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Module - 5 Lecture – 2 Velocity and Acceleration Analysis

Today, we continue our discussion on velocity and acceleration analysis of planar mechanism, so the use of velocity and acceleration diagrams.

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Before, we take up a little more complicated problem of velocity and acceleration analysis, I would like to develop a very useful concept involving the so called Coriolis acceleration. Very often, we find in mechanisms one point of a rigid body number i is constrain to move along a given path in another moving rigid body number j.

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In such a situation, it will be usual to consider instantaneously two coincident points: one, belonging to the first body that is I and the other in the second body that is number j. In such a situation as we shall see that one component of acceleration, which is not trivial become very important. Consideration of that particular component, which we call Coriolis component of acceleration, is very important.

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Let us consider two rigid bodies shown in this figure namely number 2 and number 3. As we see that there is a slot in this body number 2 and there is a spin in this body 3 the centre of, which is always constrain to move along the slot. So, we consider two rigid bodies namely number 2 and 3. We consider instantaneously two coincident points say P_2 and P_3 . At this instant, P_2 and P_3 are coincident, but P_2 belongs to body 2 and P_3 belongs to body 3. This is the configuration of these two bodies at the time instant t. After some time delta_t that is t equal to t plus delta_t the body 2, which is moving has taken up this configuration. From here, at the time t, at the time t plus delta_t body 2 has taken up new configuration and it is rotated through some angle and it is moved to some distance. Body number 3 has also moved and the centre of spin in body 3, P_3 as currently in this location, which is P_3 prime and the point P_2 which belongs to body 2 that is a point on this centre line of the slot has come here, which I call P_2 prime. So, P_2 and P_3 are the two instantaneously coincident points P_2 belonging to body 2, P_3 belonging to body 3. During this time delta_t P_2 has moved to P_2 prime and P_3 has moved to P_3 prime.

If we want to find out the velocity of the point P_3 , as the displacement of the point P_3 during this time delta_t is given by the vector P_3 to P_3 prime. If I say delta_s that is the displacement of the point P_3 is equal to the vector P_2 , P_2 prime the point P_2 has gone from P_2 to P_2 prime, this vector that is the displacement of the point P_2 plus P_2 prime P_3 prime. If we think of an observer who is sitting on body 2, if we ask this observer who is attached to body 2, what is the displacement of the point P_3 ? Then, his answer will be the point P_3 at time t was here, now it has moved to P_3 prime. So P_2 prime, P_3 prime is the displacement of the point P_3 as seen by an observer sitting or attached to body 2. This is the notation we use delta_{P3} slope 2, which means that is the displacement of the point P_3 as seen by an observer attached to body 2.

Now, If we divide both sides of this vector equation by the time delta_t and take the limit as delta_t goes to zero, we get velocity of the point P_3 is velocity of the point P_2 , these are the absolute velocities of the P_3 and P_2 respectively plus velocity of the point P_3 as measured by an observer sitting on 2. This is a relationship between the velocities of these two instantaneously coincident points namely P_2 and P_3 . Institution might suggest that a similar relationship will also hold good for the acceleration of these two points, that is acceleration of P_3 is acceleration of the point P_2 plus acceleration of the point P_3 as measured by an observer attached to the body 2. However, this is wrong this is not correct, in fact we get another term, which is twice of $\text{omega}_2 \operatorname{cross} V_{P3/2}$, this last term is called the Coriolis component of acceleration.

Let me repeat, by considering two instantaneously coincident points P_2 and P_3 , where the path of the point P_3 in body 2 is known, that is along in centre line of the slot V_{P3} is V_{P2} plus $V_{P3/2}$, where as a_{P3} is a_{P2} plus $a_{P3/2}$ plus Coriolis component of acceleration, which is twice omega₂ cross $V_{P3/2}$. Now, this $V_{P3/2}$ and $a_{P3/2}$ are nothing but the velocity and acceleration of point P_3 as measured by an observer who is sitting or attached to body 2. Velocity of the point P_3 as seen by this observer on body 2 is always tangent to the path. If the path is known, then $V_{P3/2}$ is tangent to the path. But $a_{P3/2}$ has two components: one, tangent to the path and the other is normal to the path. The component of the acceleration $a_{P3/2}$ along to the normal to the path we already know is $a_{P3/2}$, if I take the normal to the path this component is $V_{P3/2}$ squared divided by the radius of curvature of path at that instant that is, the normal acceleration as we know V square by rho where rho is the curvature of path.

All these relationships will be very useful for kinematic analysis of mechanisms, where we will see that one point of a rigid body is constraint to move along a slot or along a given path in another moving rigid body. We have not proved why this Coriolis component of acceleration comes. I will take a simple example to demonstrate the existence of this component.

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To demonstrate the existence of Coriolis component of acceleration let us again consider two rigid bodies namely body 2, which is this rod, which is hinged at the point O2 and another rigid body namely 3 that is this block, which is constant to move along the rod. The path of P₃ we consider two instantaneously coincident points here, which we call P₂ and P₃. P₂ is a point on this rod and P₃ is a point on this block. P₂ and P₃ are coincident at this instant. What is the path of P_3 on 2? If we fix body 2, I can see the block P_3 can only go along this rod, so the path of P_3 on body 2 is this vertical line, which is along the rod. $V_{P3/2}$ will be along this vertical line. Let us consider, if $omega_2$ is the angular velocity of this rigid body 2 that is this rod is rotating counter clockwise and in time deltat rotates through the angle delta_{theta}. During this time delta_t, body 3 has moved from here to there. That is, P_3 has gone to P_3 prime, where as P_2 the point on the rod has moved to P_2 prime, because it is rotating about the point O_2 . Now, If we look for the acceleration of the point P_3 , we can talk of acceleration of the point P_3 along 2 by expressing its two components: one, along the rod and other is Perpendicular to rod. What you see, if we ask a person sitting on body 2, what is the movement of this point P_3 prime because the observer is always sitting on body 2 he or she thinks that P₃ is moving only along the rod, no motion perpendicular to the rod of P_3 prime is perceived by that observer. But he or she misses this transverse distance. According to the observer sitting on body 2, P₃ the transverse displacement is only this much. But as we see from the figure the transverse displacement

is this plus this, it is this plus a_{P3} prime, which the observer misses. Let us calculate what that is?

We see A_{P3} prime this distance, as we see during this time delta_t the rotation of this rod number 2 is delta_{theta} and this distance P_2 prime to P_3 prime, it is the movement of the block 3 along the rod, which is the velocity with which it is moving on the rod which is $V_{P3/2}$ into delta_t. During this time delta_t it is moving with the velocity $V_{P3/2}$ so, this is the moment from P_2 prime to P_3 prime and this angle is delta_{theta}. The equation becomes A_{P3} prime is equal to $V_{P3/2}$ into delta_t into delta_{theta}. Where delta_{theta} is nothing but omega₂ into deltat, AP3 prime is equal to VP3/2 into deltat into omega2 into deltat because it is rotating with constant angular velocity $omega_2$ even if it is not constant during time delta₂ due to angular velocity $omega_2$ the rotation $omega_2$ into delta₁. This comes out $V_{P3/2}$ into omega₂ into delta_t square. This extra transverse displacement that the point P₃ prime has, which the person or the observer sitting on body 2 missed is given by this. This extra displacement as we see is propositional to delta_t square, which means this is due to some acceleration in the transverse direction, which this observer is missing and what is the magnitude of that acceleration? That is magnitude a Coriolis is equal to twice of $V_{P3/2}$ into omega₂. Because, with this acceleration I can cover a distance half a Coriolis into delta_t square during the time interval delta_t which gives me back this A_{P3} prime. So, let me go through this argument on slope. The point P₃ has come to P₂ prime and according to the observer sitting on body 2, P_3 has gone only from P_2 prime to P_3 prime. The transverse direction displacement is P_2 to P_3 prime.

As we see there is an extra displacement in the transverse direction from the point P_3 which is given by A_{P3} prime and A_{P3} prime can be covered in time delta_t, if there is an acceleration with transverse direction of magnitude twice $V_{P3/2}$ into omega₂ and this is what we call Coriolis acceleration. As a result we can write, acceleration of P_3 is equal to acceleration of the coincident point P_2 plus whatever acceleration is along the slot, which this observer sees for the point P_3 that we denote by $a_{P3/2}$ and the component, which this observer misses which is this a Coriolis and that we can easily see by looking at the sign of the cross product twice omega₂ cross $V_{P3/2}$. $V_{P3/2}$ is along the slot omega₂ vector is perpendicular to the plane of motion and this cross product will give the magnitude of

Coriolis acceleration as this. This last term is called as Coriolis. At this stage let me consolidate whatever we have seen about these two coincident points.

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We identify two bodies namely body number i and body number j, that is the link number i and link number j. Suppose, it is the path of a point in body j, a point in body j I call P_i and it is the path of P_i in body i is known. This is very important, whether it is the path of P_i in body j is known or whether it is a path of P_i in body i is known. I am assuming the numbering in such, that it is the path of P_i in body number i is known. Then, I can find the velocity of the point V_{Pi} is velocity of the point P_i , which is instantaneously coincident with P_i plus velocity of P_i in i that is, as measured by an observer attached to body i and this velocity V_{pji} is tangential to this path of P_j in body i which is known. For acceleration, I will have acceleration of P_i is acceleration of P_i plus acceleration of P_i as seen an observer on body i plus a Coriolis, which is twice omega_i. Angular velocity of the body i is known cross V_{Pii} . This I write a_{pi} plus. There are two components of a_{Pii} the path of P_i body i is known these are two components: one, is normal to the path and the other, is tangential to the path. Normal component I write as apji the normal component n plus acceleration P_{ii} the tangential component, which is tangent to the path and normal component, is normal to the path plus a Coriolis, which I write twice omegai cross V_{Pii}. After the velocity analysis is completed as we see this normal component will be completely determined. Because the normal component is nothing, but V_{Pji} square divided by rho of that known path along the normal direction towards the centre of curvature plus other two terms as usual a_{pji} t plus a Coriolis that is, twice omega_i cross V_{pji} . As we will see later these two equations: first one, for the velocity and second one, for the acceleration will be very useful for carrying out kinematics analysis of planar mechanisms particularly, when one rigid body is guided along a given path in another rigid body. We shall solve the problem to show you the use of these two equations for carrying out the kinematics analysis.

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At this stage we consider the kinematic analysis of a slotted lever quick return mechanism, which we have seen earlier. This is the slotted lever quick return mechanism used in stepping machines. In this mechanism there are six links one is this rotating link on which, that is link number 2, fixed link is link number 1, this is link number 3 this circular block is link number 3, which is hinged to body 2 at this point and this is link number 4 the slotted lever, this is link number 5 and this tool holder of the tool is the link number 6, which has a prismatic pair in the body number 1. We see this mechanism moves at this particular point of body 2 or body 3 is always moving along the axis of this slotting body 4. If we call this point a_2 or a_3 , two coincident points: one, belonging to body 2 and the other is also belonging to body 3, a_2 and a_3 they always remain coincident.

If we consider a point a_4 , which is instantaneously coincident with a_2 and a_3 at this instant but belongs to this body 4, a_2 and a_3 they always remain coincident but a_4 moves in a circle, but a_2 and a_3 is moving in a different circle. a_4 is moving on a circle with this pointer center. a_2 and a_3 always remain coincident, but a_4 , a_2 and a_3 they have relative movement, a_4 is a point belonging to this slotted lever. That is the concept of coincident point's a_2 and a_3 always remember coincident, because this is the block center where body 3 is hinged to body 2, a_4 is a coincident point belonging to body 4, which is going in a circle with this point as center and this as radius. Whereas, a_2 and a_3 are going on a different circle so they always do not remain coincident. For the purpose of kinematics analysis it will be very useful to consider the velocity and acceleration relationship between these two instantaneously coincident points' a_2 and a_4 . This is the problem which will be analyzing now. Suppose the angular velocity of this body 2 that is the bull gear is given and assumed to be constant. Let us find out, at this configuration what is the velocity and acceleration of the tool holder.

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Let me first, draw the kinematic diagram of this slotted lever quick return mechanism. This is the fixed link number 1, this is the bull gear, the input link number 2, this is the block link number 3, this is the slotted lever link number 4, this is the tool holder link number 6 and this link is number 5, which connects the tool holder to the slotted lever.

Let me call this point O_2 the point fixed on link 2, this point O_4 that is the point fixed on link 4, this point let me call A and this point let me call B. The statement of the problem is the angular velocities of this input link omega₂ is given and let us assume it to be constant. Of course even it is not constant we can prescribe alpha₂ as well. Omega₂ is given and assumed to be constant that is alpha 2 for this problem let us take to be zero. We have to determine the velocity and acceleration of the point B, which is the same as the velocity and acceleration of the tool holder that links 6. To solve this problem let me first identify three coincident points at this block center. P_2 belongs to link 2, which always remain coincident with point P₃, which belongs to the block. P₂ belongs to link 2, P₃ belongs to link 3, but they always remain coincident, so I can write them to be the same point. We consider an instantaneously coincident point P₄, which at this instant is here, where P_2 and P_3 are but, P_4 belongs to link 4. We should note the difference that P_2 and P_3 moves on a circle with O_2 as the center, where as P_4 moves on a circle with O_4 as the center. It is the path of P_2 in body 4 is known that is along this line O_4A . The path of P_2 in 4 is O_4A , this is a straight line path the curvature of this path is infinity. To do the velocity analysis we start as usual from the input end.

We know velocity of the point P₂ is omega₂ cross O_2P_2 , which is completely known because of vector of O_2P_2 is known and also angular velocity vector omega₂ is known. As you have seen because, the path of P₂ in body 4 is known I can also write V_{P2} is V_{P4} plus velocity of P₂ in body 4. Because P₂ and P₄ are coincident we have already shown these relations. Let me see in this vector equation what are the known quantities are and what are the unknown quantities? Velocity of P₂ is completely known, I known its magnitude and its acceleration. But velocity of P₄ I do not know the magnitude, but I know the direction because that is perpendicular to O₄P₄, that is Perpendicular to this line O₄A. The direction is known but the magnitude is unknown. Velocity of P₂ in body 4, because the path of P₂ in body 4 is along O₄A so the velocity must be along that direction O₄A, here also I know the direction, but not the magnitude. As we see in this vector equation there are only two unknowns denoted by these two crosses, I can represent it by a velocity diagram. Let me try to draw it to some scale. Velocity of P₂ is perpendicular to O₂P₂ let me draw by this vector. This, as I said we represent by small letter o₂ and p₂ representing vector V_{p2}. The Pole of this velocity diagram also represents the fix point O_4 . Now, velocity of P_4 I know the direction that is perpendicular to O_4A , I draw a line through O_4 , which is perpendicular to O_4P_4 are O_4A . This vector must represent V_{P4} and $V_{P2/4}$ is along O_4A that is perpendicular to this line. I drop a perpendicular from P_2 to this line, this is perpendicular. To satisfy, this vector equation what I determine is this point must be P_4 . This point we denote by P_4 because o_4p_4 represents the velocity V_{P4} and this vector p_4p_2 represents $V_{P2/4}$.

We have list up to this stage of the velocity analysis. We notice that there are three points on this link 4 O_4 , P_4 and A. For O_4 I have this o_4 , for P_4 I have this small p_4 , the velocity of the point A I use the concept of velocity image and draw a diagram similar to this line O_4P_4A I write o_4p_4a such that, o_4p_4 by o_4a is O_4P_4 by O_4A . I just represent the capital letter by the corresponding small letters. Here, as we see O_4P_4A are in one line so here also small o_4p_4a are in one line and the ratio is also the same that is O_4P_4 by O_4A is o_4p_4 by o_4 . I determine the point 'a' and this o_4a determines the vector V_A , velocity of the point A. To determine the velocity of the point B we go as follows.

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A and B are the two points on the same rigid body 5, I can write V_B is V_A plus omega₅ cross AB. A and B are two points on the same rigid body 5, so their velocities are related through this equation, V_B is equal to V_A plus omega₅ cross AB. In this vector equation as

we know velocity has been completely determined I know both the magnitude and the direction. For V_B I know the direction, which is horizontal because the point B is constant to move along a horizontal path due to the prismatic pair between 1 and 6. Omega₅ plus AB I do not know the magnitude, because I do not know the magnitude of omega₅, but I know the direction it is perpendicular to vector AB. I know the direction, but do not know the magnitude. For V_B I know the direction horizontal, but I do not know the magnitude.

Again in this vector equation I have only two crosses that is two unknowns, I should be able to complete the velocity diagram. For V_B , because it is horizontal I draw a line, which is horizontal through the point O_2 or the pole of the velocity diagram. This is V_A we have already got O_4a , V_B must be horizontal so, I draw a horizontal line and omega₅ plus AB is perpendicular to AB that is in this direction through A, I draw a line perpendicular to AB. Wherever in this line, which is Perpendicular to AB must represent omega₅ cross AB. And wherever it intersects the horizontal line drawn through the Pole of the velocity diagram, determines the point b. As we see V_A plus omega₅ plus AB gives me V_B . O_2b determines the velocity of the point B that is the velocity of the slider V_6 to the same scale in which this velocity diagram has been drawn.

So, the velocity diagram is complete. We have shown you the use of the velocity image by locating the point 'a' we have also shown the use of the coincident points namely P_2 and P_4 because the path of P_2 in body 4 was known that was along O_4A . At this stage complete velocity analysis has been done so, I can find all the angular velocity like omega₄ or omega₅. Because, as we have seen this vector of a, b represents this vector omega₅ cross AB. Because, AB is known, this vector is known I can determine the only unknown omega₅. Similarly, o_4p_4 is nothing but if I consider o_4p_4 that is the velocity of the point P_4 , which is omega₄ cross O_4P_4 because I know O_4P_4 is this distance and I know this velocity. I can also determine the angular velocity omega₄. Omega₃ is same as omega₄, because there is only a prismatic pair between 3 and 4, so they must have same amount of rotation there can be only relative translation. So, omega₃ is same as omega₄. So, this completes the velocity analysis of the six-link mechanism given in the input angular velocity omega₂. Everything else has been written and once the velocity analysis is completed, all these angular velocities omega₃, omega₄, omega₅ is known we are in a position to start the acceleration analysis. Now, we shall carry out the acceleration analysis of the same slotted lever quick return mechanism.



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The kinematics diagram we had already drawn and we are saying omega_2 is given and let us assume it to be constant as before and objective is to find the acceleration of the point B that is the acceleration of the body 6, which is a slider. Again, we consider the coincident points P₂ and P₄.

Now, acceleration of the point P_2 is omega₂ square P_2O_2 . Because this point is fixed so P_2 is going in a circle the acceleration of P_2 is towards O_2 that is, P_2O_2 the direction and omega₂ square. If there are alpha₂ then I will have A transverse component that is perpendicular to O_2P_2 , which is alpha₂ cross O_2P_2 but alpha I have taken to be zero this is the total acceleration of the point P_2 , which is completely known.

Now, we know considering the two coincident points P_2 and P_4 it is a_{P2} I can write a_{P4} plus acceleration of P_2 in body 4 plus the Coriolis acceleration, which is twice omega₄ cross $V_{P2/4}$. In this vector equation, let us see what the quantities which are known and what are the quantities which are unknown? a_{P2} is completely known from the input given by acceleration. a_{P4} has two components. One is, P_4O_4 which I can write a_{P4} . There

are two components one is, omega square P_4O_4 plus alpha₄ cross O_4P_4 . These two vectors together decide a_{P4} . Then, we have $a_{P2/4}$, this has two components one is, $a_{P2/4}$ normal to the path of P_2 in body 4 plus $a_{P2/4}$ tangential to the path of P_2 in body 4 plus the Coriolis acceleration that is twice omega₄ cross $V_{P2/4}$. This is the complete expression for a_{P2} .

Now, let us see on the right hand side which vectors are completely known and which are still unknown. Because we have already determined omega_4 and P_4O_4 this vector is known so both magnitude and the direction are known. Alpha₄ cross O_4P_4 this is perpendicular to O_4P_4 so, I know the direction that is perpendicular to the line O_4P_4 but I do not know alpha so, the magnitude is unknown. For this vector the magnitude is unknown but the direction is known. $a_{P2/4}$ n that is the path of P_2 in body 4, which is the straight line O_4A . Because it is a straight path the radius of curvature of this path is infinity and this as we discussed earlier, if we write separately $a_{P2/4}$ n is nothing but $V_{P2/4}$ square by rho, along the normal to the path toward the center of curvature.

Now, for this particular case, because the rho is infinity because the path is a straight line the radius of curvature of path is infinity this rho is infinity, this is zero. This goes to zero now $a_{P2/4}$ t that is tangential to the path. Because the path is a straight line, the path itself is tangent so I know the direction along the straight line O₄P but the magnitude is unknown. Whereas, Coriolis component because we have completed the velocity analysis we have already determined $V_{P2/4}$ and we have already determined omega₄, this quantity is completely known I know both its magnitude and its direction. So, what we see is that in this vector equation I have one cross here and another cross there so there are only two unknown quantities. Consequentially, this vector equation can be drawn in the vector diagram. We, by considering two instantaneously coincident points P2 and P4 we reach this acceleration equation between a_{P2} and the Coriolis component $a_{P2/4}$ and a_{P4} . Because there are two unknowns in this vector equation let me draw the acceleration diagram, vector diagram. The acceleration of a_{P2} is this way, which is to some scale I represent this is O_2 and this is P_2 , which is the acceleration of the point P_2 . Omega₄ square P_4 is completely known and that is parallel to this line towards O₄ so I can draw to the same scale let it come up to this point. This I call P₄ prime alpha 4 plus O₄P₄ is perpendicular

to this line but the magnitude is not known so, I draw a line perpendicular to this and the point P_4 will lie on this line such that, O_4P_4 , O_2 and O_4 is the pole of the diagram O_4P_4 will denote the acceleration of the point P_4 . Now, twice omega₄ cross $V_{p2/4}$ is completely known and that is perpendicular to this O_4A because $V_{p2/4}$ is along this O_4A .

Let us look at this term, which is the Coriolis component of acceleration, which is completely known. If we go back to the relevant portion of the velocity diagram what we see is that V_{P4} is in this direction O_4P_4 , which means omega₄ is counter-clockwise. And this vector p_4p_2 represents $V_{p2/4}$ that is this vector. This is cross product because omega₄ is counter-clockwise then $V_{p2/4}$ is in the upward direction along O_4A . This vector turns out to be perpendicular to O_4A and in this direction in the same of omega₄ is rotated through 90 degree.

If, I transfer this term to the left hand side it comes with a negative sign. So, here I draw parallel to this line with the magnitude twice omega₄ cross $V_{p2/4}$. Now, this term $a_{p2/4}$ is tangential, which is along O₄A, from this point I draw a line which is parallel to O₄A and this line which we have drawn earlier to represent this vector, these two lines intersect here. This point I can call P₄, this vector is alpha₄ cross O₄P₄. This vector represents $a_{p2/4}$ tangential and this vector represents the Coriolis component of acceleration. Now, we see that acceleration of P₂ this vector is the sum of one, two, three, four vectors namely this one represented by this, this one is represented by this, this one is represented by this and the last term is represented by this (Refer Slide Time: 50:47). Once, I determine P₄ that means I know the acceleration of point P₄, which is represented by O₄P₄, this is acceleration of the point P₄. Once, we know P₄ using the concept of image I can easily get to the point A as explained earlier such that this point I write P₄ and this point I write a, such that O₄P₄ by O₄A is same as O₄P₄ by O₄A and o₄P₄ a lie on the same straight line.

This o_4a represents the acceleration of the point a_A . Once, we get the acceleration point A, the acceleration of the point B can be determined very easily. It is just like a slider crank, once we know the acceleration of the point A and acceleration of B is in horizontal direction, we can find the acceleration of point B. Just following this method which I

explained with reference to the slider crank mechanism. This I would like to leave it to you as an exercise.