**Kinematics of Machines**

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**Module - 6 Lecture - 2**

In our last class, we have seen different types of problems of dimensional synthesis of linkages. Today, we will get into the details of the geometrical method of solving these problems. We will start these problems with reference to a 4R-linkage.

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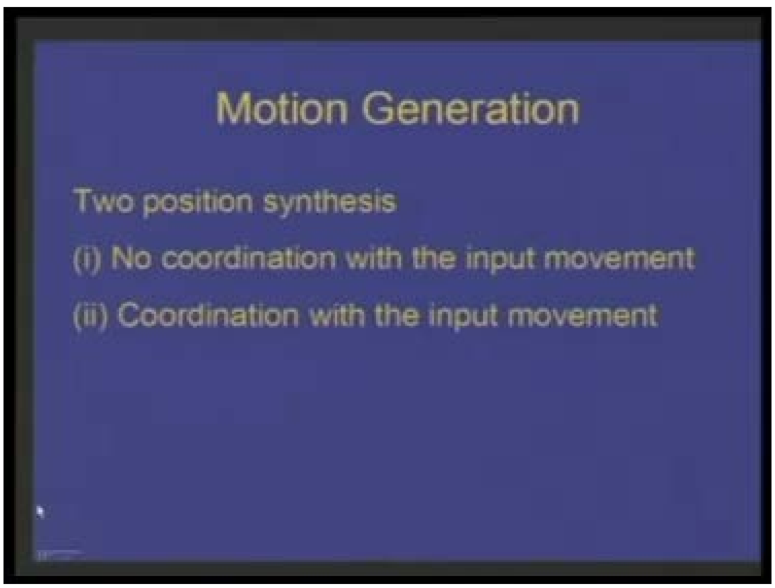


First problem we discuss is that of motion generation. By motion generation, we mean how to guide the coupler of a four bar through various specified positions. First, we discuss the simplest problem of two position synthesis. That means, the coupler has to be moved from position one to another prescribed position number two. These two position synthesis can be as I said earlier, without any coordination with the input movement. That means, we have satisfied so long the coupler takes up the prescribed positions numbered one and two. We are not bothered about what is the corresponding movement of the input link? Then we shall discuss how to solve the same problem of two position synthesis coordinated with the input movement. In the second type of problem as the coupler moves from position one to position two, it is desired that the input link should be moved by a prescribed amount say $theta_2$.

After solving this problem, we get into three position synthesis that means the coupler has to occupy three specified configurations namely I, II and III.

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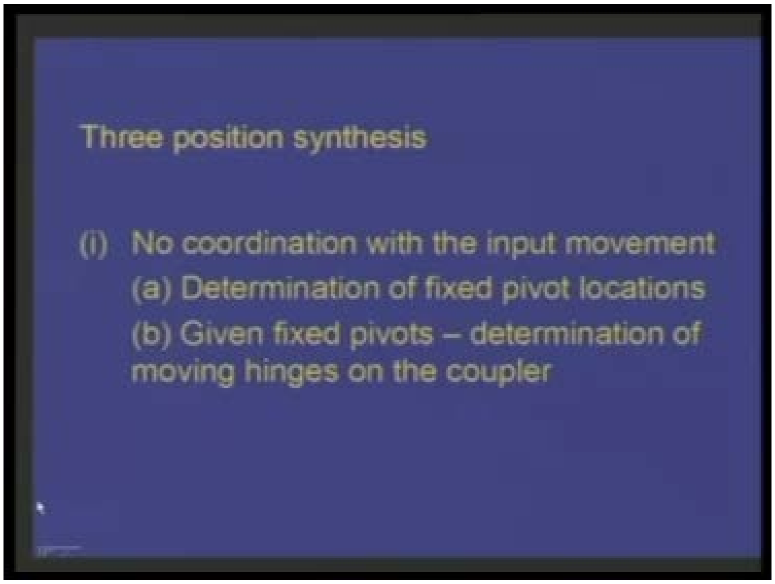


Here we will discuss without any coordination with the input movement. That means, we are happy, so long the coupler passes through these three positions and we are not bothered about what is the corresponding movement of the input link. Under this heading of three position synthesis without coordination with the input movement, we can classify the problem into two headings: one is determination of fixed pivot locations, that means, we shall choose the moving hinges A and B on the coupler AB. We have to determine, what must be the locations of the fixed pivot say $O_2$ and $O_4$, so that the coupler passes through the three prescribed configurations.

Next, we shall discuss the given fixed pivots, how to determine the location of the moving hinges on the coupler. This second problem is very often more important from practical considerations, because as we know the fixed pivots have to be located on the frame, and if they are left unknowns, may be those locations of the fixed pivots come at very uncomfortable or unsuitable positions. We choose the fixed pivot locations from the frame according to our convenience and we determine rather the locations of the required moving hinges on the coupler A and B.

Let me start with the first problem namely II position synthesis without and with the coordinated input movement.

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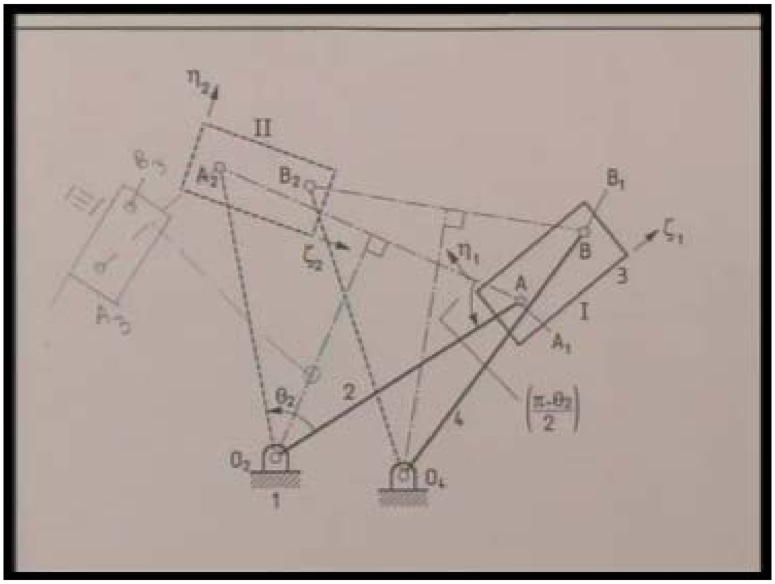

To explain the geometric method of solving an II position synthesis problem, let us consider this diagram. Here, the first position of the coupler, which is the rigid body is defined by these two axis zeta$_1$ and eta$_1$. These two axis completely defines the position of the coupler, that is what we call the first position I. It is desired that this coupler should move from this position I to position II and the corresponding positions of these two axis takes up eta$_2$ and zeta$_2$. That is to design a 4R-linkage, such that by moving that linkage, we will be able to transfer the coupler from position I to position II. Right now, we are not interested in the corresponding movement of the input link. To solve this problem, the first step is to choose location of A and B perfectly arbitrarily in this first configuration. Let us say, A and B are the locations of the moving hinges on the coupler and they are chosen arbitrarily.

In the second position the corresponding movement of A brings it to $A_2$ and B goes to $B_2$. We must notice that the locations of A and B are fixed on this coupler, in position I, A and B. In position II, the same two points are here as $A_2$ and $B_2$. The only job left is to determine the fixed pivot locations namely that of $O_2$ and $O_4$. What we do, I know the length $O_2A$ remains constant, so A and $A_2$ must lie on a circle with the centre at $O_2$. Similarly, the point B and $B_2$ must lie on a circle with center at $O_4$. We join A and $A_2$, B and $B_2$, draw perpendicular bisector of these two lines namely $AA_2$ and $BB_2$. We can choose $O_2$ and $O_4$ on these two perpendicular bisectors respectively. If I choose $O_2$ here, then $O_2A$ will be same $O_2A_2$, by rotating the link $O_2A$ it will become position $O_2A_2$. Similarly, if I choose $O_4$ anywhere on this perpendicular bisector, $O_4B$ will be equal to $O_4B_2$, by rotating the link $O_4B$, B will be moved to $B_2$. That is the simplest solution of an II position synthesis, without any coordination with the input movement.

We have also specified that, as the coupler moved from position I to position II, the corresponding movement of the input link $O_2A$ must be theta$_2$ in the counter-clockwise direction. In the previous solution, where $O_2$ could have been anywhere on this perpendicular bisector, now the location of $O_2$ is specified. To make this angle theta$_2$ between $O_2A$ and $O_2A_2$, we draw here at the point A, a line at an angle pi minus theta$_2$ by 2 and that line intersects the perpendicular bisector which was drawn earlier, at $O_2$. That decides the location $O_2$. This four bar linkage namely $O_2ABO_4$ is moved and then as $O_2A$ rotates through an angle theta$_2$, the coupler will move from configuration I to configuration II.

It is very easy to see from this triangle, that this angle is pi minus theta$_2$ by 2, so will be this angle because $O_2A$ is same as $O_2A_2$, so the third angle will be theta$_2$. Such that the sum of the three angles of this triangle comes out as pi. This is the solution for II position synthesis coordinated with the input movement. $O_4$ can be still taken anywhere, because we are not interested in the movement of the output link namely $O_4B$.

If you want to add another third position as III, then $A_2$ goes to a specific location say here, which I call $A_3$ and $B_2$ goes here, which I call $B_3$. Once we have chosen A and B arbitrarily on the coupler, then I know A, $A_2$ and $A_3$ must be on a circle. Similarly, B, $B_2$ and $B_3$ must be on another circle. The centers of these two circles should determine the location of the fixed pivot namely $O_2$ and $O_4$. It is very easy to draw a circle passing through A, $A_2$ and $A_3$, three points. Similarly, it is very easy to draw another circle, passing through these three points namely B, $B_2$ and $B_3$, that will uniquely locate the fixed pivots $O_2$ and $O_4$. Previously we had drawn a perpendicular bisector of $AA_2$, which was this line. Now, I have to draw a perpendicular bisector of $A_2A_3$ and if these two lines meet at this point, that will determine the location of $O_2$. $O_2$ will be the center of the circle passing through A, $A_2$ and $A_3$.

Similarly, we have to find the center of the circle passing through B, $B_2$ and $B_3$ and that will determine the location of $O_4$. This is the three position synthesis for the 4R-linkage, when the moving hinges A and B were chosen arbitrarily on the coupler and the fixed pivot location $O_2$ and $O_4$ can be decided. As I said earlier, more often or not, it is the location of $O_2$ and $O_4$, which I have chosen according to the convenience, we have to determine the desired location of the moving hinges A and B on the coupler. We shall solve this problem in the next step.

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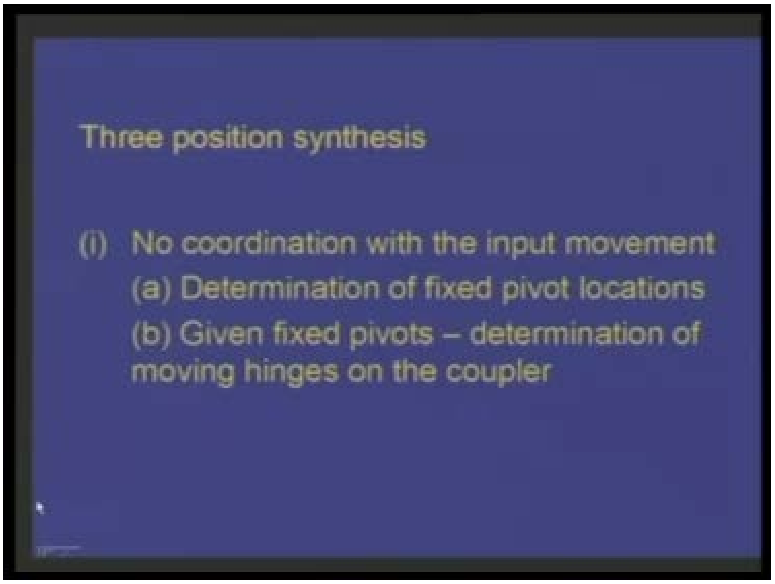


Let me now explain, how to determine the moving hinges on the coupler, such that its three position synthesis problem can be solved with given fixed pivot locations. Towards this end, we shall use the principle of kinematics inversion. If you remember in a mechanism the relative movement between various links remains independent of kinematics inversion.

What do you mean by kinematic inversion? In the same mechanism, we use a different fixed link and the original fixed link is allowed to move. We are interested only in the relative movement between various links and the kinematic inversion leaves this relative movement untouched, independent of whichever length we hold fixed. To determine the moving hinges on the coupler, we consider the same relative movement between the coupler and fixed link, but we hold the coupler fixed and allow the original fixed link or the ground link or the frame to move. We shall now explain this with the help of an example.

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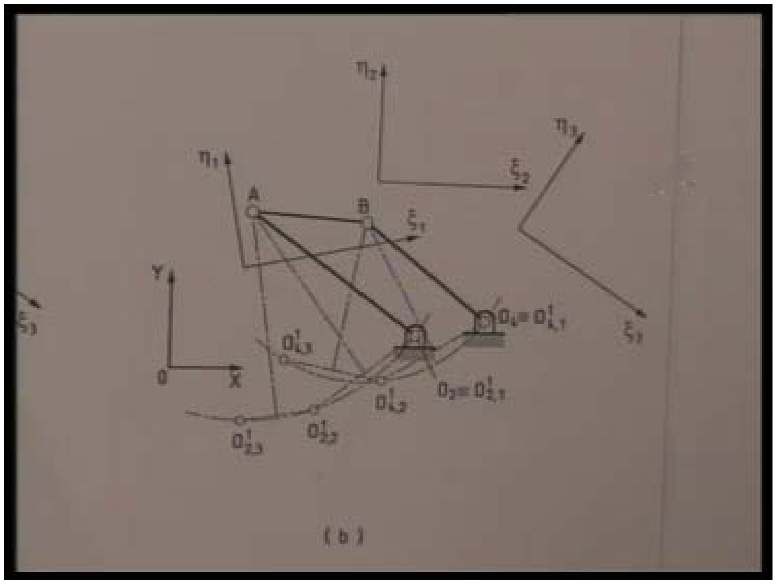

Let the three positions of the coupler are prescribed by $eta_1$ and $zeta_1$, this is position I. These coordinate axis are fixed on the coupler. The second position of the coupler is given by position II, when this eta zeta takes off the position $eta_2$ $zeta_2$. Similarly, this is the third position which is $eta_3$ and $zeta_3$. The problem is that, if we choose the locations of the fixed pivot at $O_2$ and $O_4$, where should be $A_1B_1$, that is the moving hinges in the first configuration, such that as we move this 4R-linkage, the coupler moves from position I to II and then from II to III.

To solve this problem, as I said earlier we shall use the principle of kinematic inversion. To locate $A_1B_1$, what we do, we fix the coupler at this first position. Let $O_2$ $O_4$ move, keeping the same relative movement as prescribed here. Let me do this inversion by using a tracing paper, I use a tracing paper and put it here and draw this $eta_2$ and $zeta_2$ and the position of $O_2$ and $O_4$, I mark on this tracing paper. I have drawn this eta two and zeta two axis on the tracing paper as you can see here and mark the position of the given fixed pivots $O_2$ and $O_4$. The same relative movement I consider, as if the coupler has not moved from the first position. This is the position, I coincide $eta_2$ with $eta_1$ and $zeta_2$ with $zeta_1$.

The second position, if I hold the coupler fixed in the first position, then $O_2$ and $O_4$ moves to these two positions, this is $O_2$ and $O_4$, having the same relative positions in the second configuration, but I have fixed the coupler in the first position. These two positions we mark on the paper by piercing with the point. This is the position of $O_4$, this is the position of $O_2$. I name these positions as follows: I call it $O_4$ in the second configuration, but inverted on the first configuration, so I call it $O_{4,2}$ one. This is really I could have said, this is $O_{4,1}$ 1 and this is $O_{2,1}$ 1. Holding the coupler fixed in the first position, if I allow the fixed link to move, then $O_4$ comes here and $O_2$ comes here, which I call $O_{2,2}$ 1.

I hope it is clear, that I get the inverted position of these fixed pivots if I hold the coupler fixed in the first position. Then, we take the tracing paper again, draw $eta_3$ and $zeta_3$ and mark this $O_2$ and $O_4$. This is the relative position of $O_2$ and $O_4$ with respect to coupler in configuration. If I make a kinematic inversion with the coupler fixed in the first position, that is $eta_3$ and $zeta_3$ I make coincide with $eta_1$ and $zeta_1$ respectively. Then, $O_4$ moves here and $O_2$ moves here, this position I call $O_{4,3}$ 1 the third position inverted on the first position. Similarly, this is $O_{2,3}$ 1. If we fix the coupler and allow the fix link to move, the same relative configurations as given by the moving coupler with respect to the fixed link. Then, the fixed link moves, such that $O_2$ goes from $O_2$ 1 to $O_{2,2}$ 1 to $O_{2},_{3}$. Similarly, $O_4$ goes from $O_{4},_{1}$ 1 to $O_{4,2}$ 1 and $O_{4,3}$ 1, by holding that coupler fixed. If the coupler is fixed, then A does not move, A is fixed and because $AO_2$ distance is same then $O_2$ moves on a circle with A as center. We have to determine the center of the circle passing through this $O_2$ 1, $O_2$ 2, $O_{2},_{1}$ 1, $O_{2},_{2}$ 1 and $O_{2},_{3}$ 1. This is one point, this is one point and this is one point. It is very easy to determine the center of a circle passing through these three points and that will be $A_1$.

Similarly, the center of the circle passing through $O_{4},_{1}$ 1, $O_{4},_{2}$ 1 and $O_{4},_{3}$ 1, center of that circle will be set here and that will be $B_1$. We have located the moving hinges required position of the moving hinges such that if $O_2A_1$ $B_1O_4$ and this 4R-linkage are moved, the coupler goes from position I to position II to position III, with $O_2O_4$ has the fixed pivots. As the principle of kinematics inversion forms the basis of procedure for dimensional synthesis, let me repeat it once more the solution shown in this. What we have done, we are using a tracing paper $eta_1$ $zeta_1$, $eta_2$ $zeta_2$ and $eta_3$ $zeta_3$ they specify the three desired positions of the coupler. $O_2$ and $O_4$ have been chosen according to the (Refer Slide Time: 19:41). Our task is to determine the location of the moving hinges A and B. We want this A and B in the first position of the coupler. We make a kinematic inversion, hold the couplers fixed in the first position and allow the fixed link to move and determine where do these points $O_2$ and $O_4$ to keep the same relative movement between the coupler and fixed links, which remains invariant for a given (Refer Slide Time: 20:13).

Now, you note $O_2$ I call $O_{2,1}$ 1, this subscript one refers to the configuration and superscript one refers where we are inverting. That means, which position we are holding fixed. The superscript in this problem will be always 1, because we are always inverting on the first position, which we are holding fixed. Let me explain, how I get this inverted position. What we do, we use a tracing paper and mark this second configuration defined by $eta_2$ and $zeta_2$ axis. The relative position of this fixed pivot in the second configuration here is $O_2$, here is $O_4$. If I now have the same relative movement, but hold the coupler fixed in its first configuration. That means, $eta_2$ $zeta_2$ I coincide with $eta_1$ $zeta_1$. We see the $O_2$ moves here and $O_4$ moves here such that the same relative position corresponding to the second configuration is maintained. $O_2$ comes here and $O_4$ comes here. This position I call $O_{2,2}$ 1 and this position I call $O_{4,2}$ 1. Again we note superscript 1 refers to the configuration one, where we are fixing or holding the coupler fixed. These two refers to the configuration, which is being inverted, it is the second position which we invert.

For the third position we draw on the same tracing paper, $eta_3$ and $zeta_3$ and mark these $O_4$ and $O_2$. The same relative position between the coupler and the fixed length is maintained, but the coupler is fixed in its first position, so I move this and make it coincide eta zeta with $eta_1$ and $zeta_1$. As we see $O_2$ moves here, $O_4$ moves there. This position I call $O_{2,3}$ 1 and this position I call $O_{4,3}$ 1. Rest is simple, I determine the center of the circle by drawing these two perpendicular bisectors of $O_{2,1}$ 1, $O_{2,2}$ 1 and $O_{2,2}$ 1, $O_{2,3}$ 1. These two perpendicular bisector intersect here, which is the center of the circle passing through these three inverted positions of $O_2$ namely $O_{2,1}$ 1, $O_{2,2}$ 1 and $O_{2,3}$ 1, this is what I call A. Similarly, the center of the circle passing through the three inverted positions of $O_4$ is B and that is the other moving hinge on the coupler. The problem is solved, that $O_2ABO_4$ is the required 4R-linkage which when moved the coupler moves from position I to position II and position III. This is the three position synthesis of motion generation for the coupler of a 4R-linkage with the fixed locations of the ground pivot.

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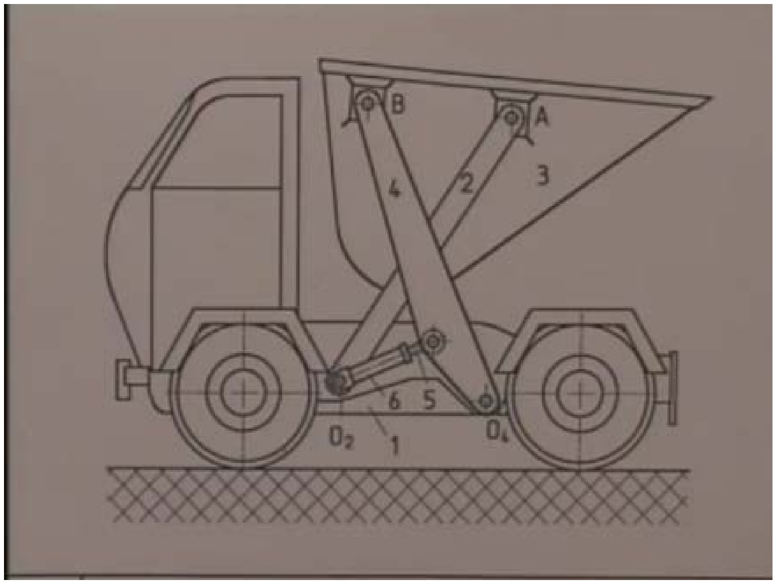


Let us take an example of this three position synthesis, with reference to this steeper dumper. As we see here, the dumper bin that is link number 3 is driven by this link number 4, which is driven in turn by this hydraulic cylinder piston arrangement, which are link number 5 and 6. As the cylinder expands, this link 4 turns clockwise, here we have a 4R-linkage namely $O_2ABO_4$. This 4R-linkage is driven by moving link 4 by this hydraulic cylinder and as a result this dumping bin goes from this configuration to some other configuration and the final dumping configuration. Our job is to locate the suitable positions of A and B for given locations of $O_2$ and $O_4$, these are the fixed pivots which must be on the body of this steeper dumper. The A and B have to be determined, such that this dumping bin passes through the three given configuration and that is the problem. Let us see, how we solve it by using the principle of kinematic inversion.

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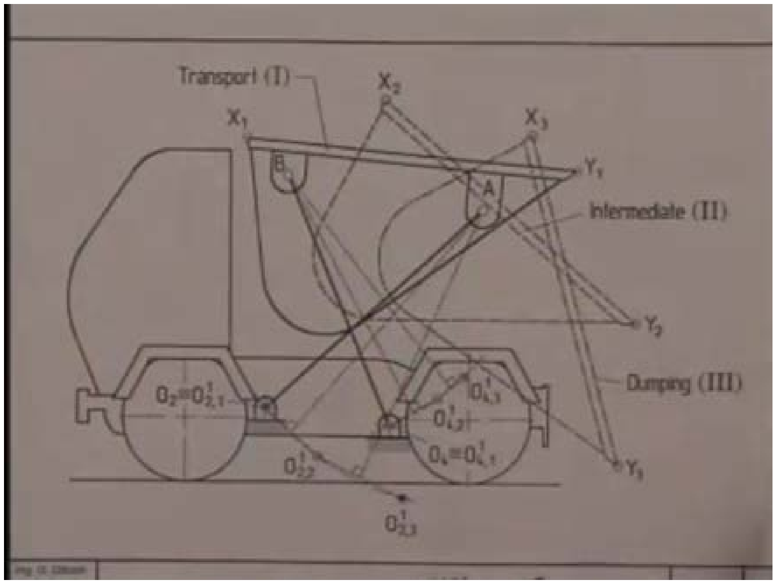


Let us now, discuss this example of steeper dumper and the dumping bin has to take up this first configuration, which is the configuration during the transporter movement. This is another configuration, which we call the intermediate configuration and this is the third configuration, which I call the dumping configuration. The task is for given locations of $O_2$ and $O_4$, we have to determine where should A and B lie on this dumping bin, such that when we move this 4R-linkage, that dumper goes from transport to intermediate to dumping configuration. The thing to note, that in plane motion of a rigid body the configuration of the rigid body is completely defined by any two points on it. Here, we take those two points at corner $X_1$ and $Y_1$. You can forget about the rest of the dumping bin and we only concentrate on these two points namely $X_1$ and $Y_1$. That is, they are location in the first configuration, then $X_2$ and $Y_2$ that is their location in the intermediate configuration and $X_3$ and $Y_3$ is the positions in the dumping configuration.

The dumper bin is completely defined by X and Y passing through these three positions $X_1$, $X_2$, $X_3$ and $Y_1$, $Y_2$, and $Y_3$. As I said, I will make a kinematic inversion to locate A and B in the first configuration, that is transport configuration. We hold the dumper bin fixed in this first configuration. That means, $X_1$, $Y_1$ will not move, rather we allow $O_2$ and $O_4$ to move keeping the same relative movement, that is kinematic inversion. To carry out the inversion I use a tracing paper and mark this point $X_2$ and $Y_2$ and mark $O_4$ and $O_2$. If we hold the dumper fixed in first configuration, then we make $X_2$ coincide with $X_1$ and $Y_2$ coincides with $Y_1$. We see that, this $O_4$ has moved to this position and $O_2$ has moved to this position. These two positions, we call $O_{2,2}$ 1 and $O_{4,2}$ 1. To invert the third position, we mark $X_3$ and $Y_3$ and $O_2$ and $O_4$. If I invert it on the first position that is $X_3$ goes to $X_1$ and $Y_3$ goes to $Y_1$. As we see, this $O_4$ has come here, which I call $O_{4,3}$ 1 and $O_2$ has come here, which I call it $O_{2,3}$ 1. Thus, we have located the three inverted positions of $O_2$ namely given location, which is $O_{2,1}$ 1 and these two inverted positions $O_{2,3}$ 1 and $O_{2,2}$ 1. Similarly, for the given location $O_4$, which is $O_{4,1}$ 1 and the inverted positions $O_{4,2}$ 1 and $O_{4,3}$ 1. The center of the circle passing through these three inverted positions of $O_4$ determines B. The center of the circle passing through these three inverted positions of $O_2$ determines A. If I join $O_2A$ and $O_4B$, we get the required 4R-linkage, and which will drive the dumper bin through these three configurations I, II and III.

We have just seen how to solve the problem of motion generation for three position synthesis. We have noticed that the whole method depends on the fact that three points lying on a circle has a unique center. If we want to extend this method to more than three accuracy points, what happens? You remember what we did? Suppose we wanted to locate the fixed pivots after choosing the moving hinges A and B. We use the fact that A moves on a circle with $O_2$ as center, so $A_1$, $A_2$, $A_3$, these three positions of the point A determines $O_2$ as the center of the circle passing through $A_1$, $A_2$, $A_3$. Similar thing is true for the location of $O_4$, which is the center of the circle passing through three positions $B_1$, $B_2$, $B_3$.

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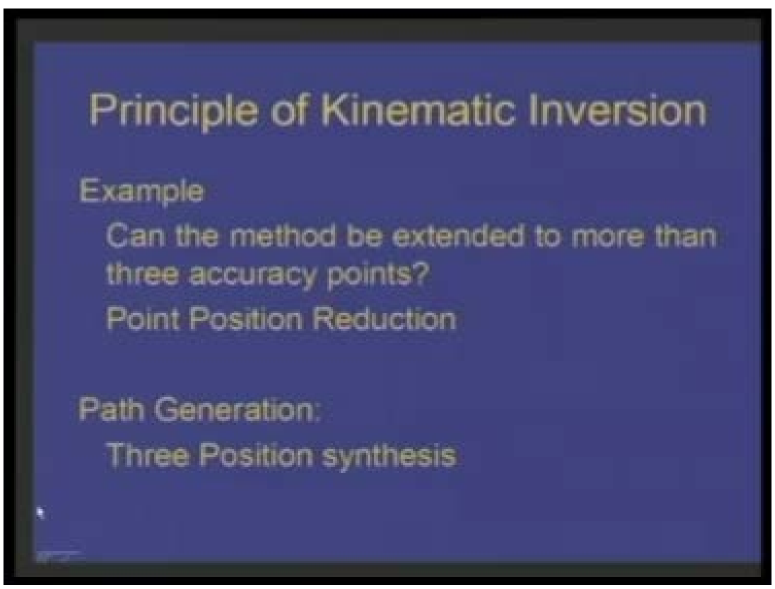

Then the question is can we extend this method to more than three accuracy points. Because then $A_1$, $A_2$, $A_3$, $A_4$ which are all four arbitrary positions, there is no guarantee that they will lie on a circle, so there is no way we can determine the center. However, there is a method which is beyond the scope of this course, so we will not discuss that in detail, but we can think of that there must be a solution if we keep the following fact in mind. We see the location of A was perfectly arbitrary I could have taken A anywhere on the plane of the coupler. There are special locations of this point A, such that as the coupler moves to four arbitrary positions, A will lie on the circle. This method can be extended to given four accuracy points. In this course, we will discuss some limited version of this by saying using a method of point position reduction that we will take up much later.

The method can also be extended to five position synthesis, but then A cannot be chosen arbitrary. For three positions, A was chosen anywhere on the coupler plane. B is chosen anywhere on the coupler plane. For four position synthesis A have to be chosen on a particular curve, for five position synthesis they are only some discrete fixed locations for the choice of A. As I said, this general discussion of four and five position synthesis is beyond the scope of this course. We will discuss later on some limited version for four position synthesis by a technique, which is known as point position reduction.

At this stage, let me now get into the second type of problem namely the problem of path generation. For path generation, we start our discussion with three position synthesis, because two position synthesis, as we have seen is pretty obvious.

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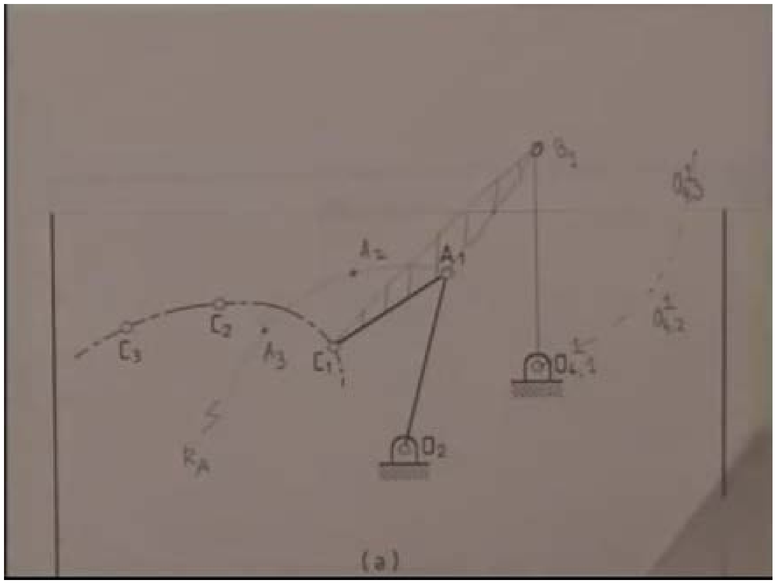


For three position synthesis of path generation, the problem is posed as follows: let a particular coupler point C has to pass through three positions namely $C_1$, $C_2$ and $C_3$. Actually, this is the desired coupler curve, but we are not interested in the intermediate positions, we have defined a three position synthesis problem that a coupler point C of a 4R-linkage must pass through $C_1$, $C_2$ and $C_3$. To solve this problem, we choose $O_2$ and $O_{4,}$ the locations on the fixed pivots arbitrarily anywhere according to the convenience of the problem. Then, we also choose $A_1$ arbitrarily. $A_1$ is the moving hinge on the input region. The only thing remaining is to determine, where is the other moving hinge on link number four which I call B that is $B_1$. $O_2A_1B_1O_4$ is the required 4R-link and A, B, C is the coupler. As these 4R-linkage moves, this particular coupler point C should pass through $C_1$, $C_2$ and $C_3$ and we have to determine, where must be the location of the other moving hinge $B_1$.

To do this, we again make a kinematic inversion, as we see we have to determine the point B on the first position of the coupler that is A B C $B_1$. We hold the coupler fixed in first position and allow the fixed link $O_2O_4$ to move, since $B_1$ is remains fixed and $B_1O_4$ distance is constant, so $O_4$ must move on a circle with $B_1$ as the center. Our job is to determine, what we call the inverted position of this fixed hinge $O_4$, as $O_4$ which we call $O_{4,1}$ 1. We have to determine $O_{4,2}$ 1 and $O_{4,3}$ 1 corresponding to the three positions of the linkages. To do this we proceed as follows: first, we determine the location of this moving hinge A corresponding to the other two configurations. As we see because $O_2A$ distance is constant, A must lie on this circle. This is what we call the path of A say $k_A$, A lies on this circle with $O_2$ as center and $O_2A_1$ as radius. Since, the distance $A_1C_1$ is also fixed, this is one of the coupler lengths. We determine $A_2$ by making $C_2A_2$ as $C_1A_1$, this $C_1A_1$ is same as $C_2A_2$ and A must lie on this circle, so I can determine the location of the point $A_2$. Similarly, corresponding to the third position since AC distance remains constant I draw a circular arc with $C_3$ as center and CA as the radius and that intersects the path of A at this point, which I call $A_3$. AC is one part of the coupler, ABC is the coupler. We have to determine $B_1$, by holding the coupler fixed at its first position that is at $A_1C_1$. We have to determine the inverted positions of the fixed hinge $O_4$, so what we do? We use a tracing paper and mark $A_2$, $C_2$ and $O_4$. The same relative position is maintained, but at the first configuration, that means $A_2C_2$ I make coincide with $A_1C_1$, $A_2$ with $A_1$ and $C_2$ with $C_1$ and as we see $O_4$ moves here. We mark this point on our drawing sheet, this point we call $O_4$ corresponding to the second position but inverted on the first position we call it $O_{4,2}$ 1. Again, now we consider the third configuration that is we mark $C_3$, $A_3$ and $O_4$. Let me now show how to find out the inverted position corresponding to the third configuration.

This is $C_3$ I put it as a cross and this is $A_3$ and this is the location of $O_4$, so if I invert it on the first position of the coupler that means, I make $C_3$ coincides with $C_1$, $A_3$ coincides with $A_1$. This $C_3$ I am making coincides with $C_1$ and $A_3$ with $A_1$ and as we see $O_4$ has moved there, so I marked this point on my drawing sheet and this I call $O_{4,3}$ 1. If the coupler is held fixed, corresponding to these two relative movements on these three configurations, the location of $O_4$ is here, here and here (Refer Slide Time: 39:42). If the coupler is held fixed at its first configuration that is the point B does not move, it remains on $B_1$ and $O_4$ moves in a circle with $O_4B$ as the radius. If I locate the centre of the circle passing through these three positions, this is the circle which passes through these three inverted positions of $O_4$ and the center is determined somewhere here, which I call $B_1$.

We have determined the linkage $O_2$ $A_1$ $B_1$ $O_4$ and C is the coupler point and this ABC is one rigid body and this is another link which I can join as another rigid link. $O_2$ $A_1$ $B_1$ $O_4$ with $C_1$ as the coupler point and if this linkage moves as $A_1$ goes through $A_2$, $C_1$ will take up the position $C_2$ and as $A_2$ goes through $A_3$, $C_1$ will take up the position $C_3$. We have solved the problem of path generation with three accuracy points.

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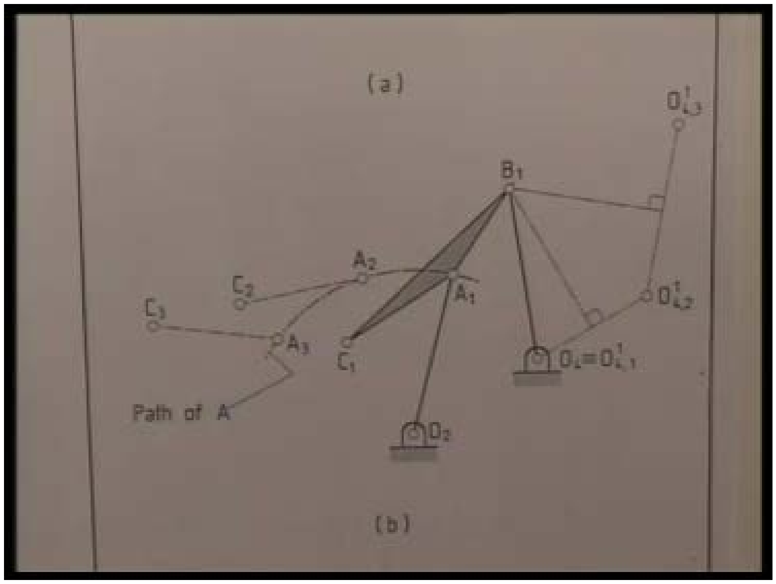


Let us now see at the solution of the problem which has been solved just now. As I said $C_1$, $C_2$, $C_3$ are the three required points of the path of the coupler points C, these are given to us. $O_2$ and $O_4$ have been chosen arbitrarily according to the convenience. We also choose $A_1$ arbitrarily. The only point is to determine, where should be $B_1$ the other moving hinge. What we have done, first we determine the path of A, which is a circle with $O_2$ as center and $O_2A_1$ as radius. On the path of A, I locate the position of A corresponding to the second and third configurations as $A_2$ and $A_3$. This is easily done, since I know this distance AC being a rigid link does not change, so with $C_2$ as center, I draw a circular arc with AC as radius and that intersects the path of A at $O_2$. Similarly, with $C_3$ I get $A_3$. We make a kinematic inversion to locate the point $B_1$. Please note that, because we want to locate the point $B_1$ which is on the coupler, I hold the coupler fixed such that $B_1$ does not move and it is finally obtained as a center of the circle passing through three points. With $B_1$ fixed, it is this point $O_4$ which moves on a circle with $B_1$ as center and $B_1O_4$ as radius. To locate the inverted positions of $O_4$, we have done as explained earlier, let me repeat it. We mark $A_1$, $C_1$ and $O_4$. This is the relative positions of A, C and $O_4$ in the first configuration, so I call this $O_4$ as $O_{4,1}$ 1. Corresponding to the second configuration, the relative positions we mark $A_2$, $C_2$, and $O_4$. These are the relative configurations of the points A, C and $O_4$. We are holding the coupler fixed, that means, $A_2C_2$ does not move rather they remain at $A_1C_1$.

If they remain at $A_1C_1$ as we see this point $O_4$ comes here, which I call $O_{4,2}$ 1. Similarly, we mark $A_3$, $C_3$ and $O_4$. These are the relative positions of A, C and $O_4$ in the third configuration. Because we are inverting on the first configuration, I take $C_3$ to $C_1$ and $A_3$ to $A_1$ and wherever $O_4$ goes that I call $O_{4,3}$ 1. I hope it is clear how to obtain the inverted positions of $O_4$ and the center of circle passing through these three inverted position of $O_4$ is at $B_1$ as determined by these two perpendicular bisectors. We design the linkage $O_2$ $A_1$ $B_1$ $O_4$ with $C_1$ as the coupler point, if this linkage when moved $C_1$ passes through $C_1$, $C_2$, $C_3$.

Again, I emphasize please note this notation $O_{4,2}$ 1, the subscript two refers to the second configuration which is being inverted on this superscript that is the first configuration. Similarly, $O_{4,3}$ 1 corresponds to third configuration and inverted on the first configuration. I would suggest the students can try by inverting say on the second configuration rather than on the first configuration. That means, we locate the point $B_2$ rather than $B_1$, we say that the coupler does not moves from $A_2$, $B_2$, $C_2$ and we invert the first position on the second and the third position on the second, when $O_4$ will be $O_{4,2}$ 2 and the other two positions we will call $O_{4,1}$ 2 and $O_{4,3}$ 2 and the center of the circle passing through these three inverted positions will locate $B_2$.

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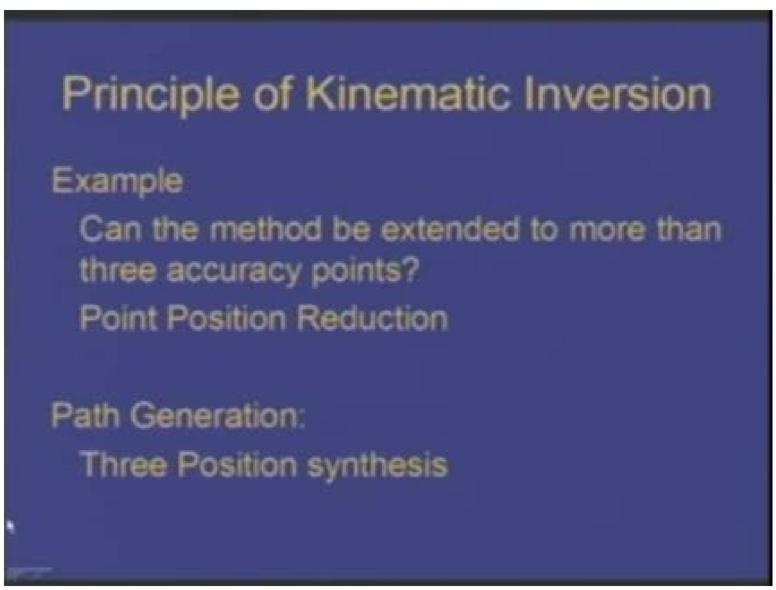


Let me now summarize, what we have learnt today. Today, we have discussed the graphical method for three position synthesis with reference to motion generation and path generation problem. We treated the coupler as the moving link and for motion generation it is the position or orientation that is the configuration of the coupler, which were prescribed and has to be satisfied. In the path generation problem, it is a particular point on the coupler which has to take three positions which you are specified and we have to come up with the 4R-linkage. We must have noticed, the most important point of this graphical method is the principle of kinematic inversion.

The students should note, inversion should be done on which link, that means which link should be held fixed and on what configuration, we have a choice. We can choose the inversion to be made on the first, second or third configuration it does not matter. But it is the choice of the link which has to be chosen depending on the problem. For example, in both motion and path generation problem, we have to held the coupler fixed and allow the fixed link to move. In our next lecture, we will discuss function generation problem and we will see that we have to make a different kind of kinematic inversion, to solve that problem.