Kinematics of Machines

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Module – 1 Part – 2

In our last lecture, we have explained what we mean by a planar mechanism. We have also defined kinematic analysis and synthesis of mechanism. This course is entirely based on kinematic analysis and synthesis of planar mechanisms. So, it will not be a bad idea to recapitulate a little bit about planar mechanisms.

In a planar mechanism, all points of the mechanisms move in parallel planes. A single view along a direction perpendicular to these planes reveals the true motions of all the points of the mechanism. In fact, all these parallel planes are projected on to a single plane which is called the plane of motion. Let us also see what happens during plane motion of a rigid body.

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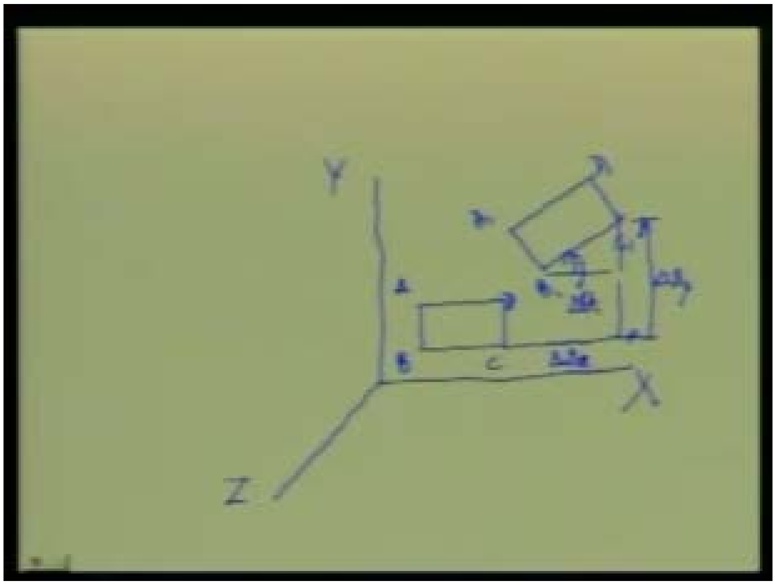

Suppose, XY is the plane of motion of the rigid body and Z-axis is perpendicular to this XY plane. If a rigid body, ABCD, is moving in this XY plane and takes up configuration $A_1B_1C_1D_1$. Then, the configuration of this rigid body, during this plane motion can be completely described by 3 independent coordinates. For example, the motion of any particular point, say C, as it goes from C to C_1 - this we call translation along X-axis given by delta S_x. The movement of C along the Y-axis during this motion is given by the change in Y coordinate say delta S_y, which is the translation in the Y direction. The rotation of this rigid body is taking place only about an axis parallel to the Z-axis and the amount of rotation can be described by the change of orientation of any line, say the line BC, which is horizontal has now become inclined and this motion is delta theta.

Since 3 independent coordinates namely, delta S_x, delta S_y and delta theta, can completely describe this motion of the rigid body, it has 3 degrees of freedom. Unconnected rigid body in plane motion has 3 degrees of freedom. However, as soon as this rigid body is connected to another through a kinematic pair, 1 or more of these 3 degrees of freedom are restricted or lost. So, let us see, what kind of kinematic pairs are allowed for a plane motion of a rigid body.

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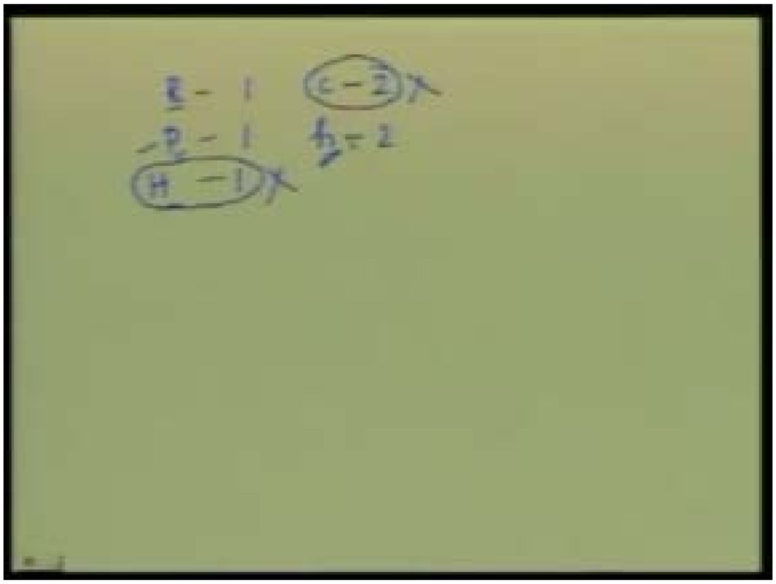

If we recall, the revolute pair has 1 degree of freedom, a prismatic pair has 1 degree of freedom and a screw pair has 1 degree of freedom; whereas, a cylindrical pair has 2 degrees of freedom and a higher pair has 2 degrees of freedom. Since, there are only 3 degrees of freedom available, at most up to 2 degrees of freedom can be restricted. So, it appears we can have either a revolute pair or a prismatic pair or a screw pair or a cylindric pair or a higher pair. However, a little thought would convince us that during a connection, say a helical pair and a cylindrical pair, the point on the moving body does not remain in a plane; it moves along a helix; so these two pairs are not allowed in a planar mechanism. That leaves us only with a revolute pair, prismatic pair and higher pair. Not only that for all the points to remain in one plane, axes of all these revolute pairs must be parallel and perpendicular to the plane of motion. At this stage, let us define certain words which will be used very often while describing kinematic analysis and synthesis of mechanism.

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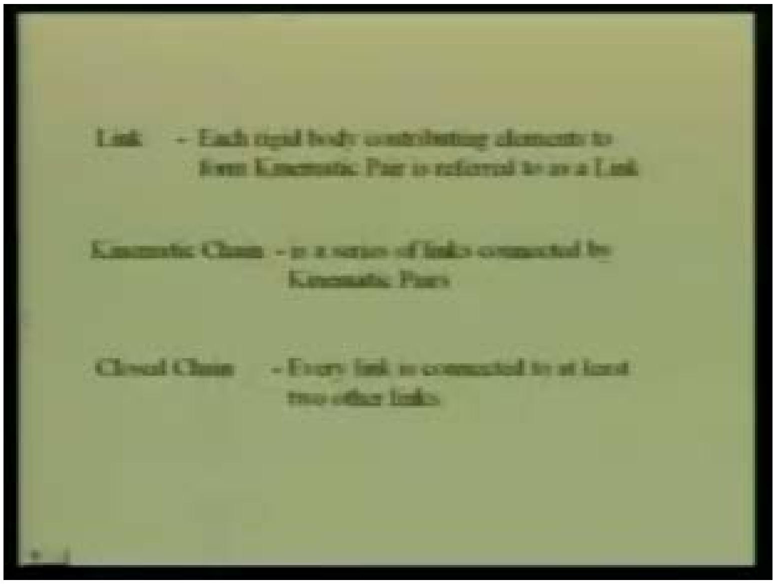

Link - Each rigid body contributing elements to form Kinematic Pair is referred to as a Link

Kinematic Chain - is a series of links connected by Kinematic Pairs

Closed Chain - Every link is connected to at least two other links

Let us start with the definition of link. Each rigid body contributing elements to form kinematic pair is referred to as a link. Kinematic chain is defined as a series of links connected by kinematic pairs. Such a kinematic chain is said to be closed, if every link is connected to at least 2 other links. That means, it may be connected to 3, 4 or 5 links, but definitely at least 2 other links. If a particular link or some links are connected to only 1 other link, then the chain is said to be an open chain. These links are also classified according to the following description:

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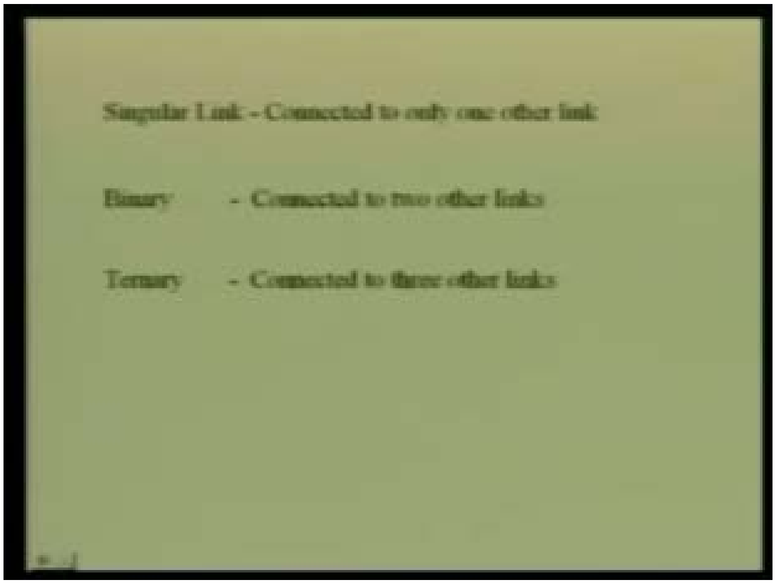

Singular Link - Connected to only one other link

Binary - Connected to two other links

Ternary - Connected to three other links

We call a link, a singular link if it is connected to only 1 other link. If a link is connected to 2 other links, then we call it a binary link. We call a ternary link which is connected to 3 other links. Similarly, quaternary and other higher order links can be defined depending on the number of other links that it is connected to. From these definitions, it is clear that a closed chain cannot have any singular link, because in a closed chain the minimum order of the link must be a binary link.

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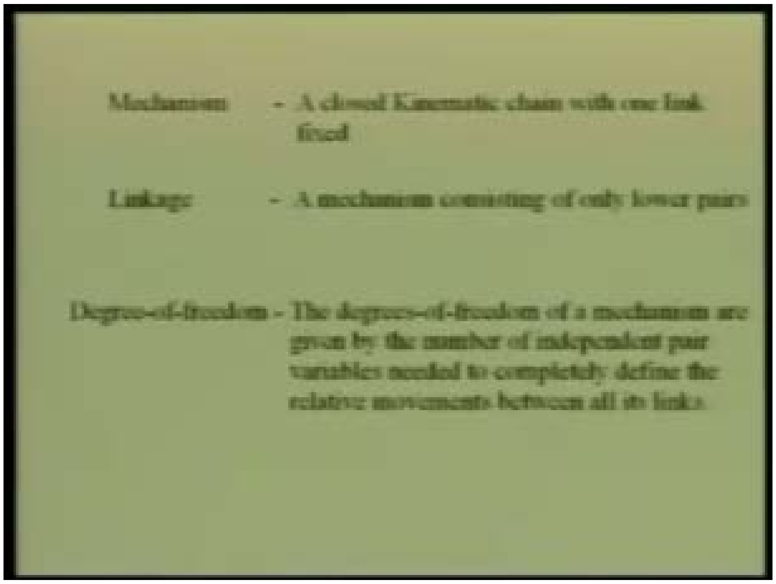

Mechanism - A closed Kinematic chain with one link fixed

Linkage - A mechanism consisting of only lower pairs

Degree-of-freedom - The degrees-of-freedom of a mechanism are given by the number of independent pair variables needed to completely define the relative movements between all its links.

At this stage, we can define a mechanism as a closed chain in which 1 link is fixed. By this fixed link, we mean the frame of reference or the foundation of the machine or mechanism. We must remember that the relative motion of all other links with respect to this frame, which is of importance, but this fixed link in real life may not be actually fixed. For example, if we talk of a windshield wiper mechanism which we have seen in the earlier lecture; here, the relative movements of various mechanisms, particularly those of the wiper blades with respect to the body of the automobile, or the windscreen which is a part of the body of the automobile is of importance. So, the body of the automobile can serve the purpose of the fixed link. Even if the automobile is moving, we still consider the body of the automobile as the fixed link.

Similarly, we can talk of the steering mechanism of an automobile. Here, the rotation of the steering wheel has to be converted into the rotation of the wheels on its axes. Both these axes and the steering wheel are mounted on the body of the automobile. So, movement of the body of the automobile is of no importance. We are interested in converting the rotation of the steering wheel into the rotation of the wheel, and that relative motion is independent whether the body of the automobile is moving or fixed. Consequently, in such mechanisms we will consider the body of the automobile as our fixed link. Another word which is very commonly used as a synonym for mechanisms is a linkage. In some books, the linkage is defined as a mechanism consisting only of lower pairs. If there is a higher pair, then we do not call it a linkage; we rather call it a mechanism. In our last lecture, we define the degrees of freedom of a kinematic pair.

Let me define the degrees of freedom of a mechanism. The degrees of freedom of a mechanism are given by the number of independent pair variables needed to completely define the relative movements between all links. A mechanism is said to be constant if the degrees of freedom of the mechanism equals the number of independent input. If there is a single input and the degree of freedom of the mechanism is also 1, then we call it a constant mechanism. Similarly, if the degree of freedom of the mechanism is 2 and we have 2 independent inputs, then also we say that the mechanism is constant. By constant mechanism, we mean that the output can move in a unique fashion. We shall demonstrate it by three models of arc moving machinery or tipper dumper having 1, 2 and 3 degrees of freedom. Obviously, in a planar mechanism, we can have at most 3 degrees of freedom.

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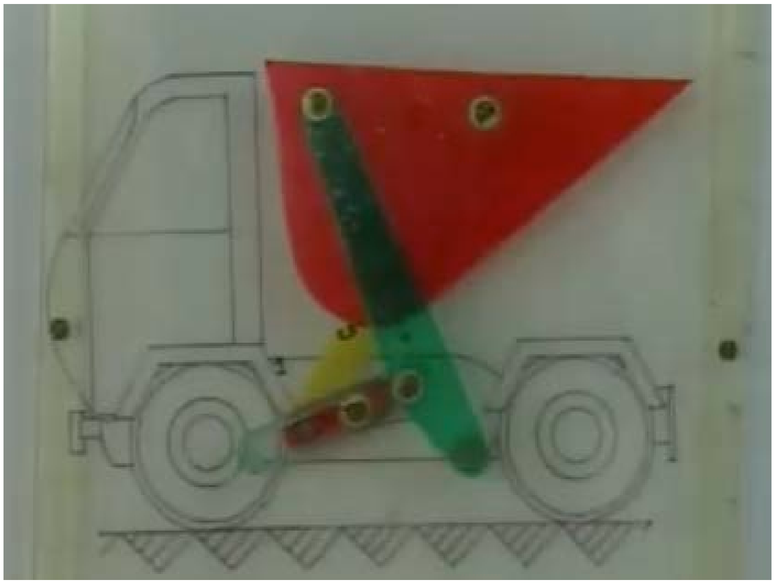

As an example of a single degree freedom mechanism, let us look at this model of a tipper dumper. This has 5 moving links and one fixed link which is the body of the dumper. Here, the input motion is provided by this hydraulic actuator which tilts the dumper bin. The linear motion of the actuator is being converted to the dumping motion of the bin. This has single degree of freedom. It means that for a given input, say this much, this position of the dumper bin is unique (Refer Slide Time: 11:15 min). We cannot change anything, so long we do not change the input motion. In other words, for this input motion of the actuator only 1 of the 3 independent coordinates of the dumper that is XY of a particular point and the rotation theta, only 1 of these 3 variables can be specified. Other two are automatically fixed. That means, I can take the X-coordinate of this particular point of the dumper anywhere and accordingly the needed input is decided - what is needed, but Y-coordinate and theta, that is the orientation of the dumper bin are automatically decided.

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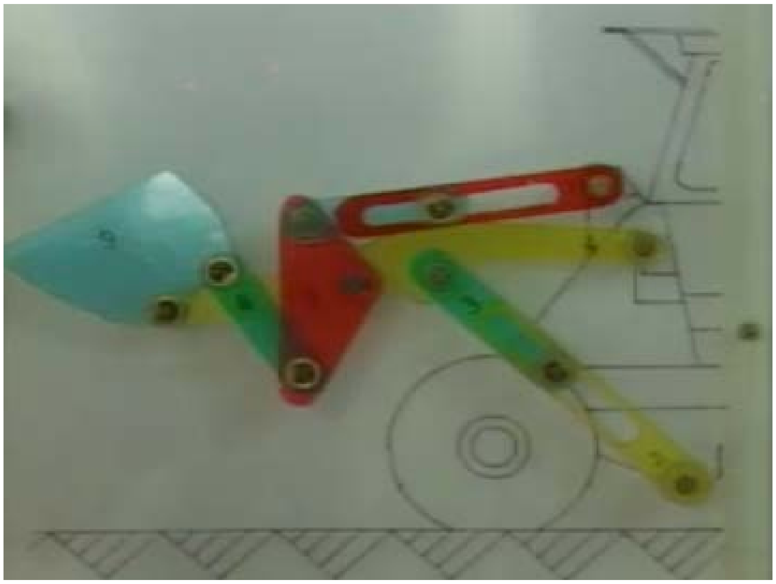

As an example of 2 degrees of freedom mechanism, let us look at this model of a front-end loader. This mechanism has eight moving links and fixed link is again the body of the machine. This is the second link, if I call that as the first link, the body of the machine that is the fixed link. This is the third link, fourth, fifth, sixth, seventh, eighth and ninth. This output bin is the link number 9. Here, we have 2 independent inputs provided by these 2 hydraulic actuators. Since it has 2 degrees of freedom, I can control 2 of the coordinates of this output bin. Say X-coordinate of a particular point and the rotation theta or the orientation theta of this output bin by controlling independently, these 2 input motions (Refer Slide Time: 13:11 min). We might notice that this movement (Refer Slide Time: 13:21 min) is not changing the orientation of the bin so much. It is mainly changing the Y-coordinate of this particular point; whereas if this input motion is provided, this point (Refer Slide Time: 13:32 min) is not moving so much, it is the orientation of this bin which is being primarily controlled. So, this is an example of 2 degrees of freedom mechanism with 2 independent inputs. That is why it is also a constant mechanism. By that, we mean if we fix up both the inputs then the output bin takes up a particular location and orientation.

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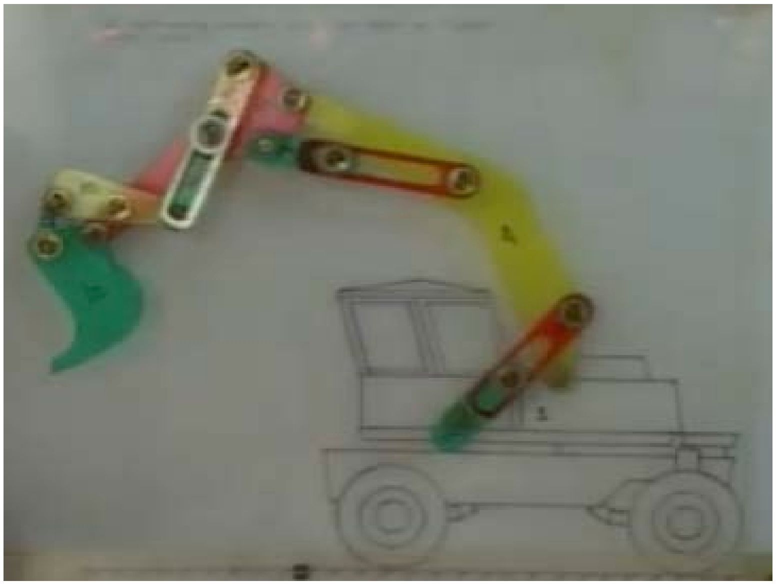

As an example of a mechanism with 3 degrees of freedom, let us look at this model of arc moving machinery. Here again link number 1 - the fixed link - is the body of this stuck and there are 11 other moving links 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. This twelfth link is the output link of the bin of this arc moving machinery. There are 3 hydraulic actuators that means there are 3 prismatic pairs here, rest of the pairs are all revolute pairs. As these 3 hydraulic cylinders are moved, the output bin can move in this plane of motion. In fact, there are 3 independent inputs. So, all the 3 motions that is - two translation and the rotation of this output bin can be independently controlled. As we might notice that if this hydraulic cylinder is moved, not much change in the orientation of the bin is taking place. Same is true even for this actuator (Refer Slide Time: 15:36 min), whereas the third actuator is mainly changing the orientation of the bin without causing much change in its location.

We have just seen 3 examples of constant planar mechanisms. The first one had single degree of freedom with 1 input. The second one had 2 degrees of freedom with 2 independent inputs and the third one has 3 degrees of freedom with 3 independent inputs. Since the number of inputs and the number of degrees of freedom of the mechanisms were same, all the mechanisms were constant. That means, when all the inputs are given a prescribed value, all the links have specific configuration.

However, sometimes we also use mechanisms which are not constant, that means the number of input is not equal to the number of degrees of freedom.

A typical example is the differential gear box of an automobile. Here, we know the input is only one, that is the popular shaft which is coming from the gear box. But if the car has to take a turn, then the 2 wheels must rotate at different speeds, because we need to have 2 independent speeds depending on the radius curvature of the turn from the same single independent input, we must have a mechanism with 2 degrees of freedom. So the differential, which we will study later, has 2 degrees of freedom but with a single input. Consequently, such a differential gear box does not constitute a constant mechanism. For the study of kinematics, we need to know only the type of pair, their relative location, their sequential order and that is all. All other geometric features or dimensions or inertial parameters are completely irrelevant so far as the study of kinematics is concerned. Consequently, for the study of kinematics, we always make a skeleton diagram which removes all irrelevant parameters and return only those geometric features which are essential to study the relative movement, in other words, which govern the relative movement. This skeleton diagrams are called the kinematic diagrams. To draw these kinematic diagrams we follow certain conventions. We now discuss all these conventions.

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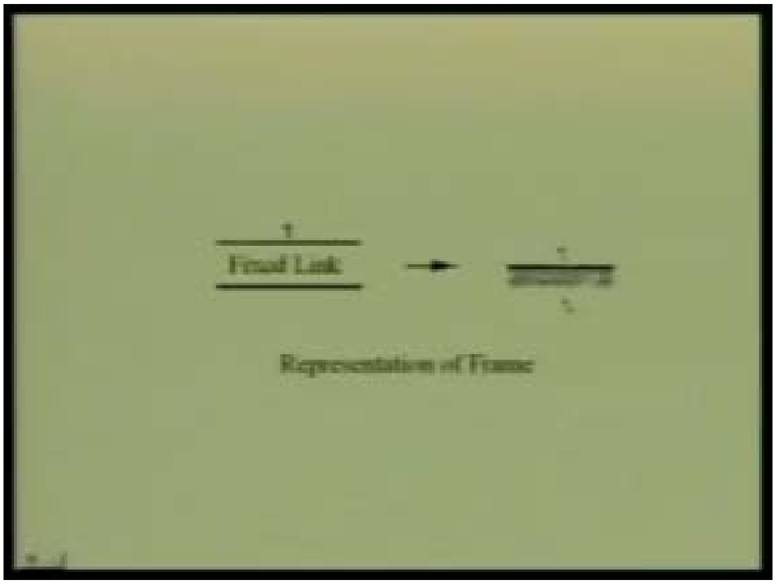

Representation of Frame

As we have said every mechanism has a fixed link and the fixed link is given the number 1 and it is indicated as fixed by having these hatch lines.

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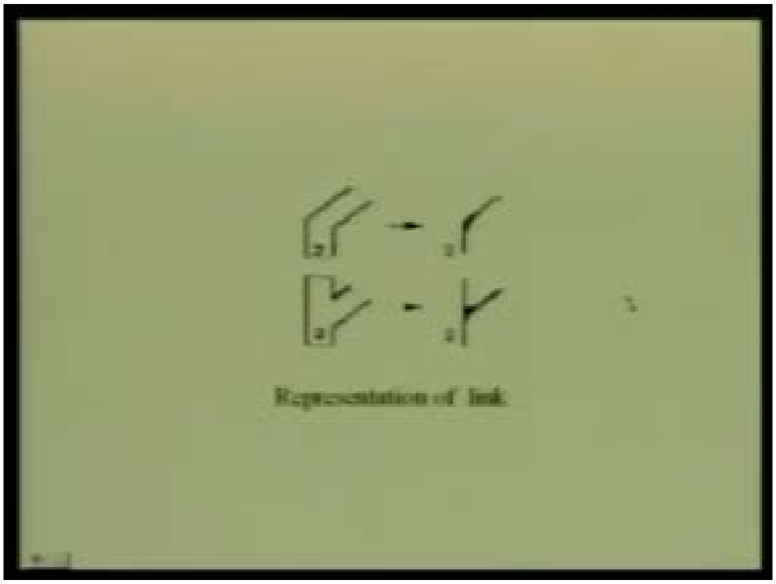

Representation of link

All the numbers are linked sequentially starting from 1. This diagram shows that even if the link has some width or some height which are completely irrelevant so far as the kinematics is concerned, so in the skeleton diagram, we represent these links with say number 2 by a line joint. This link, having such a bifurcation will be represented by this line joint.

The next slide shows how we represent a revolute pair in a kinematic diagram.

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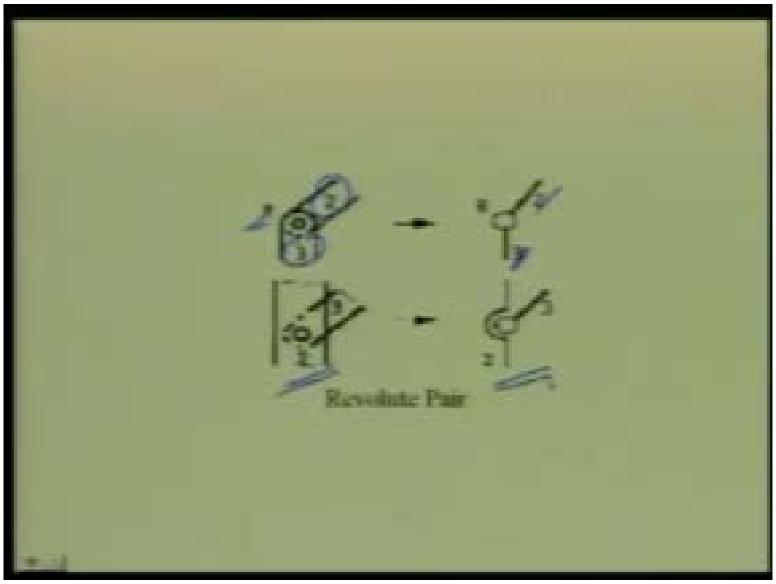
Revolute Pair

Suppose link number 2 and link number 3 are connected by a revolute pair. In the kinematic diagram, link 2 is represented by this line and link 3 is represented by this line are connected by a revolute pair. The revolute pair is represented by a small circle and the centre of the circle denotes the axes of the revolute pair. For example, if the actual construction looks like this, the kinematic diagram will be shown in this. The link 2 which extends beyond the revolute pair, as seen here, is represented by this line diagram.

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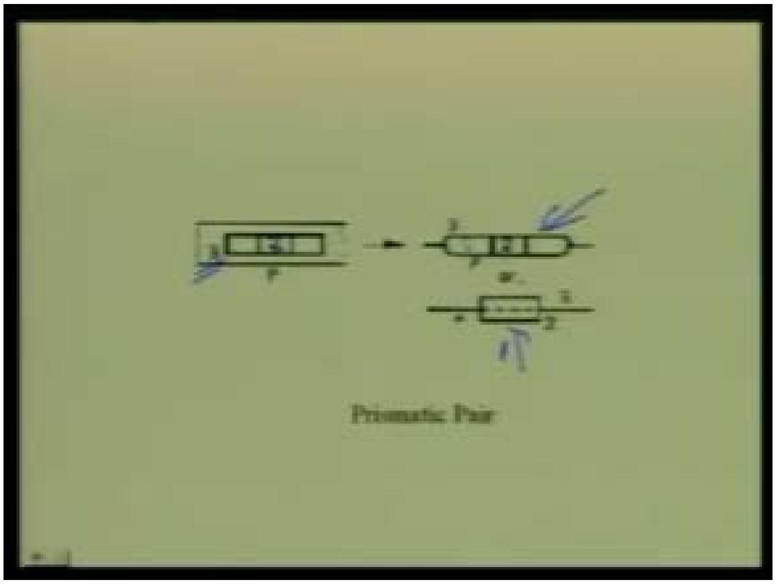
Prismatic Pair

Prismatic pair is between body number 2 and body number 3. The block which has the body 2 has relative translation along this slot. In the kinematic diagram, we may show this prismatic pair between body 2 and 3 by this line diagram or sometimes to make it completely unambiguous, we represent it like this. There is a prismatic pair between body 2 and body 3.

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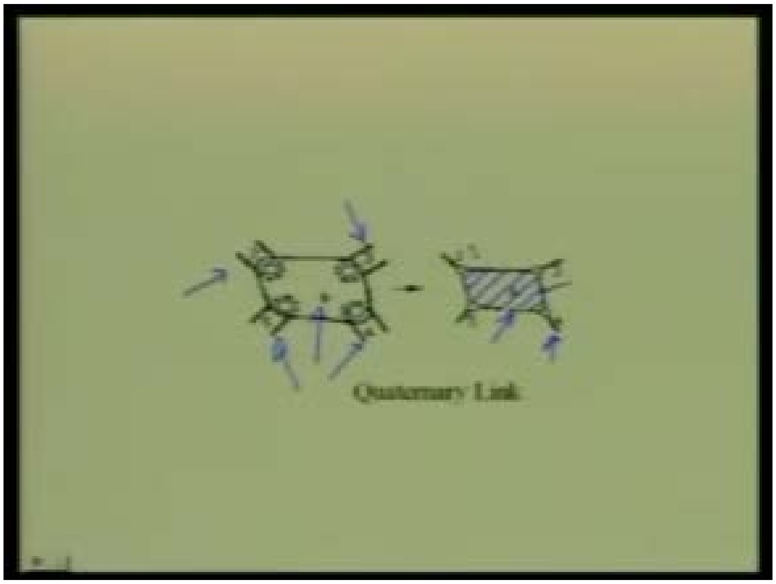

Quaternary Link

This diagram shows how higher order links must be represented. Here as we see, link number 2, 3, 4 and 5 are connected to a quaternary link, which is number 6. Since link number 6 is connected to 4 other links, this is a quaternary link. In the kinematic diagram of the line sketch, if it is represented like this, then there is ambiguity (Refer Slide Time: 21:04 min). One may interpret that at this revolute pair, 3 binary links - this one, this one and link number 4 - are being connected. To avoid this ambiguity, we should hatch (Refer Slide Time: 21:22 min) this link number 6 to represent that this is a single rigid body and this is a quaternary link connected to 4 other links.

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3R-1p Linkage

This diagram shows how to represent a linkage where there is prismatic pair. We have a 4-link mechanism. Link number 1, the fixed link is connected to the link number 2 by a revolute pair at this location (Refer Slide Time: 21:47 min). Link number 3 is connected to link number 2 by another revolute pair at this location (Refer Slide Time: 21:55 min). There is a prismatic pair between link number 3 and link number 4 and the direction of the prismatic pair is given by this line (Refer Slide Time: 22:05 min). Link number 4 is again connected to the fixed link number 1 by a revolute pair. In the kinematic diagram, since the axis or locations of the axis of the revolute pairs are important, we have to represent these 3 revolute pairs as it was in the original diagram.

However, the prismatic pair has only direction that is represented by the axis of this cylinder. For a prismatic pair, the location of the axis is not important, only the direction is. So here, what we do, link number 3 which is connected to link number 2 by a revolute pair, we draw the direction of the prismatic pair to the location of this revolute pair. And link number 4 has the prismatic pair with link number 3, the direction of the prismatic pair has been retained as in the original diagram and all other revolute pairs, their locations have been retained. This is the kinematic diagram which does not have any extra irrelevant geometric dimensions which was present in this original diagram. So it has 2 kinematic dimensions namely, the length of this link, let me call it l_2 and the distance between these 2 revolute pairs located on link number 1 which we may call l_1. So, l_1 and l_2 are the only 2 relevant geometric dimensions and the direction of this prismatic pair which is parallel to this direction that is important and that has been detailed. This is what we call a kinematic dimension of the original mechanism.

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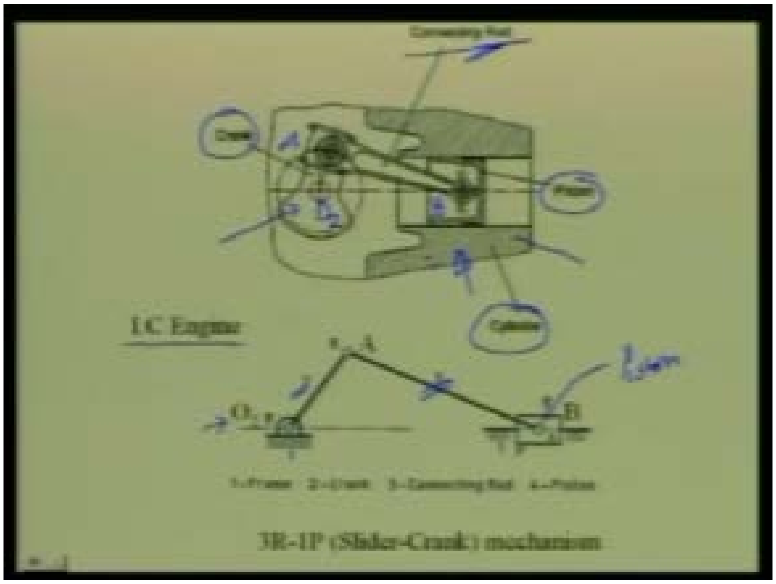

3R-1P (Slider-Crank) mechanism

As a further example, let us talk of the mechanism which is used in an IC engine. As we know here, the cylinder block which is a part of the automobile frame, in which this piston executes to and fro motion (Refer Slide Time: 24:18 min). These small dots as we can see are nothing but the piston rings which moves along with the piston (Refer Slide Time: 24:28 min). In fact, they are the same rigid body as the piston.

Then, the piston is connected to this connecting rod. So, what we know as the gudgeon pin, at this location, say we name it as B. The other end of the connecting rod, which is the beginning of the connecting rod, is connected to this crank which is mounted on to the crank shaft. Now to draw the kinematic diagram of this mechanism, we should first try to determine, how many different rigid bodies are involved? One is the cylinder which is the fixed link. Two is the piston which has the prismatic pair along the horizontal direction. Three is the connecting rod which has a revolute pair if the piston at this location B is the crank which has a revolute pair with that connecting rod at this location, let me call it A. So, for the kinematic diagram which is drawn below this diagram, has three revolute pairs: one at $O_{2,}$ which is the location of the crank shaft; one at A, between the crank and the connecting rod and at B between the connecting rod and the piston. The piston and the cylinder have a prismatic pair. 4 denotes the piston, 3 denotes the connecting rod and 2 denotes the crank. O_2 A B is a kinematic diagram of this mechanism which is normally called a slider-crank mechanism.

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Sometimes, certain kinematics spheres can be disguised due to certain practical considerations. As an example, let us look at this model of a punching press driven by this eccentric. Here, the eccentric is connected to the motor shaft eccentrically and this is the connecting rod which is driven by the eccentric and the punching tool is connected to the connecting rod. The continuous rotation of the motor is converted into to and fro oscillation of the punching tool. It is very obvious, that the punching tool has a prismatic pair with the frame of the pairs. There is a revolute pair between the punching tool and the connecting rod; there is a revolute pair between this eccentric and the crank shaft. What is the kinematic pair between the connecting rod and this eccentric? (Refer Slide Time: 27:23 min)

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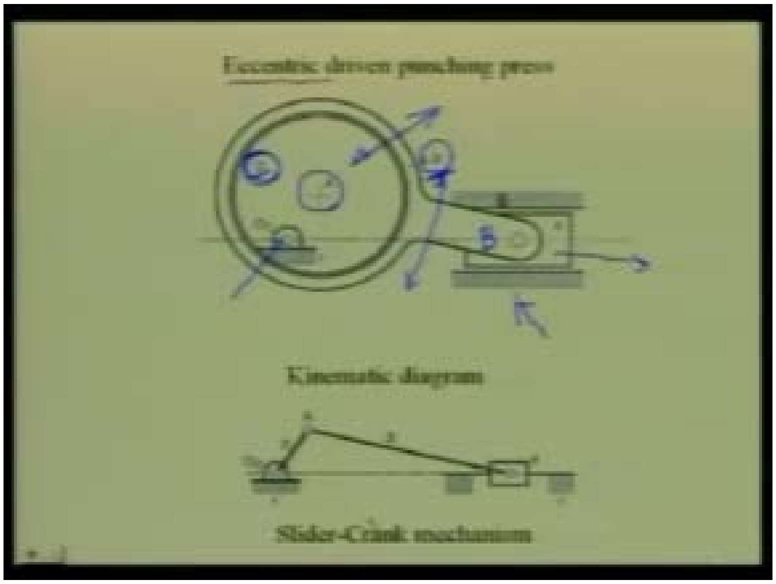

Eccentric driven punching press

Kinematic diagram

Slider-Crank mechanism

Now let us look at this schematic diagram of the punching press, the model of which we have just now seen. This punching press is driven by this eccentric wheel which is number 2. This represents the punching tool and this fixed link 1 represents the frame of the machine. The connecting rod is represented by this rigid body 3. It is easy to identify the prismatic pair between punching tool and the frame of the machine. There is a revolute pair between the punching tool and the connecting rod at this location. This is the shaft which rotates the eccentric number 2 (Refer Slide Time: 28:15 min). So, there is a revolute pair between the frame of the machine and this link number 2 at O_2.

The question is to determine the kinematic pair between these two links namely 2 and 3. To decipher the kinematic pair, the best thing is to forget about all other connections and hold one of these two links, say link number 2 fixed and try to move link number 3. It is obvious that link number 3 cannot move in the plane of motion by any translation in any direction, because, this circular eccentric is placed in a hole in this connecting rod. However, if we hold link to fixed, then link 3 can definitely be rotated about an axis perpendicular to this plane of motion. This rotation takes place at the centre of this circular eccentric that is at A. Consequently, the kinematic diagram of this punching press driven by an eccentric will look as shown in the next picture. As it is clearly seen, it is a simple slider-crank mechanism which revolute pairs at $O_{2,}$ A and B and a prismatic pair between frame of the machine, that is link number 1 and the punching tool that is link number 4.

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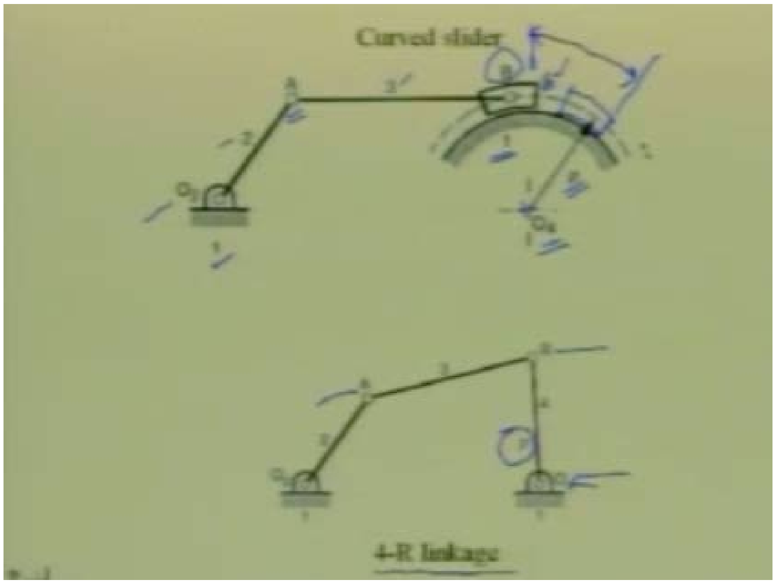

The next figure shows another folding mechanism 1, 2, 3 and 4. There is a revolute pair at O_2 between link 1 and 2, at A between link 2 and 3 and at B between link 3 and 4. 4 is the curved slider and it is sliding over the cylindrical surface which is also fixed, that is given by the number 1. The question is what is the kinematic pair between link 1 and 4? It may appear because it is a curve slider that there is a sliding movement and there is a prismatic pair between 1 and 4. However, that is wrong. If link 4 moves on this link 1 and takes up this configuration, it is clearly seen that this line has undergone rotation by this amount (Refer Slide Time: 30:55 min). So, the relative motion between link number 1 and link number 4 is of rotation. There is a revolute pair and this rotation, the axis is at the centre of curvature of this circular portion, that is the centre of this circular portion which is O_4 and the radius of this circle is say rho.

Consequently, the kinematic diagram for this mechanism is as shown here by a 4 R linkage with a 4 revolute pairs at O_2, A, B and O_4 and the length of this link 4 is nothing but rho which is the radius of this circular portion.

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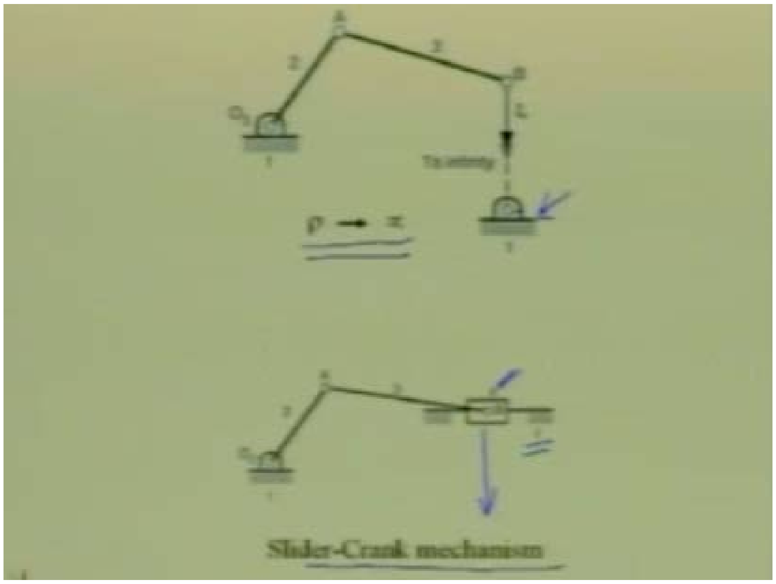

Slider-Crank mechanism

Now in the previous figure, if the radius of curvature rho is allowed decreasing indefinitely and becomes infinite, then the rotary motion of link 4 is converted into translatory motion of link 4. So, as rho goes to infinity, this revolute pair which goes to infinity boils down to a prismatic pair between link 1 and link 4. Again, we get a slider-crank mechanism, if the radius of curvature of the curved slider tends to infinity.

In other words, we can say that if the prismatic pair is nothing but the limiting case of a revolute pair, where the revolute pair is located at infinity, but along the direction which is perpendicular to the direction of relative sliding, which in this case is horizontal. So, this prismatic pair is equivalent to a revolute pair as shown in this diagram, with this revolute pair at infinity. But, it must be on a vertical line passing through B.

Next, we discuss a very important principle in kinematics which is known as kinematic inversion. As we shall see later, this kinematic inversion is very useful particularly for kinematic synthesis. We have already defined mechanism as a closed kinematic chain with one of its links fixed. Kinematic inversion means that the process of fixing different links of the same kinematic chain to produce distinct mechanism.

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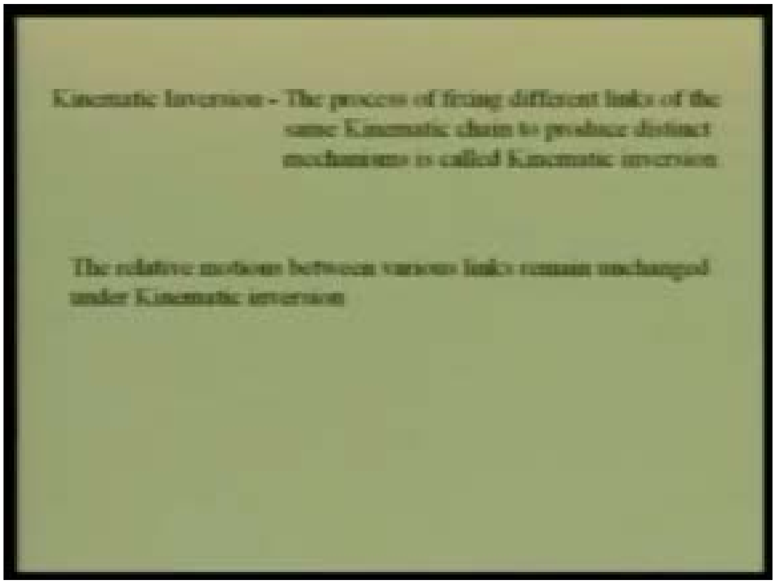

Kinematic Inversion - The process of fixing different links of the same Kinematic chain to produce distinct mechanisms is called Kinematic inversion

The relative motions between various links remain unchanged under Kinematic inversion

Mechanism is a closed kinematic chain with one of its links fixed, but I can fix any link fixed at a time. So from the same chain, we get a different mechanism. The most important thing about this process of kinematic inversion is that the relative motions between various links are independent of the kinematic inversion. So long as the chain is same, the relative motion between various links remains the same, independent of which particular link is kept fixed. We shall explain this principle of kinematic inversion through an example.

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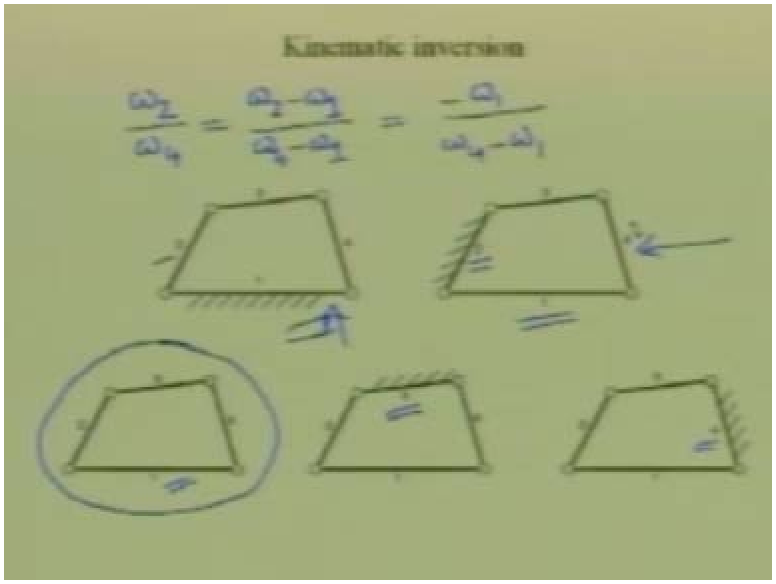

Kinematic inversion

$$\frac{\omega_2}{\omega_4} = \frac{\omega_2-\omega_1}{\omega_4-\omega_1} = \frac{-\omega_1}{\omega_4-\omega_1}$$

Let us consider a 4R kinematic chain as shown in this diagram (Refer Slide Time: 34:25 min). From the same chain holding 1 or 2 or 3 or link 4 fixed one at a time, we can get 4 different mechanisms. Like this by holding link number 1 fixed; it is the same chain, but if we hold link number 2 fixed, then we get this mechanism (Refer Slide Time: 34:41 min). Similarly, we get two other different mechanisms respectively holding link 3 or the link 4 as the fixed link (Refer Slide Time: 34:49 min).

Let me now explain, what we mean that the relative motion is independent of the kinematic inversion. As both of these mechanisms were obtained from the same kinematic chain by a process of kinematic inversion, the relative motion between various links of these two mechanisms will be same. For example, if we say the ratio of the angular velocity of link number 2 to that of link number 4, that is omega$_2$ by omega$_4$, which is the same as the relative velocity of link number 2 with respect to link number 1 divided by angular velocity of link number 4 minus omega$_1$. Since, omega$_1$ is 0, link number 1 is fixed. This relative angular velocity remains the same. So, for this mechanism omega$_2$ is 0. So it will become minus omega$_1$ divided by omega$_4$ minus omega$_1$. So, the ratio of absolute angular velocities of link number 2 and 4 in this mechanism is the same as this ratio minus omega$_1$ by omega$_4$ minus omega$_1$ of this mechanism at this particular configuration. This principle as I said earlier, of kinematic inversion is very important for the entire study of kinematics, especially, when we study of kinematic synthesis.