Kinematics of Machines

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Module – 06 Lecture - 3

Let us start our discussion on the third type of three position synthesis that is function generation.

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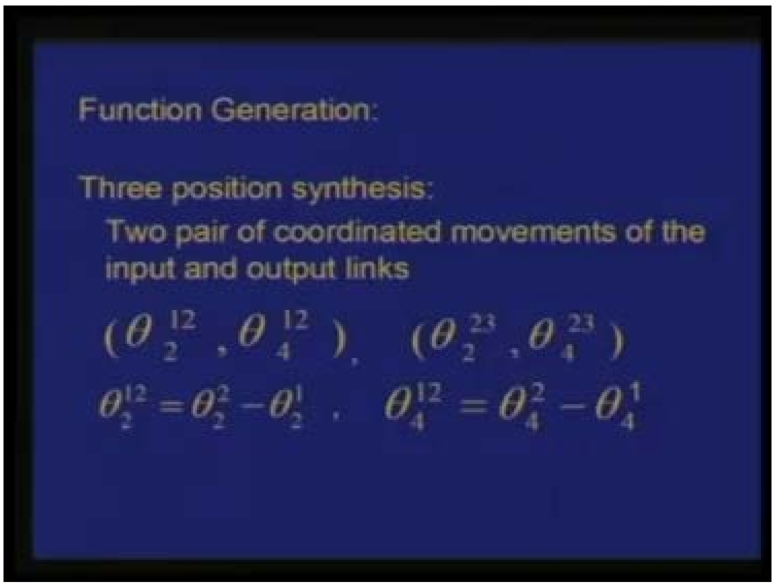

By function generation, we mean how to maintain the specified relationship between the input and output movements. We shall take the example of a 4R function generator with three position synthesis. By three position synthesis, we mean two pairs of coordinated movements of the input and output links, namely theta$_2$ one two and theta$_4$ one two where theta$_2$ represents the orientation of the link number 2 which is the input link and theta$_4$ represents the orientation of link number 4 which is the output link. This theta$_2$ and theta$_4$ can be measured from some reference line, say the line of frame O_2O_4. Now, let us look at this superscript, theta$_2$ one two that is the changing the value of theta$_2$ from position 1 to position 2 that is, theta$_2$ one two is nothing but theta$_2$ two minus theta$_2$ one. The superscripts one and two refer to the two configurations 1 and 2. For two pairs of coordinate movement, this is one pair that has a linkage moves from configuration 1 to 2 and theta$_4$ one two. Similarly, theta$_4$ two minus theta$_4$ one, that is the output movement from configuration 1 to configuration 2. So that is the prescribed value for this theta$_2$ one two and theta$_4$ one two so, we have to satisfy by designing the 4R linkage. Similarly, the movement of the input and the output links from configuration 2 to 3 described by theta$_2$ two three and theta$_4$ two three are also prescribed which have to be satisfied. Let me now explain this symbol with reference to a figure and then explain how to graphically synthesis the 4R linkage such that these two pairs of coordinated movements are achieved.

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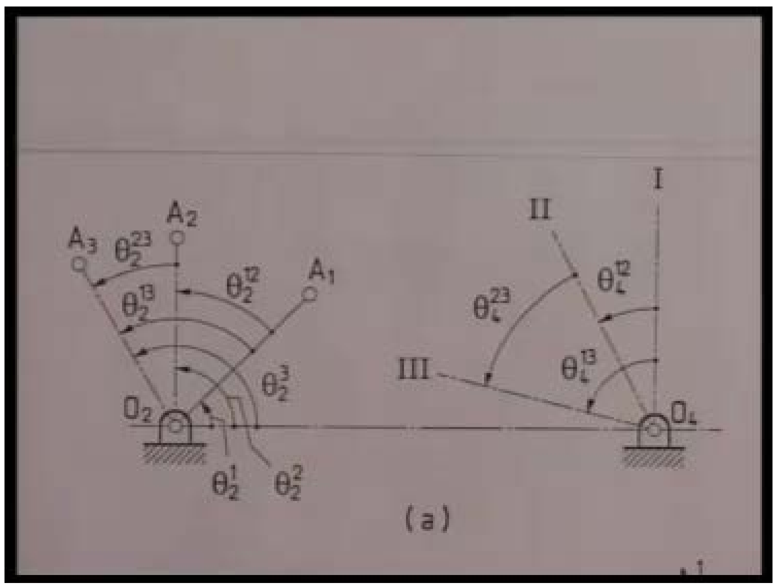

This figure explains the symbols that we have just now talked about for three position synthesis of a 4R function generator. Let O_2 and O_4 be the two fixed pivots that define a line of frame O_2O_4. The input link is O_2A. Now as this input link moves from position 1 to 2, the rotation is theta$_2$ one two. If we measure theta$_2$ from O_2O_4, then this angle that O_2A_2 makes with O_2O_4, we call it as theta$_2$ two, the superscript two refers to the second configuration. Similarly, the angle made by O_2 A_1 is theta$_2$ one, so theta$_2$ one two is theta$_2$ two minus theta$_2$ 1. This is the movement of the input link as the linkage goes from first to second configuration. Corresponding movement of the output link is also prescribed as theta$_4$ one two. Similarly, as the linkage goes from the second to the third configuration that is the crank pin goes from A_2 to A_3, the rotation is theta$_2$ two three. That is theta$_2$ three minus theta$_2$ two, the difference of these two angles, that is the movement of the input link from second to third configuration that is theta$_2$ two three that is also prescribed and we desire that the corresponding movement of the output link should be theta$_4$ two three. We can easily see that this whole angle which is theta$_2$ three minus theta$_2$ one, that is this angle I can say theta$_2$ one three. I ask to note the superscript one three means theta$_2$ three minus theta$_2$ one, two three means theta$_2$ three minus theta$_2$ 2 and so on.

To solve this problem, that is to satisfy these two pairs of coordinated movements theta$_2$ one two must generate theta$_4$ one two and theta$_2$ two three must generate theta$_4$ two three. We proceed as follows. We assume the locations of O_2 and O_4, the fixed pivots according to our convenience. We also choose the crank pin A_1 arbitrarily. The only thing remaining is to determine the location of B_1, the other moving hinge such that O_2, A, B, O_4, the 4R linkage is complete. To do this again, we apply the principle of kinematic inversion. If we want to locate the point B_1, then we should hold link 4, that is the follower link fixed at its first configuration and allow this fixed link O_2O_4 to move such that the same relative movements are maintained. Accordingly, we find the inverted positions of the point A that is A_1, A_2 and $A_{3.}$ Since we are holding link 4 fixed, that is the point B is not moving from B_1, the point A must move in a circle with BA as radius. As we have seen earlier, first step is to determine the inverted position of A_1, A_2, A_3, holding the link 4 fixed at its first configuration.

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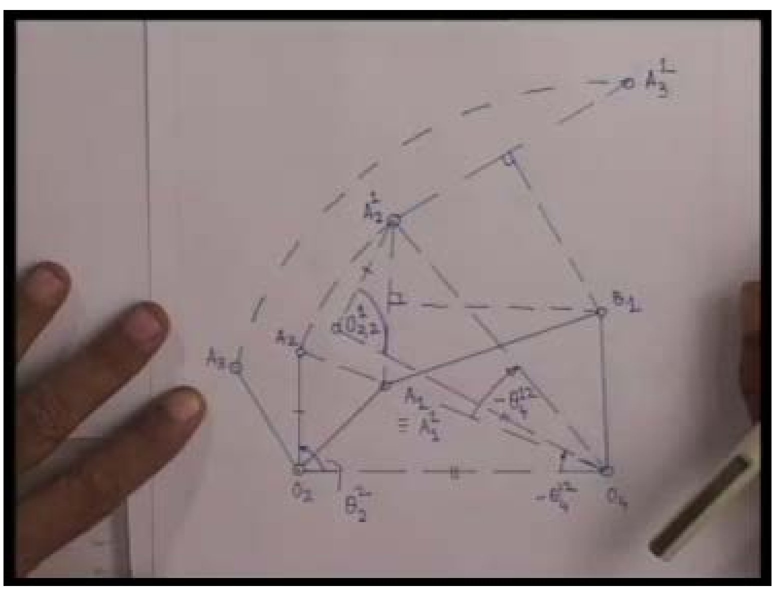

To do this we proceed as follows. Let me now first choose the fixed hinges O_2 and O_4 arbitrarily. This is O_2 and this is O_4. We also choose the first position of the crank pin A arbitrarily, here which I call A_1 because the rotation of the input link O_2A is prescribed, I can find the second position of the crank pin which I call A_2. So the rotation of the input link from first to second configuration which is given to us is the angle between these two lines namely, O_2A_1 and O_2A_2. This is the angle of rotation $theta_2$ one two. Similarly, the third position is also known because $theta_2$ two three is prescribed and A_2 comes to $A_{3.}$ This is the third position of the crank pin A_3 and the angle between O_2A_2 and O_2A_3 is $theta_2$ two three. This point is O_4. So the only thing remained is to locate the point B_1 such that the $theta_4$ one two and $theta_4$ two three, the coordinated movements of the follower can occur.

To do this, we have decided to hold the follower fixed at its first position. Accordingly, applying the principle of kinematic inversion, that is to maintain the same relative motion, the fixed link O_2O_4 must rotate by an angle minus $theta_4$ one two. O_4B has been held fixed, so O_4O_2 moves in the direction opposite to $theta_4$ one two which was counter clockwise. So this angle is minus $theta_4$ one two. By minus I mean clockwise, because $theta_4$ one two was counter clockwise. So this is the inverted position of O_2, inverted position corresponding to the second configuration inverted on the first configuration The angle between O_2O_4 and O_2A_2, which is this angle that is $theta_2$ two, if I draw that I can find the inverted position of A_2 inverted on the first configuration. The superscript always refers to the configuration which has been held fixed. Here O_4B_1 has been held fixed, that is the follower is held fixed at its first configuration. So A_1 is identical to A_1 one. If we look at these two triangles namely, O_4, O_2, A_2 and O_4, O_{22} one and A_2 one, but we see that this length is same as this length, because that is the length of the fixed link. This length is same as this length, because that is the crank length and this angle is same as this angle because both of them are $theta_2$ two (Refer Slide Time: 10:39).

Consequently, these two triangles O_4, O_2, A_2 and this triangle O_4, O_{22} one and A_2 one, these two triangles are identical, because two sides and the included angle are same. We get O_2, O_4, $A_2{}^1$ is same as O_4A_2. In fact what is happened? This whole triangle has been rotated by the angle minus $theta_4$ one two. So we can say, I can go from A_2 to A_2 one by rotating O_4A_2 through an angle minus $theta_4$ one two. This is the principle of getting the inverted position of A_2 inverted on first configuration. Exactly the same logic can be applied to get the inverted position of A_3 by rotating O_4A_3 about the point O_4. This I will call A_3 one and O_4A_3 is rotated by minus $theta_4$ one three. So we get the three inverted positions A_1 which is A_1 one, A_2 one and A_3 one. B_1 is nothing but the center of the circle passing through these three points, so I can easily determine where B_1 is.

Draw a perpendicular bisector of this line. This is the perpendicular bisector. Similarly, drop a perpendicular bisector of this line A_2 one, A_3 one, this is the perpendicular. So these two perpendicular bisectors meet at B_1, that is B_1 is the center of the circle passing through A_1 one, A_2 one and A_3 one. We have designed the required linkage as in this first configuration as O_2, A_1, B_1, O_4. So we get the required 4R linkage to generate the two prescribed pairs of coordinated movements of the input and the output linkage O_2 A_1 B_1 is the required linkage.

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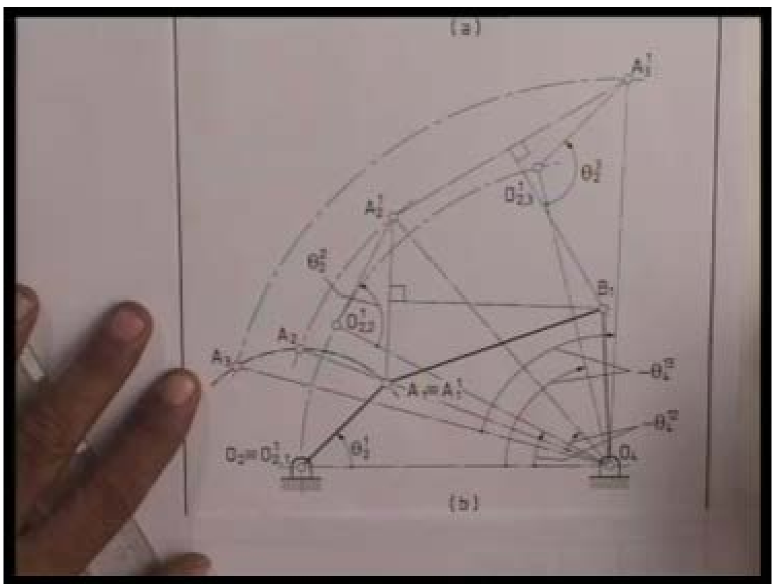

Let me go through the procedure once more. O_2 and O_4 were chosen arbitrarily. A_1, that is the location of the crank pin at the first configuration were chosen arbitrarily. From the given movements from the first to the second configuration of the crank, I can locate A_2 and A_3 on this circle with O_2 as center and O_2A_1 as the radius. A_1 O_2 A_2 is the angle theta$_2$ one two. A_2 O_2 A_3 is the angle theta$_2$ two three. Now, we apply the principle of kinematic inversion. Hold the follower fixed at its first configuration, which means the fixed link will rotate in the opposite direction by minus theta$_4$ one two and minus theta$_4$ two three to get the corresponding inverted positions. So O_2 goes to this point which I have called O_{22} one, that is the second position inverted on the first configuration. By rotating this link O_4O_2 about O_4, through an angle minus

$theta_4$ one two, now $theta_2$ two is known, that is the angle between O_2O_4 and O_2A_2. So I draw the same angle $theta_2$ two between this line O_4O_{22} one and O_2A_2, one. I get the point A_2 one. Then using simple geometry, we could prove that I could have obtained A_2 one simply by rotating O_4A_2 through an angle minus $theta_4$ one two. O_4A_2 is the line, I am rotating through an angle minus $theta_4$ one two and we automatically get the inverted position A_2 one. Similarly, by rotating O_4A_3 through an angle minus $theta_4$ one three, I get the inverted position of A_3 as A_3 one. Finally B_1 is obtained at the centre of the circle passing through these three points namely, A_1 one, A_2 one and A_3 one. Next, we will take an example of a slider crank mechanism such that, that can be used as a function generator that is the output link that is the slider movement is coordinated with the movement of the input of the crank.

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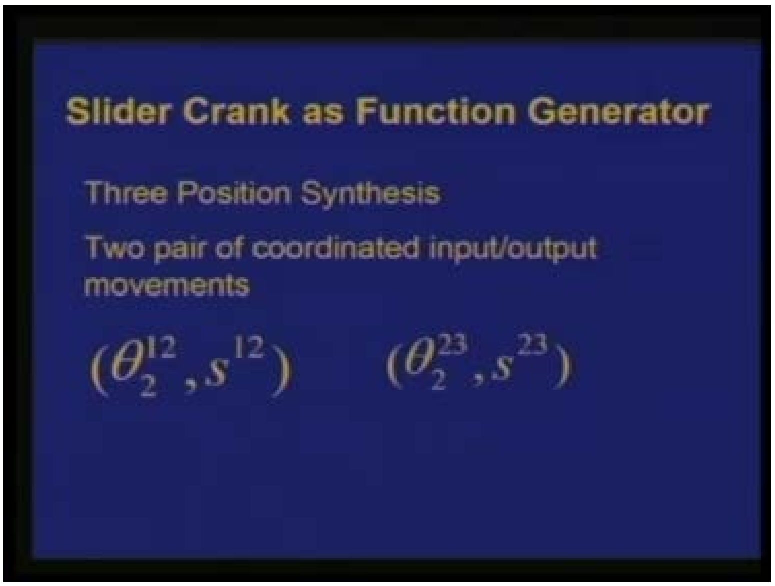

Let us now discuss the slider crank as the function generator where we treat the slider as our output link and crank as the input link. If we talk of three position synthesis as we have seen in the case of a 4R linkage, we mean two pair of coordinated input output movement. That means we have to design a slider crank such that the crank rotation $theta_2$ from position 1 to 2, that is $theta_2$ one two must generate a slider displacement in a particular direction which we shall take it to be horizontal say s one two. So this pair is prescribed $theta_2$ one two must generate s one two. Similarly $theta_2$ two three that is from the second to the third position must generate slider displacement s two three. So, these two pairs of coordinated input output movements are prescribed and we have to design the required slider crank mechanism.

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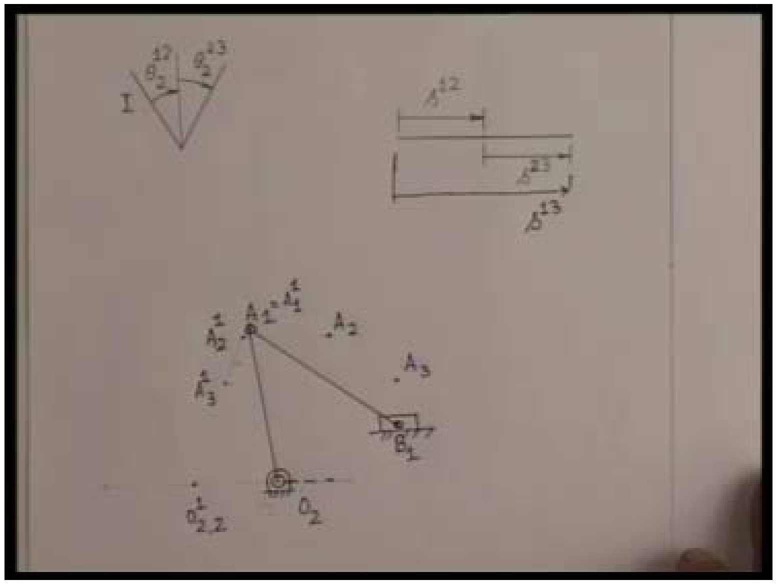

This diagram explains the two pairs of coordinated movements for a slider crank, $theta_2$ one two, this amount of clockwise rotation of the crank must produce s one two this much of horizontal displacement of the slide. Similarly, $theta_2$ two three that is from position 2 to position 3 of the crank this amount of rotation should generate s two three. This is the amount of horizontal sliding movement of the slide. Our job is to find the required slider crank mechanism. As before, we assume that crank step location O_2, according to our convenience we choose it arbitrarily. We also choose the crank length and the crank pin corresponding to this first configuration, if I call it configuration 1, we choose A_1 at this location. So we call this as A_1. Using this information, $theta_2$ one two and $theta_2$ two three, I can rotate O_2A_1 and get the corresponding locations of A which I call it as A_2 and A_3.

The only thing remaining is where should be the slider corresponding to the first configuration that is the other moving hinge connecting rod and the slider which we call as B_1. To determine B_1, we must hold the slider fixed and consider the kinematic inversion. The same relative movement between the crank and the slider, but we hold the slider fixed and allow the fixed link to move. So if we hold the slider fixed at the first position then corresponding to this movement of the slider, the fixed hinge will move opposite to that by the same amount. So it should come here. This we call as O_2, second position inverted on the first position. We also know that the angle between the horizontal line through O_2 at the A_2 position, second position. We draw here at the same angle and we get the inverted position of A_2, we call it as A_2 one. A_1 and A_2 one are coming out to very close.

As we have seen in case of the 4R linkage, A_2 one can be easily obtained by sliding A_2 by minus s one two. A_2 move to the left, because s one two is at right, minus s one two is to the left, so I move horizontally the point A_2 and I get A_2 one. Similarly, this whole distance s one two plus s two three, this whole distance is called s one three that is to the right. So holding the slider fixed, we move the fixed link and the crank to the left. So A_3, I move by minus s one three to the left and get to this point to which I called A_3 one. So holding the slider fixed at the first configuration B_1, because the connecting rod is a rigid member, the length AB remains fixed. So A_1 which is nothing but A_1 one, A_2 one and A_3one. We have got three points, here A_1 one, here A_2 one and here A_3 one.

We can easily find the centre of the circle passing through these three inverted positions of A and we get the point B_1. Accordingly, I have designed the required slider crank mechanism as O_2, A_1, and B1. O_2 A_1 B_1 is the required slider crank mechanism such that when A_1 goes to A_2, B_1 will move to the right by s one two and as A_1 goes to $A_{3.}$ This slider will move again to the right by magnitude s two three. As we have seen, we assume arbitrarily the location of the crank shaft O_2 and the crank pin A_1. Only determine the slider location corresponding to the first configuration at B_1 this revolute pair. They solve the problem also by choosing arbitrarily B_1 and find A_1 that is what we will do in the next example. The same slider crank mechanism, I will try to design but assuming the location of B_1 arbitrarily and find the required position of A_1. I would ask the students to solve the 4R function generator. The same way instead of choosing A and determining B, one can also solve the problem by choosing B and determining A. Of course, then we have to hold the input link that is the crank fixed and apply the principle of kinematic inversion, as we shall explain just now with respect to this slider crank mechanism.

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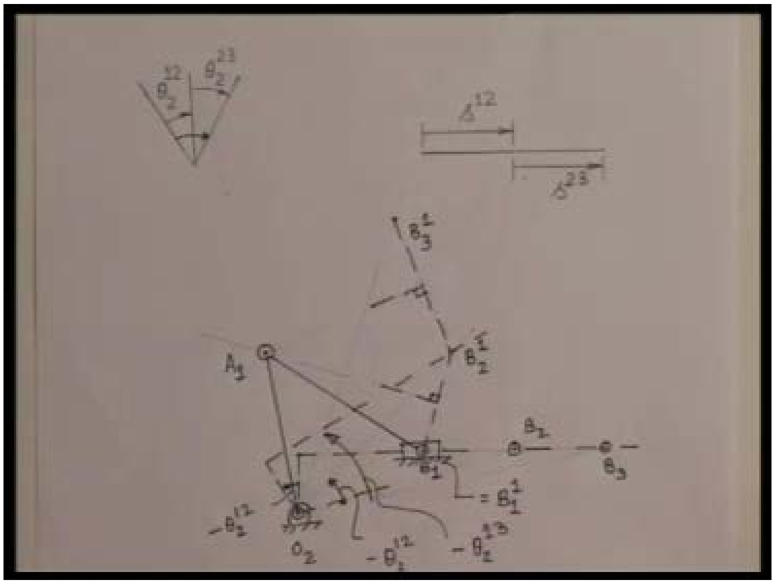

Let me now say, this design of this slider crank mechanism to generate these two pairs of coordinated movement as we have seen in the previous problem. It is the same problem, only thing is we solve it a little differently by assuming the crank pin location arbitrarily which we call it as O_2. We assume the other moving hinge which is connecting to the slider at the first position is here which I call it as B_1.

Generate these two movements s one two and s two three, B_1 must move to B_2 and B_3. B_2 B_3 is s two three. Only remaining task is to locate the crank pin A_1. So to determine that, we hold the crank fixed at its first position, such that the point B moves on a circle with A_1 as the centre. So we have to first find the inverted positions of B_2 and B_3, if we hold the crank fixed at its first location and apply the principle of kinematic inversion.

As we seen the fixed link is represented by this tag. This is the fixed link on which the slider is moving to the right. So if we hold the crank fixed then this link should rotate by minus $theta_2$ one two. The crank is held fixed, so O_2 point is not moving. This fixed link is rotating counter clockwise by $theta_2$ one two, which I call it as minus $theta_2$ one two. So the fixed link is now represented by this line and point B_2 is at a distance from here by so much. This is the distance of B_2 from this point, so from this point I take this distance and I get to this point B_2 which is nothing but B_2 one inverted position with the position 1 being held fixed. So that we can easily do by rotating O_2B_2, so the same angle. This is O_2B_2 is rotated by minus $theta_2$ one two. Similarly by rotating O_2B_3, this is the line O_2B_3, this was the line O_2B_2. This whole angle be minus $theta_2$ one three, which is $theta_2$ one three is this angle which is clockwise. So I rotate counter clockwise and we get to the point by rotating O_2B_3, I get to the point B_3 one. B_1 is nothing but B_1 one, B_1 is same as B_1 one.

In the center of the circle passing through these three points, B_1 one, B_2 one and B_3 one, determine the locations of A_1. We draw perpendicular bisector of these two lines as usual. We draw the perpendicular bisectors. This is perpendicular and this is also perpendicular. These two lines intersect to determine the center of the circle passing through B_1, which is B_1 one, B_2 one and B_3 one and we get A_1. We have designed the required slider crank mechanism O_2, A_1, B. We have got O_2, A_1, B_1 as the required slider crank mechanism, which will generate these two pairs of coordinated movements. Only difference is that I arbitrarily assume the location of B_1 and determine A_1. In the previous example, we assume the location of A_1 and determine the corresponding location for $B_{1.}$

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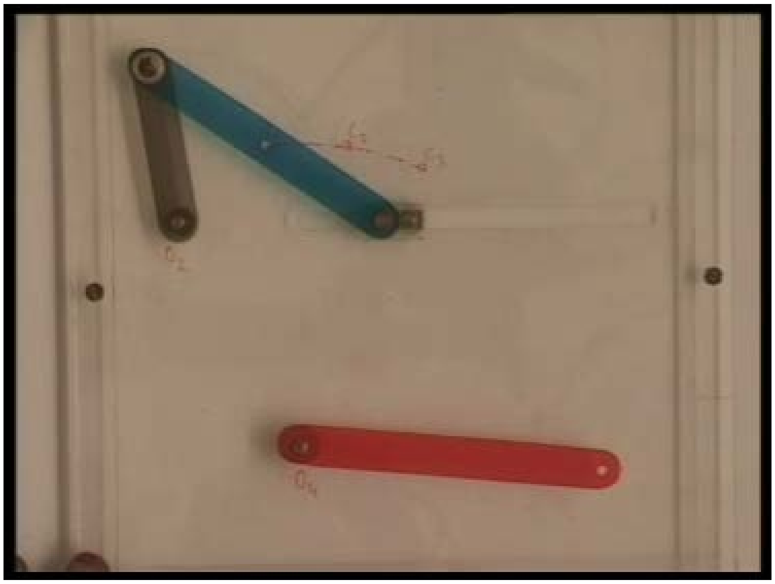

Now, we have explained the three position synthesis for path generation and function generation. We can attempt to solve another problem by a two state synthesis. Suppose the problem as follows: We have to design a 4R linkage such that the coupler curve is approximately a straight line. We talk of three position synthesis, obviously that problem we could have done by the method explained earlier. Here, we are saying, it is not only a three position synthesis for a coupler curve which is approximately a straight line. We also want to coordinate with the input movement that means as the coupler point goes from first position to second position, the crank must rotate by a specified amount. Similarly, as the coupler point goes from the second to third position, we want the crank to rotate by again a specified amount.

To solve this problem of coupler curve generation, of course an approximate straight line curve generation coordinated with the input movement can be solved in two stages which will be explained now. To solve this problem, first we design a slider crank mechanism. This is the slider crank mechanism, such that this pair of coordinated movements from first to second configuration and second to third configuration can be designed. This we have done just now, if we specify $theta_2$ one two, $theta_2$ two three and the corresponding s one two, s two three, I can always design this slider crank mechanism.

After designing this slider crank mechanism as a function generator for two pairs of coordinated movements, we talk of the mid point of this connecting rod. At the third configuration, it is here. At the second configuration, the same mid point of the connecting rod is here and at the first configuration, it is here. We mark this midpoint of the connecting rod corresponding to three specified configurations as C_1, C_2 and C_3. The center of the circle passing through these three points C_1, C_2 and C_3 is here, so we make that as our O_4 for the required 4R linkage.

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For that purpose, we remove this slider and take this fourth link and put a hinge between those midpoints of the original connecting rod connected by this rigid link whose center is at O_4, which is hinge at O_4. This is the other fixed link. We have got a 4R linkage namely, O_2, A, B, O_4. This is designed from the previous slider crank, only the slider has been removed.

If we move this linkage, we see that this point of the coupler is still following the originals of straight line but only approximately. We can notice that this coupler point, though not guided by the slider as before, the slider is totally detached but this 4 bar linkage is also driving this coupler point through the axis of this slot by the previous slider was moving. We have generated a coupler curve synthesis problem or path generation problem coordinated with the input movement. As it goes from position 1 to position 2, this rotates by a specified amount as it goes form position 2 to position 3, again it rotates by a specified amount which was dictated by the original slider crank. So this is what we called a two stage synthesis. So all the methods that have been taught so far can be cleverly used to design more complicated mechanism rather mechanisms with more complicated motion requirements.

Let me now repeat this two stage synthesis problem what we did. First we designed a slider crank mechanism such that it acted as a function generator. The slider displacement was coordinated with the crank rotation. Then we took the midpoint of this connecting rod at these three configurations, which are C_1, C_2 and C_3. For the three positions which have been generated by this slider crank as a function generator. Then we removed the slider and considered the midpoint of this connecting rod. Corresponding to these three positions, it is at C_1, C_2 and C_3. The center of the circle passing through these three points is at O_4, so we added a fourth link with O_4 as the fixed hinge and connecting it with C_1. So we get a 4R linkage with this coupler point generating an approximate straight line as we have seen earlier. As it goes from position 1 to position 2, the movement of the crank is exactly same as it had with the slider crank.

Similarly the movement of the crank from C_2 to C_3 is again $theta_2$ three which is coordinated. Now the question is why did I take the midpoint of the connecting rod? I could have taken any point on this connecting rod and can consider the three positions of that particular point and think of the surface.

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