Kinematics of Machines

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Module-3

Lecture-4

In this lecture, we will continue our discussion on displacement analysis of planar linkages by analytical method. Today, we shall start our discussion with an example. Let us look at this figure which is a kinematic diagram of a 10-link mechanism.

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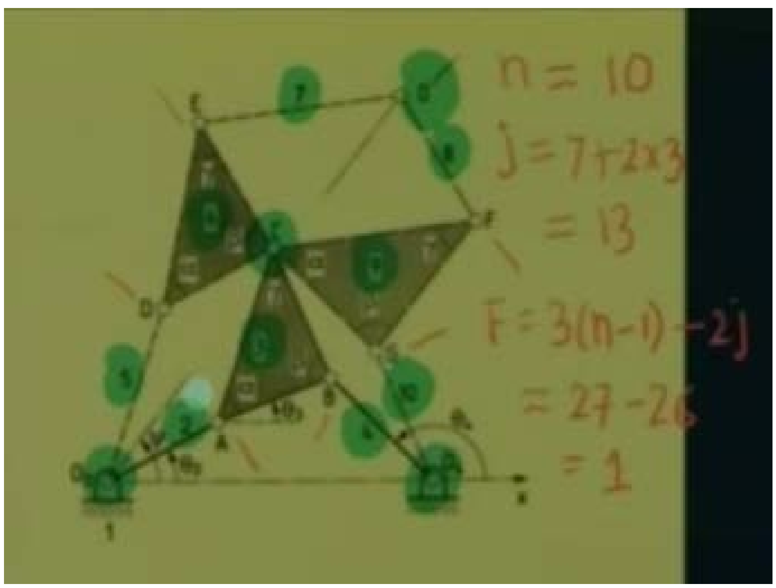

This is the fixed link number 1, then O_2A is the link 2, ABC is this ternary link, link number 3, link number 4 is O_4B, link number 5, link 6 is again a ternary link, link 7, link 8, link number 9, which is again a ternary link, and link number 10 so we have ten links. now let us look at the kinematic pairs.

As we see, at this point O_2, three links are connected namely 1, 2 and 5 so this is a second order hinge. Similarly, at O_4 three links are connected namely 1, 4 and 10 so this is again a second order hinge. Finally, at C three ternary links namely 3 6 and 9 are connected so this is again a second order hinge. Let us now calculate the degrees of freedom of this 10-link mechanism.

We have already seen the total number of links, n is 10. The number of kinematic pairs j is equal to, we have seven simple hinges namely at O, F, G, B, A, D, and E. So, j is 7 plus as already mentioned, there are three second order hinges, one at O_2, one at O_4 and the other at C. So, 2 into 3, that is, j is 13. So, the degree of freedom F is 3 times (n – 1) – 2j which is 3 into 9 is 27 – 2 into 13 is 26 that is equal to 1.

We see it is a single degree of freedom mechanism that means, whenever any link moves all other links move in a unique fashion. The question is, as this mechanism moves let us see how this point O moves. To find that we will use the links as link lengths vectors and try to find the vector O_2O.

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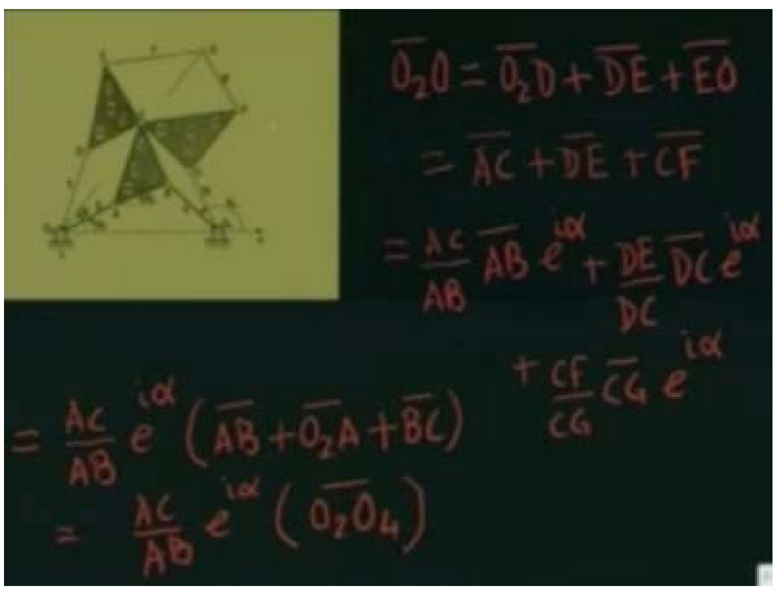

Here as we see, the vector O_2O we can write as vector O_2D plus vector DE plus vector EO. One has to note that in this particular mechanism O_2ACD is a parallelogram. Similarly, O_4BCG is also a parallelogram and OECF is another parallelogram. That means, the link AC is same as the link O_2D; link length EO is same as the link length CF and so on. Not only that, the three ternary links namely link number 3, 6 and 9 consists of three similar triangles as indicated by these three angles namely alpha, beta, gamma in each of these triangles. These three triangles ABC, CGF and CED are three similar triangles.

Let me write the vector O_2D as same as the vector AC because they always remain parallel and of equal length plus DE plus EO which is same as CF because CF and EO always remain parallel and of equal length. Now, the vector AC can be written as AC by AB into vector AB that takes care of the magnitude of AC. However, the vector AC is at an angle alpha in the counter clockwise direction from the vector AB. So I write, e to the power of i alpha. So, the first term vector AC can be represented as AC by AB into vector AB multiplied by e to the power i alpha. Now, the vector DE can be written as DE by DC into vector DC that takes care of the magnitude of DE. But the vector DE is again at an angle alpha from the vector DC so we multiply it by e to the power i alpha.

Then CF, I can write as CF by CG into vector CG that takes care of the magnitude of CF and again to take care of the direction, I multiply it by e to the power of i alpha. Now as we see, as these three triangles are similar triangles; AC by AB is same as DE by DC it is also same as CF by CG, as all these three ratios are same, so I can take any one of them say, AC by AB, that I can take common from all these three expressions. Same is true for e to the power i alpha, so that also I take common, that leaves me with vector AB from the first term; then vector DC which is same as the vector O_2A because DC is same length as O_2A and they always remain parallel.

So here, instead of DC, I write the same vector O_2A that leaves me with the vector CG which is same as BC because CG and BC are of equal length and they are even parallel. So I write this as BC which means the vector O_2O finally comes out as AC by AB e to the power of i alpha and summation of these three vectors namely O_2A plus AB plus BC which is nothing but the vector O_2O_4. So this vector O_2O which is vector O_2O_4 into AC by AB into e to the power of i alpha.

As the mechanism moves, all the vectors change but O_2O_4 never change because O_2 is a fixed point; O_4 is a fixed point, so the vector O_2 O_4 is always on X-axis without changing its length. Neither the length AC nor length AB changes, because those are the rigid link lengths. Same is true for this angle alpha which is again the angle alpha of this ternary link. Consequently, as the linkage moves the vector O_2O never change which means the point O never moves in this mechanism. Though this is single degree freedom mechanism all other points move as the mechanism moves but the point O never moves.

Just now we have seen that in the 10 link planar mechanism with specific dimensions there is one point which was not moving though the overall mechanism had a single degree of freedom.

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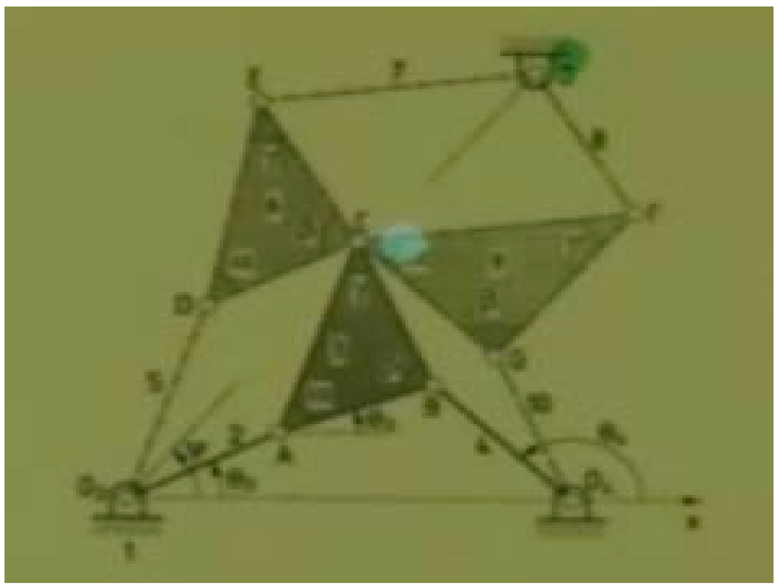

Consequently, if we fix that point O which was not moving, let us fix it with the fixed link by putting a hinge thus converting this hinge at O to a higher order hinge connecting three links namely 1, 7, and 8. As a result, now we have an assembly of links where there are three 4 bar links: one 4 bar link consisting of 1, 2, 3, and 4 with the coupler point at C; there is a second 4 bar linkage consisting of link 1, 8, 9, and 10 with the same coupler point C; and the third 4 bar linkage consisting of link number 1 that is the fixed link, link 7, link 6 and link 5.

Now, all these three 4 bar linkages have the same coupler point C and this assembly moves in a unique fashion. In other words, it means there are three different 4 bar linkages we can generate the same coupler curve at the point C. As a result, this gives a wider choice to the designer to choose one of these three linkages to produce the same coupler curve. This is now what I will demonstrate with a model.

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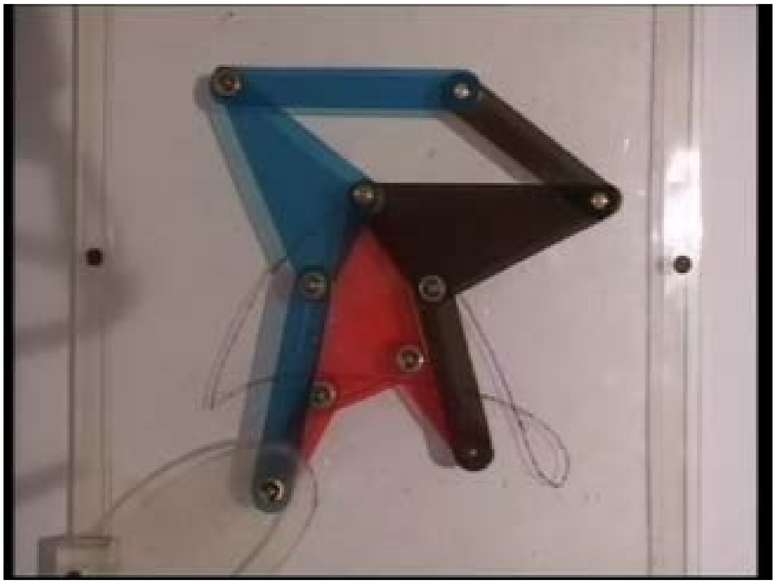

Let us now look at the model of the 10 link mechanism which we have just discussed. There is a fixed link and there are three second order hinges, all connected to the fixed link which we marked previously as O_2, O_4 and O. There are three gray moving links, there are three red moving links, and there are three blue moving links. We also note that, this link length is equal to this link length and this link length is equal to this link length. Thus, this point O_2A and this was C and this was D probably, these form a parallelogram.

Similarly, we have a parallelogram here and we have a parallelogram there. These three ternary links, this red ternary link, the gray ternary link and the blue ternary links, they are similar triangles. That means, this angle is equal to this angle, this angle is equal to this angle, and this angle is equal to this angle.

We have seen as a consequence of these special dimensions, this 10-link mechanism move in a unique fashion because it has single degree of freedom and this coupler point C can generate this coupler curve. Whether I use only this 4 bar linkage or this 4 bar linkage or this 4 bar linkage all these three 4 bar linkages generate the same coupler curve and this gives the designer a wider choice to choose a particular one which may be convenient for the purpose.

Now, we shall discuss some useful results for 4R-linkage which are most commonly used and these results will be obtained analytically.

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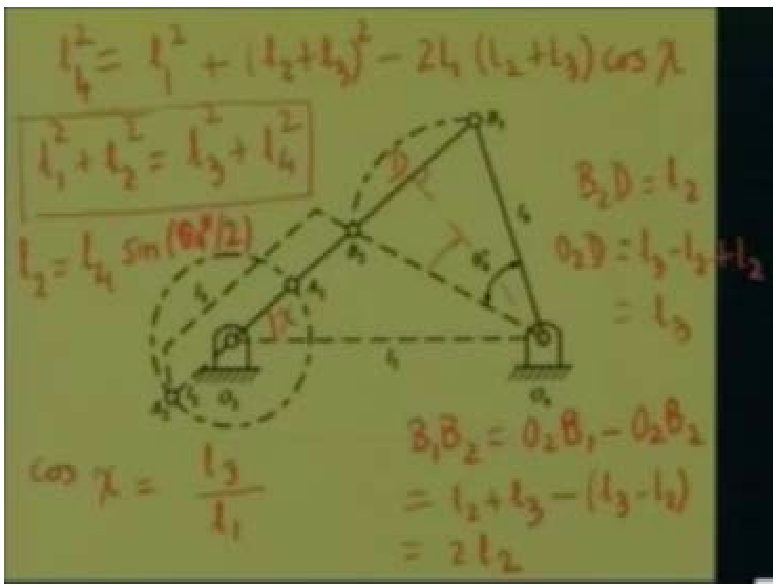

For example, let us consider this 4R-linkage namely, O_2, A, B, and O_4. As shown in this diagram, at this configuration $O_2A_1B_1O_4$, the two links O_2A_1 and A_1B_1 are collinear.

Consequently, this link O_4B_1 has taken one of its extreme positions. It cannot go further to the left. As this link, the crank rocker mechanism, this crank O_2A rotates, there is another configuration when O_2A_2 and A_2B_2 again become collinear and the corresponding configuration of O_4B that is, and this O_4B_2 is the other extreme position of this follower link, which is link 4.

We are considering a crank rocker mechanism and we see that, as the follower goes from O_4B_2 to O_4B_1, during this movement, the crank rotates from O_2A_2 to O_2A_1. That means, it moves through an angle theta$_2$ star. During the return from B_1 to B_2, the crank rotates from A_1 to A_2. So it rotates to an angle 2 pi minus theta$_2$ star. If the crank rotates at constant speed, then the time taken for the follower motion during the B_2B_1 and B_1B_2 are not same.

Normally, we would like to have a quick return that is returning from B_1 to B_2, it rotates through an angle 2 pi minus $theta_2$ star and the follower motion that is B_2 to B_1, it rotates through an angle $theta_2$ star, which is more than pi. The quick return ratio can be defined as $theta_2$ star divided by, we can write q r r, quick return ratio will be $theta_2$ star divided by 2 pi minus $theta_2$ star.

Our objective is, to determine the relationship between the various link lengths namely, the fixed length l_1, the crank length l_2, the coupler length l_3, and the follower length l_4 so that we can determine whether there is any quick return effect or not. Towards this end, we consider this figure, which has been drawn for a mechanism without any quick return. One extreme position is O_2, A_1, B_1, O_4 and the other extreme position is O_2, A_2, B_2, O_4. During these extreme positions, that the crank and the coupler are always collinear. This is the outer death center which is $O_2A_1B_1$ and this is called the inner death center when O_2A_2 and A_2B_2 are opposite to each. The angle between them, once I can say 0 degree, in the other case it is 180 degrees.

So, for this configuration as we see, the angle between O_2A_1 and O_2A_2 is pi. That means there is no quick return. It takes pi amount of rotation of the crank for the forward motion and again another pi amount of rotation for the return motion. This $theta_4$ star gives you the swing angle of the rocker. Let us now derive what is the relationship between various link lengths. If we consider the triangle $O_2B_1O_4$ what do we see? O_2O_4 is of length l_1 and O_2B_2 is of length l_2 plus l_3 and O_4B_1 is l_4.

Let us say, this angle is let me denote it by chi. So considering the triangle $O_2O_4B_1$, I can write, l_4 square is l_1 square plus ($l_2 + l_3$) whole squared minus twice l_1 into ($l_2 + l_3$) cosine of the angle chi. Now, to determine the angle cosine chi, this value, let me draw a perpendicular from O_4 to the line B_1B_2. $O_2B_1B_2$ is an isosceles triangle because this length O_4B_1 is always equal to O_4B_2, so this perpendicular bisector meets B_1B_2 at the mid point. Now, let me call this point say, D.

We can easily see that, B_1B_2 is $O_2B_1 - O_2B_2$. Now, link length O_2B_1 is $l_2 + l_3 - O_2B_2$ is A_2B_2 is l_3 and O_2A_2 is l_2, so this is $(l_3 - l_2)$. So we get, 2 l_2, so B_1B_2 is 2 l_2, so half of that B_2D will be l_2. So I can write B_2 D is l_2. So O_2D, which is $O_2B_2 + B_2D$ and O_2B_2 is this so, $l_3 - l_2 + l_2$ which is l_3. So cosine of this angle chi is O_2D divided by O_2O_4, so cosine chi is l_3 that is, O_2D divided by O_2O_4 that is, $l_{1.}$ So, we substitute this l_3 by l_1 here and if we substitute this, we can easily show that we will get l_1 square plus l_2 square will be same as l_3 square plus l_4 square.

Thus, for a 4R-linkage, to have no quick return effect that is, without any quick return effect, the link lengths of a 4R-linkage must satisfy this relationship between its link lengths. l_1 is the fixed link length, l_2 is the crank length, l_3 is the coupler length and l_4 is the follower length. Not only that, we can also see that, B_2D which we have got as, l_2 is nothing but l_4 sine of theta$_4$ star by 2. So, this angle is theta$_4$ star by 2. So there is another relationship for such a linkage without quick return that the crank length must be follower length into sine theta$_4$ star that is the swing angle of the rocker divided by 2.

So, these are the two very important relationships which can be used while designing a mechanism. Let us now look at the model of this crank rocker linkage without quick return that is we have just discussed.

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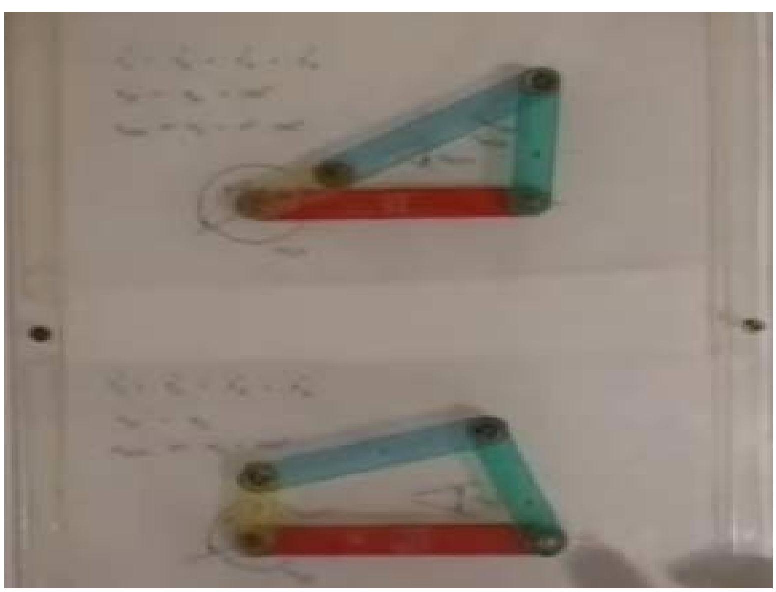

This is a crank rocker linkage where l_1 square + l_2 square is l_3 square + l_4 square. This is the one extreme position, where the crank and the coupler have fallen in one line; this is one extreme position of the follower. Now, as it rotates to 180 degree, again the crank and the coupler becomes collinear giving rise to the other extreme position of the follower. As a result, this 4R-linkage, if the crank rotates at uniform speed the forward and return motion of the follower takes equal time and there is no quick return.

Now, let us look at the model of this crank rocker linkage, where l_1 square + l_2 square is not the same as l_3 square + l_4 square As a result, the extreme position of the follower that it takes is due to the unequal rotation of the crank. One extreme position is here, when the crank and the coupler has fallen in one line. Now from here, it rotates through this angle, again the crank and the coupler falls in one line, giving rise to extreme position of the follower.

So here, as we see the follower does not take equal time during its forward and return motion. There is some quick return effect depending on of course, whether I am clockwise or counter-clockwise. Here, $theta_2$ from here to there is more than here to there, so if I rotate it clockwise, you can see the return is quicker.

Let us now look at another model where again it is a crank rocker linkage but l_1 square + l_2 square is much less than l_3 square + l_4 square. Consequently, the quick return effect will be much more predominant. For example, this is one extreme position and the other extreme position is taken here. So, the angle that the crank rotates is more than pi and the angle through which it returns is much less than pi. So if we move the crank at uniform speed, the quick return effect is much more predominant. The follower is taking much longer to come from right to left extreme and much less time to go back.

Now, that we have discussed both graphical and analytical methods of displacement analysis. Let me take you through an example on how both these methods can even be combined while designing a particular mechanism.

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As an example, let us consider this wind shield wiper mechanism, which is a 6-link mechanism. This figure shows the mechanism to a particular scale, for example, this distance is 50 mm, that is the scale of this diagram. Let us now see this mechanism. This has O_2ABO_4, this part is a crank rocker mechanism and then this link 4 is extended beyond O_4 and we have another 4R mechanism namely, O_4, C, D and O_6 and the second 4R mechanism that is O_4CDO_6 is in the form of a parallelogram. O_4C is same as length in O_6D and length CD is same as the length O_6O_4. This wiper blade is an integral part of the coupler of this parallelogram linkage that is link number 5. The wiper blade is same as link number 5. The first part of the question is what is the wiping field? That means, as this crank O_2A is driven by a motor, what is the field that is wiped by this wiper blade?

Now to solve this problem, what we do? First, we use a little bit of graphical method. We study only this 4 bar linkage namely, O_2ABO_4 and determine the extreme positions of this link O_4B that is, link number 4. For that, we know as usual, B is going along a circle with O_4B as radius. This is the circle with O_4 as center and O_4B as radius that we call the path of B say k_B. The extreme positions of B will be taken up when the links O_2A and AB become collinear. So the farthest point B can go is, when the distance of B from O_2 is O_2A plus AB. So, I take O_2 as center and draw a circular arc with l_3 plus l_2 as radius and let that intersect k_B at this point, this I call $B_{2.}$

Other extreme position of B will be taken up again, when AB and O_2A will be collinear and the distance of B from O_2 will be equal to $l_3 - l_2$. So I draw a circular arc with O_2 as center and l_3 minus l_2 as radius and let that intersect k_B at this point B_1. So O_4B_2 is one extreme position of link 4 and O_4B_1 is the other extreme position of link 4. As we see, the link 4, this extension O_4C is not in line with O_4B. There is an angle delta which has been prescribed as 16 degree.

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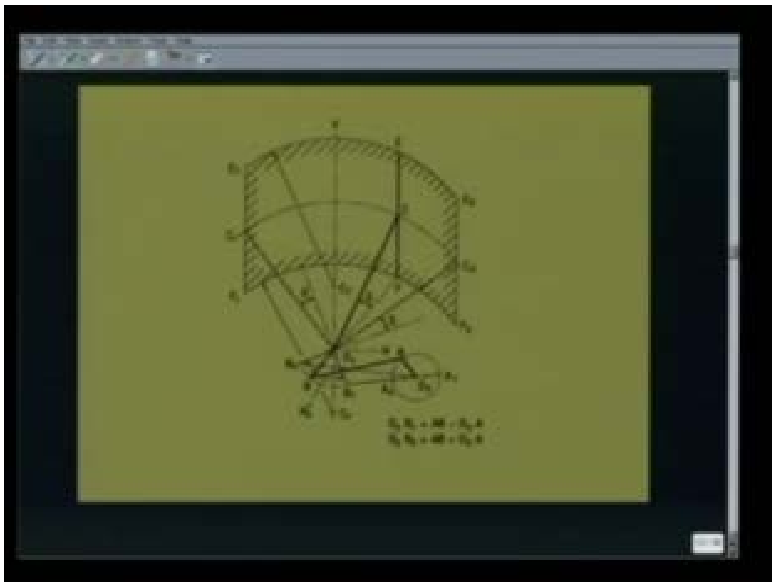

So we can draw the extreme positions of, we have already determined B_1 and B_2 as explained earlier. When corresponding to B_2, I draw this line O_4C_2 at an angle delta which was 16 degree and corresponding to B_1. Again I draw at an angle delta 16 degree to get C_1. So, O_4C_2 and O_4C_1 are the two extreme positions of the link 4.

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Now the question is, as we know, because the second part of the mechanism was a parallelogram, this line CD always remains horizontal and the wiper blades always remain vertical, because there is no rotation of the coupler of this parallelogram linkage.

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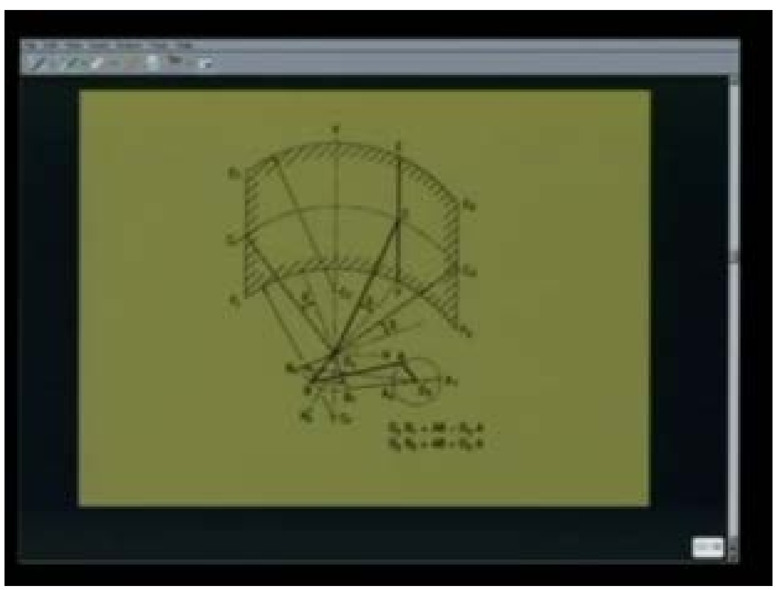

We have already determined the extreme position C_1 and C_2, so I can draw the wiper blade sets E_2F_2 and other extreme positions when C is at C_1 as $E_1F_{1.}$ So these are the two extreme positions E_2F_2 and E_1F_1 for the wiper blade. To determine the wiping field, we see that, because it is a parallelogram linkage, the point C which goes in a circle with O_4 as center and O_4C as radius. Because it is a parallelogram linkage, all the points of the coupler move in identical curves.

That means, the curve generated by the points E or F that is, the end of the wiper blades also will be similar circles, exactly of same radius as O_4C. Only thing the center of the circle will be shifted from O_4 to CE for the point E and O_4 to CF for the point F. That is, CECF is same as E_2 F_2 and E_1 F_1. So, with center as CE and radius as O_4C which is same as CEE_1, I draw this circular arc. Similarly, with center as CF, I draw this circular arc, and these are the two extreme positions and the wiping field is what has been shown by this hashed lines. So we have obtained the field of wiping for this particular mechanism.

Let me repeat, first, we said determine the path of B which is the circle with O_4 as center and O_4B as radius. On this circular path, I locate B_1 and B_2 using the relation O_2B_1 is AB – O_2A, O_2B_2 is AB plus O_2A. Once I got the extreme positions of link 4, I draw O_4C_2 and O_4C_1 corresponding to O_4B_2 and O_4B_1, because link 4 is a rigid link, the same angle delta is maintained between the line O_4B and O_4C. Once we get the extreme positions of the point C, I draw the wiper blades which always remain vertical because of the parallelogram linkage as E_2F_2 and $E_1F_{1.}$ Because of the parallelogram linkage, all the coupler points generate same circular arc as O_4 C. Only thing, the center of the circle is shifted in a symmetric fashion from C_2 to E_2 that is O_4 to CE; C_2 to F_2 that is O_4 to CF; these are all on the same vertical line. As a result, we get the complete field of wiping as generated by this particular wiping mechanism.

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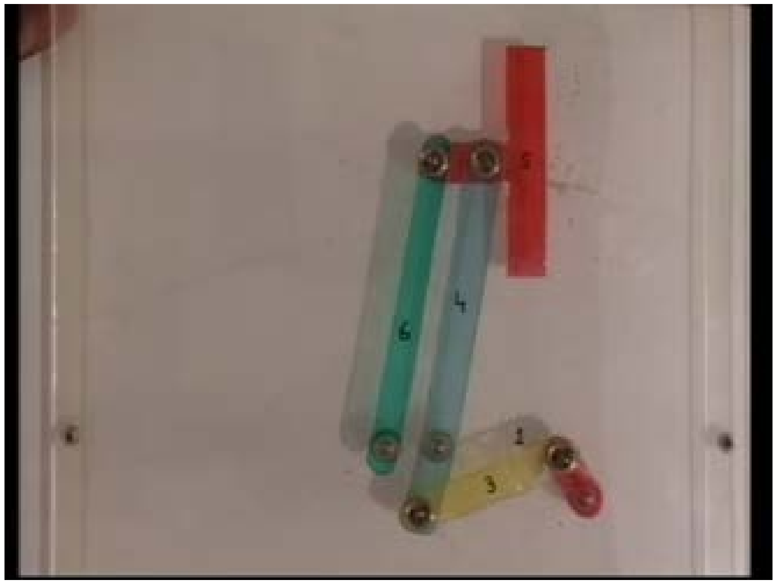

We demonstrate the same wiper mechanism that we have just now studied. This is the same wiper mechanism consisting of a crank rocker, consisting of link 2, link 3, and link 4. O_2 and O_4 are the two fixed hinges. There is another parallelogram linkage starting from here link 4, link 5, and link 6. This link length is same as this link length and this link length is same as this link length. So it is a parallelogram, and we should note that because it is a parallelogram linkage, the coupler always remains parallel to itself and it never changes its orientation.

So, as the motor here rotates, the wiper blade goes from left to right generating a field of wiping, but the coupler blade always remains vertical.

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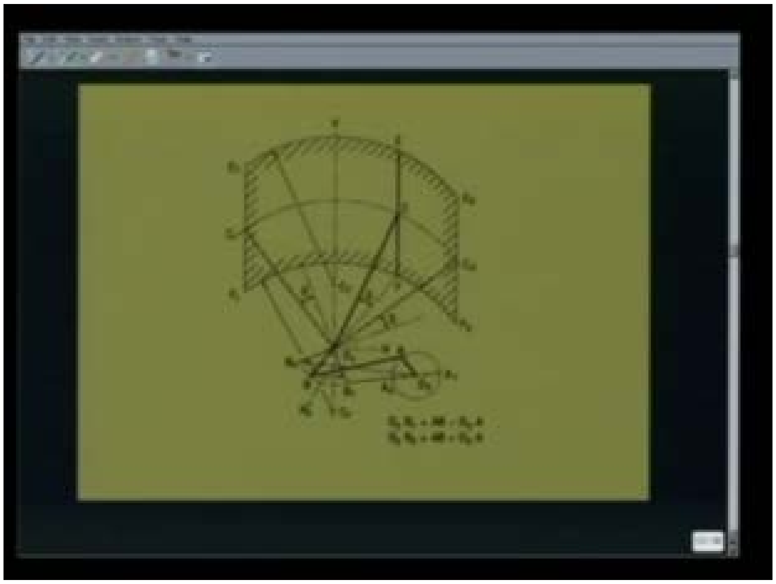

Now that we have obtained this field of wiping for this particular given mechanism, let us observe that this wiping field that is $E_1E_2F_2F_1E_1$. This field of wiping is not symmetrical about the vertical line passing through O_4. This O_4V is the vertical line passing through O_4. But the field of wiping is more on the right and less on the left. So, as a designer, maybe we can make a very little change to make this field of wiping symmetrical about this vertical line. So, the second part of the problem is retaining all other link parameters same, change only the angle delta which was given a 16 degree. Change only this angle delta to make the field of wiping symmetrical about the line O_4V. To solve this problem what do we do? We find, what is the angle of oscillation of this rocker link O_4B? That is, from O_4B_1 to O_4B_2 that is the theta$_4$ star which we call the angle of swing. This theta$_4$ star is not symmetrical about the vertical line as a result the field of wiping is also not symmetrical about the vertical line.

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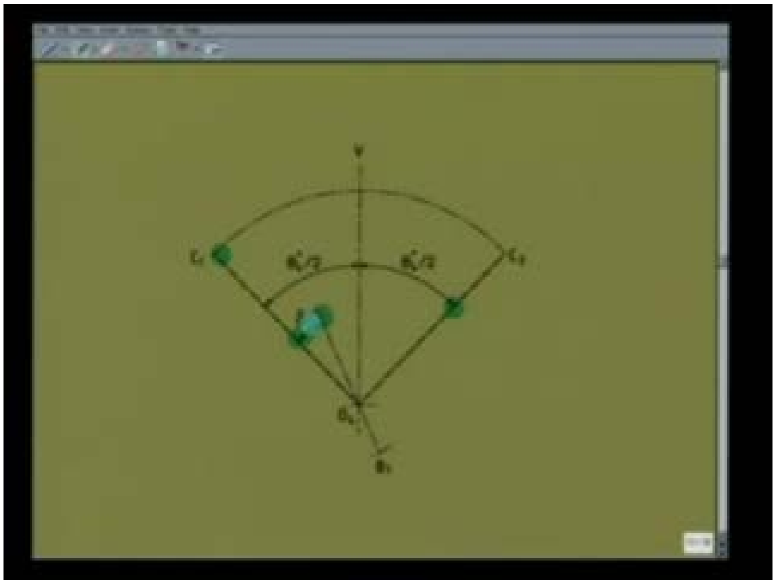

This $theta_4$ star we measure.

And then, $theta_4$ star by 2 and $theta_4$ star by 2 are symmetrical about the vertical lines O_4. We have already seen that B_1 which is the extreme position of the link O_4 B which we have obtained earlier, here this B_1,. Now O_4 C_1 must be like this to make the field of wiping symmetrical because now I have made O_4C_1 and O_4C_2 symmetrically placed about the vertical line just at an angle $theta_4$ star by 2 and $theta_4$ star by 2 because $theta_4$ star is entirely decided by all the link lengths that I cannot change. But now, this is O_4 B_1 and this is O_4 C_1. The extension of O_4 B_1 and this line O_4 C_1, the angle is delta. This is the angle delta, which should be provided rather than what we had earlier at 16 degree.

Now, the third part of the problem we can have some more specifications. For example, we define the width of this wiping field. That is, this horizontal distance C_1 C_2. So, let me post the problem that modifies this design.

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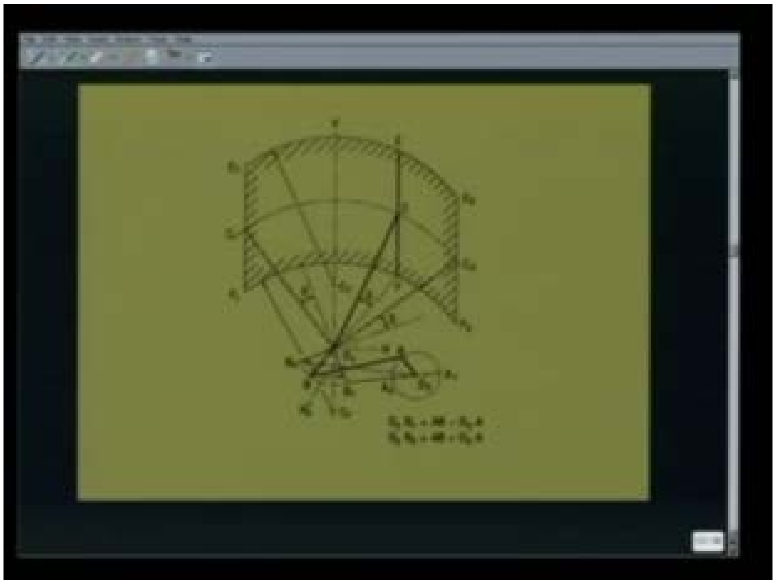

But you are allowed to change only the crank length O_2 A, this coupler length AB and the angle delta to satisfy three requirements. Namely, the wiping field should be symmetrical about the vertical line O_4B, the width of the wiping field that is, the horizontal distance between these two extreme positions E_1F_1 and E_2F_2 is say 450 mm to the same scale and there should be no quick return. That means, O_2A_1 and O_2A_2 corresponding to these two extreme positions of the follower the crank angle should be 180 degree.

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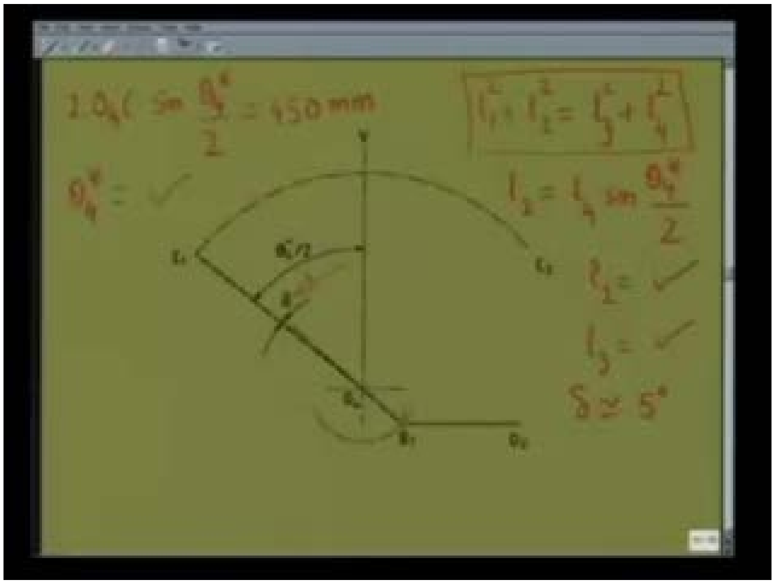

So we have no quick return for which we need l_1 square + l_2 square is same as l_3 square + l_4 square. Now, the width C_1C_2 we can easily see, it is 2 times O_4C into sine of the angle theta$_4$ star by 2. Because this angle is theta$_4$ start by 2, so this horizontal distance is O_4C sine theta$_4$ star by 2, twice of that is the width of the field of wiping. This has been specified as 450 millimeter. The length O_4C has not been changed, so you have already given in the design. Substituting that value of O_4C, I can find the theta$_4$ star value, so we determine theta$_4$ star from this equation with the given value of O_4C. Now that we know theta$_4$ star, if you remember for no quick return, we also had a relationship that l_2 must be l_4 sine theta$_4$ star by 2.

Now l_4 that was the link O_4 B is not allowed to be changed. So now that l_4 is given, theta$_4$ star we have obtained, so I can obtain the crank length O_2A as $l_{2.}$ So I have obtained l_2. l_1 has not been changed, l_4 has not been changed and l_2 we have obtained, so using this relationship for no quick return, I can get the only remaining unknown that is l_3. So, we have designed the 4R-linkage O_2ABO_4. To obtain the required value of delta, we just with the new lengths l_2, l_3, l_1, l_4 were unchanged. I again obtain the path of B, which is this circle. O_2B_1 as we know was $l_3 - l_{2.}$ From O_2, I draw a circular arc O_2B_1 as $l_3 - l_2$.So I get the extreme position $O_4B_{1.}$

The extreme position O_4C_1 is already known so the angle between the extension of O_4B_1 and O_4C_1 determines the required value of delta which as we see is much less than the original value which was 16 degree. This delta, if you draw it correctly comes out around 5 degree. We have modified the design to satisfy three requirements namely the field of wiping has to be symmetrical about the vertical line O_4B has to be of a particular width and also there should not be any quick return effect such that the wiper blade takes equal time in the forward motion and return motion. It should not go very fast in one direction and very slowly in the other direction which will definitely disturb the driver.

Now that we have completed our discussion on displacement analysis both by graphical and analytical method let me start with a very important index of a good mechanism. As you know, the mechanism has to satisfy the geometric requirements, but satisfying the geometric requirement is not all. For a real life mechanism, it must move freely and this free running quality of a mechanism is quantified by a parameter, which is called transmission angle. Let me now discuss the concept of transmission angle and show how to calculate the transmission angle at least for 4 link mechanisms.

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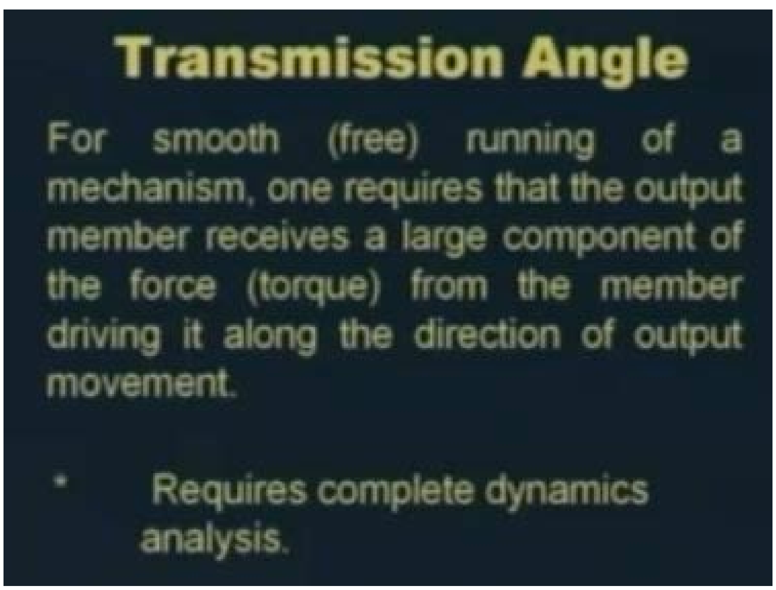

Let me repeat, for smooth running of a mechanism, one requires that the output member receives a large component of the force or torque from the member driving it along the direction of output movement. This will ensure that the mechanism runs freely. Not only satisfies the geometric requirements or kinematic requirements, it must have this quality of smooth or free running. To ensure this smooth and free running, one needs to have a complete dynamic analysis that will be discussed much later.

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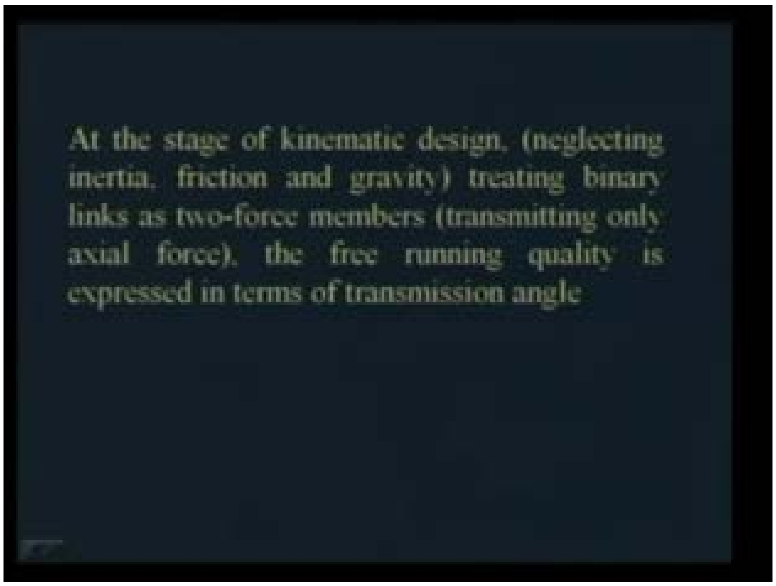

However, even at this stage of kinematic design, what we do?

We neglect inertia, friction, and gravity and treat all the binary links as two-force members that is, transmitting only axial force. With this assumption, the free running quality of a mechanism can be expressed in terms of what is called transmission angle. Let me explain this concept of transmission angle for a 4R crank rocker linkage.

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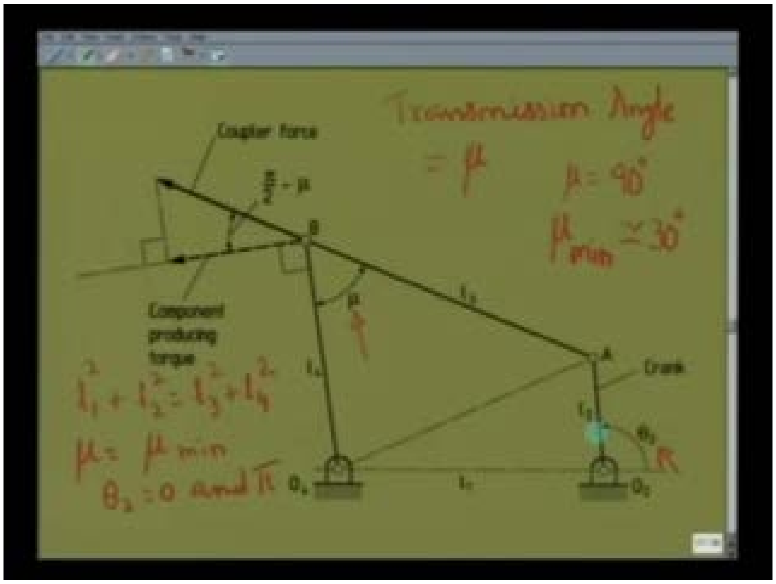

This diagram shows a crank rocker linkage namely, O_2, A, B and O_4 and let O_2A be the crank. As we said, if we assume that coupler AB is a two-force member, then the entire force that AB exerts on the output member O_4B is along the line AB. So this is the direction of the coupler force. However, it is only this component which is perpendicular to the follower, produces torque to drive the coupler. So, you have to ensure that, this angle is as small as possible.

For defining the transmission angle, it is defined as the acute angle between the coupler that is AB and the follower O_4 B. This is what is shown as the angle mu. So, we define the transmission angle is equal to mu which is the acute angle between the coupler and the follower.

Obviously, the base possible value of mu is 90 degree. Then the entire coupler force is used to produce torque about O_4, to drive the follower. However, as the mechanism moves, this angle mu changes, but one is to ensure that mu does not fall below a particular minimum value and normally, minimum value of mu prescribed around 30 degree.

Next, I will show, because we are only interested in ensuring the minimum value of mu, can you find out for what crank position that is, for what value of theta_2 the minimum transmission angle offers? Because this angle mu keeps on changing with the crank position it depends on theta$_2$. It can be easily shown that, if it is a crank rocker linkage without any quick return that is l_1 square + l_2 square is l_3 square + l_4 square, then mu attains it minimum value that is, mu min, when this angle theta$_2$ is either 0 or pi. That is the crank is along the line of frame O_4O_2. That is theta$_2$ is either 0 or pi. To get this result, that mu attains its minimum value for theta$_2$ equal to 0 and pi, if there is no quick return, that is this l_1 square plus l_2 square is equal to l_3 square plus l_4 square, this relationship holds good.

It is very easy to show, if we consider the length O_4A. O_4A. I can write in terms of l_3, l_4 and mu. Considering the triangle O_4AB, I can write O_4A square is equal to l_4 square plus l_3 square minus twice l_3 l_4 cos mu. Same way, I consider again the triangle O_4O_2A and write O_4A square as l_1 square plus l_2 square minus twice l_1 l_2 into cosine of pi minus theta$_2$, that is plus twice l_1 l_2 cos theta$_2$. To obtain an expression for the transmission angle mu, let us consider the triangle O_4AB.

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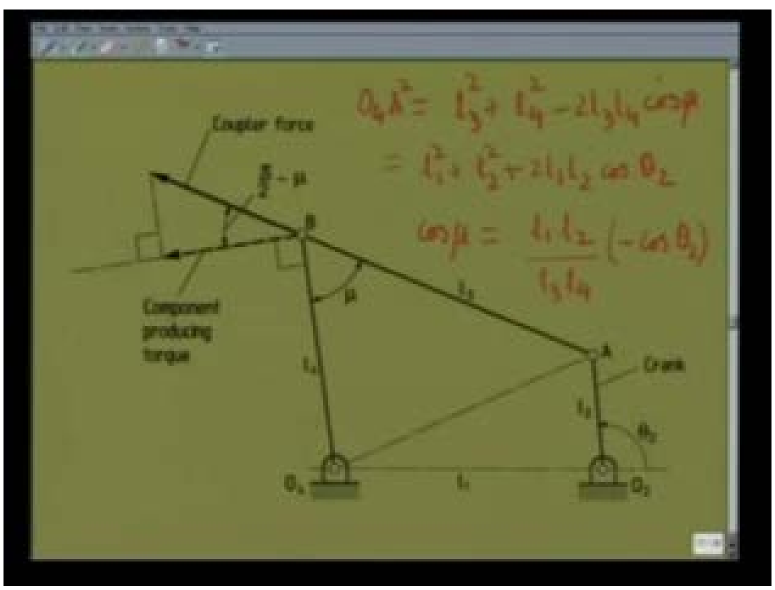

Then we can write O_4A square is equal to l_3 square plus l_4 square minus twice l_3 l_4 cosine mu. Same way, if we consider the triangle O_4AO_2, then I can write again, O_4A square is l_1 square plus l_2 square plus twice l_1 l_2 cosine theta$_2$. If this crank rocker linkage has no quick return effect that means, l_1 square plus l_2 square is same as l_3 square plus l_4 square, then I can get expression for cosine mu as l_1 l_2 divided by l_3 l_4 into minus of cosine theta_2. For mu to be maximum, we see that the values of theta_2 can be either 0 or pi. Because we have to remember, if this angle is more than 90 degree then I will take pi minus this angle between the coupler and the follower as my transmission angle. Transmission angle is defined as the acute angle between the coupler and the follower.

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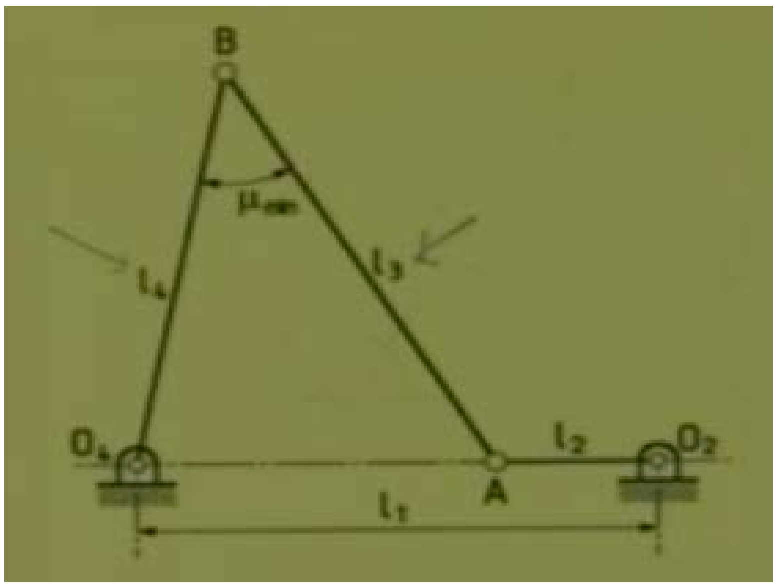

Here in this diagram, angle happens to be acute so it is mu, but if when this angle becomes obtuse, then I have to take 180 degree minus this angle as my transmission angle. So, transmission angle is minimized when theta_2 is either 0 or pi for this particular situation. That means, the crank falls in line with the frame that is O_4O_2. This will show now two models, but if there is quick return effect then the minimum transmission angle occurs either at theta_2 is equal to 0 or at theta_2 is equal to pi. Only when, there is no quick return effect then it will be at both locations theta_2 equal to 0 and pi.

This figure clearly shows that, when the crank O_2A along the line of frame O_4O_2. The transmission angle that is the angle between the coupler and the follower is at its minimum value, this is what we call mu min, because it is already acute.

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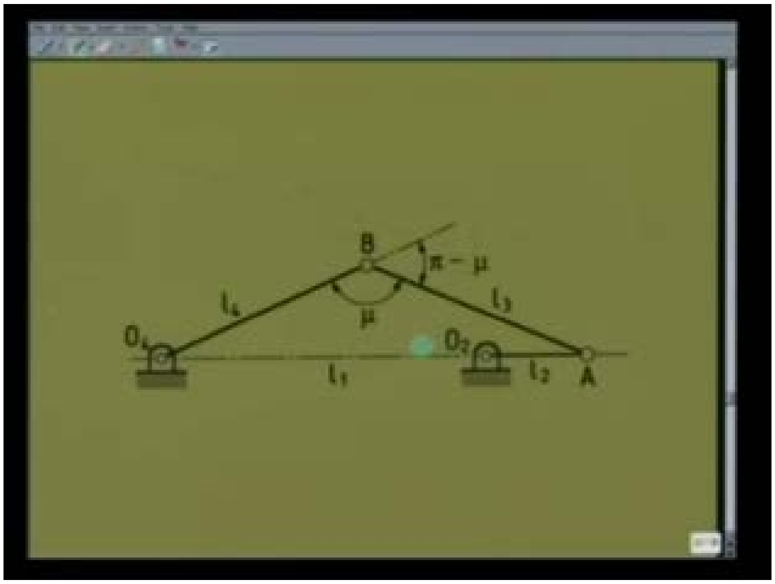

In another situation, we see that the crank O_2A is again along the frame line O_4O_2 and the angle between the coupler and the follower that is, this angle is more than 90 degree. So here, we will define pi minus mu as my transmission angle. It will always take the acute angle and this angle is minimised when O_2A is along the line of frame. So there are two situations: either O_2 A is in this direction when this angle between the coupler and the follower is more than 90 degree and the transmission angle is pi minus mu; the other situation is, when O_2A is along the line of frame and the coupler and the follower makes an acute angle and that itself is the minimum transmission angle, mu min. This now, we will demonstrate through models.

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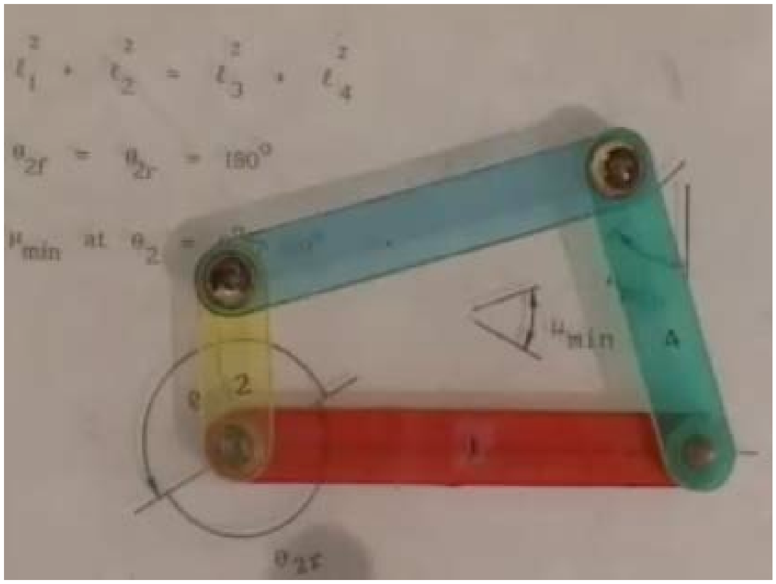

We consider this model of a crank rocker linkage without any quick return effect. That is, l_1 square + l_2 square is l_3 square + l_4 square. For such a linkage, as I told you, the minimum transmission angle occurs when the crank is along the line of frame. This angle between the coupler and the follower is its minimum value. However, again when the crank falls along the line of frame, this angle is maximised, that is the transmission angle which is defined as the acute angle between the coupler and the follower that I can see by the extension of the coupler and the follower, that angle is minimised. The minimum transmission angle occurs at two configurations: one is this and the other is this.

When the transmission characteristics are very bad, because most of the tool of the coupler is not going to drive the follower, this perpendicular component of this actual force is minimum. Here, it is very good. When it is 90 degree, all the coupler force is trying to drag the follower. Here, the transmission is very good; here the transmission is worst; and here the transmission is worst.

Now, let us look at this another model, where l_1 square plus l_2 square is not the same as l_3 square plus l_4 square. Here, the minimum transmission angle occurs only for this configuration, when the crank and the frame are along the same line. The angle between them is maximised so the transmission angle is minimum. Here, the transmission quality is poor and here, the transmission quality is very good, when the angle is close to 90 degree. Let us look at this model again where, l_1 square + l_2 square is not the same as l_3 square + l_4 square. So here, the minimum transmission angle occurs only for this configuration when the crank is along the line of frame and the angle between the follower and the coupler is very small giving rise to very poor transmission characteristics. However, in this configuration, when again the crank is along the line of frame, the angle is quite large and this is not the minimum transmission angle.

So, if there is no quick return, then the minimum transmission angle occurs either here or when this angle is 180 degree, but if there is no quick return, then it happens both at this position and at the 180 degree position as shown earlier. We now explain the concept of transmission angle with reference to a slider-crank mechanism.

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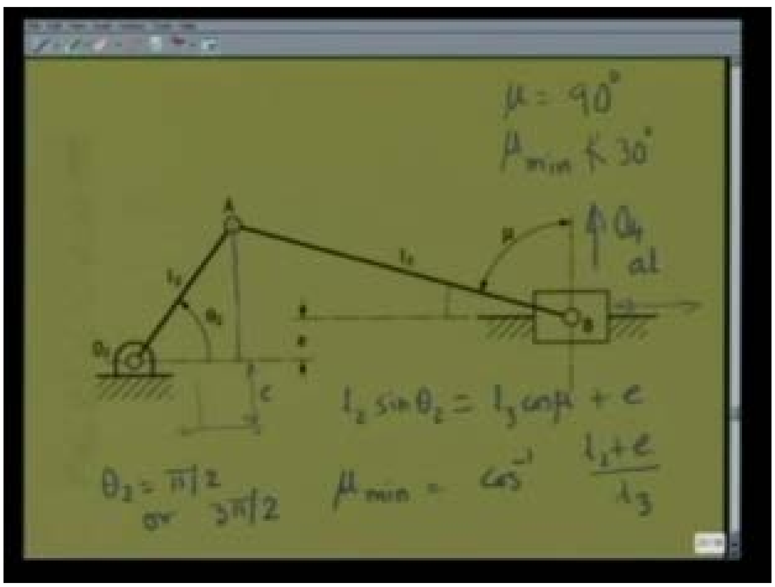

This figure shows a slider-crank mechanism, where the slider is at B undergoing horizontal translation and O_2A is the crank. It is obvious that, if the connecting rod AB is horizontal then the entire connecting rod force is in the direction of movement of the slider.

Consequently, we define the transmission angle as mu that is the angle between the connecting rod and a direction perpendicular to the line of movement. The most desired value of mu is 90 degree, but to ensure smooth free running of the mechanism, mu min, minimum value of mu should not fall below say, around 30 degree. Now we can find out, for what value of crank angle that is $theta_2$ the minimum transmission angle occurs. For that, we see this vertical distance is l_2 sine $theta_2$, which is also same as l_3, this is 90 degree minus mu, so that is l_3 cos mu plus e. As $theta_2$ changes, l_3 l_2 and e are constants so, mu changes. The minimum value of mu will take place depending on whether e is this way or suppose the offset was below this line, the direction of sliding is like this, then this distance would have been called e depending on whether e is upward or downward, one can easily find that mu min will be cos inverse l_2 plus e by $l_{3.}$ That occurs either at $theta_2$ equal to pi by 2 or 3 pi by 2, depending on the direction of e.

This I leave for the students to decide for themselves and that will add to the understanding. One can also see, this slider-crank mechanism add a 4Rlinkage with O_4B the hinge O_4 at infinity. Then you see, O_2ABO_4 is the equivalent 4R-linkage and mu is nothing but the angle between the coupler AB and the follower O_4B. This concept of transmission angle we have used is same for both the 4R-linkage, crank rocker linkage, and this slider-crank mechanism.

Let me now summarize what we have learnt today. We continued our discussion on displacement analysis of planar mechanisms by analytical methods. Then we obtained certain important results so far as 4R crank rocker linkages are concerned, with reference to its quick return effect and transmission angle and also where the minimum transmission angle occurs, which has to be ensured for a free running of a design mechanism. We have also seen through an example, that how we can combine both graphical and analytical methods to improve the design or modify an existing design.