Kinematics of Machines

Prof. A. K. Mallik

Department of Civil Engineering

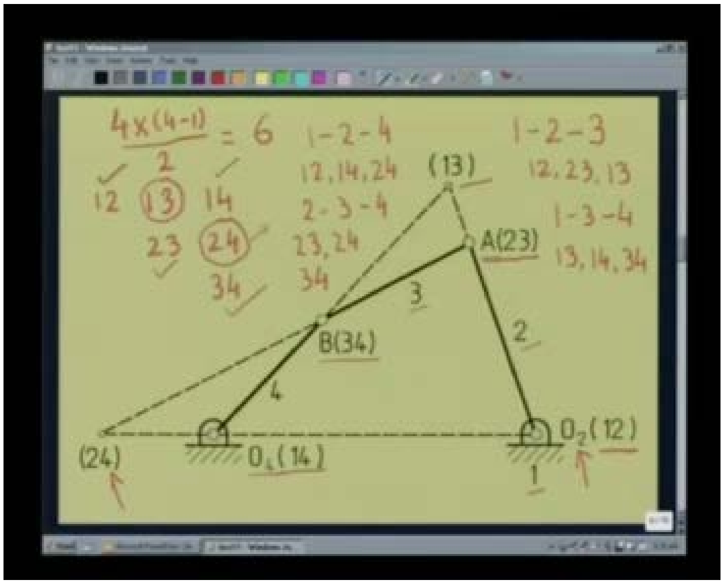
Indian Institute of Technology, Kanpur

Module 4 - Lecture - 2

Today, we continue our discussion on relative ICs of planar linkages and then we shall see how this relative instantaneous centre can be used for the purpose of velocity analysis of these linkages. To start with let me take an example of the simplest 4R linkage.

As we see in this figure, we have a 4R linkage namely O_2, A, B, O_4, because we have four links, there are n into n minus 1 by 2, that is six relative ICs.

(Refer Slide Time: 00:59)



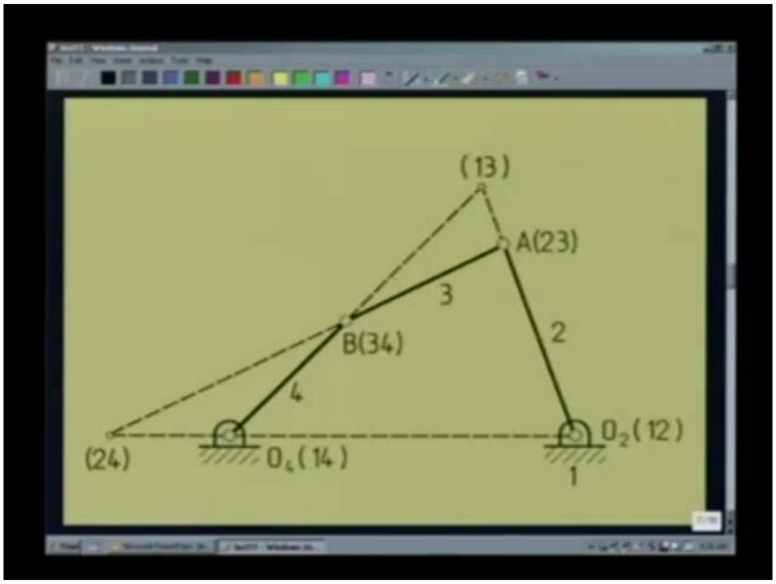
Let me list all the six relative ICs. They are: 1 2, 1 3, 1 4, 2 3, 2 4 and 3 4. So our first task is to locate all these six relative instantaneous center's of this four link mechanism. Out of these six, as we said in the last lecture, the location of 1 2 is at O_2, because link number 1 and link number 2 are connected by a revolute pairs at O_2. So O_2 gives us the location of the relative instantaneous center 1 2. Similarly, 2 3 is at A, because of same reason that links 2 and 3 are connected by a revolute pairs at A. So A gives us the location of 2 3. Following the same argument, we get 3 4 at B and 1 4 at O_4. So out of the total six numbers of relative instantaneous centres, four of these are located by just by inspection. We have still to determine two more instantaneous centers namely, 1 3 and 2 4. To determine these two, we invoke the Arnold kinetic theorem of three centres.

For example, if we say three rigid bodies namely 1, 2 and 3 are in relative motion with respect to one another, consequently, the three relative instantaneous centres that is 1 2, 2 3 and 1 3 will be collinear. Out of these three, as we see, we have already determined 12 and 2 3. So 1 3 must be on the line joining 1 2 and 2 3 that is line joining O_2A. This line O_2A must contain the third instantaneous centre namely 1 3.

Next, let us consider three rigid bodies namely 1, 3 and 4 in relative motion. Consequently, the 3 relative instantaneous centres that is 1 3, 1 4 and 3 4 must be collinear. Out of these three, we have already determined 1 4, we have already determined 3 4. So 1 3 must be on the line joining these two relative is instantaneous centres, 1 4 and 3 4 that is 1 3 also lies on the line OB. So we have the set 1 3 lies on the line O_2A and also lie on the line O_4B. As a result, 1 3 lie at the intersection of these two lines. So an extended that is here and we locate 1 3. Out going the same following the same principle, we can also determine the last one that is 2 4. For this, we consider three rigid bodies 1, 2 and 4. As a result, 1 2, 1 4 and 2 4 are collinear. 1 2 is here at O_2, 1 4 is here at $O_{4.}$ So 2 4 must lie on line joining O_2 and $O_{4.}$ Then we can consider three rigid bodies namely, 2, 3 and 4. Consequently, Arnold kinetic theorem says three instantaneous centres, 2 3, 2 4 and 3 4 also are collinear. Out of these three, we have already located 2 3 at A, 3 4 at B. So 2 4 must lie on the line joining AB. As a result, 2 4 is located at the intersection of the line $O_2O_{4,}$ that is the line joining 1 2, 1 4 and the line AB that is the line joining 2 3 and 3 4. As a result, we get the location of 2 4 at here, at the intersection of O_2O_4 and AB.

So we determine some of the relative instantaneous centre just by inspection of the kinematics pairs and the rest by application of Arnold kinetic theorem. Once we have determined all these six relative instantaneous centres, we are in a position to carry out the velocity analysis. Let as look at this picture.

(Refer Slide Time: 07:21



In this figure, all the six relative instantaneous centres have been located as explained just now. For velocity analysis, let say the angular velocity of the input link say that of two is given. The problem is input angular of velocity omega$_2$ given and our task is to determine the angular velocity of the other two moving links, say link number 3. That is the line AB and link number 4 that is the line O_4B. So at this particular configuration, if omega$_2$ is given, then question is what are omega$_3$ and omega$_4$? Obviously, this will be 2 (true) only for particular configuration. To do that, we carry out the velocity analysis as follows. If omega$_2$ is given then we can find the velocity of the point V_A which is omega$_2$ cross O_2A. Since omega$_2$ is given and O_2A is known, we can find the velocity vector V_A from the input angular velocity. Now point A happens to be the relative instantaneous centre 2 3 that means if we consider this point A to be a point on link 2, whatever its velocity. If we consider being point on link 3 then it will have the same velocity.

Now let us consider the point A as a point on link 3 and link 3 is rotating at this instant about the point 1 3. So velocity of A, if we consider it to be a point on link 3, which is rotating with angular velocity omega$_3$ about the point 1 3. That is we have to take this vector from the point 1 3 to A. Now these two are same. These two equations both give V_A and the only unknown is omega$_3$. So we can determine omega$_3$. To determine, say output angular velocity that is omega$_4$ that is of the line O_4B we consider the point 2 4, which we have already located. Now 2 4 is a point whose velocity will be same whether we consider it to be a point on body 2 or we consider it to be a point on body 4.

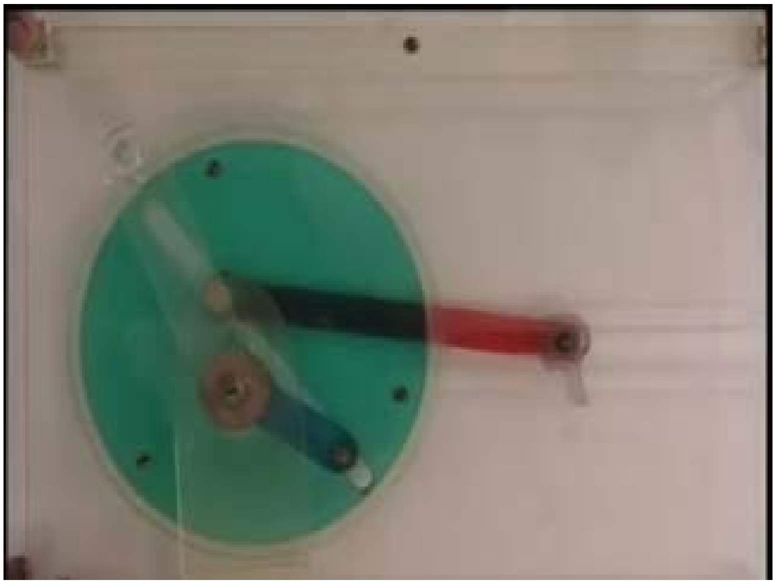
If we consider 2 4 to be a point on body 2, which is rotating about O_2, so velocity of this point 2 4 considering it to be a point on body 2 is omega$_2$ cross the vector O_2 to the point 2 4, because link 2 is rotating about the point O_2, which is 1 2. As omega$_2$ is given, we can find out what is the velocity of this point for counter clockwise velocity of link 2, it will be in this direction perpendicular to this line and the magnitude we can find out.

Now let me consider the same point 2 4 as a point on body 4 and we must get the same velocity. So same equation we can write now. V_{24} considering it to be a point on link 4 is omega$_4$ and link 4 is rotating about 1 4, that is O_4. So velocity will be the vector O_4 to the point 2 4. Now V_{24} is already known, only unknown is omega$_4$. So we can determine omega$_4$. This is what we mean by the velocity analysis of a linkage, if the input angular velocity is given, we obtain all the relative instantaneous centres and use them judiciously to determine all the unknown quantities. Suppose we are interested in finding the velocity of this particular point on link 3, we have already found omega$_3$. So link 3 is rotating about 1 3 so the velocity of this point will be omega$_3$ into…, if we call this point P, velocity of the point P will be omega$_3$ cross the vector, the link 3 is rotating about the point 1 3, so 1 3 to the point P, whatever this vector. Omega$_3$ is already known, this vector is known. So we can find the velocity of the point P. For example, in this particular problem with omega$_2$ counter clockwise, the velocity of the point A is in this direction, perpendicular to O_2A and link 3 is rotating over this point so omega$_3$ comes out as clockwise from these two equations. Omega$_3$ transfer tests clockwise, so velocity of P will be in this direction perpendicular to 1 3-P this vector and in the clockwise direction. Now let me show with a model that this 1 3 and 2 4 and in the model, it will be also clear what is the significance of this relative instantaneous centres.

(Refer Slide Time: 14:13)

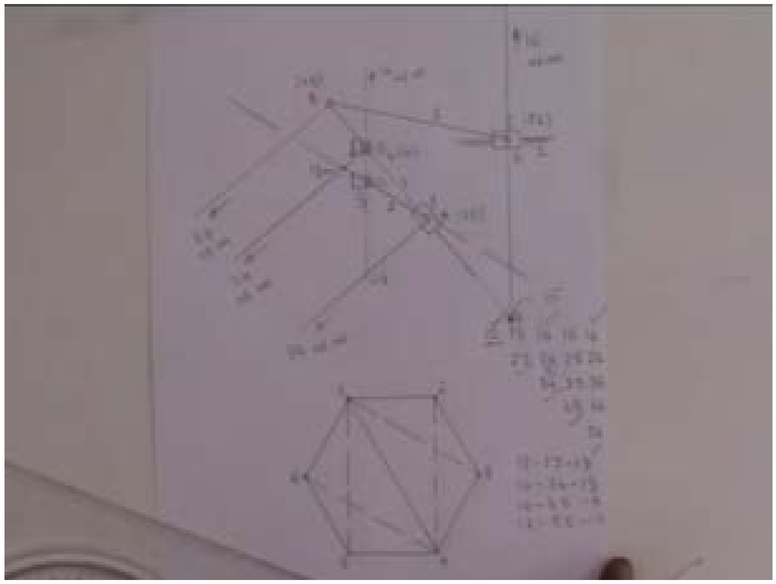
Let us know consider this model of the 4R linkage. As we see, there are three moving links, namely this blue link, say link number 2. This gray link that is the coupler, say link number 3 and this red link that is the output link, say link number 4. So this is according to our notation, O_2, A, B, O_4. Applying Arnold kinetic theorem, we have seen that 1 3 is located at the point of intersection of these two lines, O_2A and O_4B. So these two lines intersect here, which we call 1 3. Similarly, at the point of intersection of the two lines O_2O_4 and AB, we get the instantaneous centre 2 4. So this is 1 3 and this is 2 4. Now if we move this linkage around this configuration, because this point is P_{13} that means link 3 is instantaneous rotating about this point. This point does not move. This point has zero velocity. As we will see when move this linkage, all other points in this link 3 is moving around this configuration this point has zero velocity is hardly moving. To see the significance of 2 4, let me consider this point, a coincident point on link 2 and link 4. That means one point on link 2 is exactly just below this point, a point on link 4. These two points on link 2 and link 4 at this instant has same velocity. That is why we will call it 2 4 and if we move this, we see this is the hole which was coincident with this P_{24}. Right now this is different configuration, so their location is else where, but at that configuration, this hole on link 4 and the point on link 2, we cannot distinguish because they move with same velocity. Here we can see them distinctly. This is the hole which was coincident to this point at this configuration. So around this configuration, these two points, one belonging to body 2 and the other one belonging to body 4 move with same velocity.

(Refer Slide Time: 16:58)



At this stage, let me carry out another example of six link mechanism. Let us consider the model of this six link Whitworth quick return mechanism, which we have seen earlier. Here, the input link is this link, this is O_2A. This is the input link which rotates at the constant speed and consequently the output link that is this tool holder reciprocates along a straight line. Our job is given the six link mechanisms at a particular configuration. Say this configuration, if first task is to locate all the relative instantaneous centres, then second step is given the input angular velocity at this configuration, say omega$_2$ is given what is the velocity of the cutting tool or this tool holder? That is this slider which will be our link number 6. So this is the problem that now we are going to solve.

(Refer Slide Time: 17:56)



This is the kinematics diagram of the Whitworth quick return mechanism, the model of which we have just now seen. O_2A is link number 2, which is the input link, that is hinge to this block 3 and the 3 is moving on the slotted lever 4 which is hinged at the point O_4. Then link 4 is connected to link 5 by a revolute pairs at the point B and link 5 is connected to the tool holder that is the link number 6 at the point C. So our first task is to obtain all the relative instantaneous centres. As we see, there are six links 1, 2, 3, 4, 5 and 6 links. So if there are the six links then the total number of relative instantaneous centres is 6 into 5 divided by 2, that is 15. So first of all, let me list all the 15 relative instantaneous centres, namely 1 2, 1 3, 1 4, 1 5 and 1 6 then 2 3, 2 4, 2 5, 2 6, 3 4, 3 5, 3 6, 4 5, 4 6 and 5 6. So this is the complete list of all the 15 relative instantaneous centres.

At the second stage, let us now try to locate all those relative instantaneous centres, which can be easily obtained by inspection of the kinematics pairs. For example, 1 2 is easily located at the revolute pairs between link 1 and link 2, that is at O_2. Similarly 2 3 is easily located at the revolute pairs connecting link number 2 and 3, that is at this point A. 1 4 is easily located at O_4 because that is where the revolute pairs between link number 1 and link number 4 is. Link number 4 and 5 are connected by revolute pairs at B, so 4 5 is at B. So let me try to write this..., here, we have 1 2, here we have 1 4 at O_4. At B, we have 4 5, at A we have 2 3. Next we have revolute pairs, link number 5 and 6 at C, so that is 5 6. C is 5 6. We have located 1, 2, 3, 4, 5 revolute pairs and the corresponding five relative ICs. The other two kinematics pairs in this linkage are a prismatic pairs between link 3 and 4 along this slotted lever. So the relative IC 3 4 will lie at infinity in a direction perpendicular to the direction of sliding that is perpendicular to the line O_4A. So we can draw a line which is perpendicular to O_4A and 3 4 lies on this line at infinity.

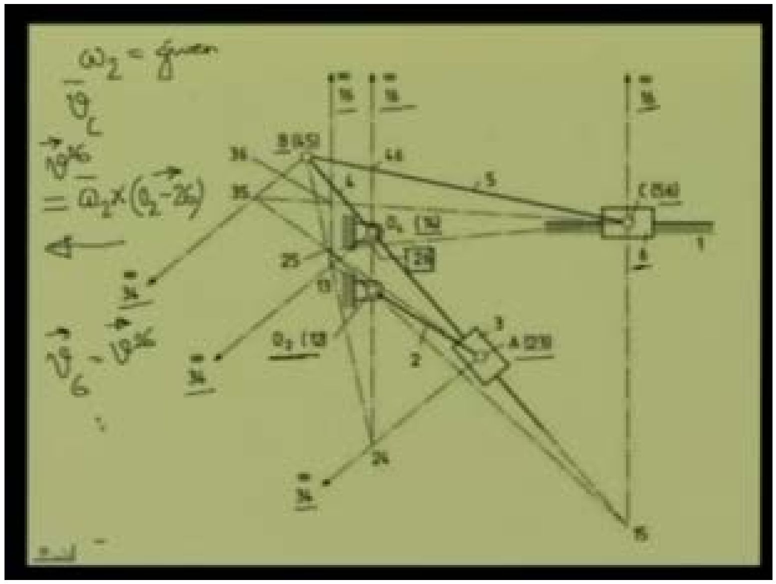
We should remember that all parallel lines meet at infinity, so one can also say that if we draw a line which is parallel to this line that 3 4 lies on this line as well, because these two lines are parallel and 3 4 lies at infinity and two parallel lines meet at infinity, so 3 4 lies also on this line. All parallel lines meet at infinity, so 3 4 at infinity, 3 4 at infinity and all this parallel lines. We have located the relative IC 3 4. 6 is in perfect translation in the horizontal direction with respect to the fixed link 1. So 1 6 lies at infinity in a direction perpendicular to the horizontal that is vertical. So if we draw a vertical line, all these vertical lines, we can write 1 6 at infinity along this vertical direction. So we have located 1 6, because of the seven kinematics pairs, we have just by an inspection located 7 out of this 15 instantaneous centres, rest 8 can be determined only by invoking Arnold kinetic theorem, because there are so many numbers, it will be quite confusing if we do it haphazardly, it is better do it little more systematically.

To do it systematically, we put this link numbers 1, 2, 3, 4, 5 and 6 at equal spacing on the circumference of a circle. This diagram is in fact called as circle diagram. Here, links are represented by these numbers 1, 2, 3, 4, 5, 6. If this relative instantaneous centre 1 2 has already been determined, then we join 1 and 2 by a straight line 1 2. This line represents the relative instantaneous centres 1 2 and if the line has been drawn, that means this has already been located. So let us draw all the 7 relative instantaneous centres that we have already determined. We have determined 1 4. So we join 1 and 4. We have determined 1 6, so we join 1 and 6. 2 3 so we join 2 3. 3 4 so we join 3 4, 4 5 and 5 6. So corresponding to the 7 relative ICs, which have already been obtained by mere inspection of kinematics pairs, we join these 7 lines. Now to apply Arnold kinetic theorem systematically, we should consider a quadrilateral in this figure. For example, we have already completed two quadrilaterals, namely 1, 2, 3, 4, all the four sides are known. Then the diagonals of the quadrilateral can be decided at the second stage. For example, we can determine say this diagonal 1 3. This is as follows: we consider that 1 2, 2 3 and 1 3 are collinear. We have already drawn 1 2 and 2 3, so 1 3 must be lying on this line. This is what we called as the Arnold kinetic theorem, that 1 2, 2 3, 1 3 are collinear. 1 2 is here. 2 3 is there. So 1 3 must lie on this line and 1 3, this diagonal as we see 1 4 and 3 4. So we consider 1 4, 3 4 and 1 3. We have already determined 1 4 here and 3 4 is on this line at infinity. So 1 3 must be on this line as well and 1 3 must be on this line as well. So the point of intersection of these two lines gives me the location of relative IC 1 3. Same way, we can determine 2 4. 2 4 we can consider as lying on a line joining 2 3 and 3 4 and also on the line joining 1 2 and 1 4. This is 1 2, this is 1 4, so 2 4 must lie on this line and 2 3 is here, 3 4 is here, so it must lie on that line.

The intersection of this vertical line which contains 1 2 and 1 4 and this line which contains 2 3 and 3 4 determines the relative instantaneous centre 2 4. This process can be continued like considering this quadrilateral with sides have already mean determined. In the next step, we can consider the two diagonals that are 4 6 and 1 5. So we have also determined 1 3 in the second step, so let me put a cross. In the second step, we have determined 1 3 and 2 4 and in this step, we can consider 1 4 and 4 5 and 1 5 are on one line. 1 4, 4 5, 15 is on one line and also 1 6, 5 6 and 1 5 on one line. As we see, this is 5 6 and 1 6 is on the vertical line. So 1 5 must lie on this vertical line and also 1 4 is here, 4 5 is here, so 1 5 must lie on the line joining O_4B. So if we extend this line, at this intersection of these two lines we can get 1 5. After the second step we see lot of quadrilaterals has been filled up.

For example, now we can determine 2 6, because I know 1 2 and 1 6 and we also know 4 6 and 2 4. Then in the third step, we can determine the rest of the unknown relative ICs like 2 6, 2 5, 3 5 and so on. The diagram will become complicated. I will show the complete diagram, where all the relative instantaneous centres have been located following the same procedure.

(Refer Slide Time: 31:13)



Following the procedure just now explain finally, we obtain all the relative instantaneous centres of this six link mechanism as show in this figure. We note O_2, that is 1 2, A which was 2 3, B which was 4 5, C which was 5 6 and O_4 that was 1 4. These are the five R pairs, immediately locate five relative instantaneous centres. Corresponding to the prismatic pairs between 3 and 4 along this O_4A, we have 3 4 in a direction perpendicular to O_4A and lying at infinity, so all this parallel lines 3 4 lie at infinity along this direction. Similarly, 6 is in horizontal translation with respect to link 1 so relative instantaneous centre IC 1 6 is at infinity in the vertical direction. That is on all vertical lines I can say, 1 6 lies at infinity. Following the procedure just discussed, we locate all the rest of the relative instantaneous centres like 1 3, 2 4, 1 5, 4 6, 3 5, 3 6, 2 5 and 2 6. After obtaining all the relative instantaneous centres, we are in a position to carry out the velocity analysis of this mechanism. That means, for this configuration, if one prescribes the input angular velocity omega_2 which is given and we have to determine, what is the velocity of the cutting tool that is link number 6? What is the velocity of link 6 or velocity of the point C, which is same as the sliding velocity of the cutting tool?

Now to determine this, we argue it out as follows. Let us consider this relative instantaneous centre 2 6, which is lying here. Now by definition, this point the velocity of this point, if we consider it to be a point on body 2 or if we consider it to be a point on body 6, it must be same. That is the definition of relative instantaneous centre 2 6, because $omega_2$ is prescribed, we can find out the velocity of this point 2 6. If we consider it to be on body 2, it is $omega_2$ cross. Body 2 is the rotating about O_2 that is here up to the point 2 6, this vector. So we can find out the velocity of this point 2 6 by considering it to be a point belonging to body 2. $Omega_2$ is given link number 2 is rotating about 1 2 and this point 2 6 is here, so we get the velocity. If $omega_2$ is counter clockwise, the velocity of this point 2 6 is horizontal and to the left, because this line is vertical. So we get the velocity of this point 2 6 is along horizontal direction and towards left and the magnitude is $omega_2$ into this distance O_2 into 2 6. Let me consider now the point 2 6 as the point on link 6 and it must have the same velocity. Link 6 is a slider, which means it is in pure translation which means all points on link 6 must have the same velocity. So V_6 is same as V_{26} because all points on link 6 have the same velocity. So immediately we get the output velocity V_6.

So in today's lecture, we have seen how to determine all the relative instantaneous centres of planar linkages by considering the different kinds of kinematics pairs and by successively applying Arnold kinetic theorem of three centres. This method is definitely going to give you result so long you have not more than six links in a mechanism which is sufficient for the purpose of this course. So for a six link mechanism, there are 15 relative instantaneous centres and all of which can be determined. Once we have determined all the relative instantaneous centres, I have also explained, how we can use the location of this relative instantaneous centre for the purpose of a velocity analysis. That is given the input velocity, we can find the velocity characteristics of all other links present in the mechanism. One thing we can remember, in the last example we saw that the output link was a slider. So the velocity of all the points of a slider is same, because it is in pure translation that fact was very cleverly utilized to find out the output velocity, when the input velocity was prescribed.