Kinematics of Machines

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Module - 5 Lecture - 1

Velocity and Acceleration Analysis

The topic of today's lecture is velocity and acceleration analysis of planar mechanisms. Through velocity analysis, we obtain the velocity characteristics of all the links of a mechanism when the input velocity is specified. Similarly, through acceleration analysis we obtain the acceleration characteristics of all the links of a mechanism when the input acceleration is specified. Two points must be mentioned at this stage: one, to carry out the acceleration analysis velocity analysis must be completed first. Second, is that both these velocity and acceleration analysis are carried out for a particular configuration of the mechanism. Just like in displacement analysis, both graphical and analytical methods can be used to carry out the velocity and acceleration analysis. In today's lecture, we will present the graphical method.

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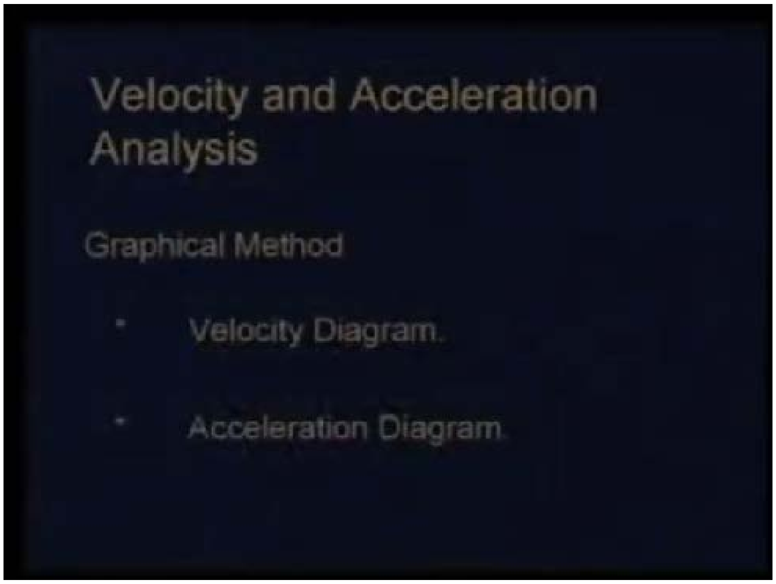

In this graphical method, we draw what is known as velocity and acceleration diagram.Because.it is a planer mechanism that the velocity vectors are represented by a two dimensional vector.

Velocity of various points of the mechanism will be drawn as vectors after choosing a scale. That is, the length of the vector will be adjusted depending on the scale of the diagram. Suppose, if we choose one centimeter represents so many meters per seconds velocity, similarly for acceleration diagram, I will choose a scale such that one centimeter in the diagram will represent so many meter per second square acceleration. So, we discussed about the velocity diagram and acceleration diagram. To bring out the salient features of drawing these velocity and acceleration diagram let me start with a very simple example.

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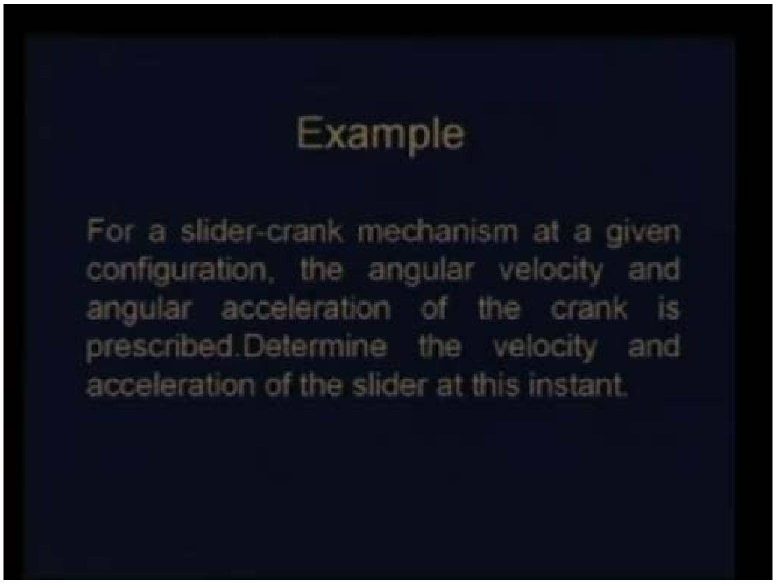

For a slider-crank mechanism at a given configuration, suppose the angular velocity and angular acceleration of the crank are prescribed. Our objective is to determine the velocity and acceleration of the slider at this particular instant that is at this particular configuration.

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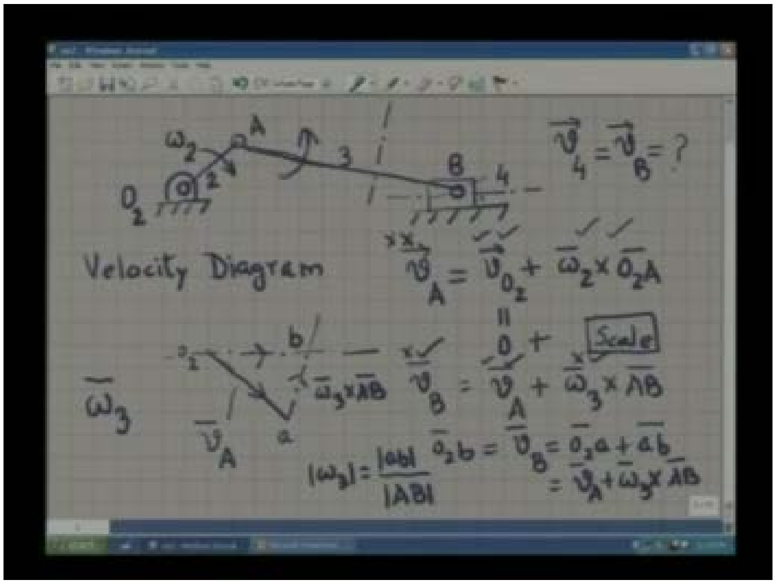

So, let us draw a kinematics diagram of a slider-crank mechanism. Let us say O_2A is the crank, AB is the connecting rod and link number 2 is the crank, link number 3 is the connecting rod and the slider is link number 4.The problem is that, if we specify angular velocity of this input link say the crank at this instant is omega$_2$ and I want to determine what is the velocity of the slider that is V of the slider which is body 4. It is same as the velocity of the point B. This is the particular configuration and the answer of those will be valid only for this particular configuration.

To carry out the velocity analysis, let me say how do I draw the velocity diagram? We start from the input n that is link from number 2. As we noticed that the two points namely A and O_2 belong to the same rigid body is link 2. We have already seen, because A and O_2 belong to the same rigid body. I can write velocity of A is velocity of O_2 plus vector omega$_2$ cross the vector O_2A. This is a vector equation and all these quantities are vectors and because it is in two dimension, each of these vectors has two unknowns namely magnitude and acceleration. Such a vector equation is equivalent to two scalar equations and we should be able to solve two scalar unknowns from this equation. So, before we start drawing the diagram I should ensure that this vector equation has only two unknowns.

For example, here V_{O2} the velocity of this point O_2 is zero, so both its magnitude and directions are known, because this is nothing but identically zero. Now here, the input velocity $omega_2$ is specified, the vector O_2A is completely known so one can easily calculate this vector $omega_2$ cross O_2A, so here also both magnitude and the directions are known. I put a tick to indicate that this is known and if it is unknown then I will put a cross. For example, we do not know anything about the velocity of the point A. Neither the magnitude nor the directions are known. But from this vector equation as we see there are only two unknowns indicated by these two crosses, so I can use this equation to solve both these unknowns. Once, I confirm that I can use this vector equation let me represent it by a vector diagram.

We draw the velocity of the point A, which is zero plus $omega_2$ cross O_2A, this vector I know the magnitude that is O_2 into O_2A, because the vector O_2 is perpendicular to the plane of motion and O_2A lies in the plane of motion. So the angle between them is 90 degrees, the magnitude of the cross product is nothing, but the magnitude of the product O_2 and O_2A. And the direction of this vector is perpendicular to O_2A in the sense of $omega_2$ that is how we determine the direction of the cross product.

So, I can draw velocity of A which is perpendicular to the line O_2A. So I draw velocity of the point A represented by this vector, while doing so I have to choose a scale of the diagram because the velocity of A may turn out to be some meter per seconds square. That one centimeter in the diagram represent so may meter per second velocity. So, this is vector V_A. Now, I name the points in this vector diagram which I call velocity diagram as follows. This vector represents the velocity of the point A, I indicate this point by small a. This capital A is the point A and the velocity of that V_A is represented by o_2a both o_2 and 'a', I write in small letters. The velocity of a is completely known. Let us go up to the unknown destination that is V_4 or V_b so this motion transfer points. Now, I consider two points namely B and A, which belong to the same rigid body namely 3. So, I can write velocity of the point B is velocity of the point A and both A and B belong to same rigid body namely 3, I can write $omega_3$ cross AB.

This is again a vector equation where velocity of A we have completely determined, I know its magnitude and direction. Velocity of B I do not know the magnitude, but I know the direction, which must be horizontal. Because of this prismatic pair, the point B always moves in the horizontal direction So, I know the direction of velocity B. Omega$_3$ cross AB I do not know its magnitude of this omega$_3$, the magnitude is unknown. But I know the direction of this vector omega$_3$ cross AB, because I know the direction of both omega$_3$ and AB, so the direction is known. Again in this vector equation, I have two unknowns one here the magnitude of omega$_3$ cross AB and the magnitude V_B and that can be determined from this vector equation.

Let me represent this vector equation graphically. I know that the velocity of the point B is horizontal, so it must be represented by a horizontal vector passing through O_2 and omega$_3$ cross AB is perpendicular to the line AB that is in this direction, which is perpendicular to the line AB. Through a I draw a line, which is perpendicular to the line capital AB. This intersection of these two lines must determine the point small b such that O_2b vector represents the velocity of the point capital V_B, which is as we see O_2a vector plus vector ab, where O_2a represents velocity of the point A and the line AB represents omega$_3$ cross AB. I put a vector sign here and a vector sign there. So O_2b represents velocity of B to the same scale which we have used earlier.

We have determined the velocity of the point B that is the velocity of the slider given in the input velocity omega$_2$. Before we go from the acceleration analysis, we must obtain all the information regarding the velocity characteristics. For example, now we determine omega$_3$ because I know this vector ab represent omega$_3$ cross AB. I can determine the magnitude of omega$_3$. Because I know the magnitude of the vector ab what velocity are representing and the magnitude of the vector AB. According to this diagram the velocity of the point b, the difference of the velocity 'b' and 'a' is in the upward direction, which means the angular velocity omega$_3$ must be counter-clockwise. We have determined both the direction and the magnitude of the angular velocity of link 3 that is the connecting clockwise. At this stage, we can start the acceleration analysis.

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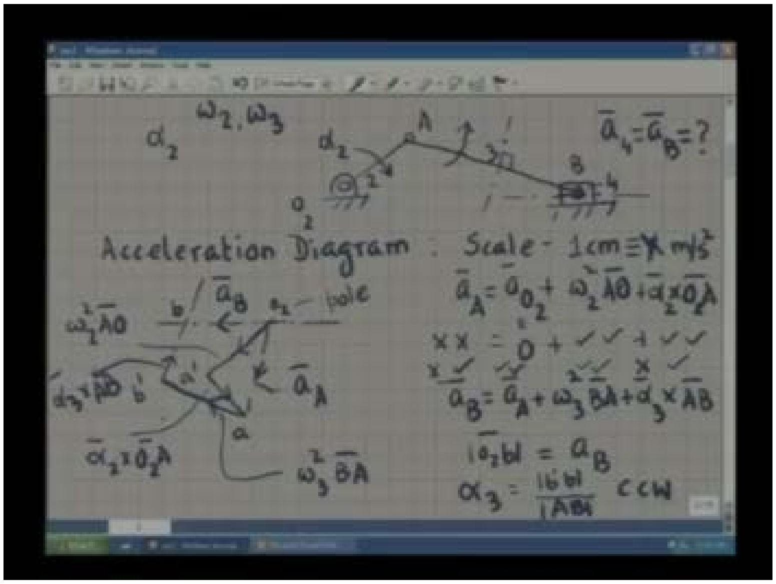

Suppose, we also give alpha$_2$ let me draw diagram again O_2, A, B. We have completed the velocity analysis for given omega$_2$. We have already determined omega$_3$. So omega$_2$, omega$_3$ are known at this stage and we also specify alpha$_2$ let us say in the clockwise direction. The question is that what will be the acceleration of this slider 4 or the acceleration of the point B. Given alpha$_2$, I have to find what is the acceleration of the point B.

Now, to draw the acceleration diagram we choose a scale. That is, the entire vector will be represented to a scale of one centimeter in the acceleration diagram we will represent some X meter per Second Square. This X is at our choice we will choose a suitable value of X to draw a diagram. Again we start from the input n and reach the output n that is the B through the motion transfer points. First, we determine A and O_2 at two points on the same rigid body 2, I write a_A is a_{O2} plus omega$_2$ square AO plus alpha$_2$ cross O_2A. This relationship we have derived earlier that the acceleration relation within two points on the same rigid body with angular velocity is omega and angular acceleration is alpha. In this vector equation a_{O2} is zero, because the point O_2 is fixed so it is completely known that this is zero. Omega$_2$ has been given, AO is known, I know this vector completely both the magnitude and direction. Alpha$_2$ is given, O_2A is known and this vector also I know both the magnitude and the direction. Whereas, a_A I neither know the magnitude nor know the acceleration.

So, this vector equation has two unknowns because it is only two unknowns I can use this vector equation to draw the vector diagram. First, I draw the acceleration of ‘a’ has two components $omegas_2$ square AO, this is the direction of the vector AO, I start from this point acceleration diagram and I draw this vector to represent $omega_2$ square AO then $alpha_2$ cross O_2A, which is perpendicular to the line O_2A. This line is parallel to O_2A, so I draw the perpendicular direction, this is $omega_2$ square AO and this vector is $alpha_2$ cross O_2A. These two vectors when added gives you the acceleration of the point A, at this point I start with O_2 and at this point I write A. Because, the vector O_2A represents the total acceleration of the point a, it has two components O_2A prime and a prime a.

Once we have obtain the acceleration of the point A, I can determine that of the point B by writing a_B as a_A, now we are considering, two points A and B on the same rigid body 3, so I add $omega_3$ squared vector BA plus $alpha_3$ cross AB. I have already determined the acceleration of the point ‘a’ completely, $omega_3$ we have determined from velocity analysis both magnitude and direction, so I can determine this vector completely both magnitude and directions are known, as $alpha_3$ is unknown, but I know the direction of $alpha_3$ plus AB, which is perpendicular to AB. So I know the direction but not the magnitude and acceleration of B again, because the point B is moving only along with a horizontal path the acceleration of B must be horizontal, so the direction is known, but the magnitude is unknown.

We see again in these vector equations there are only two crosses that is there are only two unknowns so I should be able to use this vector equation to draw the vectors diagram. So I draw $omega_3$ squared BA that is a vector, which is parallel to AB from a, I go to a point I draw this vector where I do not know the magnitude because we are not using any particular numbers. Let this vector $omega_3$ squared BA to the chosen scale becomes so much, I call this point b prime. This is the vector, which is $omega_3$ squared BA. Acceleration of the point B is horizontal, so I draw a line horizontal through the point O_2 and $alpha_3$ cross AB is perpendicular to B that means the direction is known it is perpendicular to AB. At b prime I draw a line, which is perpendicular to AB. This perpendicular line drawn through b prime, perpendicular to the line AB drawn through the point b prime is this line and the horizontal line drawn to O_2 is this line.

This equation suggest that, the intersection of these two lines I can denote by small b and O_2B will represent the acceleration of the point b, then only this equation will be satisfied. We see $a_{A,}$ Which is O_2A prime plus a prime a, then $omega_3$ squared BA, then $alpha_3$ plus AB will give us O_2B, which is AB. This vector represents $alpha_3$cross AB We have completed the acceleration diagram and using the scale of the diagram, I can find O_2b as acceleration of B. Magnitude of O_2b is the magnitude of the acceleration of point B on that of the slider. If this diagram is correct then acceleration of the slider at this particular configuration is toward left, because O_2b is toward left. We can also determine the other unknowns, the angular acceleration of link 3 $alpha_3$ because using this vector b prime b, which is $alpha_3$ cross AB, AB is a Magnitude is known so after finding what is b prime b, I can find the only unknown that is $alpha_3$.

If this diagram is correct then what we find is that the tangential component of the difference of acceleration of B with respect to A is perpendicular to AB represented by V_B and that is in the upward direction. Which means $alpha_3$ is counter-clock wise then only the tangential component of acceleration B with respect to A will be in this upward direction. $Alpha_3$we can find both the magnitude, which is magnitude of the vector b prime b divided by the magnitude of vector AB. In this particular diagram it is in the counter-clockwise direction. We have carried out the complete velocity and acceleration analysis of the slider crank mechanism.

I would like to point out certain special features going back to the velocity diagram, we start the diagram from the input end, which is the link 2. I start with the point O_2 this point is called pole of the diagram. It is the velocity pole of the diagram and we should also notice that we are using capital letters in this configuration diagram or kinematics diagram, whereas we use small letters for the velocity diagram and there is the corresponded capital A small a, capital B small b, capital O_2 small o_2. As we join all these points up to the velocity pole that is O_2B represents the velocity of the point capital B, O_2A represents the velocity of the point capital A.

Similarly, in the acceleration diagram, this O_2 is the pole capital A small a, capital B small b. O_2a vector represents acceleration of the point capital A and if I join small b with the pole of the diagram O_2 that is O_2b represents the acceleration of point capital B. If we maintain this convention then it will be very easy, particularly when the things get a little more complicated when there are too many vectors to determine velocity of a particular point of the mechanism. I just locate the corresponding small letter in the velocity or acceleration diagram and join that with the pole to determine the velocity or acceleration of that particular point. I hope this simple example, makes the procedure clear and for more complicated problems we need to develop a few more concepts.

We have just now solved a simple example to show how we carry out the velocity and acceleration analysis of planar mechanisms. At this stage, we will develop two more concepts which will be very useful particularly when we have a little more complicated problem.

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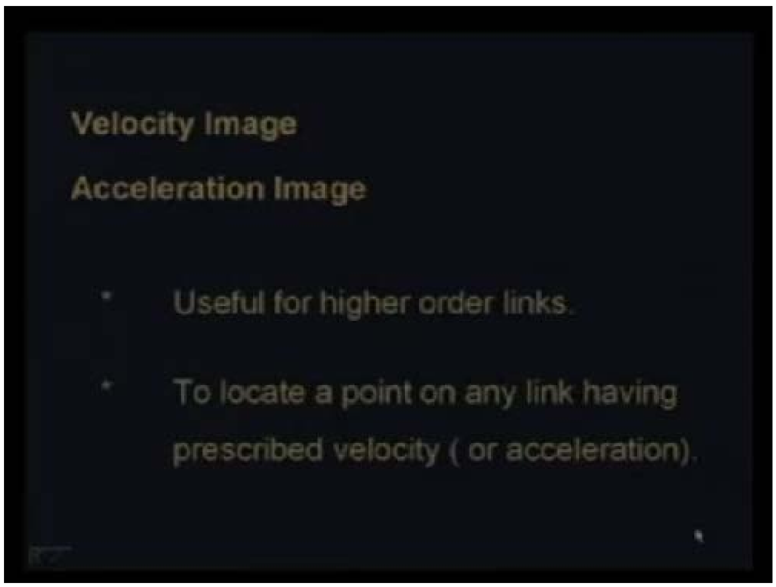

We start with what is known as velocity image of a particular link. Similarly, we develop what is known as acceleration image of a particular link. These image concepts will be very useful particularly if there are higher order links present in a mechanism. We have seen in a mechanism all links need not be binary there can be ternary, quaternary or other higher order links. When such higher order links are represented, then the concept of this velocity image and acceleration image will be very helpful. Another use of this image concept is, if we consider an inverse problem that means we have to locate a point on any link having prescribed velocity or acceleration at a particular configuration. What we have done earlier is given in the input velocity we have determined the velocity or acceleration of given points of the mechanism. Suppose, for a given input velocity and acceleration we want to determine, which particular point of a link will have prescribed velocity or acceleration then again this image concept will be very useful.

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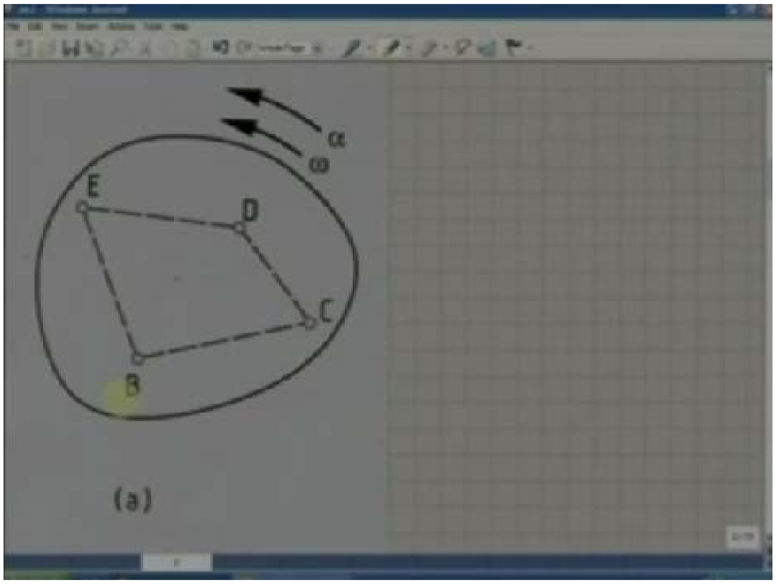

Let me develop, this concept of velocity and acceleration image. Towards the development of velocity image, let me also consider a link or a rigid body, which is quaternary and it is connected to four other links at this point B, C, D and E. The other four links which are connected here has not been shown. This is a quaternary link connected to four other links at this point B, C, D and E and the angular velocity of this particular link is omega.

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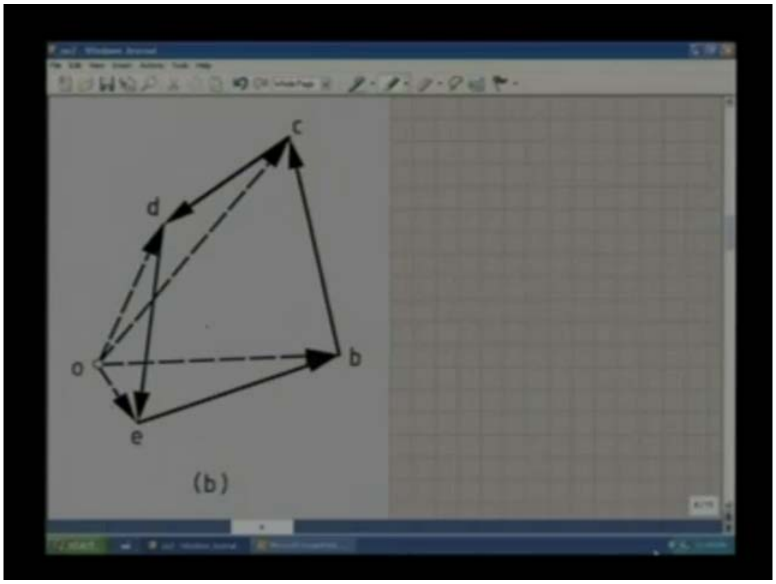

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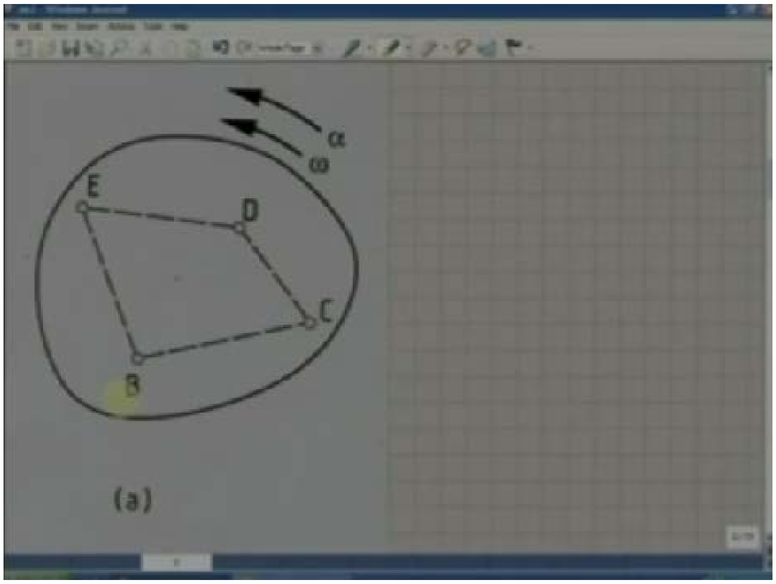

If we draw the velocity diagram, we know that there is something called velocity pole and this o is the velocity pole and b, c, d, e these are the small letters representing these capital points, capital B, capital C, capital D, capital E.

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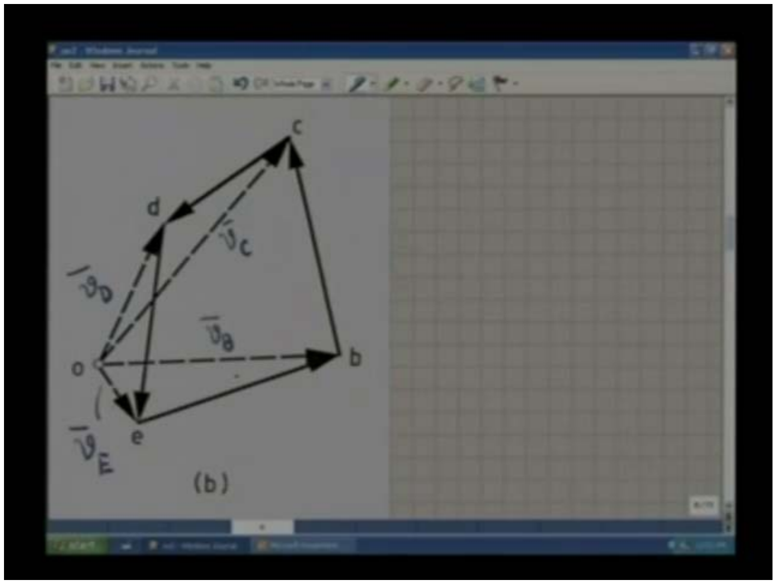

So, If we join this point small b with the velocity pole that is the vector ob represents velocity of the point b, the vector ob. Similarly, the vector oc represents velocity of the point Capital C, od represents the velocity of the point capital D and oe represents velocity of the point capital E.

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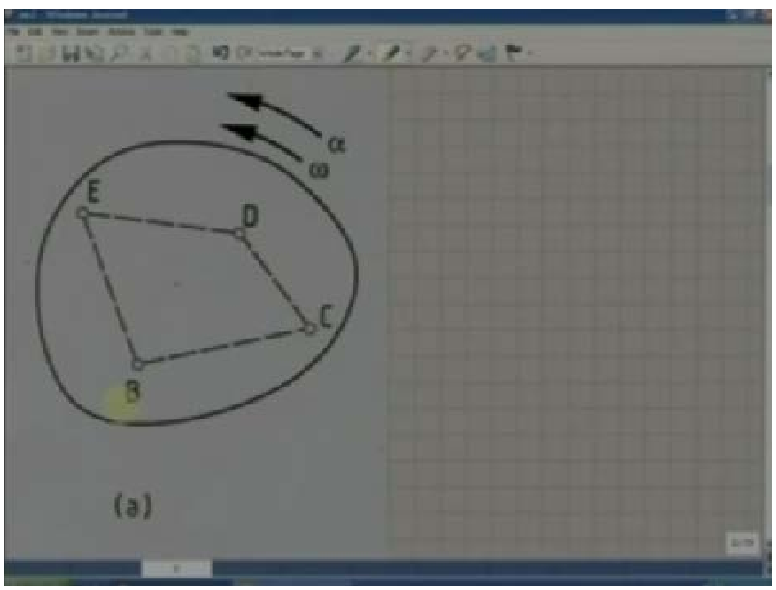

In the velocity diagram this small bcde, this quadrilateral is called the velocity image of this quadrilateral consists of capital BCDE.

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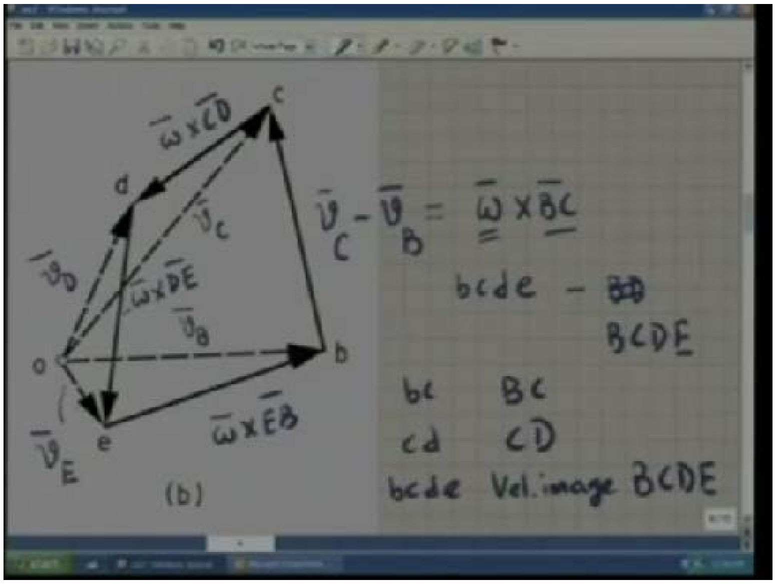

Let us see, what does this vector bc represents? oc is velocity of the point c, ob is the velocity of the point b, so vector bc represents the difference of the velocity of the point capital C and the velocity of the point capital B. The vector bc is nothing but V_C minus V_B. Because B and C belong to the same rigid body, whose angular velocity is omega as assumed earlier, this must be equal to omega cross BC. Similarly cd represents omega cross CD, which is nothing but od minus oc, that is the difference of velocity of d and the velocity of c. Similarly eb will represent omega cross EB and de will represent omega cross DE.

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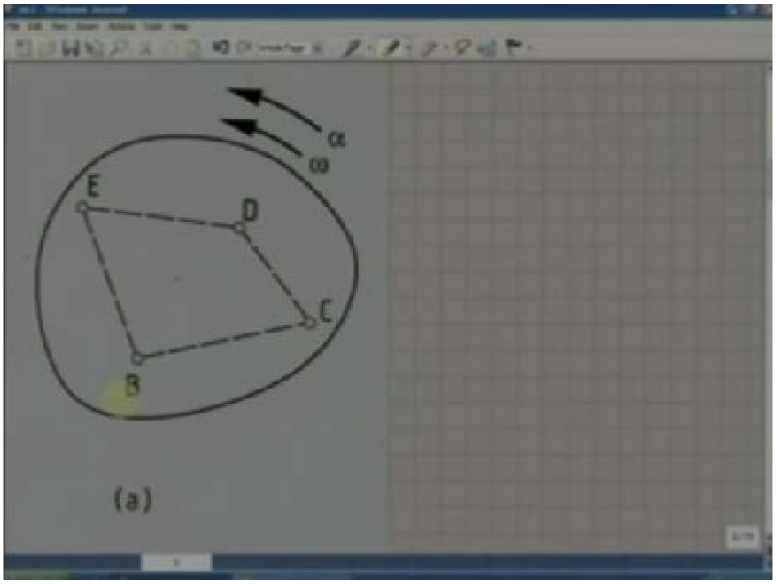

In the configuration diagram we had a quadrilateral BC, CD, DE, EB.

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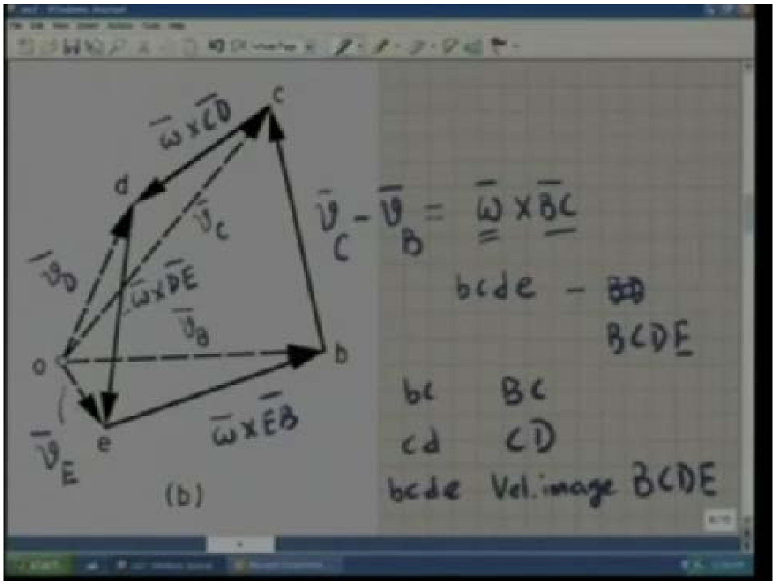

We go back to the velocity diagram we again have bc, de and let us see the ratio of these sides of these two quadrilaterals in the velocity diagram, in the acceleration diagram. Small bc is the omega cross BC, small cd is omega cross capital CD, small de is omega cross capital DE and eb is omega cross capital EB. Thus, this quadrilateral bcde is nothing but a scale drawing of the quadrilateral BCDE. Small bcde is called the velocity image of capital BCDE, which is nothing but a scale drawing and scale of the diagram is this factor omega and small bc is perpendicular to capital BC, because this is omega cross bc. This figure bcde each side is perpendicular to the corresponding to the capital sides that is bc is perpendicular to capital BC, cd is perpendicular to capital CD and so on and this perpendicular is in the sense of omega.

If we consider, this quadrilateral BCDE and the corresponding in quadrilateral velocity diagram bcde, it is the scale drawing the original capital BCDE and it is rotated through an angle 90 degrees in the sense of omega. If omega is counter-clockwise, this bcde we can see to be rotated 90 degrees in the counter-clockwise sense, because omega is counter-clockwise. The velocity image is a scale drawing of the original diagram capital BCDE drawn to a scale and rotated through 90 degrees in the sense of omega and small bcde is called the velocity image. This is called the velocity image of capital BCDE. If we know, the two points of this quadrilateral bc then I can easily determine the other two points de by making a scale drawing. The same will be true even for the acceleration diagram.

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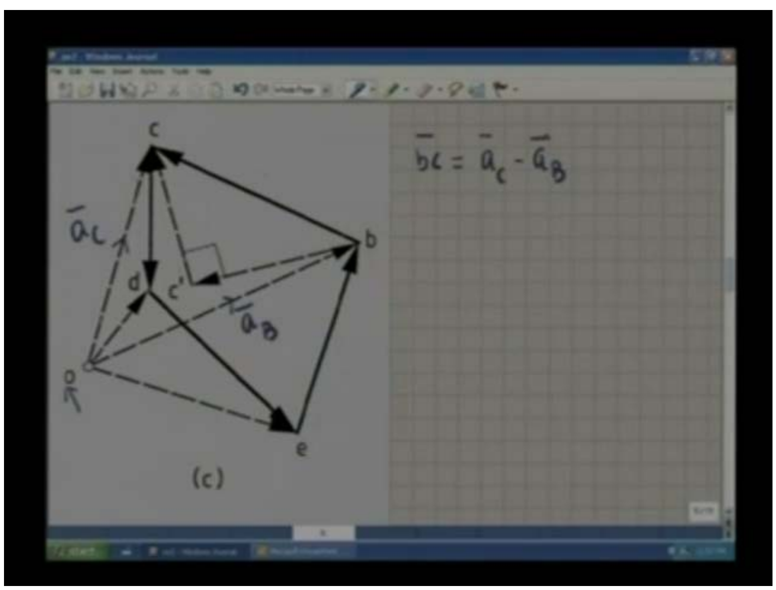

This is the acceleration diagram and this is the pole of the acceleration diagram o. ob represents the acceleration of b and oc this vector represents acceleration of c. The vector bc, which is ac minus ab, where c and b are two points on the same rigid body whose angular velocity is omega and angular acceleration is alpha.

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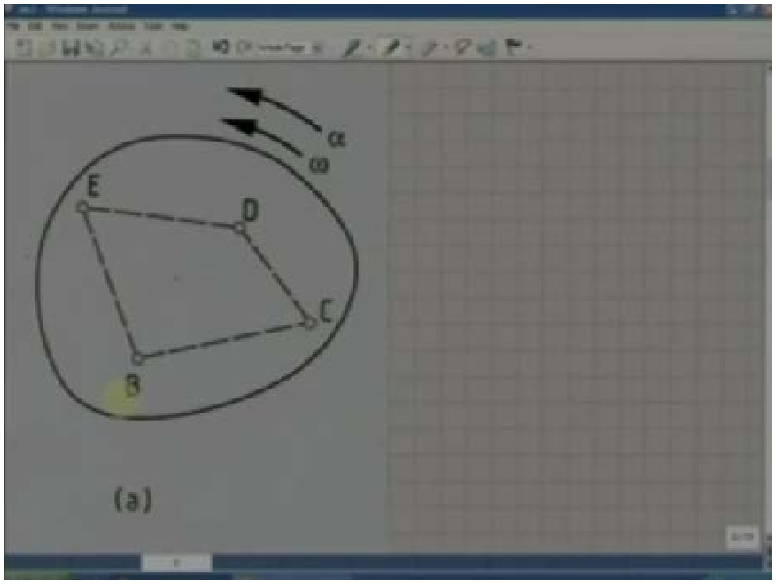

We know omega squared acceleration of C with respect to B omega squared in the CB direction.

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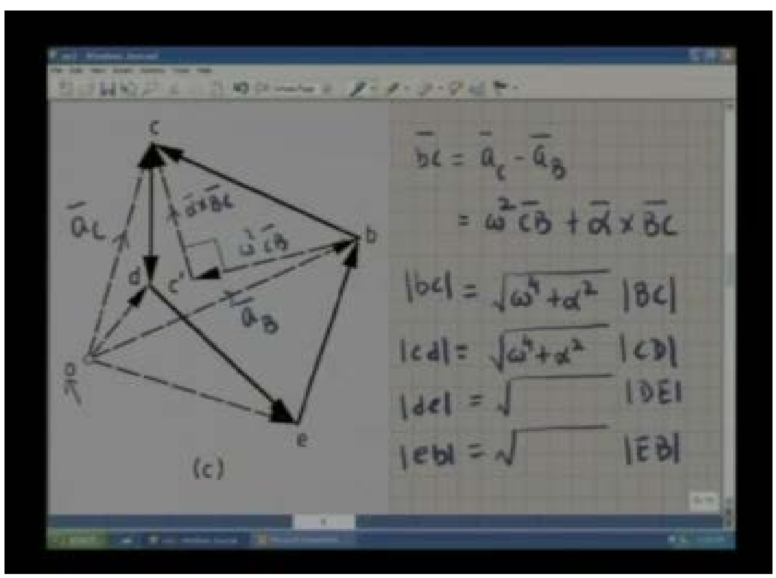

The vector bc is omega squared CB plus alpha cross BC. As we see bc prime omega squared CB and c, prime c this vector is alpha cross BC. Summation of these two vectors gives me the vector bc. It is easy to see, because this vector is parallel to bc and alpha cross bc is perpendicular to bc. These two vectors are at right angles to each other as seen here. Because these two vectors are at 90 degrees I can find by Pythagoras' theorem, that this is omega to the power of four plus alpha squared into magnitude of BC. bc prime is the omega squared into BC the magnitude, c prime c is the magnitude is alpha times bc by Pythagoras' theorem I get the magnitude of bc is squared root of omega to the power of four alpha squared into magnitude of bc.

The same way can we can prove exactly the similar way that magnitude of cd vector will be omega four plus alpha squared into magnitude of vector capital CD and will be true for other two vectors namely de and eb. They are also multiplied by the same factor and the corresponding capital letters DE and EB.

What we see the quadrilateral consisting of small letter bcde each side is proportional to the corresponding side of the quadrilateral consisting of the capital BCDE. Small bcde is nothing but a scale drawing, the scale factor is given by squared root of omega four plus alpha square.

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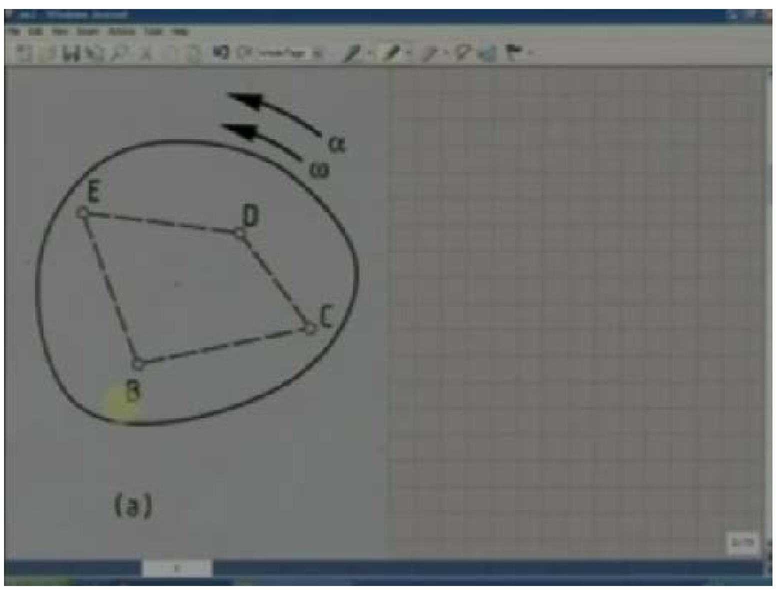

This is the configuration diagram or space diagram BCDE.

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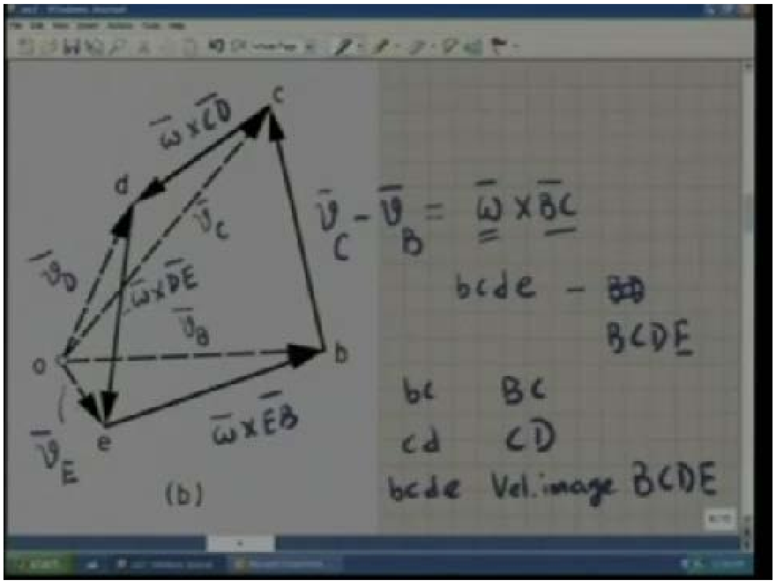

This is the velocity diagram bcde.

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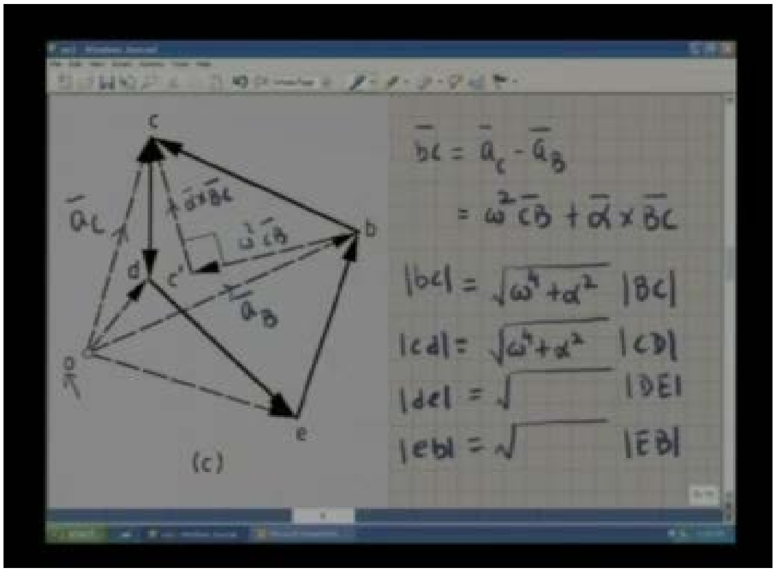

This acceleration diagram bcde and this small bcde is always a scale drawing of capital BCDE. The small bcde here is called the acceleration image of capital BCDE.

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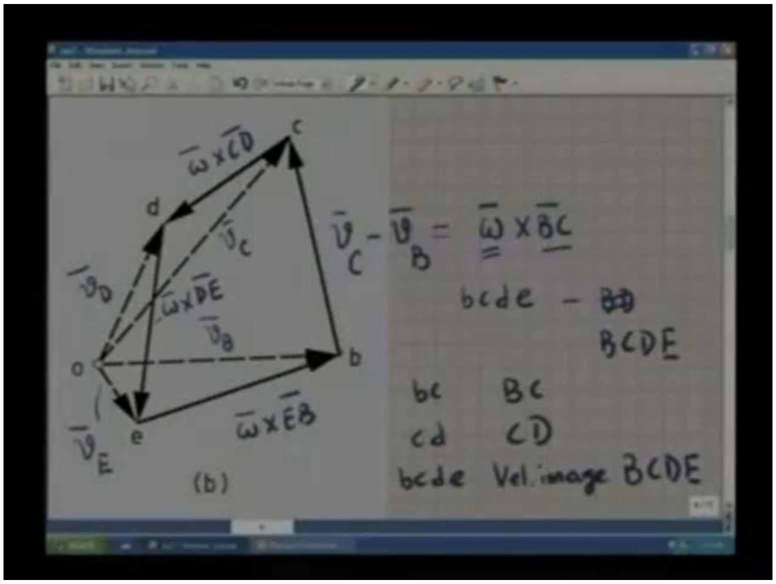

Like in the velocity diagram, this small bcde is the velocity image of capital BCDE. This is the concept of velocity and acceleration image and we shall see its utility in a later example.

What we will find is that, if the velocity or acceleration of two points of rigid body are known then any third, fourth or fifth point I need not do an elaborate analysis I can use this concept of image, which is nothing but a scale drawing of the original configuration or kinematic diagram.

At this stage let me now summarize, what we have learnt today. We have just started the process of velocity and acceleration analysis of planar mechanisms by drawing velocity and acceleration diagram. We have seen that we have to start from the input n where the input motion characteristics like velocity and acceleration given and then go successively through all the motion transfer points to reach the output end, where velocity and acceleration have to be determined. We have shown it through an example of the slider-crank mechanism.

Then, we have also developed a very useful concept like velocity and acceleration image, which will be useful for mechanism having higher order links. What do we mean by image? If we use the same convention of using small letters for the velocity and acceleration diagram and capital letters for the configuration diagram. Then for a particular rigid body the velocity diagram and the acceleration diagram we get a scale drawing of original higher order links. The scale factor is omega for the velocity diagram and squared root of omega four plus alpha square for the acceleration diagram, where omega and alpha are the angular velocity and angular acceleration of that particular higher order link. The velocity image is rotated at an angle 90 degree in the sense of omega and acceleration image is also rotated from the corresponding space diagram and that angle is not very important we can easily find it out as an exercise.