a slider-crank, that is, the input-output correlation and the slider displacement to rotation of the crank for a slider-crank mechanics.