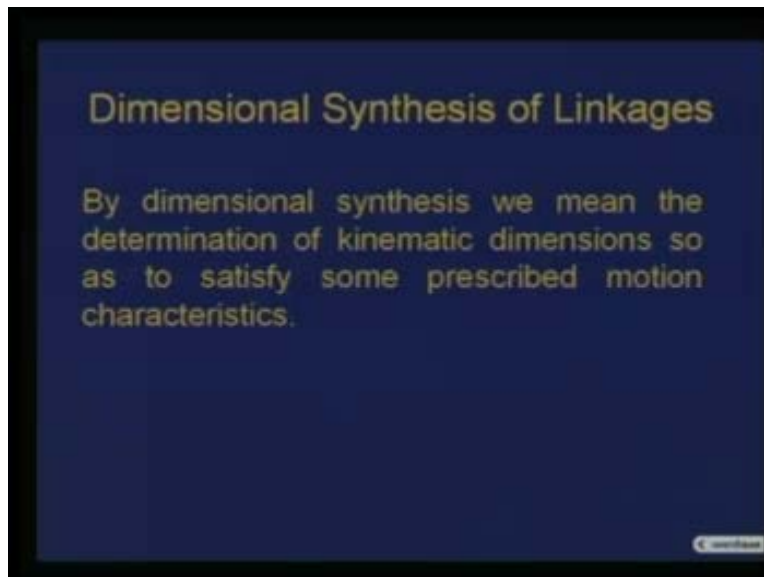


Kinematics of Machines
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Module - 6 Lecture - 1

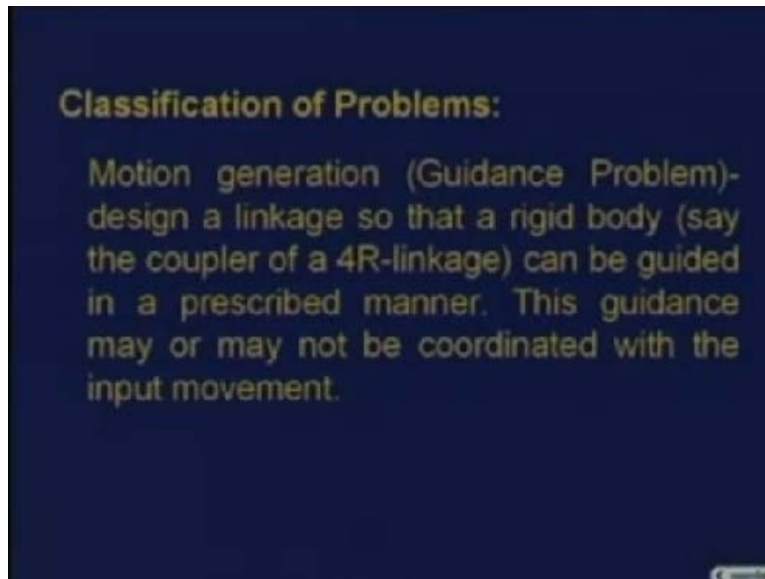
In today's lecture, we start a new topic namely dimensional synthesis of linkages. By dimensional synthesis of linkages we mean, how to determine the kinematics dimensions of a linkage so as to satisfy some prescribed motion characteristics.

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We have already chosen the type of linkage. We only have to determine the kinematic dimensions such that, this particular linkage will be able to satisfy some prescribed motion characteristics. The prescribed motion characteristics can be done in various different ways and accordingly, we classify these problems of dimensional synthesis of linkages. For example, the first class of problems can be called motion generation.

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By motion generation, which we may also call a guidance problem, what we mean, is we have to design the linkage such that a rigid body, for example that rigid body may be the coupler of 4R-linkage, which is a floating link, which can be guided in a prescribed manner. That means we want the coupler to go through various prescribed configurations that are guiding the coupler in a specified manner.

This guidance of the coupler may or may not be coordinated with the input movement. The requirement may be such that, I do not mind what the input movement is, so long as the coupler is guided in a prescribed manner. In another type of problem, we may need that as the coupler goes from one configuration to another in the specified manner, the input link must move by a prescribed amount. In that case, we say that this is a guidance problem or motion generation problem coordinated with the input movement.

Let me now clarify this particular type of problem with the help of some models.

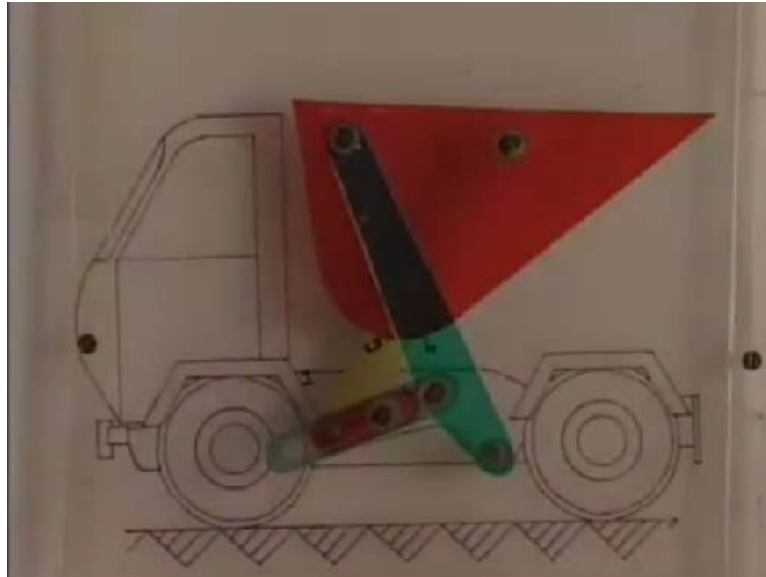
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As an example, let us look at the model of this four bar linkage. Here, this is one of the links, which is connected to the fixed hinge, this is another link which is connected to the fixed hinge and this is the coupler. Our objective is to use this coupler as the desk of a writing position. Right now, the desk is folded such that, the student can sit here and there is enough space between his leg and the next row of desks. This coupler can be moved such that, it comes to the writing position and now the student can use this coupler as his writing desk. As we see, here is a stopper such that, the coupler cannot move anymore and when the job is done, we can fold this writing desk and take it away by leaving a seat.

This is a motion guidance problem that I want the coupler of this four bar linkage to move from this configuration to this configuration in a prescribed manner. There is no need to coordinate this movement because our objective is to have it locked at the writing position and we can take it away to the folded position. This is an example of a guidance problem or a motion generation problem without any coordination with the input movement.

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Next, we see the model of another four bar linkage, which is used as a keeper dumper. Let us now look at the model of this keeper dumper. Here, the dumping bin is the coupler of a 4R-linkage, consisting of this yellow link and this green link. This yellow link is connected to the fixed hinge mounted on the body of the truck. Similarly, this green link is connected to the fixed hinge, which is mounted on the body of the truck. Of this 4R-linkage, this dumping bin becomes the coupler. Our objective is to make this dumping bin to move in a prescribed manner from this transport configuration to this dumping configuration.

This 4R-linkage is driven by a hydraulic actuator such that, link 4 is used as the input link and it is moved by this hydraulic actuator to move the dumping bin. This is again, a guidance problem that I want to guide this particular coupler from this transport configuration to set this, then this, then this and finally to the dumping configuration. This is again a motion generation problem without any coordination with the input movement. We will see another model of a little more involved linkage with the guidance problem. That will be the model of the garage door.

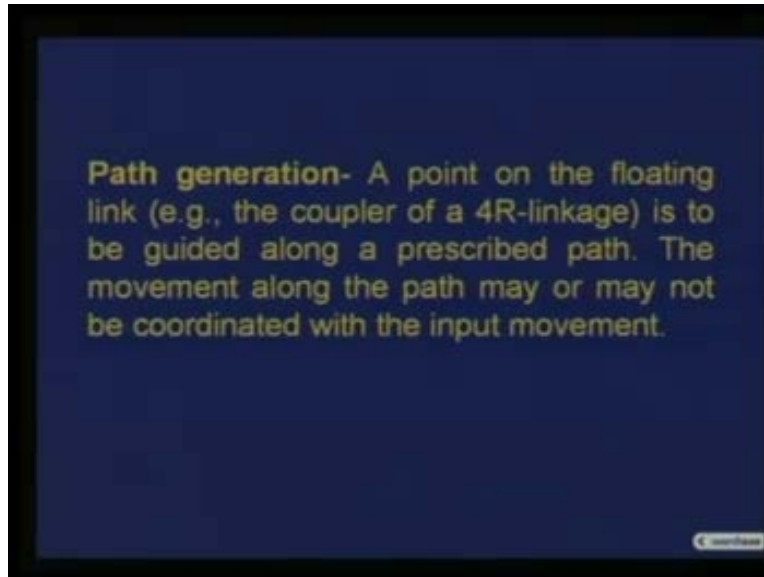
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Let us now look at model of this six-link mechanism. Here, we have the fixed links one to six. This mechanism is mounted on the side walls of the garage. The car is inside the garage, this is the roof of the garage and this link represents the door bolt. When the garage is open, the car can go out and the door bolt is parallel to the roof. To close the garage, the door bolt is moved such that, the garage gets closed. This is again a guidance problem. I want this particular rigid body, that is link number 6 to move from this closed position to the open position in a prescribed manner so that, it does not hit the body of the car. These mechanisms are mounted on the side walls, so there is no obstruction with this car. It is only the door bolt, which moves inside the garage and closes the garage or opens it. The thing to note here is that, there is no prismatic pair. The entire mechanism consists of only revolute pairs. This is the open position of the garage, when the car can move out and the door bolt is parallel to the roof of the garage and in the closed position it becomes vertical. This movement from the vertical to horizontal position must take place in a prescribed manner. This rigid body must be moving in a prescribed manner as it goes between these two extreme positions. This is another example of a guidance problem.

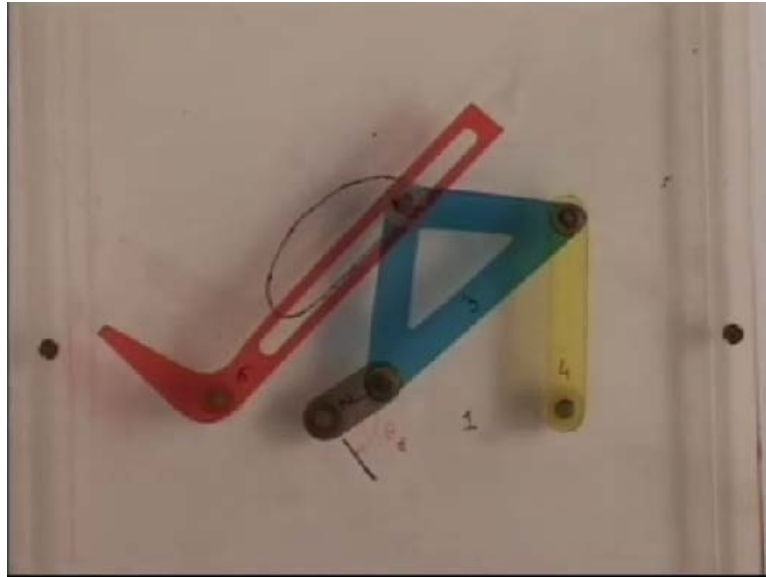
We have seen three models to explain what we understand by a guidance problem. That is, a rigid body, which was the coupler in case of two 4R-linkages, which has to be guided in a prescribed manner.

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The next class of problem is what we call a path generation problem. In this type of problem, a point on the floating link that is the coupler of a 4R-linkage is to be guided along a prescribed path. This guidance along the prescribed path may or may not be again coordinated with the input movement, just like a guidance problem. Whether we are only interested in going along a prescribed path by a particular point of the floating link like the coupler, or the movement along that path also has to be coordinated with the amount of input movement. Again, we will see some examples of this path generation problems through a number of models.

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As an example of the path generation problem, let us look at the model of this six-link mechanism, which can be used as a dual mechanism. In this mechanism, we have a 4R-linkage consisting of link number 2, link number 3 and link number 4, where 1 is the fixed link. This is a 4R crank-rocker linkage, the two revolve as a crank. The thing to note is that, at this particular coupler point, we want to generate this particular prescribed path. This path consists of an approximate straight line portion. If we have a prismatic pair here with link 5 and link 4 and there is a revolute pair between link 5 and link 4 and in link 6 there is a prismatic pair in which, this link 5 moves. So long this coupler moves along the straight line path, the output link, that is link 6 does not move and that is why, we call it a dual mechanism.

The input link 2 rotates continuously but during this movement of link 2 as we see there is hardly any movement of link 6. Link 6 almost does not move. So, link 6 undergoes a dual that is a rest period. Such a dual mechanism is very useful for various applications. Link 2 during this movement as we see, the link 6 moves, but during this period and this amount of movement of link 2, link 6 hardly moves. That is possible, only because this coupler point is going almost along a straight line portion from here to there. This is the problem of a path generation that is, how to design this 4R-linkage such that, this particular coupler point generates this prescribed path. This is a problem of path generation.

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As a further example of a path generation problem, let us look at this six-link mechanism. Here again, we have a 4R-linkage consisting of these two red links and this yellow link, which is the coupler. This particular coupler point of the 4R-linkage generates this coupler curve. This coupler curve consists of almost two circular arcs. These circular arcs are of course, approximately circular. During this movement of the coupler point along the circular arc, the center of the circle does not move. I can join another link here, which I treat as my output link. As a result, we get a dual mechanism such that, the input link 2 which rotates continuously. But, as we see the output link, that is link 6, undergoes two dual periods. It moves, but for some rotation of link 2, there is no movement of link 6. For example, from here to there, link 2 is rotating but as we observe, this link 6 is not rotating because this coupler point is going along a circular arc and this point is the center of the circular arc. There is no movement of this point. Consequently, output link 6 does not move at all. So it undergoes a dual period and we get a dual mechanism, that the input link 2 rotates continuously, but the output link 6 undergoes dual period. The path generation problem is to design this 4R-linkage such that, this particular coupler point generates this prescribed path, which consists of two approximate circular arcs.

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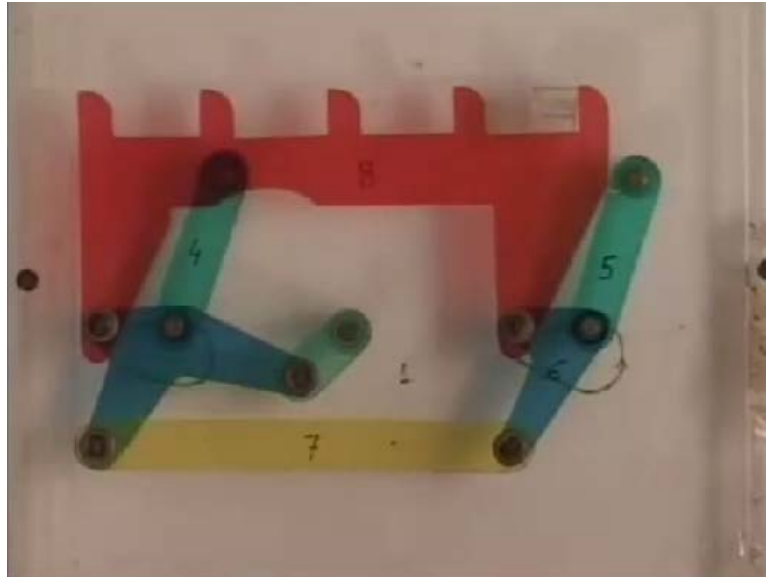
As a simpler example of a path generation problem, let us look at the model of this film drive mechanism. As we see here, we have a 4R-linkage with this red link as the coupler. This particular coupler point generates the coupler curve. It is the prescribed path through which, this coupler point has to move. If we now move this 4R-linkage, as we see that a particular coupler curve is generated by the coupler point. This end of this coupler can be used as a hook, which gets into the pockets of the film and drives the film downward. Here, it comes out of the socket goes out and again gets into another socket and draws the film downwards. This way, it can be used for deriving a movie film in a movie projector. If it goes very fast, the film also moves very fast. The critical thing is that, it must move almost vertically here to drag the film downward and then, gets out of the film and again gets into the film and drags it downward. This is another example of a path generation problem. We have to come up with this link length such that, this particular path is generated by this coupler point.

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The continuous input rotation of this red link produces intermittent rotation of this blue link. This Geneva wheel, the start and stop of the intermittent motion of this blue link is very jerky, because it does not get into the slot smoothly and does not come out of the slot smoothly. So, the start and stop of this intermittent motion of this link is jerky. To get rid of that jerky motion at the start and stop of the Geneva wheel, what we can do is, you can drive the Geneva wheel by this coupler point of this 4R-linkage. These are link 2, link 3 and link 4 and this is the coupler point. If the input is given to the crank of this crank-rocker linkage, then this particular coupler point gets into this slot and goes out of the next slot. If we have a coupler curve, such that the tangent to the coupler curve is along these two directions and then the entry and exit from this slot are very smooth. Consequently, we can use this particular coupler point to drive the Geneva wheel. This is again an example of a path generation problem because we have to generate this particular path by this point such that, the tangent to the path is along these two slots at 90 degrees.

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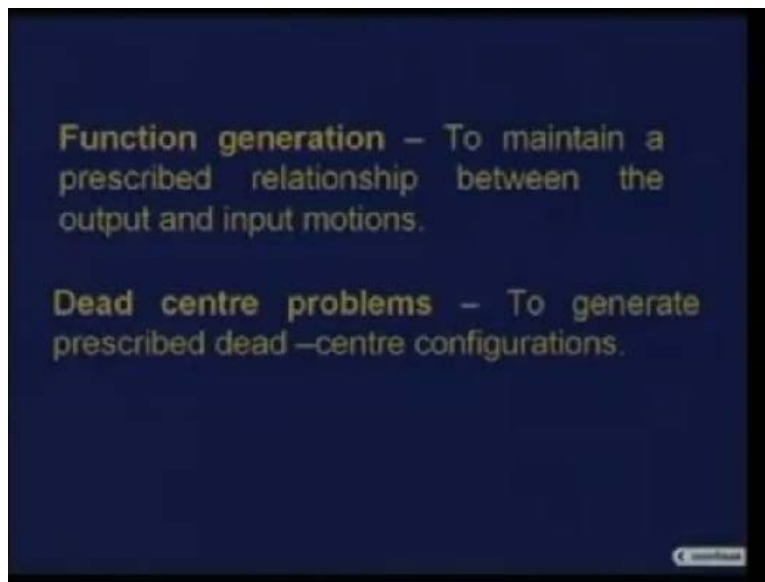
As a last example of path generation problem, let us look at this model of an eight-link transporter mechanism. Here we have a four link-mechanism, link 2 that is the crank, which is the driver. This is the coupler and this is link number 4. In this coupler at this particular point, we want to generate a curve, which has a vertical portion, has a horizontal portion and comes down. If we drive this transporter mechanism we can see the following: Suppose this is an object, which has to be moved through a number of stations. At every station it must dual. It must not move after reaching the station for some time so that, some operation can be done on this object.

If we drive this mechanism, because of the particular shape of the coupler curve which has been duplicated here, let us note the transporter movement. It has left the object, it is now being picked up and pushed. Again it is left because this coupler point is coming down. This coupler point, due to this vertical path is going up and again picking up the object. Because of the horizontal path, it is pushed horizontally to the next station. Again, it is picked up, pushed to the next station and left it. It is this particular coupler curve, which is causing the transporting movement from one station to the other for this object. If I drive this as an input link, which rotates continuously, let us look at the motion of this object. It is undergoing dual in the next station it is picked up. The design of this transporter mechanism is completely based on this particular coupler curve here,

generated by this coupler point. This is the coupler of this 4R link. This is another example of a path generation problem and its application to real life mechanism.

Through a number of models, we have just seen the application of motion generation and path generation problems towards the design of some real life mechanisms. Of course, at this stage, our only objective is to learn the method to solve the problems of motion generation and path generation. The models provided the motivation that, once we know this method, we should be in a position to apply this method towards the design of real life mechanisms. We will continue our discussion with various types of problems in dimensional synthesis of linkages with the third class of problem, which we call function generation.

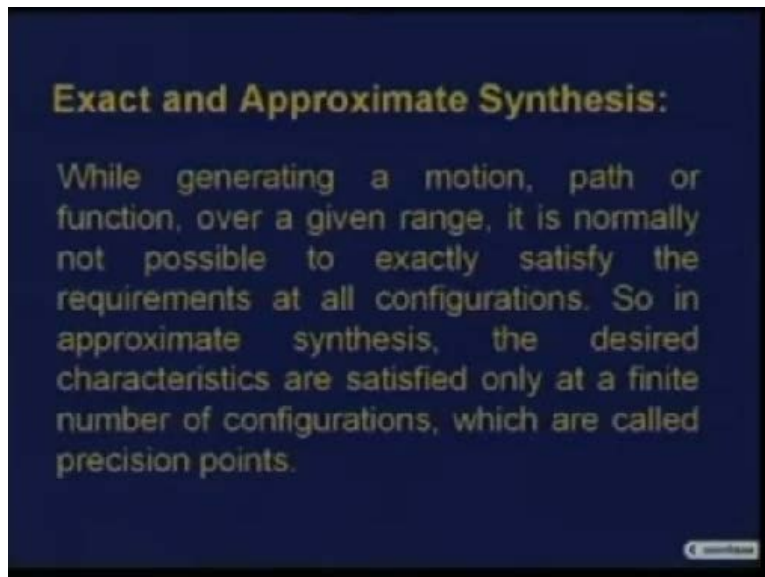
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In a function generation problem, the objective is to maintain a prescribed relationship between the output and input movements, that means to determine the kinematic dimension such that, the output movement and the input movement are co-related according to a prescribed manner. The last class of problem, we call dead center problems. In this type of problem, we have to design the linkage such that, we can generate the prescribed dead-center configurations. So, we have four classes of problems in dimension synthesis of linkages namely, motion generation, path generation, function generation and dead center problems.

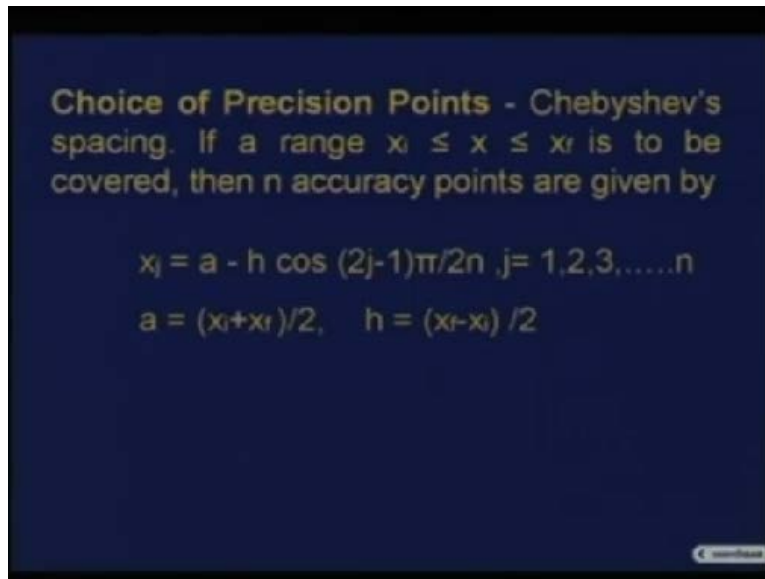
At this stage, we must emphasize that in the synthesis problem, we can talk off exact synthesis and approximate synthesis that is, the desired or prescribed motion characteristics are satisfied exactly over the entire range of movement or satisfied only approximately during the entire range of movement. More often than not, it is not possible to use the simple linkage and satisfy the prescribed motion characteristics exactly over the entire range of movement. We should have to be satisfied, while I use a simple linkage with the approximate synthesis. That means, during the entire range of movement the prescribed motion characteristics will be satisfied only approximately.

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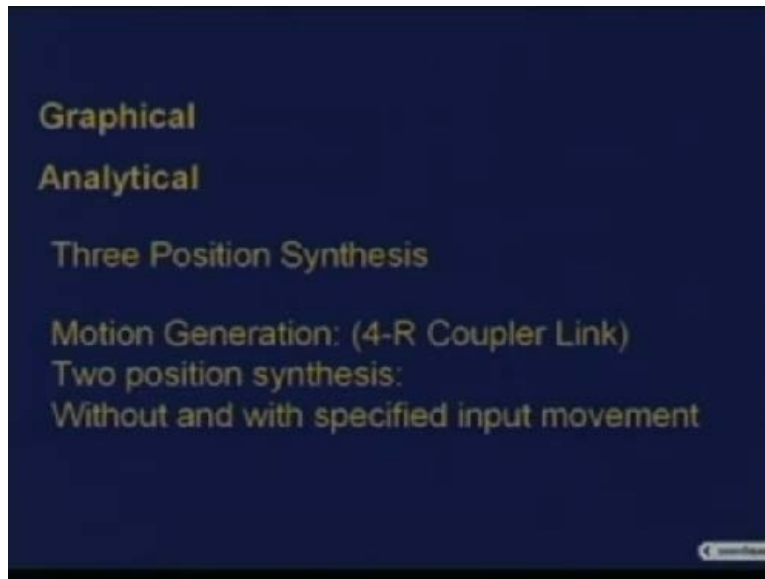
There are various approaches to this approximate synthesis and we will take up only, what we call a precision point approach. While generating a motion path or function over a given range, it is normally not possible to exactly satisfy the requirement at all configurations. In approximate synthesis the desired characteristics are satisfied only at a finite number of configurations in that entire range. These particular configurations, when the motion characteristics are satisfied exactly are called precision points.

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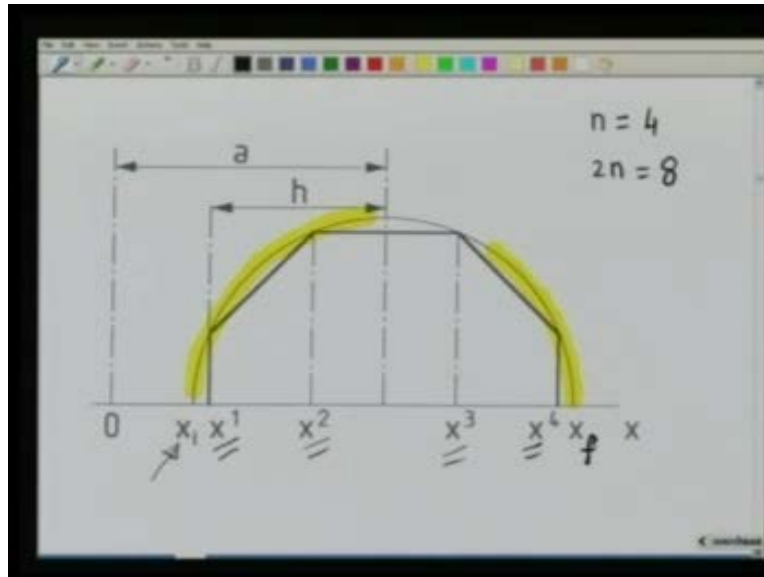
Our next task is, to choose these precision points in a better fashion. For the choice of precision points, initially we take Chebyshev's spacing. What do we mean by Chebyshev's spacing? Suppose, the motion characteristics have to be satisfied, covering a range of the variable x , starting from x_i to x_f , x_i is the initial value of x , x_f is the final value of x . If the motion characteristics have to be satisfied in this entire range of x , then we choose the n number of accuracy points, where n can be 2, 3, 4 and so on. There is a limitation on this value of n depending on the type of linkage that we use. To choose the Chebyshev's accuracy point, we take accuracy points x_j , which is equal to a minus h cosine $2j$ minus 1 into pi by $2n$, where j goes from 1 to n . That means we take n accuracy points namely x_1, x_2, x_3 up to x_n . In this expression, the variable 'a' denotes the mid-point of the range, that is x_i plus x_f by 2 and h denotes the half of the range that is x_f minus x_i by 2.

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For the dimension synthesis of linkage, just like kinematics analysis, two methods are available namely, graphical and analytical. We will start our discussion with the graphical method. In Graphical method we can start with three position synthesis. Let me now illustrate, what we mean by the three position synthesis. With reference to various types of problems, namely motion generation with or without coordination with the input movement, path generation problem with or without coordination with the input movement and the function generation problem. We shall illustrate this method assuming that the linkage we are going to design is a 4R-linkage. Before I explain the different types of problems with reference to a 4R-linkage, let me continue our discussion a little more with the Chebyshev's spacing of precision points.

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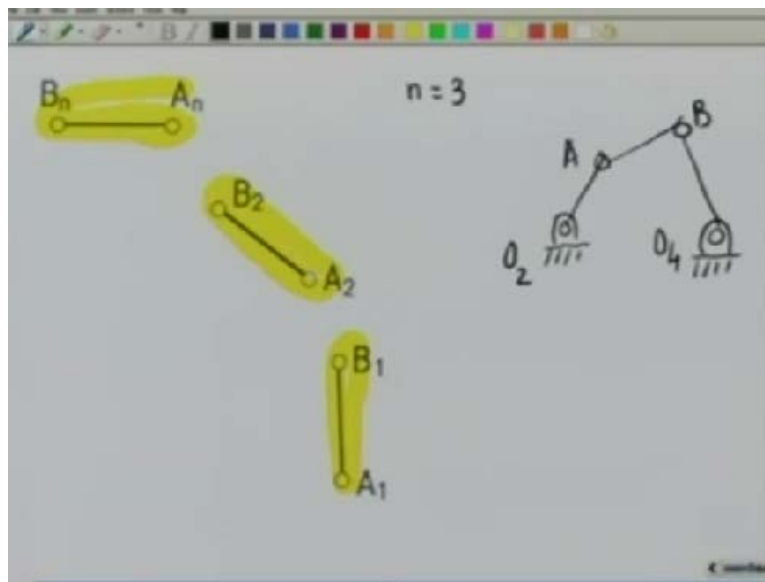
Suppose, we have to satisfy the requirement in a range of variables starting from x_i , that is x initial, to x_f , we have already seen how to choose the various precision points x_j . We have given the expression as $a - h \cos(2j - 1) \pi / 2n$, when n number of accuracy points are to be chosen in this interval from x_i to x_f , where j goes from 1, 2, and 3 up to n . If I put x^1 , I will put j equal to one there and n is already known to be depending on, whether we are going for three accuracy points or four accuracy points. Accordingly, n will be three or four. This expression can also be used to draw graphically. Whatever we get from x_j from this expression, can also be obtained from this simple drawing. We draw a semi-circle as we can see with x_i , x_f as the diameter.

This is a semi-circle drawn with x_i and x_f as the diameter. The midpoint of this interval that is, x_i plus x_f by 2 is a . Similarly, this h is the half of the range that is, x_f minus x_i divided by 2. To obtain three accuracy points in this interval, which are given as a Chebyshev's accuracy points, so that, the motion characteristics will be satisfied at these three points, namely x to the power 1, x to the power 2 and x to the power 3 and everywhere else, there will be some errors. What we have done, because n is equal to three, we have drawn a regular polygon of $2n$ number of sides, that is hexagon such that, two of its sides of this hexagon are normal to x axis. Then, the projection of these three vertices will automatically determine the Chebyshev's accuracy points given by this formula, this is x to the power 1, this is x to the power 2, this is x to the power 3. For

example, if we have n equal to 4, suppose we are looking for four Chebyshev's accuracy points n equal to 4, as before we draw a semi-circle with x_i, x_f as the diameter. Then we draw a regular polygon of $2n$ sides, that is an octagon with two of its sides perpendicular to the x axis. One side is perpendicular to the x axis and another side also is perpendicular to the x axis.

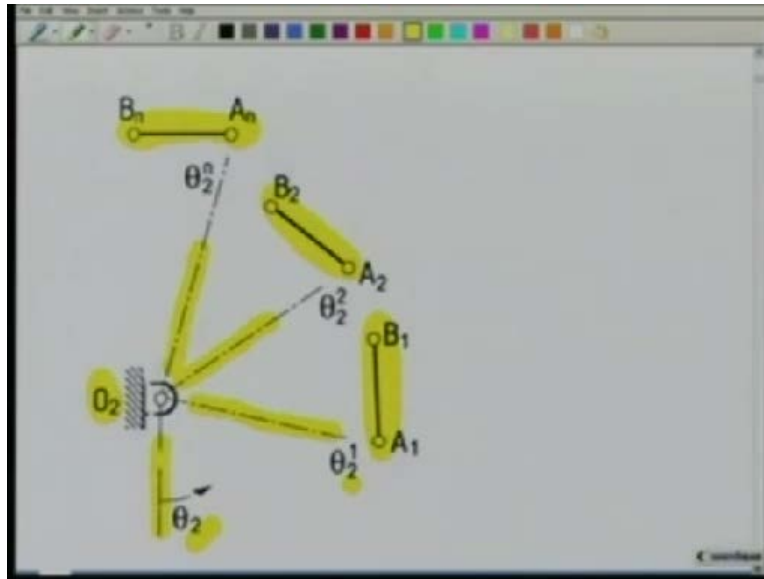
We inscribe an octagon with two sides perpendicular to the x axis. Then, the projection of these vertices of this octagon automatically determines the Chebyshev's accuracy points namely x to the power 1, x to the power 2, x to the power 3 and x to the power 4 in this range from x_i to x_f . x_f is the final value of x , x_i is the initial value of x . This is nothing but the graphical representation of the equation through which, we determine the Chebyshev's accuracy points in a given interval.

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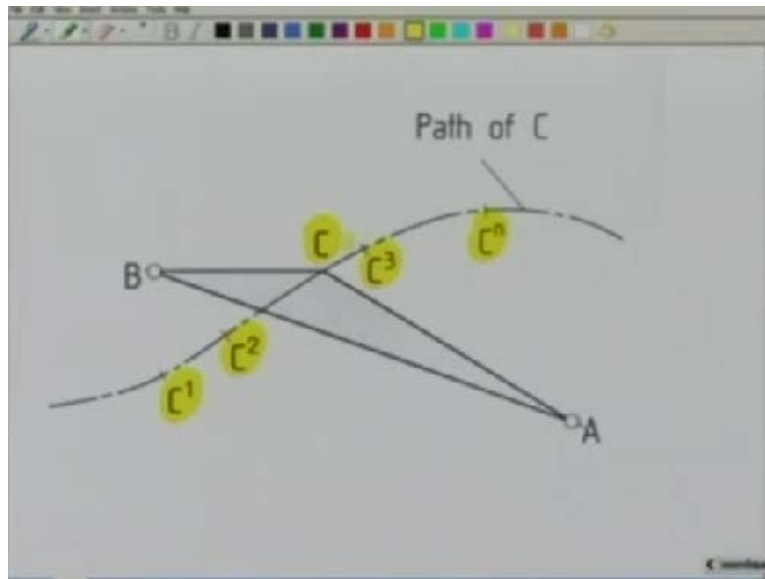
At this stage, let me say, what we mean by a guidance problem. Suppose, we are talking of a four bar linkage namely O_2, A, B, O_4 , where AB is the coupler. By motion generation problem with three accuracy points, this coupler rod AB has to take up these three positions namely A_1B_1, A_2B_2 , and A_3B_3 , that is n equal to 3. Our objective is to determine this 4R-linkage such that, the coupler AB occupies these three positions during the motion of the mechanism. This is guidance problem without any coordination with the input movement.

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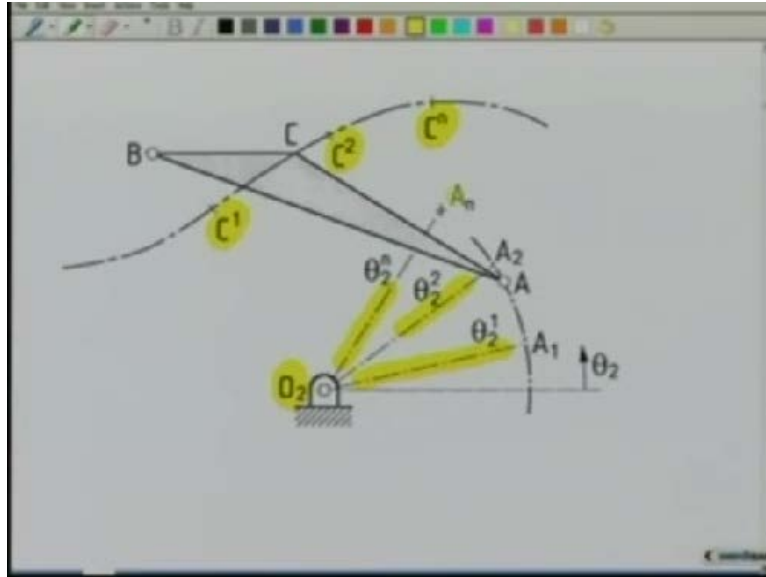
Suppose we have a guidance problem, which is coordinated with the input movement then, the same coupler AB, when it takes up the first prescribed position A_1B_1 , I want the input link, which is hinged at O_2 takes up the position given by this line that is θ_2 , the angle made from some reference line. This is θ_2 , which is the rotation of the input link O_2A . This is the guidance problem coordinated with the input movement. That means, when the coupler occupies the position A_1B_1 , the link O_2A must come along this line denoted by θ_2 one. Similarly, when the coupler goes to the second position, namely, A_2B_2 the input link must be along this line as denoted by this θ_2 two. Similarly, when the coupler is at nth position A_nB_n , which for n equal to three will be A_3B_3 , the input link must be along this line as given by θ_2 n. This is what we mean by guidance problem, but coordinated with the input movement.

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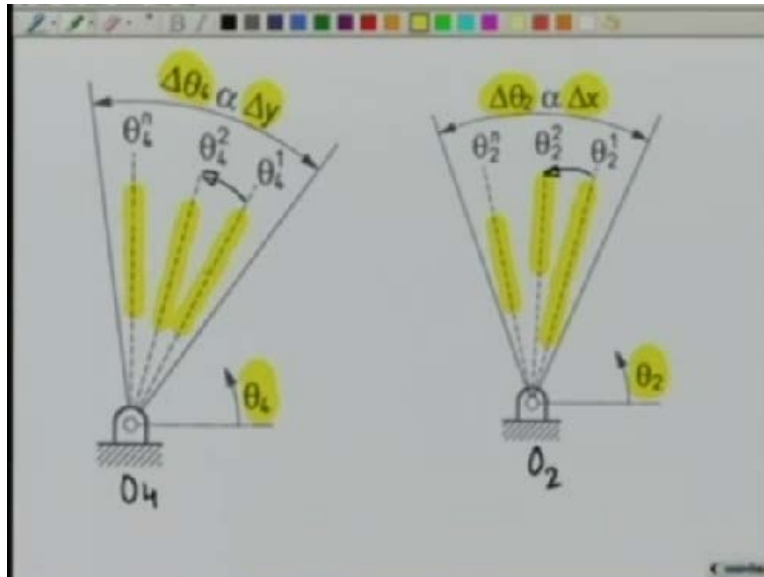
Again, we consider the same 4R-linkage with ABC as my coupler and C is the coupler point. By path generation problem, we mean that we have to design the linkage $O_2AB O_4$, the 4R-linkage we have to come up with these dimensions, such that when the linkage moves the coupler point C, passes through these four prescribed positions namely, C to the power 1, C to the power 2, C to the power 3 and C to the power 4. This is C to the power n, where n can be 3, 4 or 5 depending on the problem statement. This is path generation without any coordination with the input movement. We are not bothered about the input movement as the coupler point goes from C to the power 1 to C to the power 2 or from C to the power 2 to C to the power 3. This is path generation without any coordination with the input movement.

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This figure explains path generation coordinated with the input movement. Say, O_2 is the fixed hinge and O_2A is the input link. We mean that, when the coupler point C is at C to the power 1, I want the input link O_2A must be along this line O_2A_1 that is θ_2 one, this θ_2 is measured from some reference line. When the coupler point C goes to C to the power 2, I want the input link must be along this line that is, O_2A_2 and when the coupler point comes to C to the power n , the input link O_2A must be along this line O_2A_n . This is what we mean by path generation coordinated with the input movement. Not only the coupler point has to pass through C to the power 1, C to the power 2, C to the power n some prescribed points, but also when they pass through these points the input link must take up these configurations namely O_2A_1 , O_2A_2 , O_2A_n respectively. This is path generation coordinated with the input movement.

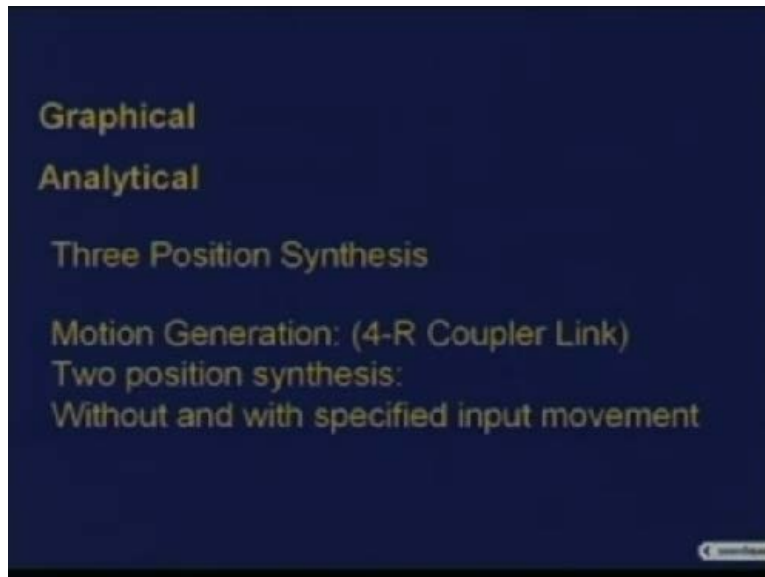
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This figure explains what we understand by a function generation problem. Again, we are talking of a 4R-linkage namely, O_2 , A, B and O_4 . O_2 and O_4 are the two fixed hinges, O_2A is the input link and O_4B is the output link. What we want is the movement of this output links, which is denoted by the change in the angle θ_4 measured from some reference line. This movement of the output link is to be coordinated according to a specific function with the input movement, which is given by the change in this angle θ_2 . That is, the change in the angle $\Delta\theta_4$, we assume, proportional to the change in the input variable Δx . $\Delta\theta_4$ is proportional to Δx and the change in the movement of the output link, that is change in the angle $\Delta\theta_4$, is proportional to the change in the dependent variable y , that is $\Delta\theta_4$ is proportional to Δy . Suppose, we are talking of three position synthesis, we want, when the input linkage along this line, the output link must be along this line. As the input link moves through this angle and occupies this position, the output link must rotate by a prescribed amount as shown here and the output link must occupy this position. Similarly, when the input link is along this line the output link must be along this line. This is what we mean by function generation, the output movement $\Delta\theta_4$ is coordinated according to a given manner with the movement of the input link that is $\Delta\theta_2$.

We have just now explained what we understand by motion generation and path generation problem with reference to a 4R-linkage that is the movement of the coupler or the movement of a coupler point.

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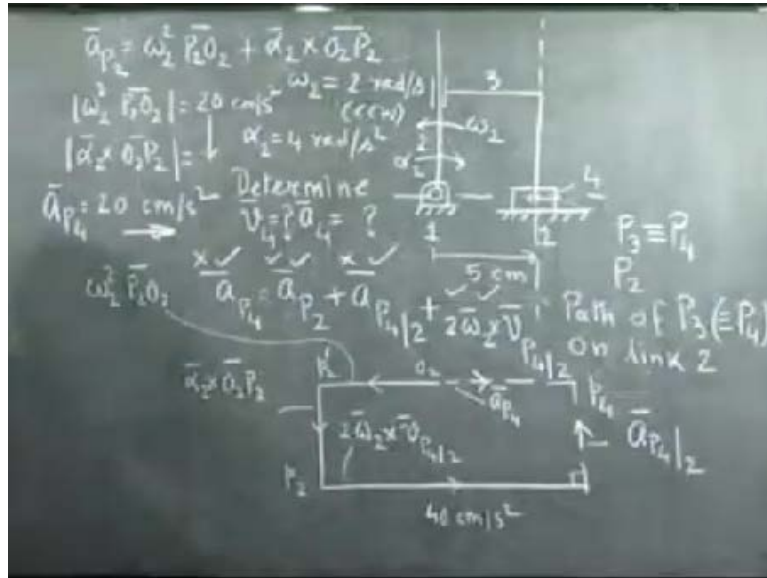


Three position synthesis means that the specified motion characteristics required, will be satisfied only at three isolated configurations. We have also seen that these three positions, where the motion requirement will be satisfied are chosen according to the Chebyshev's spacing in a given range, which ensures the error between these accuracy points will not be too much. The motion generation, we have explained with respect to 4-R coupler link, so we have explained the path generation. We can start with two position synthesis without and with specified input movement. This, we will take up in the next lecture.

Let me now summarize what we have learnt today. We have defined the ~~problem of~~ dimensional synthesis of a linkage and then classified the various types of problems which are encountered in dimensional synthesis namely, motion generation, path generation, function generation and dead-center problems. In our subsequent lectures, we will develop both graphical and analytical method to analyze these problems. We have already seen that, once we are capable of solving these problems, then these methods can be useful towards the design of real life mechanism.

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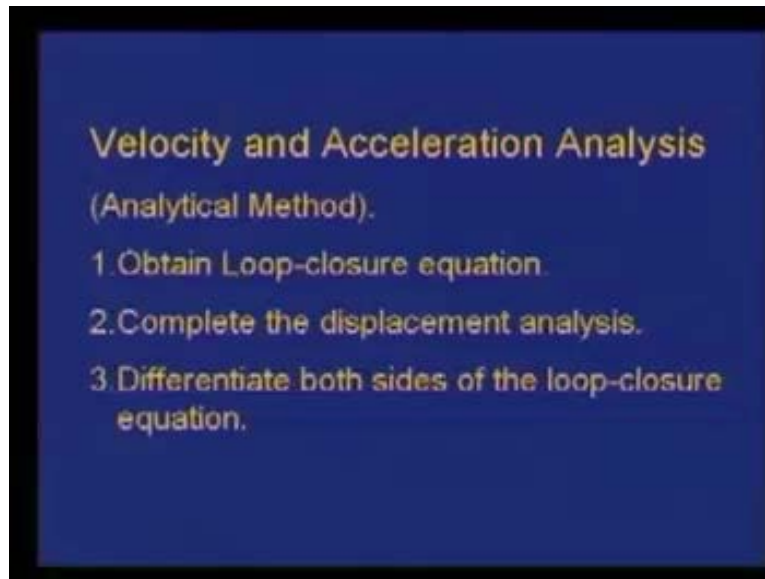
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This vector is ω_2^2 squared into p_2 . This vector is completely known. Let me try to draw this vector at p_2 . $v_{p_4/2}$ was ten, ω_2 is two. So, two into two, four into ten is 40 centimeters per second square and it is in the horizontal direction but double of this length, this is 40 centimeters per second square and this vector represents twice ω_2 cross $v_{p_4/2}$ and $a_{p_4/2}$ I know is vertical. I draw a vertical line through this point this is horizontal. So, it is a 90 degree and these four vectors, these two vectors represent this and this vector represents this and this vector is in the vertical direction and all these three vectors summed over must give $a_{p_4/2}$ which is horizontal, so I draw a line horizontal and wherever they intersect that gives me the point p_4 . According to this vector equation this $\omega_2^2 \vec{r}_{O_2 P_2}$ represents acceleration of p_4 and this represents acceleration of p_4 as seen by an observer body two.

This diagram represents this vector equation and it is easy to see because this is 40 horizontal line, this is 20, which is also horizontal, this must be 20. This $a_{p_4/2}$ I get an answer 20 centimeters per second square in the horizontal direction from left to right. This example clearly shows the power of considering the instantaneously coincident points on various links, when we have sliding joints on rotating links.

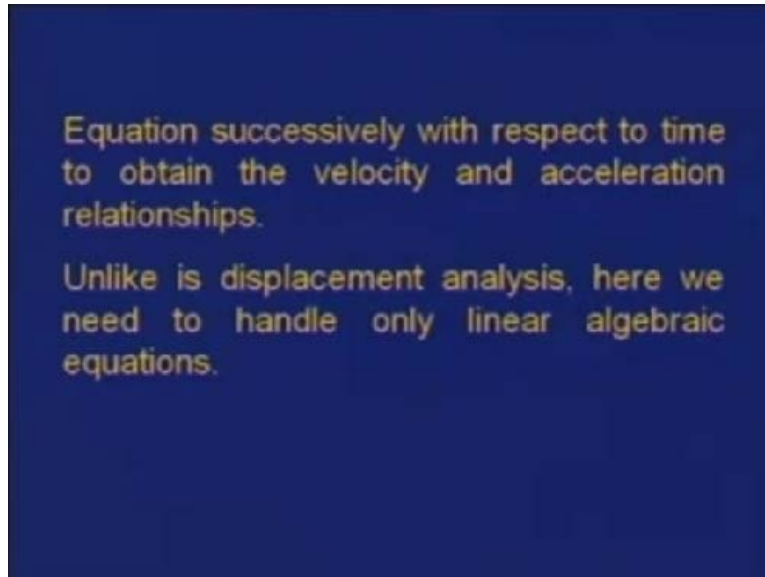
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We have completed velocity and acceleration analysis by graphical method. Let me show you how to carry out such velocity and acceleration analysis through analytical method.

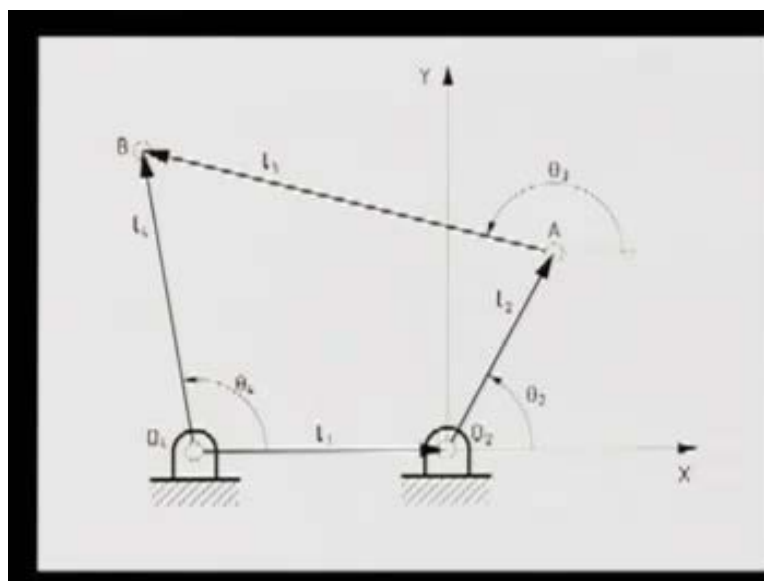
In analytical method, as in the displacement analysis, we start ~~with the loop~~ closure equation. We represent the link length and sliding displacement as vector quantities and go through each loop that is present in the mechanism and write the corresponding loop closure equation in terms of these vectors. The first step for carrying out the velocity and acceleration analysis is obviously to complete the displacement analysis. The loop closure equation is valid for all instants of time. We can differentiate both sides of such loop closer equations to complete the velocity and acceleration analysis.

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Successive differentiation of this loop closure equation with respect to time, will give us the required velocity and acceleration relationships. I would like to mention one point here, that in the displacement analysis through analytical method, we always get non-linear algebraic equation. Whereas, if the displacement analysis is complete, which is always necessary to carry out the velocity and acceleration analysis. For velocity and acceleration analysis, we always get linear equations in the unknown that means the problem is much simpler.

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As an example let me start with a 4R-linkage that means four links connected by four revolute joints. Let us look at the diagram of this 4R-linkage namely O_2 , AB, and O_4 . As in the displacement analysis, we set up a Cartesian coordinate system xy with the origin at the revolute pair O_2 . The fixed link is represented by O_4O_2 , the input link by O_2A , the coupler by AB and the follower or output link by O_4B . For this given configuration, that is, if θ_2 is given by displacement analysis, we can obtain θ_3 and θ_4 . After doing this displacement analysis, that is, knowing θ_3 and θ_4 , we should be in a position to carry out the velocity analysis. What do we mean by velocity analysis? Let us say, the input velocity that is, $\dot{\theta}_2$ the angle of velocity of link two is prescribed and we have to find out the angular velocities of link 3 and link 4, which are given by $\dot{\theta}_3$ and $\dot{\theta}_4$ respectively. Obviously everything measured in the counter-clockwise direction. Let me now write loop closure equation.

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$$\begin{aligned}
 \bar{l}_1 + \bar{l}_2 + \bar{l}_3 &= \bar{l}_4 \\
 l_1 + l_2 e^{i\theta_2} + l_3 e^{i\theta_3} &= l_4 e^{i\theta_4} \\
 l_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 &= l_4 \cos \theta_4 \quad \text{--- (i)} \\
 l_2 \sin \theta_2 + l_3 \sin \theta_3 &= l_4 \sin \theta_4 \quad \text{--- (ii)}
 \end{aligned}$$

Vector l_1 plus vector l_2 plus vector l_3 is vector l_4 . This is exactly the same as we did in the displacement analysis. Let me write these two dimensional vectors in terms of complex exponential notation, that is, l_1 plus l_2 plus e to the power $i \theta_2$. The vector l_2 can be represented by this complex exponential notation magnitude is l_2 and the orientation of this vector with the positive x axis is given by θ_2 . Similarly l_3 vector is $l_3 e$ to the power $i \theta_3$ is equal to $l_4 e$ to the power $i \theta_4$. As we know e to the power $i \theta$ always can be written as $\cos \theta$ plus $i \sin \theta$. So what we say is, we equate the

real and imaginary parts of this particular complex equation, which is same as saying equating the x component and y component of two sides of this vector equation, that way we get $l_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 = l_4 \cos \theta_4$. Equating the imaginary parts, we get $l_2 \sin \theta_2 + l_3 \sin \theta_3 = l_4 \sin \theta_4$.

This complex equation is equivalent to two real equations, which I mark as equation number 1 and 2. These equations are valid for all instants of time where θ_2 , θ_3 and θ_4 are functions of time, whereas link-lengths l_1 , l_2 , l_3 and l_4 are time independent constants. Because these equations are valid for all instants time, I can differentiate both sides of these two equations with respect to time and I can write from first equation.

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