Kinematics of Machines

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Module - 7 Lecture - 3

In our last lecture, we have explained the method of point position reduction for function generation and path generation by a 4R-linkage. Of course, the same method is equally applicable for synthesizing slider-crank mechanism. Today, we shall demonstrate the application of this point position reduction towards synthesizing an approximate dwell mechanism which is a six-link mechanism.

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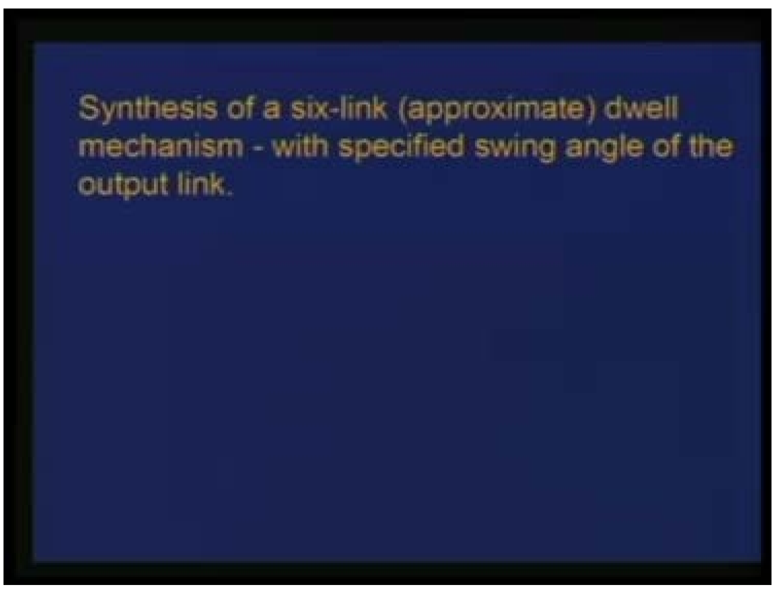

We would like to synthesize a six-link approximate dwell mechanism and with specified swing angle of the output link. That means, between the extreme position the output link should swing through a specified amount and during this continuous rotation of the input link the output link should have a dwell or should not move during some portion of the cycle. Let me explain the principle of this design with the help of the following figure.

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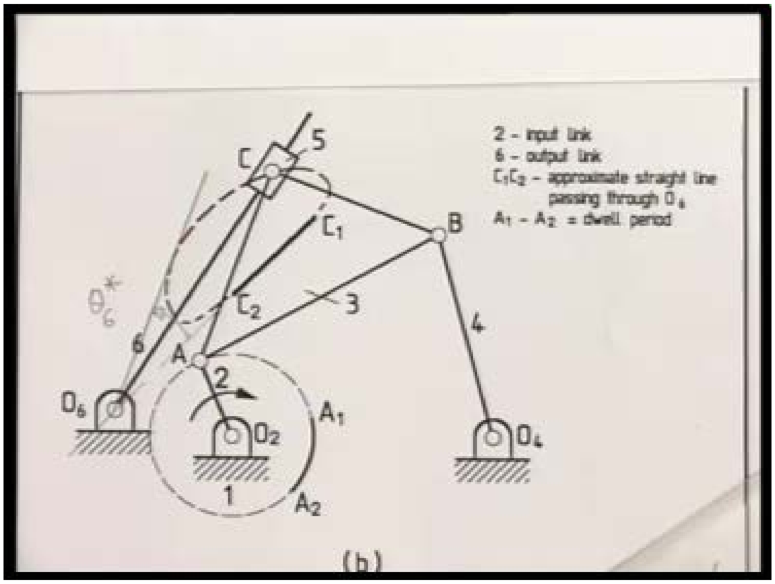

To explain the approximate dwell link mechanism, let us look at this figure: This is the kinematic stage of a six-link mechanism consisting of the six link, link number 1, link number 2, link number 3, link number 4. This 1, 2, 3, 4, that is O_2, A, B, O_4 constitutes a 4R-planar mechanism and on this coupler AB I take a point C, this coupler point. As this crank-rocker moves, the coupler points suppose to generate this particular coupler curve. As we notice, this portion of the coupler curve from C_1 to C_2 can be represented fairly by a straight line and this will be utilized to design a six-link dwell mechanism. What we do? We put another link 5 at this coupler point C through a revolute pair and the output link 6 has a prismatic pair between link 5 and this link 6. As we see, this straight line portion of the coupler curve passes through this fixed hinge O_6 where link 6 is hinged.

During the motion of this crank, as the crank pin A comes from A_1 to A_2 the coupler point C moves from C_1 to C_2, because there is a prismatic pair between link 5 and link 6 during this portion of the movement of the input link, the output link 6 does not move as the coupler point C goes along this straight line. Beyond that the output link starts moving in the counter-clockwise direction and the maximum swing angle I can obtain by drawing a tangent to this coupler curve from this O_6. So this angle, let me call, theta$_6$ star defines the swing angle of this output link. During continuous rotation of the crank of this six-link mechanism the output link 6 undergoes a dwell period when the crank pin A moves from A_1 to A_2, that is when coupler point moves from C_1 o C_2. It swings through this angle theta$_6$ star. Let me now explain this with the help of a model.

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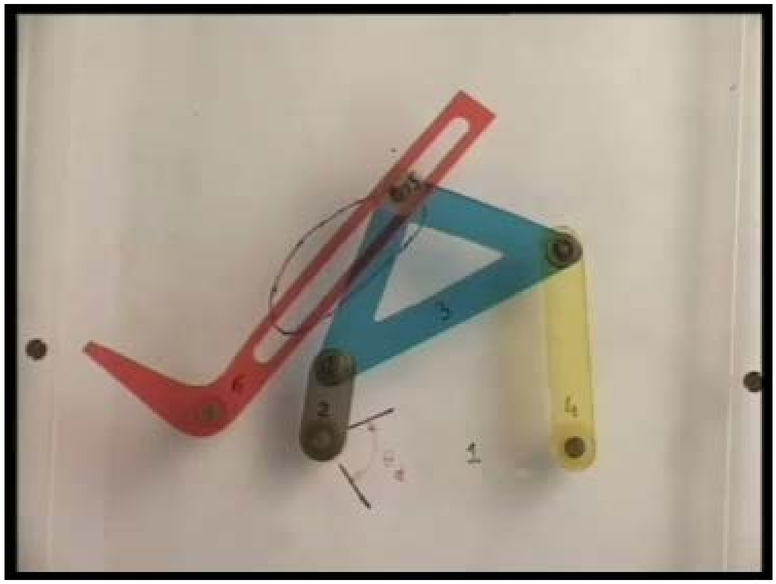

Let us look at this model of the six-link dwell mechanism. The link numbers 2, 3 and 4 constitutes a 4R-linkage and this is the coupler point C. There is a prismatic pair between this link 5 and the output link 6 which is hinged at this point which we earlier called O_6 this is O_2, A, B, O_4, C and O_6. As we give continuous input rotation to this crank-rocker linkage, what we should notice that during this interval of the rotation of the crank the output link hardly moves, because this coupler point goes along a fairly a good approximation for a straight line. As soon as this comes out of the straight line portion the output link starts moving and this is the maximum position rather the extreme position of the output link, when this line becomes tangent to this coupler curve.

Our job is to design these linkages using the method of point position reduction. First, we shall design a four-link mechanism with four points on this coupler curve namely C_1, C_2, C_3, C_4.

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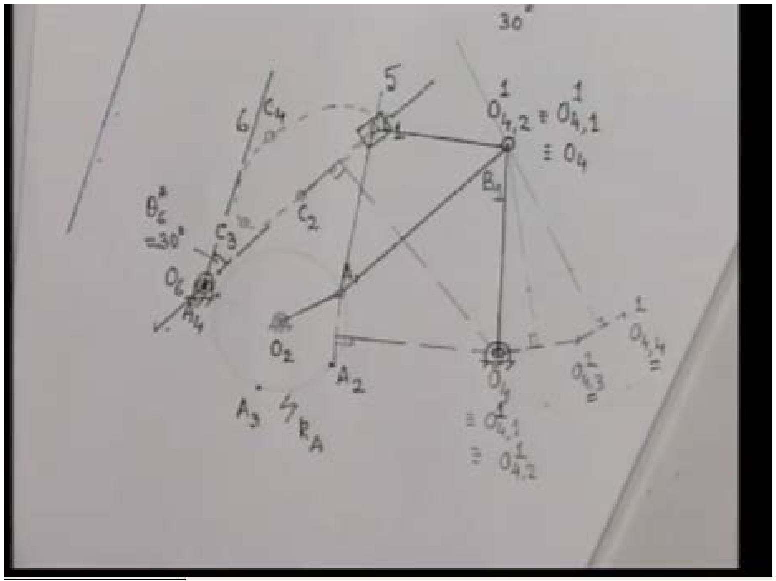

Suppose, we want to synthesize this coupler curve by allowing the coupler point to pass through these four positions namely, C_1, C_2, C_3 and C_4. We will design a four-link mechanism, rather 4R-mechanism to generate this coupler curve only ensuring that the coupler point C passes through these four positions C_1, C_2, C_3 and C_4. As explained in the last lecture, what we do, we choose O_2 and A_1, arbitrarily. This is A_1, this is O_2. Since the link length A_1 remains constant, when this link moves the path of A, I can get by drawing a circle with O_2 as center and O_2A_1 as the radius. This is the path of A, I call it k_A. Since, this link length AC_1 is rigid which does not change as the mechanism moves corresponding to C_2, C_3 and C_4, I can find the location of the point A and mark them as A_2, A_3 and A_4. This is the circle which I call k_A that denotes the path of the point A. To locate the location of A, corresponding to the second configuration when C comes to C_2 what I do? I measure C_1A_1 which does not change, so from C_2 I draw this circle and I get the location A_2.

Similarly, from C_3 using the same link length AC, I get A_3 and from C_4 I get A_4. As we know we cannot choose O_4, the other fixed hinge arbitrarily, we have to choose it such that four of the inverted positions of O_4 become three distinct locations. Towards this end we decide that $O_{4,\ 2}$ one that is the second configuration inverted on the first configuration becomes same as $O_{4,1}$ one which is same as O_4. We choose O_4 such that these two inverted position coincides.

Towards that end as we know what we have to do, we take the mid normal of A_1A_2, I draw the perpendicular bisector of A_1A_2, this is the perpendicular bisector and I draw the perpendicular bisector of C_1C_2, this is the perpendicular bisector of C_1C_2. These two straight lines meet at this point and I choose my O_4 at this location which is same as $O_{4,1}$ one. This will ensure that if second position is inverted on the first position holding the link number 3, that is the coupler fixed then $O_{4,2}$ one will also be here. Let me mark these two points namely A_2, C_2 and O_4. If I invert it on the first position that A_1 and C_1, A_2 goes to A_1 and C_2 goes to C_1, as we see the O_4 does not move and gives me the location of $O_{4,2}$ one.

Next, to get the inverted position corresponding to third and fourth configuration we go as usual A_3, C_3 and O_4. Then A_3 coincides with A_1, C_3 coincides with C_1 and wherever O_4 goes, I call that $O_{4,3}$ one. I can pierce my tracing paper and mark that point on the drawing sheet which is $O_{4,3}$ one. Then for the fourth position follow the same technique, we mark A_4, C_4 and O_4. Move the tracing paper such that A_4 coincides with A_1, C_4 coincides with C_1 and wherever O_4 goes I mark that as $O_{4,1}$ one. Since, the coupler was held fixed the point B on the coupler was not moving and O_4B is the fixed length, O_4 moves on a circle and the center of the circle passing through these three points will give me the location of B_1.

We have got the three inverted positions of O_4 as. O_4 which is same as $O_{4,1}$ one and $O_{4,2}$ one, so the three distinct locations for the four inverted positions of O_4 are here, then $O_{4,3}$ one is here and $O_{4,4}$ one is here. The center of the circle passing through these three distinct locations, I can find out by drawing the perpendicular bisector of this line and perpendicular bisector of this line. These two perpendicular bisectors intersect here which gives me the location of the other coupler hinge B_1. At this stage, we have reached up to the design of the four bars linkage namely, O_2A_1, O_4B_1, A_1B_1 and ABC is the coupler at the first configuration, that is C is the coupler point C_1. So we have reached the first phase of the design, namely: $O_2A_1B_1O_4$ and $A_1B_1C_1$ is the coupler.

At this point C_1, with a revolute pair we put the fifth link, link number 5 and as we noted that if this is the approximate straight line portion of the coupler curve then O_6 must be located on the extension of this line, O_6 will lie here. If we specify the swing angle of the output link 6 equal to 30 degree then O_6 must be so located in this line that the tangent drawn from O_6 to this coupler curve will be at 30 degree to this line. What we can do? We can draw a line which is at 30 degree, this line is at 30 degree to this line, the approximate straight portion of the coupler curve. Then, I draw a tangent to this coupler curve, parallel to this line and this line (Refer Slide Time: 15:53) is tangent to this coupler curve which is generated by the 4R-link namely O_2, AB, O_4 and this line is parallel to this line which was at an angle 30 degree to this line. So location of O_6 should be here. I put a revolute pair at O_6, then the swing angle of the follower link, that is, link number 6 has a prismatic pair with link number 5, this angle is 30 degree which we call as $theta_6$ star.

Let me go through this process all over again. We wanted to generate this particular coupler curve and we took four points on it C_1, C_2, C_3, and C_4. C_1, C_2, C_3 is to ensure that there is an approximate straight line portion. Then, we choose O_2 and A_1 arbitrarily, choose O_4 such that two of the inverted positions of O_4 namely, $O_{4,2}$ one and $O_{4,1}$ one coincide. To do that we drew the perpendicular bisector of A_1A_2 and C_1C_2 and wherever these two lines meet that locates the point O_4 which is same as $O_{4,1}$ one and $O_{4,2}$ one. Then using the tracing paper, we obtain the inverted positions $O_{4,3}$ one and $O_{4,4}$ one. The center of the circle passing through these three points was located at B_1. So, that located the B_1 and the coupler link length AB. We can also verify that this O_2, AB, O_4 is a crank- rocker such that this link A_2A_1, the shortest link, can rotate completely.

Then we knew that the other fixed hinge O_6 connected to the output link 6 must be on this line which represents approximately the straight line portion of this coupler curve. I have joined C_1 and C_2 and extended that line. The location of O_6 on this line should be such that the swing angle of the rocker is specified as 30 degree. I drew a line which is at 30 degree to this line and drew a line parallel to this line but tangent to this coupler curve that located the point O_6. I drew a tangent parallel to this coupler curve, but at an angle 30 degree to this line C_1C_2. That completes the design of this six-link mechanism.

We have just now seen how we can synthesize a six-link approximate dwell mechanism with specified swing angle of the output link.

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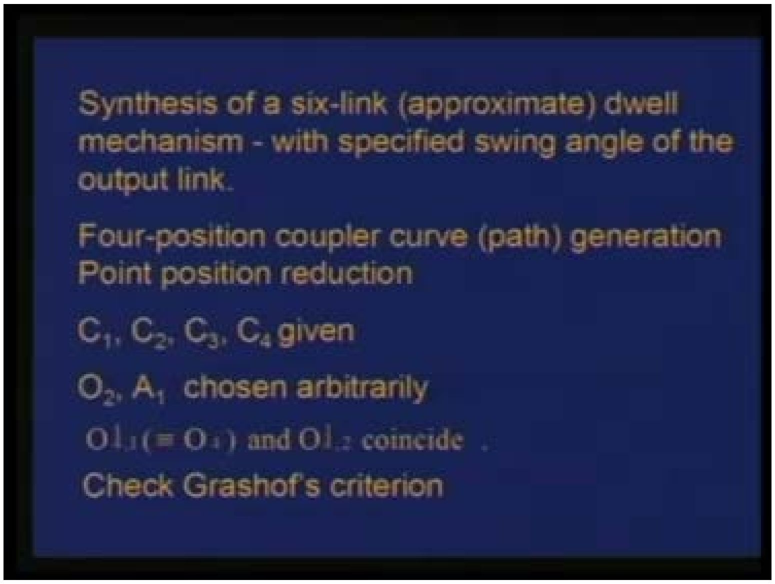

We use the four position coupler curve generation with the point position reduction technique. We took C_1, C_2, C_3, and C_4 suitably to give the desired coupler curve. Then we chose O_2 and A_1 arbitrarily and applied the point position reduction technique and O_4 is chosen that $O_{4,1}$ one and $O_{4,2}$ one coincide. Then we computed the design, first of the four-link mechanism. Then, we chose the fixed hinge O_6 suitably to give the specified swing angle of the output link.

We have to check Grashof's criterion for the design linkage O_2, AB, O_4 to ensure that we have really got a crank-rocker. Next, we discussed what we call dead-centre problems.

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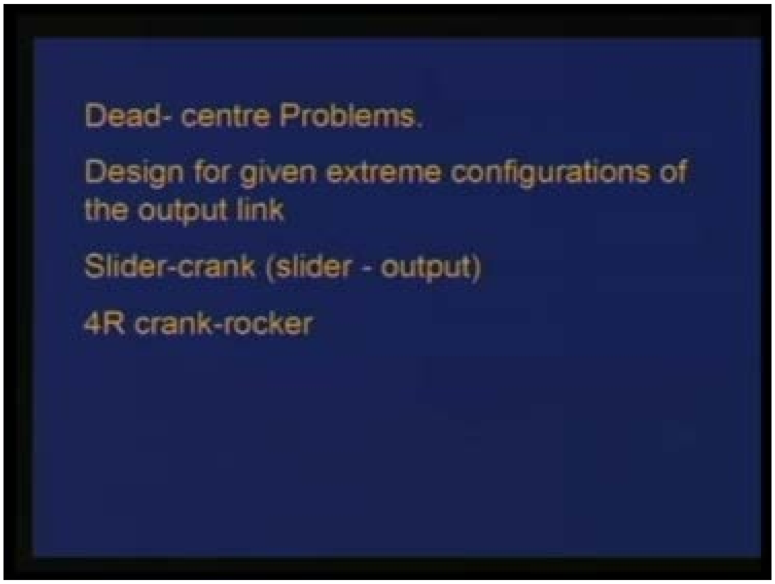

We have to design linkages for given extreme configuration of the output link. We can do it for the slider-crank mechanism with slider as the output link and also for crank-rocker mechanism where obviously rocker is the output link.

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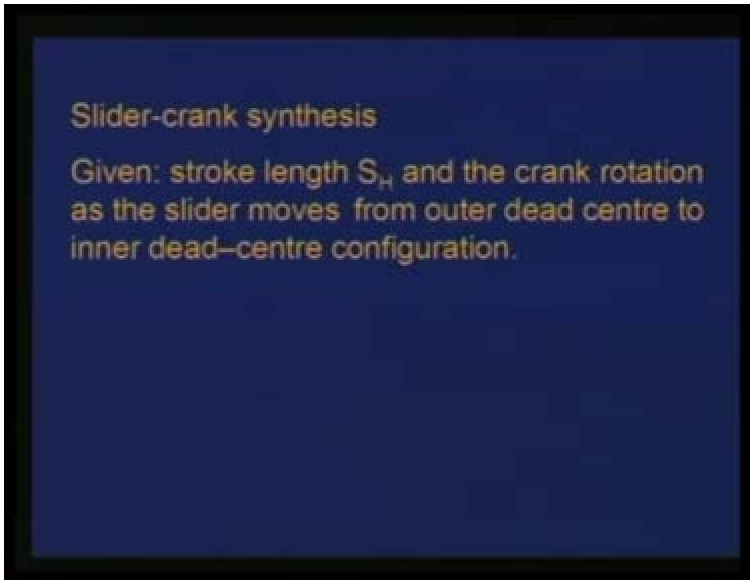

First let us discuss the slider-crank synthesis for specified dead-center configurations. For this, we are given the desired stroke length which we call S_H and the crank rotation as the slider moves from the outer dead-center to inner dead-center configuration. Let me explain this with the help of a figure:

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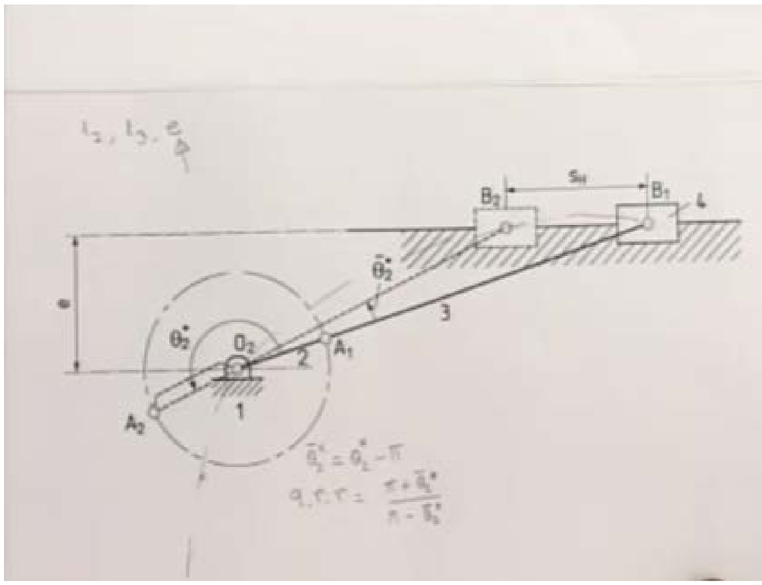

This figure shows a slider-crank mechanism 1, 2, 3, 4 in two of its extreme positions. Here, as we note the crank O_2A_1 and the connecting rod A_1B_1 have become collinear in the same direction giving one extreme position of the slider which we call the outer dead-center. Again, as the crank rotates this crank pin A_1 comes to A_2, the crank O_2A_2 and the connecting rod A_2B_2 have become collinear giving raise to the other extreme positions, which we call the inner dead-centre. At the outer dead-centre, the slider is farthest from the crank shaft and for the inner dead-centre the slider is nearest to the crank shaft. As the slider moves through B_1 to B_2 the stroke length is given by S_H. So, S_H is specified and the rotation of the crank as it moves from outer to inner dead-centre, that is the rotation from O_2A_1 to O_2A_2 given by this angle theta$_2$ star.

What is given to us is this stroke length S_H and theta$_2$ star and we have to come up with the required dimensions of this slider-crank namely, the crank length l_2, the connecting rod length l_3 and the offset-that is the distance between the crank shaft and line of movement of the slider as e. There are three kinematic dimensions, namely: O_2A_1, A_1B_1 and e, given to us is S_H and theta$_2$ star. As we see from B_2 to B_1, the return movement of the slider, during this return movement of the slider the crank rotates from O_2A_2 to O_2A_1 through an angle which is less than pi and through the movement from B_1 to B_2 the movement is through θ_2 star which is more than pi and this angle is the difference between θ_2 star and pi that is the angle between O_2B_1 and O_2B_2. So, θ_2 star bar is θ_2 star minus pi. We can also derive the quick return ratio for this slider-crank mechanism. Assuming the crank is rotating at uniform angular speed, then the forward motion is θ_2 star which is pi plus theta $_2$ star with a bar and this angle is pi minus θ_2 star bar. If the desired quick return ratio is given then also we can obtain θ_2 star bar and from there I can obtain θ_2 star. To solve this problem geometrically, what we should notice, that for a given quick return ratio this angle is fixed θ_2 star bar which we can solve from this equation and this is the stroke length S_H. As the result, the point O_2 must lie on a circle passing through B_1, B_2 and O_2 since the circumferential angle of a circle remains constant, O_2 can be anywhere on this circle. If I draw a circle through B_1, B_2 and O_2, this is the circle which passes through B_1, B_2 and O_2. If O_2 is here on this circle because it is the same chord length B_1, B_2 the circumferential angle at O_2 will remain same. Thus, we see O_2 can be chosen anywhere which will immediately change all the dimensions l_2, l_3 and e. There is no unique solution, in fact there are infinite solutions. So we have to choose one of the parameters out of l_2, l_3 and e. Out of these three kinematic dimensions, we have to choose one and determine the other two to get a unique design. If all these parameters are left unknown, then there can be infinite solutions. First, we show geometrically how you can obtain l_2 and l_3, if we assume a given value for this e.

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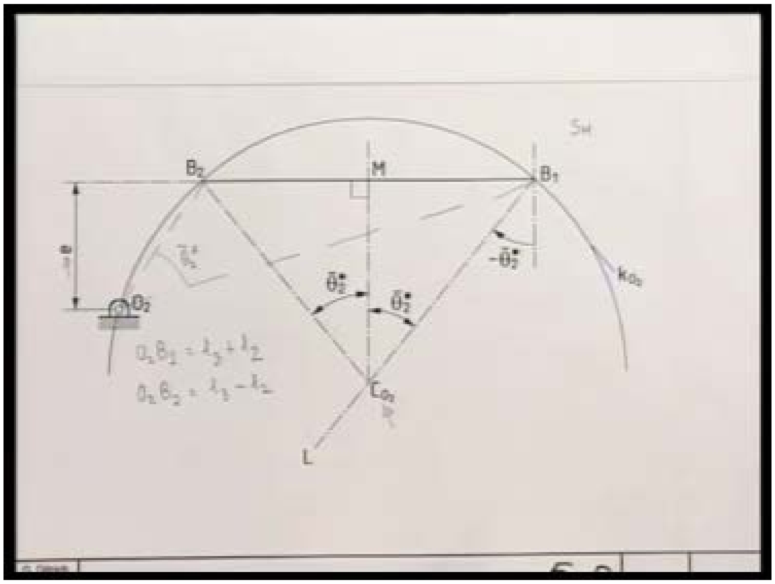

We have been given the stroke length B_1, B_2 as S_H. I draw a line B_1B_2 along the line of movement of the slider of length S_H. We have seen O_2, that is the crank shaft should be so located that this B_1B_2 subtends an angle theta$_2$ star bar at O_2. Theta$_2$ star bar can be obtained from the desired quick return ratio. Our objective is to draw a circle passing through B_1, B_2 and O_2 such that at O_2 we get a circumferential angle B_2O_2, this angle should be theta$_2$ star bar. We can do that very easily geometrically by drawing the mid normal of B_1B_2. This is the perpendicular bisector. So, the centre of the circle must be on this line and if this is the circumferential angle then the central angle must be twice of that, twice theta$_2$ star.

What I do, at B_1 I draw a vertical line draw a line at an angel theta$_2$ star bar in the clock-wise direction and where this line intersects the perpendicular bisector gives me the center of the circle as CO_2.

With this point as center and CO_2B_1 as the radius, I draw this circle which will obviously passes through B_2 and also subtend an angle. Any point on this circular arc is the same angle theta$_2$ star bar. We have assumed that the kinematic dimension the offset e so I draw this line at a distance e from B_1B_2 and wherever this line intersects the circle locates my crank shaft O_2. Once I have obtained the crank shaft O_2, I can easily find the connecting rod and the crank length because O_2B_1 at the outer dead-centre is nothing but l_3 plus l_2, where l_3 is the connecting rod length and l_2 is the crank length.

Similarly, O_2B_2 at the inner dead-centre configuration, O_2B_2 is equal to l_3 minus l_2. Out of the three kinematic dimensions to define the linkage, I assume e and determine l_3 and l_2 from the measurement of O_2B_1 and O_2B_2 and then solving these two equations. However, if l_2 or l_3 is given then it is much easier to solve the problem analytically rather than graphically. Of course, it can be done graphically but the graphical construction will be much more cumbersome. Let me do that analytically assuming that one of these two lengths l_2 or l_3 is given and find the other two dimensions namely e and l_3 or e on l_2.

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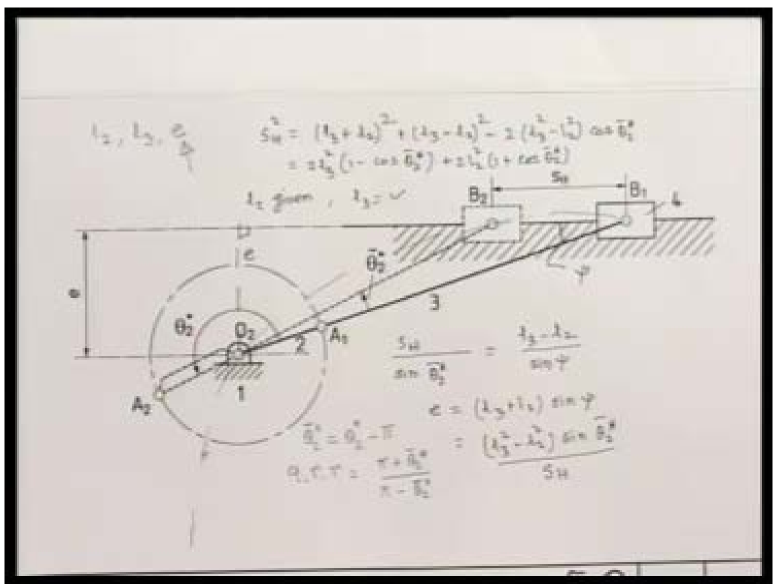

If one of the link lengths l_2 or l_3 prescribed then let us go back to this figure showing the outer dead-centre configuration and inner dead-centre configuration. S_H is given to us, theta$_2$ star bar is given to us and one of the link lengths say l_2 or l_3 is given to us. What we do? We see this triangle $O_2B_1B_2$. In this triangle, if I apply the cosine law, I can write S_H square is equal to l_3 plus $l_2$whole squared, that is, O_2 beyond squared plus O_2B_2 squared, that is l_3 minus l_2 whole squared minus twice of O_2B_1 into O_2B_2, that gives me l_3 square minus l_2 square into cosine of theta$_2$ star bar. Applying the simple cosine law for the triangle $O_2B_1B_2$, I get this equation. If I simplify, we get two l_3 squared into 1 minus cosine θ_2 star bar plus two l_2 square into one plus cosine θ_2 star bar. In this equation S_H is given to us θ_2 star is given to us from the quick return ratio and one of this either l_2 or l_3 is given. So, I can solve for the other. Suppose, l_2 is given then I can easily solve for l_3 and once we have solved for l_3 I can easily find this offset or e, for that I draw this perpendicular from O_2 to the line of B_1B_2 which is the offset e. Again, I apply the psi law in this triangle to determine this angle, let me call it psi. We get S_H divided by sine of θ_2 star bar, S_H divided by sine of the opposite angle is O_2B_2 by sine psi, O_2B_2 is nothing but l_3 minus l_2 divided by sine psi.

With l_2 given, I have already solved for l_3 so, I know this psi. This is given to us I can easily find sine psi and this e is nothing but O_2B_1 sine psi which is l_3 plus l_2 sine psi. I can easily substitute for sine psi from here and we get l_3 squared minus l_2 squared into sine θ_2 star bar divided by S_H. So given stroke length, given quick return ratio and given l_2 first we solve l_3 then I can find e or the other way round, if l_3 is given then I can find l_2 and then again I can find e. This completes the dead-centre configuration problem for designing or synthesizing a slider-crank mechanism of given stroke length and given quick return ratio.

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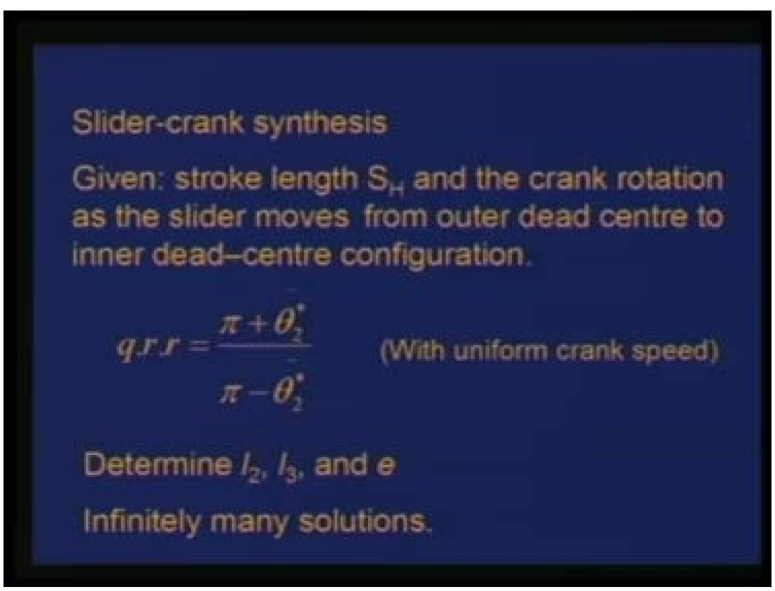

We have just now seen how we can do either geometrically or analytically synthesis of a slider-crank mechanism for given extreme configuration where the stroke length S_H is given and the quick return ratio is given assuming of course, that the crank is rotating at uniform speed. The problem was to determine the three kinematic dimensions crank length l_2 connecting rod length l_3 and the offset e. We have also seen that infinitely many solutions are possible if we leave all these three as unknown. We showed that we can either determine l_2 and l_3 assuming e or assuming l_2 we can determine l_3 and e or assuming l_3, we can determine l_2 and e.

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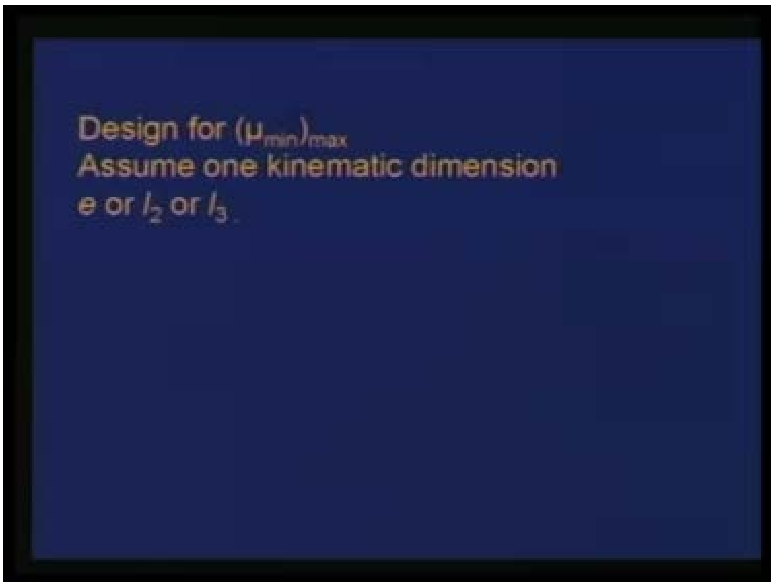

Because there are infinitely many solutions possible, the best design will be that which makes the minimum transmission angle maximum. This we can do only by trial and error. We assume one kinematic dimension e or l_2 or l_3 to get a unique design because infinitely many solutions are possible when all of these are variable, then I can go for the best design $(mu_{min})_{max}$ by trial and error.

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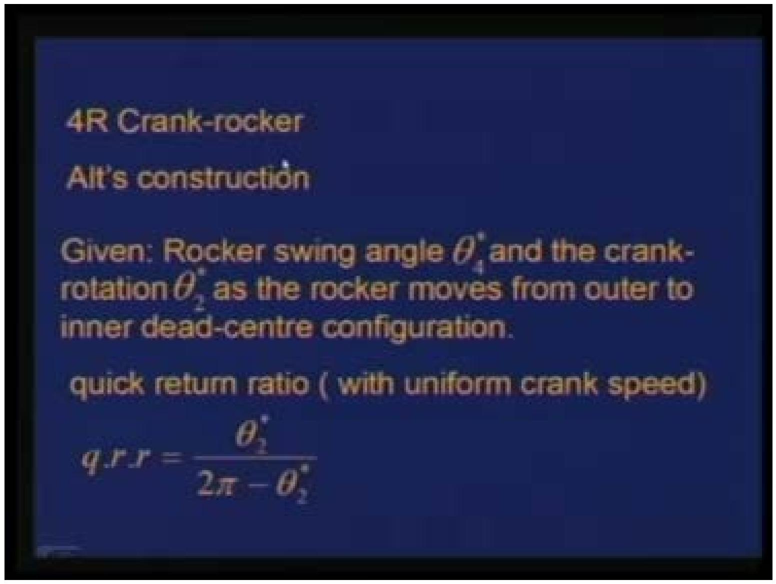

Next, we shall discuss the dead-centre problem for 4R-linkages, that is, the crank-rocker mechanisms for given swing angle $theta_4$ star and the crank-rotation $theta_2$ star as the rocker moves from outer to inner dead-centre configuration. We shall now discuss the synthesis of 4R crank-rocker mechanisms for given dead-center configurations. We shall follow the methods what is known as Hall's construction. Unfortunately, the theory behind this Hall's construction cannot be explained in this course but, we shall still discuss the method because it is the very useful technique for designing a useful mechanism that is 4R crank-rocker with rocker swing angle $theta_4$ star prescribed and the crank-rotation $theta_2$ star also prescribed as the rocker moves from outer to inner dead-center configuration. Here, as we see the quick return ratio will be defined as $theta_2$ star by 2 pi minus $theta_2$ star.

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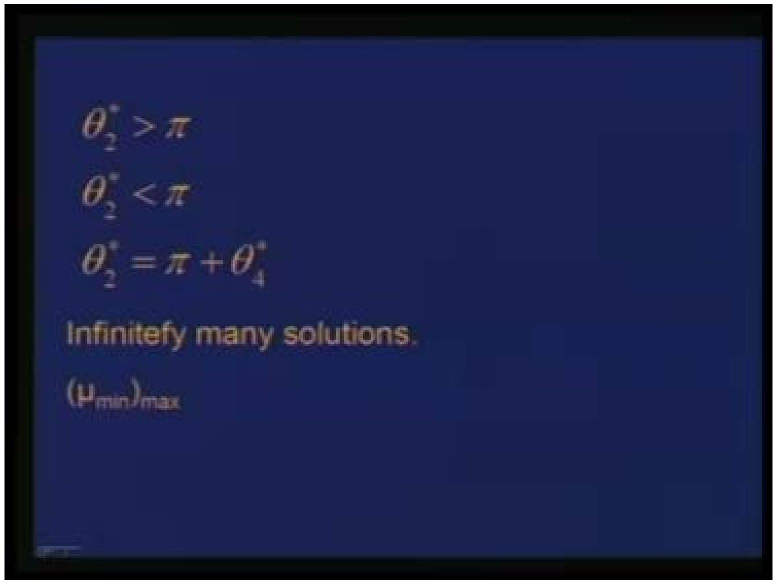

We shall discuss the cases separately for theta$_2$ star more than pi, theta$_2$ star less than pi and if theta$_2$ star is pi plus theta$_4$ star. As we shall see from the geometrical method there will be infinitely many solutions and we can choose the best design to maximize the minimum transmission angle that is to design for (mu$_{min}$)$_{max}$. We shall explain all these graphical methods with the help of figures.

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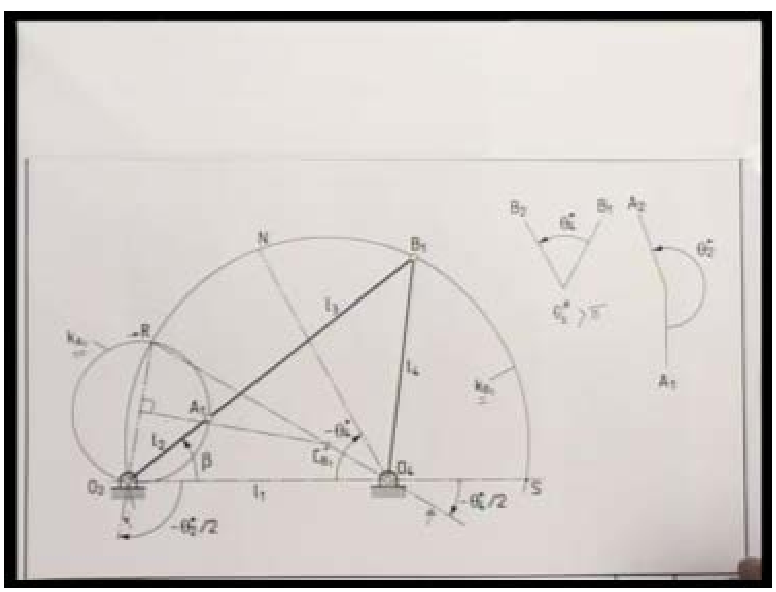

Let me now explain this Hall's construction for the design of 4R crank-rocker mechanism where the swing angle of the rocker is prescribed $theta_4$ star as the rocker moves from outer dead-center to inner dead-center. The corresponding rotation of the crank is through $theta_2$ star in the counter-clockwise direction. We are considering the case where this $theta_2$ star is more than pi. We choose O_2 and O_4, the location of the two fixed hinges, conveniently on the frame. Then, I draw a line at O_2 at an angle minus $theta_2$ star by two with O_2O_4, this is minus because this is clockwise whereas $theta_2$ star is counter-clockwise. Because this is more than pi, this angle is drawn at an angle which is more than 90 degree.

At O_4, I draw this line which is at an angle minus $theta_4$ star by two minus this clockwise because $theta_4$ star is counter-clockwise and these two lines drawn at O_2 and O_4 meet at this point R. Then, we draw a circle with O_2R as the diameter, this circle I call k_{A1}, the circle with O_2R as diameter. Next, we draw the perpendicular bisector of O_2R, that is, this line; this line is 90 degree to O_2R and passing through the midpoint of O_2. This line meets O_4R at the point CB_1. Then I draw a circle with CB_1 as center and CB_1R as radius. Obviously, this circle also passes through O_2 and this circle is k_{B1}.

Next, I draw a line at O_4 at angle minus $theta_4$ star, this is clockwise. So, I call it minus theta star this line is O_4 N intersecting k_{B1} at N and k_{B1} intersects the line O_2O_4 when extended at this point S. Then, B_1 can be chosen anywhere on this circular arc from N to S on k_{B1} but not on this side to the right of N and above S. I can choose B_1 anywhere on this circular arc and if I join O_2 with B_1 that line intersects the previous circle k_{A1} at the point A_1. Then, $O_2A_1B_1O_4$ gives me the desired 4R crank-rocker linkage at its outer configuration.

This is what we know as Hall's construction but as I said the proof of this construction is beyond the scope of this course. Let me now demonstrate what happens if $theta_2$ star is less than pi? That we will see in the next figure.

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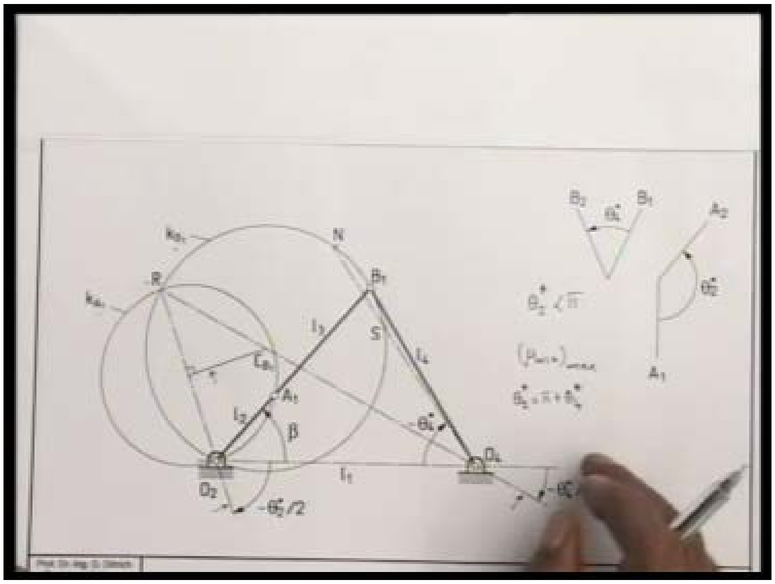

Let us look at this figure of Hall's construction, where $theta_2$ star is less than pi. Rest of the specification as before $theta_4$ star is the swing angle of the rocker from outer to inner dead-center in the counter-clock wise direction. Corresponding rotation of the crank is through $theta_2$ star in the counter-clockwise direction. Previously, we had $theta_2$ star more than pi and now we have $theta_2$ star less than pi. Just as before, we choose O_2 and O_4 arbitrarily. We draw the same two lines at O_2 at an angle minus $theta_2$ star by two with O_2O_4 and at O_4 at angle minus $theta_4$ star by two. This line with the same O_2O_4 extended. These two lines meet at R as before and I draw k_{A1} with O_2R as the diameter. Up to this point there is no change then, I draw the perpendicular bisector of O_2R and this line the perpendicular bisector meets O_4R at CB_1. With CB_1 as the center and CB_1R as the radius I draw this circle which I call k_{B1}. Again up to this point there is no change, where the $theta_2$ star is more than pi or less than pi. The construction of $k_{A1}$and k_{B1} remains as before, but now as we see, if I draw the line O_4N at an angle minus $theta_4$ star then, the line O_4N comes out like intersecting k_{B1} here again at S. Previously, this point of intersection was below O_2 O_4 and S, I considered the intersection of k_{B1} with O_2O_4. Here, N and S are both points of intersection of this line O_4N with k_{B1}. Then, the point B_1 can be taken any where on this k_{B1} on this circular arc NS. It cannot be here, it cannot be below S, it has to be taken within this range NS of the circular arc belonging to the circle k_{B1}. I have taken B_1 here and I get a design by joining O_2B_1 which intersects k_{A1}

at A_1 and O_2, A_1, B_1, O_4 gives me the desired 4R crank-rocker linkage satisfying the desired extreme positions of swing angle θ_4 star and the corresponding crank rotation θ_2 star.

As we see, because I could have taken B_1 anywhere I get infinite number of solutions and the same is to even for θ_2 star more than pi. I can parameterize this particular design by this angle beta. I can see beta can take any value if I join O_2S and O_2N. There are infinitely many solutions for various values of beta and that is true that θ_2 star is more than pi or less than pi does not matter, it always gives me infinite number of solutions. The best solution for beta can be read from a nomogram, which we are not showing here know as Folmar's nomogram and we know the optimum value of beta which will make the minimum transmission angle maximum. That means mu_{min} will be maximum for a particular value of beta and that value of beta can be read from a nomogram known as Volmar's nomogram. At this stage, I would like to point out what happens if θ_2 star is more than pi but θ_2 star is pi plus θ_4 star. To explain this, let me go back to the previous drawing.

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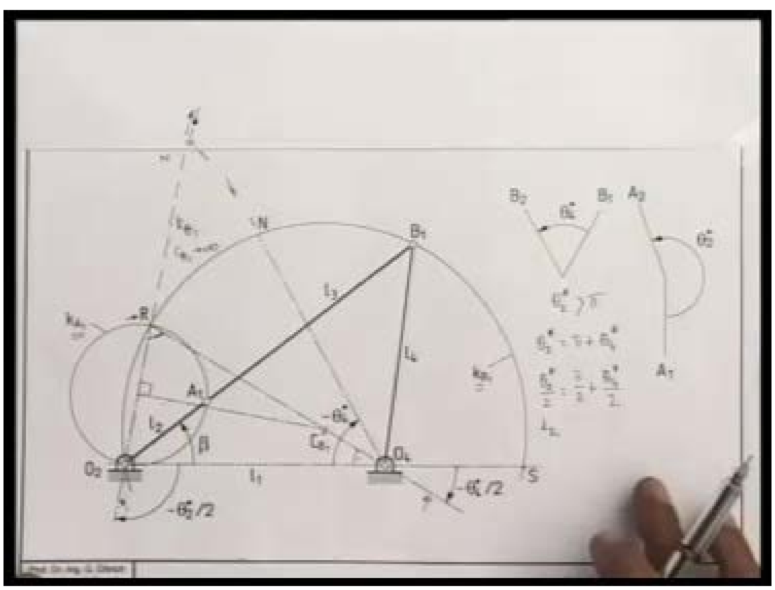

Looking at this figure which was drawn for θ_2 star more than pi, let us consider the special case, if θ_2 star happens to be equal to pi plus θ_4 star, that means θ_2 star by 2 is pi by 2 plus $theta_4$ star by 2. Look at this construction. This angle is $theta_2$ star by 2, this angle is $theta_4$ star by 2 which means this angle is also $theta_4$ star by 2. If this is pi by two plus this, then this angle will be 90 degree. So, this line will be perpendicular to O_2R. As a result, this perpendicular bisector and O_4R will never intersect because this angle also 90 degree, this angle is also 90 degree and these two lines become parallel and they will never meet. Consequently, the center C_{B1} goes to infinity and this k_{B1} with C_{B1} as the center degenerates into a straight line, straight line passing through O_2 and R. That means k_{B1} will be the straight line which is nothing but extension of O_2R. If I draw this line O_4N at an angle $theta_4$ star they will intersect at some point N and this is the circle of infinite radius k_{B1}, when C_{B1} goes to infinity because theta $_2$ star by two is pi by two plus $theta_4$ star by two. So, N is the intersection of these two lines. Then, B_1 can be taken anywhere on this line, but beyond this point N. That is B_1 will be anywhere on this line beyond this point N, say here and B_1 I can choose there and O_2A_1 will be always equal to O_2R. That means, we get an infinite number of designs but every time the crank length remains the same as O_2R because k_{A1} is this circle which has not changed. Under this situation l_2 does not change and this C_{B1} goes to infinity that means this k_{B1} degenerates into a straight line and B_1 can be taken anywhere above this point N on this line. So that is a very special.

Let me now summarize what we discussed today. Besides discussing the design of an approximate dwell mechanism having six-links starting from a coupler curve, we have also discussed the dead-center problems. That is, design for extreme configurations the outer and inner dead-centers are the two extreme configurations and we have discussed both slider-crank and 4R crank-rocker mechanisms.

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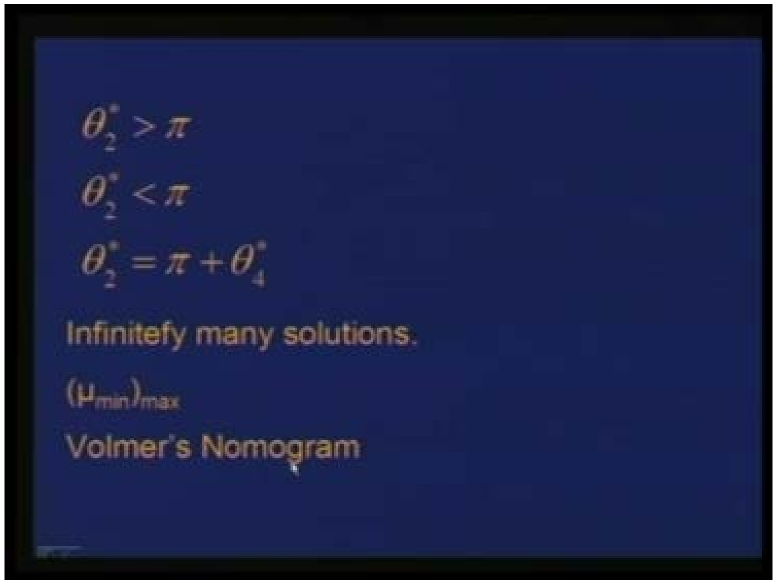

For the 4R crank-rocker mechanisms we considered three cases separately: whether $theta_2$ star that is the crank rotation as the rocker moves from outer to inner dead-center by an angle more than pi, less than pi and a special case if $theta_2$ star is pi plus $theta_4$ star. In all these cases, what we have seen that the Hall's construction give us infinitely many solutions. The optimum solutions is that for which the minimum transmission angle mu min is maximized and that design means, which is parameterized by the angle beta in a all our figures can be obtained by using an nomogram which is known as Volmar's nomogram which we have not discussed in this course.

This brings us to the end of the graphical method of kinematic synthesis of planar linkages. In our next lecture, we shall start the discussion on analytical method of kinematic synthesis of planar linkages.