Kinematics of Machines
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Module – 07 Lecture – 2
Six-Link Mechanism

Today, we continue our discussion on advance synthesis problem. In the last lecture, we have seen how a multistage procedure can be applied for the design of a six-link mechanism to be used as a fork lifter consisting only of R pairs.

As a second example, today we will discuss a six-link rail-less garage door mechanism.

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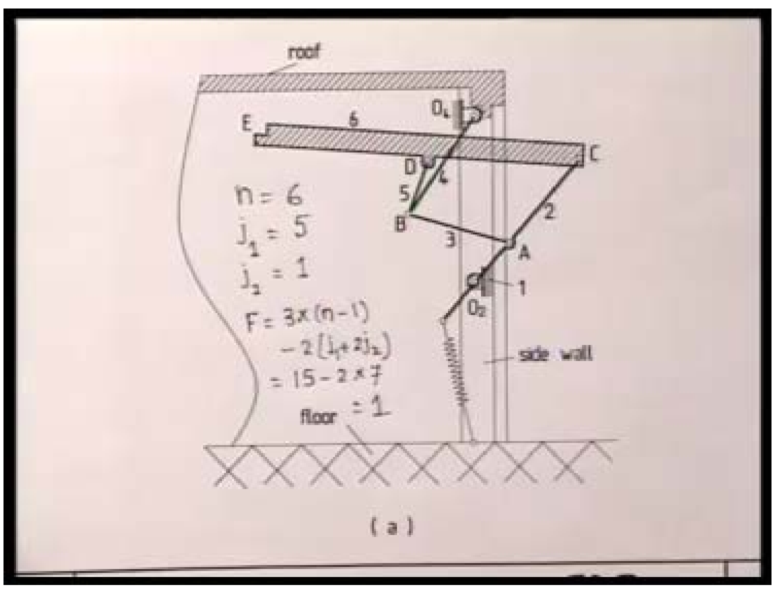

This figure shows the sketch of the proposed six-link mechanism, to be used as a rail less garage door. This member EC represents the door board. The fixed hinges or a fixed pivot of this mechanism is at O_2 and O_4 which are mounted on the sidewall of the garage, this is the floor of the garage and this is the roof. This is fixed link 1, O_2AC this is link number 2, AB is link number 3, O_4 B is link number 4 and BD is link number 5 and door board EC is link number 6. We have total number of links n is equal to 6. We have 1, 2, 3, 4, 5 revolute pairs which are connecting only two links. So j_1, simple hinges is 5 and at this revolute pair B, three links namely, 3, 4 and 5 are connected. We have a second order hinge j_2 is 1. The degrees of freedom of this mechanism using our known formula is F equal to 3 into (n minus 1) minus 2 (j_1 plus $2j_2$).

If we substitute these values, we get 15 minus 14 is equal to 1. We have a single degree freedom mechanism. Now the question could have been asked, why six-link mechanism is used? We could have used the door board as the coupler of a four link mechanism, but it was found that using only a four link mechanism, it was not possible to have the desired motion of this door board. What is desired? That the door board, when the garage is open, should remain parallel to the roof that is horizontal and in the close position, when this linkage is moved, the door board becomes vertical to close the garage. From this horizontal to vertical transition, the door board should not take too much space inside the garage, because the car is kept here and that will interfere with the car. The rest of mechanism is mounted on the side wall. We shall show the model of such a mechanism to demonstrate the desired motion that is required.

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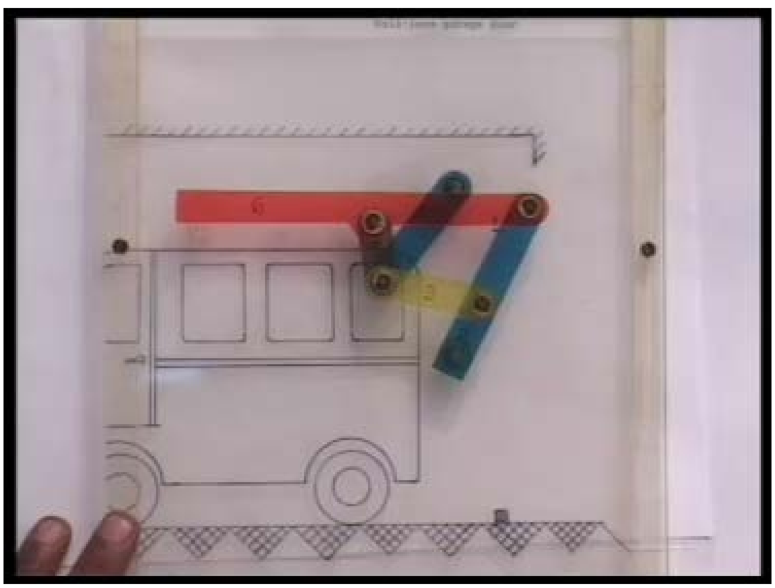

Let us now study the model of this proposed fixed link mechanism. This is the door board which is horizontal and parallel to the roof of the garage, when the garage is open. By driving this mechanism, we can close the garage and the door board becomes vertical. If we see the movement of this garage door from this open to the closed position, what should we note? That the garage board is not interfering with the car that is kept inside and the car is quite a high car. Thus, the motion of this garage door board is almost horizontal and then suddenly falls to the vertical position. Such a movement is not possible by using a simple four link mechanism with this as the coupler, the door board as the coupler. So, we go for this six-link mechanism. Now, we shall give the details of the design of this six-link mechanism and determine all the kinematic dimensions by applying the multistage synthesis procedure that we have learnt earlier.

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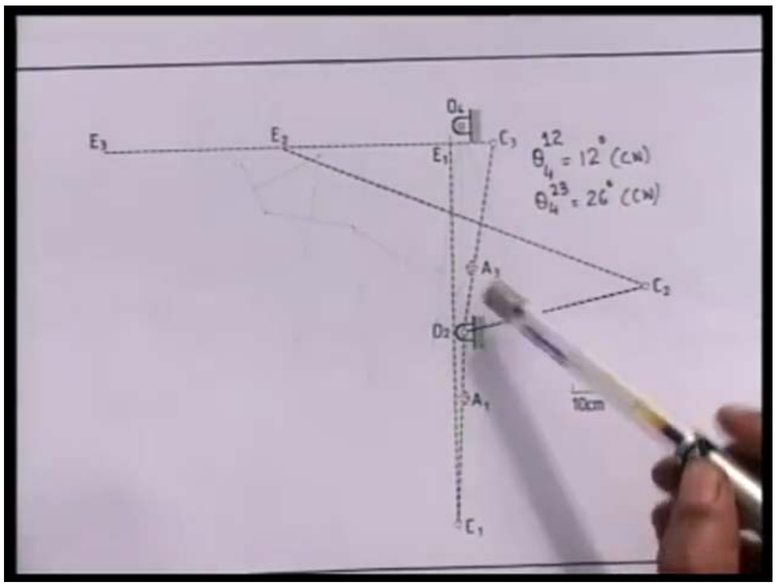

This sketch we show that O_2 and O_4 are the location of the two fixed hinges on the side wall. The location of these two hinges, we choose conveniently wherever it is possible. C_1E_1 is the door board, when the garage door is closed and it is almost vertical. C_3E_3 is the door board when the garage is open and this C_3E_3 is almost horizontal. We consider an intermediate position C_2E_2. The length of the door board is also taken as the height of the garage to be closed. It is also specified that the fourth link which is connected to O_4 up to the point B, where there was a second order hinge - three links were connected - that point we call B. This O_4B is the link 4 and it is also said that, the rotation of this fourth link theta$_4$12 from configuration 1 to configuration 2 is 12 degree clockwise. Similarly, theta$_4$23 is also prescribed that it should be 26 degree clockwise.

Our task is to determine the rest of the linkage that is the location of the second order hinge at B and also the hinge at D on this door board CE and we shall determine the linkage at the third configuration. We shall determine where B_3 and D_3 are. Let me show you the complete design specification for this particular linkage. The scale of the drawing is shown here. This is 10 cm. The complete design specification for this particular mechanism is as follows:

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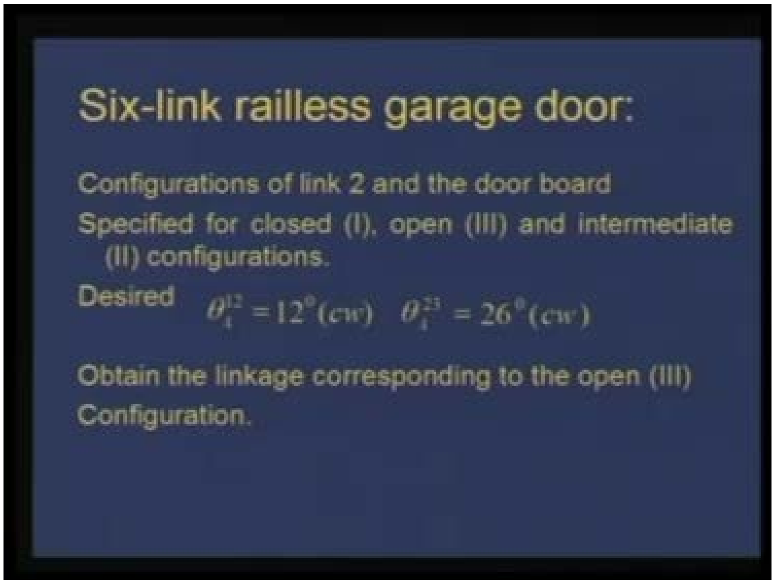

We have given three configurations of link 2 and the door board. The door board, that is link C E or link number 6 is specified for closed position which we call position 1, open which is called position 3 and intermediate - we call position 2 - configuration 2. It is desired that $\text{theta}_4 12$ is 12 degree clockwise and $\text{theta}_4 23$ is 26 degree clockwise. Our task is to obtain the complete linkage corresponding to the open configuration when the door board is parallel to roof or horizontal. All other kinematic dimensions and the locations of the hinge point have to be determined for these particular three configurations.

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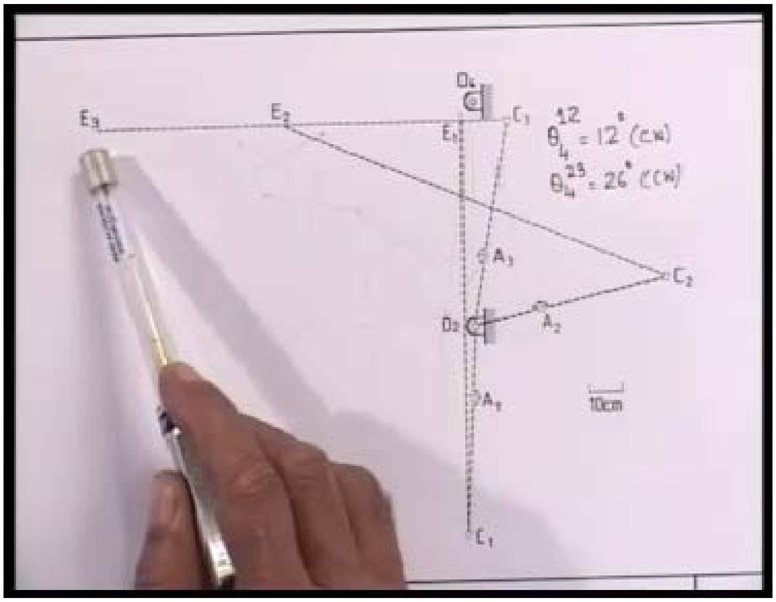

As I said earlier, O_2 and O_4 have been chosen arbitrarily, three positions of the door C_1E_1 closed, C_2E_2 intermediate and C_3E_3 open, these three configurations are given. We would like to determine that the rest of the linkage corresponding to the third or the open configuration. We achieve this design in two stages. First, we try to design the four link part of this mechanism, namely $O_2ABO_{4.}$ To obtain this four link mechanism, we conveniently choose the location of A, the hinge on this link 2, A_1 is chosen arbitrarily. So, we can immediately determine where A_2 and A_3 are. Because, this distance O_4A does not change. In the first configuration, it is here because C_2 is given so O_2C_2 is the length; this distance remains same so, I can determine A_2 and O_2C_3.

This distance remains same, so A_2 comes to A_3. So, I know that the three positions of this revolute pair at A namely A_1, A_2, and A3. Since we have to obtain the position of the hinge B, at the third configuration that is $B_{3,}$ we hold the link 4 fixed at its third configuration and make a kinematic inversion and determine the inverted positions of A_1 and A_2. A_3 is nothing but $A_3$3, because we are holding the link 4 that is O_4B fixed at its third configuration. This is A_2, we have to determine $A_2$3. It is known that theta$_4$23 is 26 degree clockwise which means, theta$_4$32 is 26 degree counter clockwise. To determine the inverted position of 2, we have to rotate O_4A_2 through minus theta$_4$32, minus theta$_4$32 is nothing but 26 degree clockwise. This is O_4A_2 and we rotate this through 26 degree. So, O_4A_2 is this line and we rotate it through 26 degree. This angle, this rotation is 26 degree because minus $theta_4 32$ is 26 degree and this point, I mark as A_2 inverted on the third position $A_2 3$. Now, A_1 is here, where is $A_1 3$? $Theta_4 31$ is equal to 26 degree plus 12 degree that is 38 degree in the counter clockwise direction because $theta_4 13$ is 38 degree clockwise direction. To get to the point $A_1 3$, I have to rotate O_4A_1 through 38 degree by minus $theta_4 31$ which is again 38 degree clockwise. This is the line O_4A_1 and I rotate this about O_4, 38 degree in the clockwise direction, this rotation is 38 degree and this point I call $A_1 3$ because it is inverted on the third position of O_4B and the rigid link AB is of constant length. So, the center of the circle passing through these three inverted positions, namely: $A_3 3$, $A_2 3$ which is here and $A_1 3$. To determine the center of the circle passing through these three points in the usual manner, we draw the perpendicular bisector of these two lines, namely: $A_2 3$, $A_1 3$, $A_3 3$ and $A_2 3$ that is this line.

Perpendicular bisector of these two lines, we can draw simple geometrically. This is the perpendicular bisector of $A_2 3$, $A_1 3$ and this is the perpendicular bisector of $A_3 3$ that is A_3 and $A_2 3$, these two lines meet here which will locate at B_3. This is where the compound hinge the second order hinge is located at the third configuration. To complete the 4-bar linkage, I have to join O_4B_3 and A_3B_3. O_2, A_3, B_3 and O_4 that is the 4-bar part of this fixed length mechanism.

Now, we have to obtain B_3 that is, the hinge which is located on the door board, because we are holding it in the third position fixed. So, I have to find out the inverted position of B. To do that, first we note where B_2 and B_1 are. Because the link O_4B is of fixed length, the point B moves on this circle with O_4 as center and B as radius and the rotation of the fourth link is O_4B from second to third position is 26 degree clockwise. So, from third to second, it will be 26 degree counter clockwise. I draw a line at 26 degree from O_4B_3, this rotation is 26 degree in the counter clockwise direction and B lies on this circle. So, I can locate B_2 and if I draw another line at 12 degree, I get B_1 because from position 1 to 2, it is 12 degree clockwise. From position 2 to position 1, it is 12 degree counter clockwise. I draw a line, this angle is 12 degree and we get the position $B_{1,}$ this is B_1. We have located that compound hinge corresponding to the three configurations, 1, 2 and 3 at B_1, B_2, and B_3.

To locate the hinge on the door board D, I have to hold the door board fixed, because BD length is a constant length. If we hold link 6, that is the door board fixed, then B goes on a circle with D as center. We make a kinematic inversion holding the door board always fixed at its third position. To obtain the inverted position of B_2 and B_1, we use our procedure of kinematic inversion and use a tracing paper.

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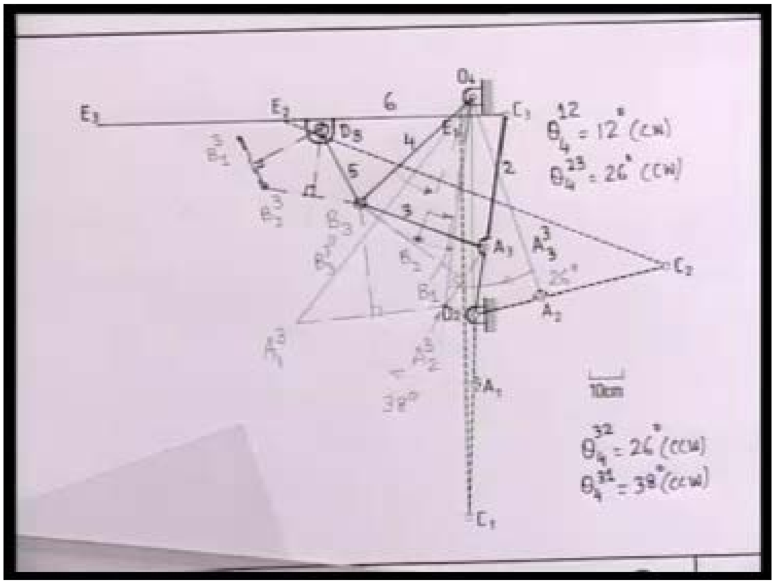

We use a tracing paper and mark the relative position of the door board. Door board is completely specified by two hinge points on it namely C_2, and E_2, and B_2 is here. We have marked as we see here three points namely C_2, E_2 and B_2 on this tracing paper. If the door board is held fixed in its third configuration, then C_2 is made to coincide with C_3 and E_2 is made to coincide with E_2 and wherever B_2 goes, this point, I mark on my drawing sheet which is B_2 inverted on the third position. B_3 is nothing but, this is same as B_3 inverted on the third position. Only thing remaining is to obtain the inverted position of B_1 which is here. This is the location of B_1. So, again using tracing paper mark C_1, E_1 and B_1. This is C_1, this is E_1 and this is the B_1. So I now make a kinematic inversion with C_3 fixed at its third position, that is C_1 coinciding with C_3, E_1 coinciding with E_3 and wherever B_1 has moved, that is the inverted position $B_1$3.

I use again, mark this inverted position on my drawing paper and I get the inverted position of B_1 which I call $B_1$3. The center of circle passing through these three points $B_1$3, $B_2$3 and $B_3$3, we determine in the usual manner. We take the perpendicular bisector of these two lines. Draw the perpendicular bisector and the perpendicular bisector of this line $B_1$3, $B_2$3. This is the perpendicular bisector of $B_1 3$, $B_2 3$ and these two perpendicular bisectors meet here, giving me the revolute pair location D_3. There is a rigid link connecting B_3 and D_3 and this D_3 belongs to the door board. I can have extension of the door board like this. We have determined O_2, A_3, B_3, and D_3. O_4 was already chosen and C_3E_3 was already chosen which completes the design of the six-link mechanism, for the door board namely: O_2, A_3, B_3, O_4, D_3 and C_3E_3 corresponding to the open configuration on the door board is parallel to the roof of the garage.

By driving this mechanism, I ensured that the door board will go from C_3E_3 to C_2E_2 and then to the closed position C_1E_1. To complete this linkage, let me make this line continuous. This line is also continuous, so, we get by this firm line that design of the representation of the entire mechanism O_2, A_3, C_3, O_4, B_3, A_3B_3 is another link. So let me say this is link number 2.

A_3B_3 is link number 3, O_4B_3 is link number 4, B_3D_3 is link number 5 and the door board C_3E_3 is link number 6. So, again we have used the multistage synthesis procedure to design this six-link rail-less garage door mechanism, where the garage door has the desired movement that is, almost horizontal movement inside the garage, then grouping them fast to take up this vertical closed position. So, we have seen how the three position synthesis technique for function generation, motion generation and path generation can be applied at various stages to even design more complicated mechanism.

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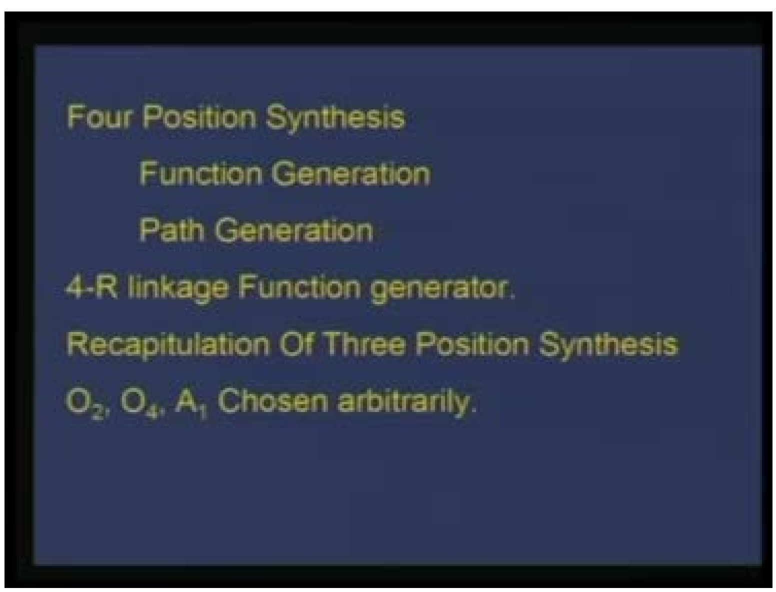

Now we would discuss how to get four position synthesis. If we recapitulate, the function generation, path generation for three position synthesis, what we have seen that, we consider the three inverted position of a particular revolute pair and then the centre of the circle passing through those two inverted positions determine the location of the other revolute pairs. However, for example, for 4-R linkage function generator, we chose O_2 and O_4, the two fixed hinges O_2 and O_4 arbitrarily and also chose one of the moving hinges the crank pin at A_1 and we determine B_1 as the center of the circle passing through three inverted positions of A_1 that is, $A_1$1 which is A_1, $A_2$1 and $A_3$1.

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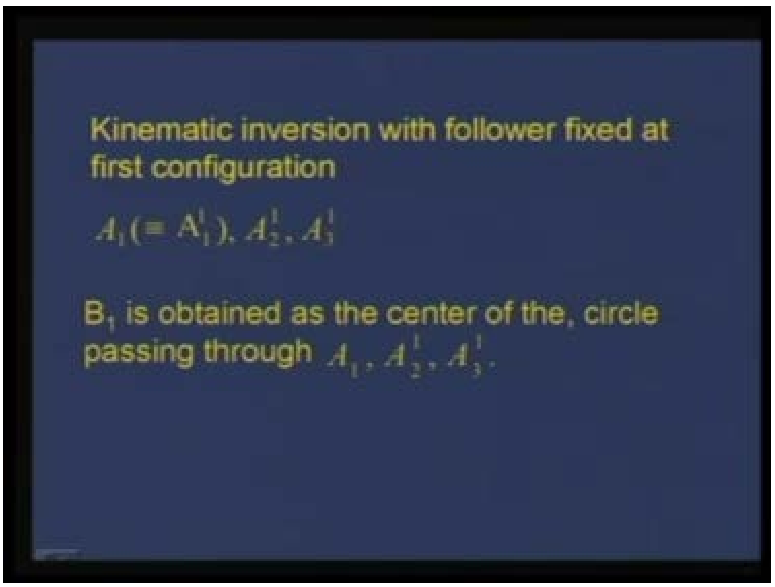

We did the kinematic inversion with follower fixed at its first configuration. Then we chose A_1 arbitrarily and found $A_2$1 and $A_3$1 using the principle of kinematic inversion. B_1 was obtained as the center of the circle passing through A_1, $A_2$1 and $A_3$1.

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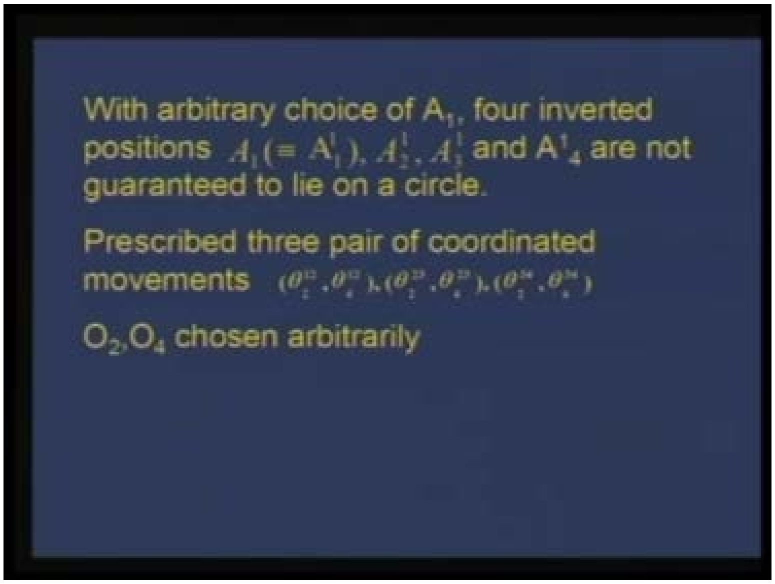

Now, with arbitrary choice of A_1, four inverted positions of A_1 that is say $A_1$1, $A_2$1, $A_3$1 and $A_4$1, there is no guarantee that they will lie on a circle, then I cannot determine the desired location of B_1.

Now, we shall explain a technique, which is known as point-position reduction for such a four position synthesis as a function generator which means, if three pairs of coordinated movements of the inputs and output links are prescribed. We have given theta$_2$12 that is the rotation of link 2 from position 1 to 2, theta$_4$12 that is the coordinated rotation of link 4 from position 1 to 2, 1 pair is theta$_2$12 theta$_4$12. Similarly, the other two pairs are theta$_2$23 and theta$_4$23. Another pair is theta$_2$34 and theta$_4$34. As this now just to say that I cannot choose A_1 arbitrarily anymore we choose O_2 and O_4 arbitrarily because those are the fixed pivots. We choose them at a convenient location on the frame of the fixed link.

The method that we are going to adopt is known as, point-position reduction such that, the four inverted positions of A_1 become only three distinct location not four distinct location. If we have three distinct locations for the inverted four inverted positions of A_1 then I can still draw a circle passing through those three points and B_1 can be determined as the center of the circle passing through those three distinct locations of four inverted positions of A_1.

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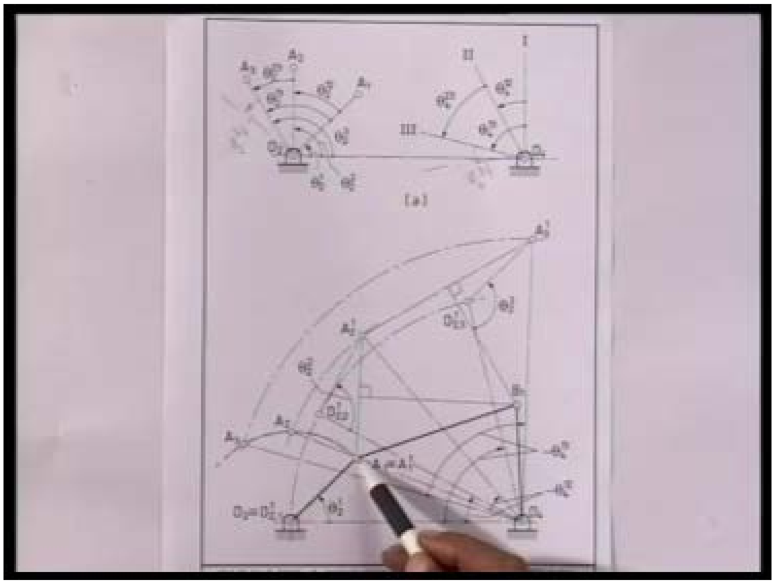

We shall explain this method with the help of an example. To explain the point-position reduction method, let me first recapitulate the three position synthesis quickly. As we have seen, these are the prescribed motion of the input link or link number 2 from position 1 to 2 theta$_2$12 and 2 to 3 theta$_2$3. Corresponding desired motion of the follower link or the output link, link number 4 is prescribed as theta$_4$12 and theta$_4$23. We solve this problem by choosing O_2 and O_4 arbitrarily. We choose them conveniently on the frame. We also choose A_1 arbitrarily. Corresponding to this prescribed movement theta$_2$12 and theta$_2$23, A_1 which goes on this circle with O_2 as center and O_2A_1 as radius. I can determine A_2 and A_3. Angle $A_1O_2A_2$ will be theta$_2$12, $A_2O_2A_3$ will be theta$_2$23.

To determine B_1, we considered the inverted positions of A_2 and A_3. We obtain the inverted position by rotating O_4A_2 through an angle minus theta$_4$12. Theta$_4$12 was counter clockwise. So, minus theta$_4$12 is clockwise by rotating about O_4, this line O_4A_2 through minus theta$_4$12, I located $A_2$1. Similarly, rotating O_4A_3 about O_4 through an angle minus theta$_4$13, theta$_4$13 is this angle which is prescribed, which is counter clockwise. So, minus theta$_4$13 is clockwise. So, rotating O_4A_3 about O_4 through minus theta$_4$13, I locate at $A_3$1. Then the center of the circle passing through these three inverted positions of A namely: A_1 which is $A_1$1, $A_2$1 and $A_3$1, I located B_1 and the design was complete as $O_2A_1B_1O_4$.

With such a arbitrary choice of A_1, if I had one more pair of coordinated movements, say theta$_2$3 4 and the corresponding movement, here is say, theta$_4$3 4. Then I could have located A_4 on this circle, but the inverted position of A_4 will be the fourth point, $A_4$1 and there is no guarantee that these four inverted positions of A namely $A_1$1, $A_2$1, $A_3$1 and $A_4$1 will lie on a circle. For four position synthesis, I cannot choose A_1 arbitrarily. How to carry out such a four position synthesis by the point-position reduction technique will now be explained with the example.

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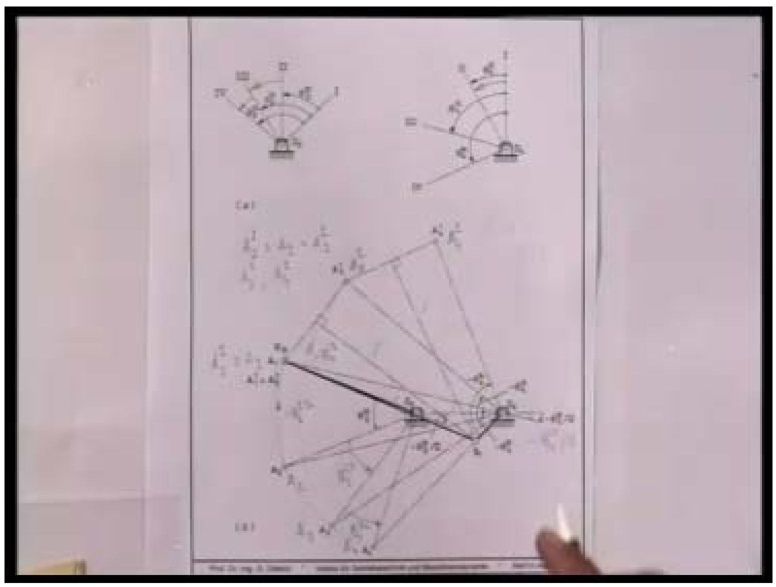

To explain the point-position reduction technique, let us take this example of a 4-R function generator for three pairs of coordinated movements, say theta$_2$12 should cause theta$_4$12, theta$_2$13 should cause theta$_4$13 and theta$_2$14 should cause theta$_4$14. We have to generate these three pairs of coordinated movement between input and output links of a 4-R function generator.

As before, we choose the fixed hinge location O_2 and O_4 conveniently. As we know A_1 cannot be chosen arbitrarily any longer. We choose A_1 in such a way that the inverted position of A_2 - which we call $A_2$1 - coincides with A_1, which is the same as $A_1$1. I will explain how to obtain this, such that $A_2$1 coincides with A_1. The choice of A_1 is guided by this. Once after this, I can always obtain $A_3$1 and $A_4$1 so, two of the inverted positions are same. So, I get three distinct locations for the inverted positions of A namely: $A_1$1, $A_2$1, $A_3$1 and $A_4$1. Then, I can determine the center of the circle passing through these three points and get the location of B_1. To make

$A_2 1$ coincides with A_1, I choose A_1 as follows: I draw a line at minus $theta_2 12$ by 2 with O_2O_4 at O_2, that is, this line. This line is drawn at an angle minus $theta_2 12$ by 2 at O_2 with O_2O_4.

At O_4, I draw a line at an angle minus $theta_4 12$ by 2. This angle is minus $theta_4 12$ by 2 and this angle as we said is minus $theta_2 12$ by 2. These two lines intersect here and I choose my A_1 here. This is the choice of A_1 which will ensure that $A_2 1$ 1 will coincide with A1. Now, once this is the choice of A_1, I can determine A_2, A_3 and A_4 by rotating O_2 A by $theta_2 12$ and then $theta_2 13$ and $theta_2 14$, these are prescribed. So, I rotate O_2 A about O_2. So, A lies on this circle and the angle between O_2A_1 and O_2A_2 should be $theta_2 12$ then between O_2A_2 and O_2A_3, this angle should be $theta_2 23$ which is also given, that is this angle.

Rotating it further, $theta_2 3$ 4 which is also given, I can locate from this location of A_1, A_2, A_3, and A_4. Now, to obtain the inverted position of A_2, we rotate O_4A_2 through minus $theta_4 14$. If I rotate this O_4A_2 about O_4 through an angle, this angle is minus $theta_4 12$, $theta_4 12$ is given, and this is $theta_4 12$ which is counter clockwise. So, I rotate it clockwise and because of this O_4A_2 equal to O_4A_1, O_2A_2 is equal to O_2A_1 and this becomes an isosceles triangle. O_2O_4 become the angular bisector of both of these isosceles triangle and because this angle is minus $theta_4 12$ by 2, the total angle is $theta_4 12$. Thus, $A_2 1$ has coincided with $A_1 1$, and then we can obtain in the usual manner. What is $A_3 1$?

I rotate O_4A_3 through an angle minus $theta_4 13$. This is $theta_4 13$ which is counter clockwise. It rotates it clockwise because this is minus $theta_4 13$ and I get what I call A_{31}. Then rotating O_4A_4 through minus $theta_4 1$ 4, this is counter clockwise, I rotate it clockwise and get the inverted position of A_4 as $A_4 1$. We see for the four inverted positions, I have only three distinct locations.

This is $A_3 1$, this is $A_4 1$, but $A_2 1$ and $A_1 1$ are coincident by inverted position of A_2, when I rotate O_4A_2 through minus $theta_4 12$. I got to the same point. So, A_1 is same as $A_2 1$. Then the center of the circle passing through these three points, that is perpendicular bisector of this line and perpendicular bisector of this line, these two lines this line and this line they intersect at B_1 and I get the required 4-R linkage, namely: O_2, A_1, B_1, and O_4. O_2A_1 and A_1B_1 in this first configuration are almost coincident because of this given data. This is what we call point-position reduction. Of course, we could have done this point-position reduction in a different manner such that two other inverted positions may coincide. Here, we have coincided $A_2$1 and $A_1$1.

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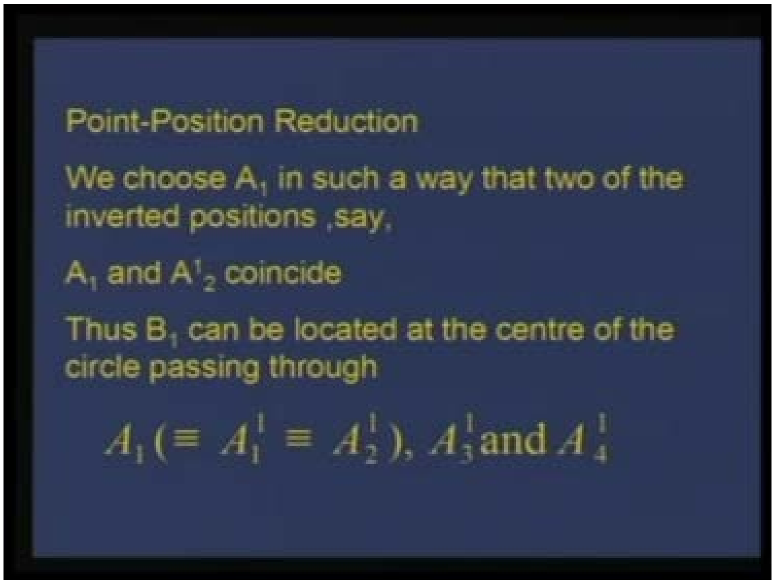

Let me now recapitulate what we did in this point-position reduction technique for function generation with for four position synthesis that is three pair of coordinated movements.

We choose A_1 in such a way that two of the inverted positions in the example, what we did, A_1 and $A_2$1 coincide and then B_1 can be located at the center of the circle passing through A_1, which is same as $A_1$1 and also coincident with A_{21}, $A_3$1 and $A_4$1. Basically, four inverted positions give rise to three distinct locations and we can always draw a circle through these three distinct positions and the center of the circle determines the locations of the other moving hinge.

It is obvious that, we could have made instead of A_1 equal to $A_2$1, or same as $A_2$1, I could have chosen A_1 in such a way that A_1 becomes $A_3$1 or A_1 becomes $A_4$1 and we get two different solutions. We have already solved with A_1, same as $A_2$1 but, I could have chosen A_1 in such a way that $A_3$1 coincides with A_1 or I could choose A_1 in such a way that $A_4$1 coincides with A_1. These are all inverted on the first configuration and I will locate B_1. Similarly, we can say I will convert on the second configuration and I will make $A_3$2 coincide with A_2 or $A_4$2 coincide with A_2 and we get two other different solutions to the same problem. So, by this point-position reduction, I can have many solutions to the same problem depending on the choice of the moving hinge A either at A_1 or at A_2 or at A_3. Now, we shall discuss this four position synthesis for a path generation problem.

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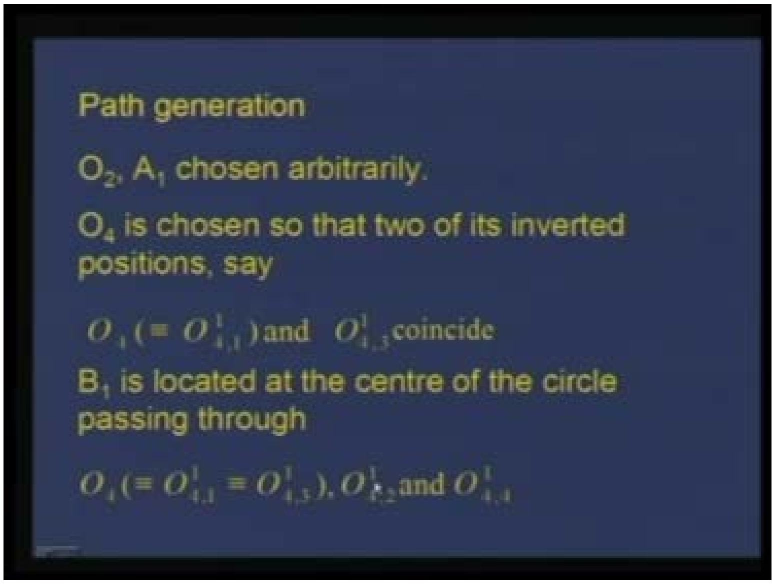

Previously, if we remember, we chose O_2, O_4 and A_1 arbitrarily for three position synthesis. Of course, for four position synthesis, I cannot choose O_4 arbitrarily because B_1 was located at the center of the circle passing through three inverted positions of O_4. Here, for four position synthesis, we will get four inverted positions and there is no other guarantee that a circle can be drawn. So, O_4 has to be chosen in such a way that four inverted positions of O_4 give three distinct locations of O_4 and I can again determine B_1. This I will explain with the help of an example. O_4 will be chosen so that two of its inverted position say $O_{4,1}1$ and $O_{4,3}1$ coincide. In the example that we shall solve, I will choose O_4 in such a way that O_4 also becomes $O_{4,3}1$ and of course O_4 is $O_{4,1}1$ because we will invert on the first configuration. B_1 then will be located at the centre of the circle passing through O_4, $O_{4,2}1$ and $O_{4,4}1$.

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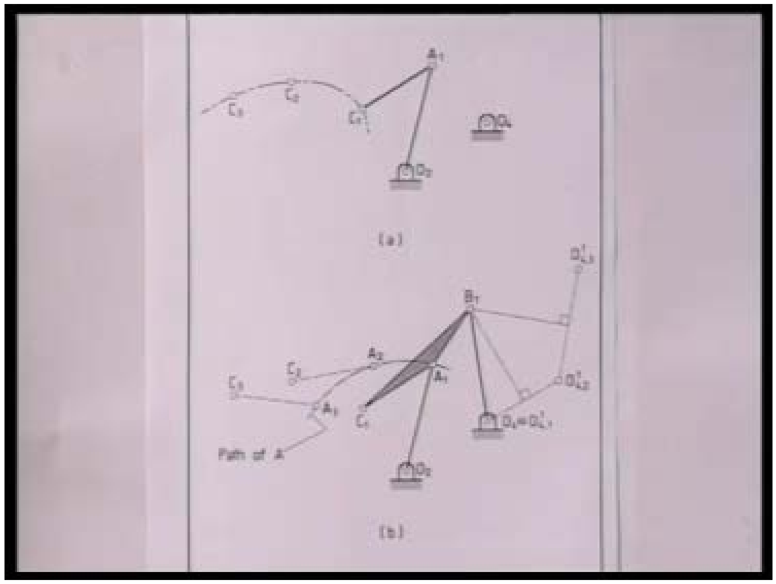

Let me now explain this with the help of an example. Let me now recapitulate the path generation problem for three position synthesis. The coupler point C has to pass through three locations namely: C_1, C_2 and C_3. To solve this problem, we chose O_2 and O_4 and A_1 arbitrarily, with this choice, O_2, O_4 and A_1 arbitrarily and C_1, C_2 and C_3 given to us. I can locate A_2 and A_3 on the path of A which is the circle passing with radius as $O_2$1 and center at O_2. I draw this circle and I can find out where A_2 and A_3 are because C_1, C_2 and C_3 are given to us. C_1A_1 is same as C_2A_2 and also equal to C_3A_3. From this given location C_2 and C_3, I can determine A_3 and A_2 because AC is of given fixed length. To determine B_1, we considered the inverted positions of O_4. To obtain the inverted positions of O_4, I hold the coupler fixed at its first configuration that is A_1C_1.

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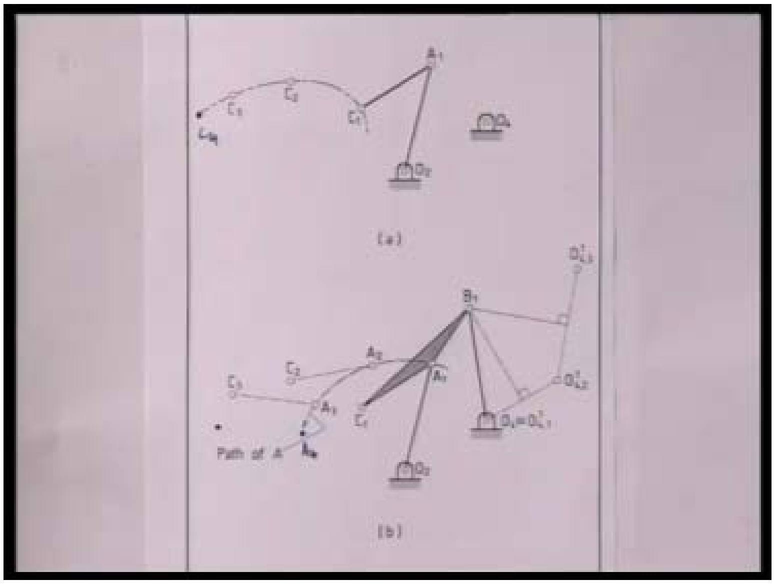

Corresponding to the second configuration, we marked A_2, C_2 and O_4. These are the relative positions of these three points in the second configuration. If I hold the coupler fixed at its first configuration that is A_2 coincides with A_1 and C_2 coincides with C_1 and wherever O_4 goes that I mark as $O_{4,2}1$. Similarly, I take a tracing paper mark A_3, C_3 and O_4 then move this tracing paper such that A_3 coincides with A_1 and C_3 coincides with C_1 and wherever O_4 goes that I call $O_{4,3}1$ because O_4B is of fixed length. I determine B_1 as the center of the circle passing through these three positions O_4 which is same as $O_{4,1}1$, $O_{4,2}1$ and $O_{4,3}1$. If we had one more point on this and I say, it is C_4. I have to design a 4-R linkage such that, the coupler point C passes through four prescribed positions namely: C_1, C_2, C_3 and C_4. Then with such an arbitrary choice of A_1, if I got A_4C_4 from this location of C_4, I could have located A_4 on this circle because, C_3A_3 is C_4A_4. I could have located A_4. But the inverted position $O_{4,4}$ 4 would have been an arbitrary point and there is no guarantee that one can draw a circle through three through such four positions of O_4.

In the point-position reduction, we choose O_2 and A_1 arbitrarily, but O_4, I do not choose arbitrarily, O_4, we locate in such a way that two of these four inverted positions coincide. I shall explain this with a help of an example.

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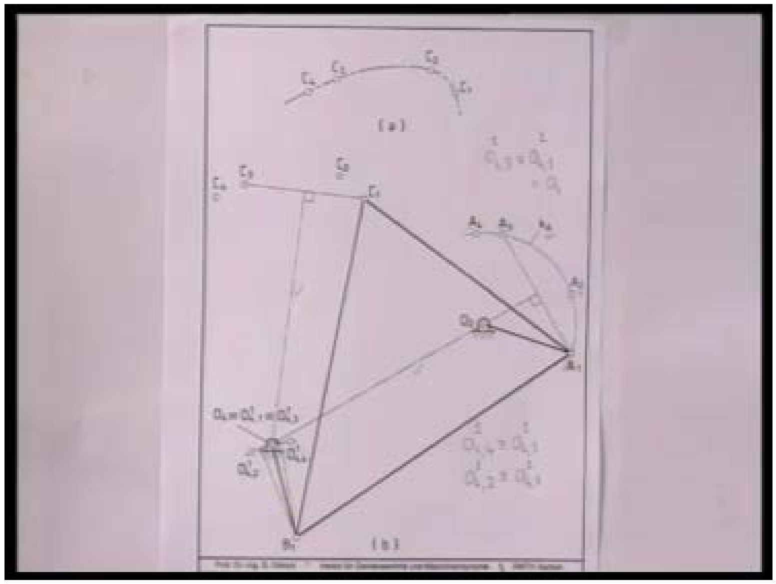

Let me now explain, this four position synthesis for the path generation. The coupler point of a 4-R linkage has to pass through prescribed four positions namely: C_1, C_2, C_3 and C_4. We shall apply the point-position reduction technique, such that two of the four inverted positions of O_4 namely: O_4, third position inverted on the first position should be same as O_4, first position inverted on the first position, which is nothing but the choice of O_4.

To solve this problem as before, I choose O_2 and A_1 arbitrarily. The path of A_1, I can draw with O_2 as center and O_2A_1 as radius, it is this circle. This is the path of A which I have marked as A_A. So, C_1 is given to me, A_1 I have chosen arbitrarily, I know this rigid length namely AC. Accordingly, I can find corresponding to the second configuration where is A_2 because A_2 has to lie on this circle and C_2 is here and C_2A_2 length is fixed. So, I can determine the location of A_2. Similarly, from C_3 on this circle with k_A, I locate the point A_3. Similarly, from C_4, using the length C_A, I determine the location of A_4. So, I get A_1, A_2, A_3 and A_4 corresponding to C_1, C_2, C_3 and C_4.

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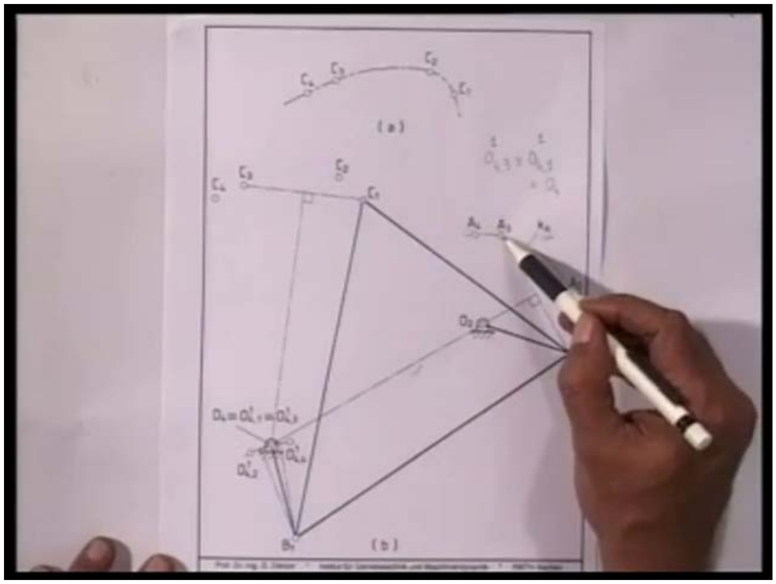

Now, O_4 is chosen on the mid normal of C_1C_3 and A_1A_3. This line is the perpendicular bisector of A_1 A_3 and this line is the perpendicular bisector of C_1C_3. These two lines meet at this point and I choose my O_4, at the intersection of these two perpendicular bisectors. If I choose O_4 here, O_4 is nothing but $O_{4\ 1}$1. It is very easy to see that $O_{4\,3}$1 will also be here. So I mark A_3, C_3 and O_4. This is A_3, this is C_3 and this is O_4. If I invert it on the first position that is A_3 coincides with A_1 and C_3 coincides with C_1.

As we see that the point O_4 does not move. So, this is also $O_{4,3}$1. It is the same point O_4 is $O_{4,3}$1. This is obvious because O_4 has been located on the perpendicular bisector A_1A_3 and C_1C_3. So O_4, A_1, C_1, this triangle moves as a rigid triangle from the first to the third configuration. Next, we determine where is $O_{4,2}$1 and $O_{4,4}$1. To do that, we follow the same procedure of kinematic inversion. I mark C_2, A_2 and O_4. This is C_2, this is A_2 and this is O_4. I make C_2 coincides with C_1 and A_2 coincides with A_1 and wherever this O_4 has moved that I call $O_{4,2}$1.

Similarly, for the fourth configuration, I mark C_4, A_4 and O_4, then make C_4 coincides with C_1, A_4 coincides with A_1 and wherever O_4 goes, the O_4 has moved here and that I call $O_{4,4}$1. So, two of the four inverted positions coincide, four inverted positions give me only three distinct points and I can always draw a circle through these three points and the center of the circle is determined here at B_1.

Now, I get the 4-R linkage path generator O_2, A_1, B_1, O_4 with C_1 as the coupler point. Once this linkage is moved, this point C will go from C_1, C_2, C_3 and C_4. We have got a four position synthesis path generation problem solved by using point-position reduction technique. As I said before, with reference to function generation problem, I could have chosen different inverted positions of O_4 to coincide. I could have inverted the whole thing, such that O_4 for the fourth position inverted on the first position coincides with $O_{4,1}1$. Similarly, one could have solved is by another choice namely $O_{4,2}1$ coincides with $O_{4,1}1$ and so on.

We get can different solutions depending on which two inverted positions, this one, this one or this one, we want to coincide. Let me now summarize, what we have learnt today.

Initially, we used the three position synthesis technique in multiple stages to achieve the design of a six-link mechanism. We discussed how to do four-position synthesis for both function generation and path generation problem with respect to a 4-R linkage. In this point-position reduction technique was used to achieve, the four position synthesis and the technique is such, that two of the inverted positions coincide such that four inverted positions gives us only three distinct locations, then I can always draw a circle to those three distinct locations and determine the center of such a circle. The same technique can be used for function generation using a slider crank with four position synthesis.

In our next class, I will take up an example to show how to use this four-position synthesis technique in multiple stages, so that we can achieve a fixed design of a six-link mechanism.