Finite Element Method

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Module - 2 Lecture - 3

In the previous lectures, we had talked about how to compute the stiffness matrix and the load vector for a given model problem using the finite element method. What were the essential features what we discussed in the previous lecture? We had defined these global basis functions $phi_{i,}$ which were given as functions of this type.

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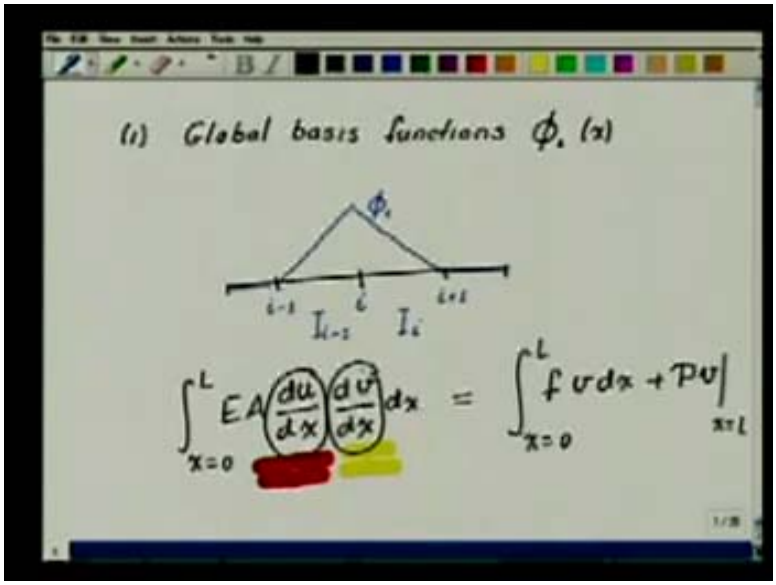

That is, the functions which at the node i had a value 1 and at all other nodes they had a value 0. These are so-called piecewise linear functions that we have formed; they were continuous you can see that they are non zero only in the elements i minus 1 and element i. These were functions with local support. One thing that we should answer before going further is why did we choose these functions?

If you remember in the previous lectures, we had talked about the problem with point loads. We saw in that case that the problem with point loads was that it has piecewise smooth functions which are

continuous at the points and the derivatives have jumps at those points. So that was the motivation for using such functions in the Rayleigh-Ritz Method. Now, it is not always obvious what kind of functions we have to use for the given problem of interest? We should have some kind of general mechanism by which you can see what is the minimum smoothness requirements on the function phi_i that we are using to form our global basis functions, before we can define global basis functions. For different differential equations you will have different phi_i's. So let us see how to do it.

If you remember that we had made our weak formulation of this type: integral x is equal to 0 to L EA du by dx and into dv by dx, where I will be using v or w as my weight functions and this on the right hand side is equal to integral x is equal to 0 to L f v dx plus as we had put an end load P v evaluated at x equal to L. So this was our weak form for the model problem that we have taken. So now let us look at this part. If I look at this part here you see that this term and this term are sitting in this integral. So we want this integral to be finite. So the integral should be finite. Now what does it mean? If this integral has to be finite then each of these terms du by dx dv by dx have to be defined in the domain of interest. So what we want is du by dx and dv by dx, they should both be defined in domain omega or I.

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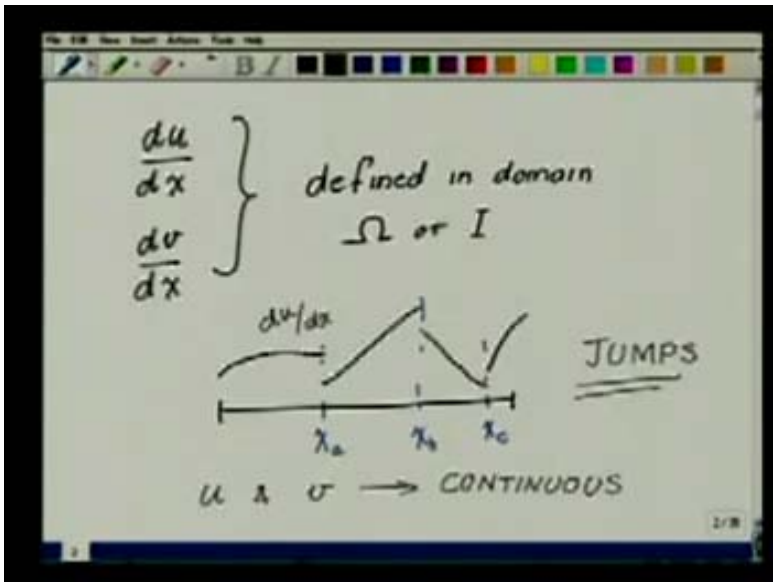

So what are the roughest or the least smooth of these functions u and v for which du by dx and dv by dx will be defined. The answer is very simple that is, if we look at this domain we can have du by dx and dv by dx defined means that they can be like this. That is at particular points I will call point x_a, point x_b,

and point x_c. At these particular points the derivatives can have jumps. This is if you remember I am writing it as du by dx dv by dx. So if this is the function that we have, then in that case the integral on the left hand side in the previous page is defined. Anything worse than that in this is if I have in a way worse than the jump means that there will direct delta at these points data derivatives that is in fact even the u can be discontinuous. In that case this integral will be infinite; it will not be defined. In order to have these integrals finite we ask the question what is the worst type of functions that will satisfy that requirement and the answer here is the functions for which derivatives have jumps at specific points. That is the reason why we say that if the derivative has jumps at specific points, then u and v are continuous in the domain. In such way that at certain points slopes change. So with this requirement we ask, what are the functions that can be used in our series representation? Because if you remember, in this series also we need functions which can capture this kind of behavior.

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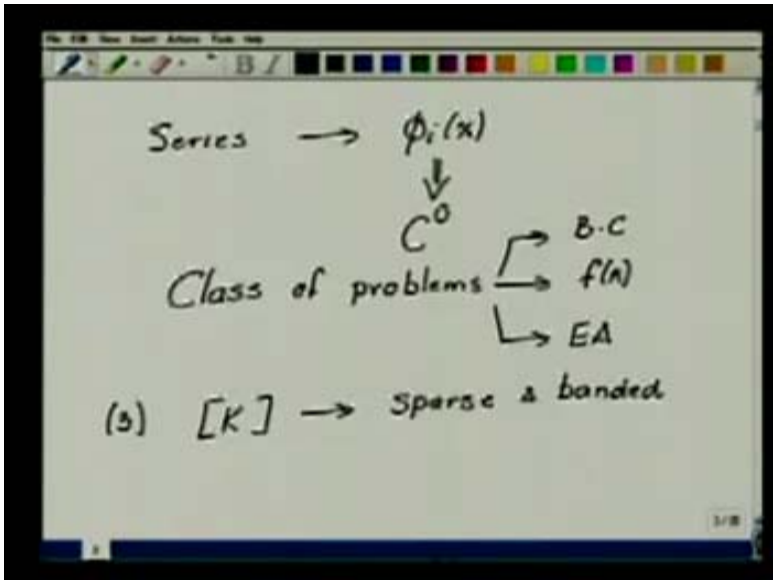

So the phi_i's that we have should also satisfy this minimum requirement of the derivative. So we want phi for this particular problem, because we want the left hand side to be defined; should also be continuous or we say it lies in C zero. That is the reason why we took phi_i's as the hat shaped functions that we are taking. We could have taken it to be smoother, but the smoother functions would not satisfy this minimum condition and you remember that whenever you are developing a method that is a method which will translate into a computer program you do not want to write the program for one particular problem; you want to write it for a class of problems.

Class of problems in this case is defined by what? - By different types of boundary conditions, different types of body force, and different types of materials. It turns out that in order to look at a big class of problems of practical interest we have to talk about the point loads. So in this case whenever we are trying to write a program or whenever we trying to define a finite element method we would like to cover the solutions for all these types of problems that are of interest and for the point load this phi_i with jump in derivatives was needed or with the material interfaces we needed and it comes out also from our minimum requirement for the left hand side to be finite. So that is the reason why one chooses one type of basis functions for a given differential equation. We will see later on when we go to the beam problem that these basis functions, the definitions of these are going to change. There again we will see what is the minimum requirements for which we have to define the basis functions.

So do not think that the whole process is arbitrary in nature. Everything comes for a particular reason and the reason is we have to go back our weak formulation and look at the left hand side, look at the right hand side and ask the question what will make this left hand side and right hand side finite. So this is one thing that I wanted to harp on and that is a very important reason, very important thing that we have to keep in mind when we are doing the finite element method. Third thing that we obtained was, because this function phi_i were locally defined that is phi_i was non zero in only elements i minus 1 and element i, so in that case what happened is the integral were not all non zero. That is the stiffness matrix was sparse. Not only sparse, it was banded that is some of the elements around the diagonal that is either side of the diagonal were only non zero everything else in the matrix was zero. Today let us go further and see how do we go and do everything from a computer based orientation.

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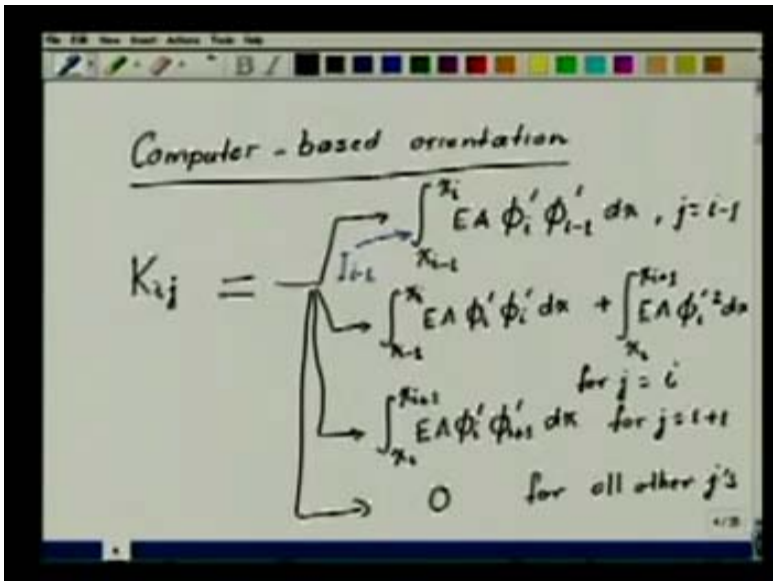

Finally, we are not going to solve this problem by hand. We would like to write a computer program which can give us the answer to this problem. So from this point of you we would like to go head. To start off, let us if you remember that I take the ij th entry of the stiffness matrix. So what does it mean?

I am looking at the ith row and the jth column of the stiffness matrix and what do we get for this entry? There are few possibilities that we have. That is for j equal to i minus 1, what do we have? that for j equal to i minus 1 I will have in the element $x_{i \text{ minus } 1}$ to x_i, if you remember it will be equal to EA phi_i prime $phi_{i \text{ minus } 1}$ prime dx for j is equal to i minus 1. This is all that is only in the element i minus 1 this part is non-zero everywhere else it is zero. Why? because if you see here phi_i is non zero in elements i minus 1 elements and element i while $phi_{i \text{ minus } 1}$ is non zero in only element i minus 1 but it is zero in element i. So element i does not contribute. Similarly let us look at the second possibility. Second possibility is, when j is equal to I, so in that case we will get integral over the element i minus 1 phi_i prime phi_i prime dx plus integral over element i EA phi_i prime d square x for j is equal to i.

Third case is when j is equal to i plus 1. Then this integral becomes x_i to $x_{i \text{ plus } 1}$ EA phi_i prime $phi_{i \text{ plus } 1}$ prime dx for j is equal to i plus 1. For all other cases it is zero for all other j's. You see that only these three entries are non zero in the ith row everything else is zero. You see that the ith row i minus Ith column entry the ith column entry i plus Ith column entry are non zero. This is the structure that we will have for all i's. Similarly what will the F_i be?

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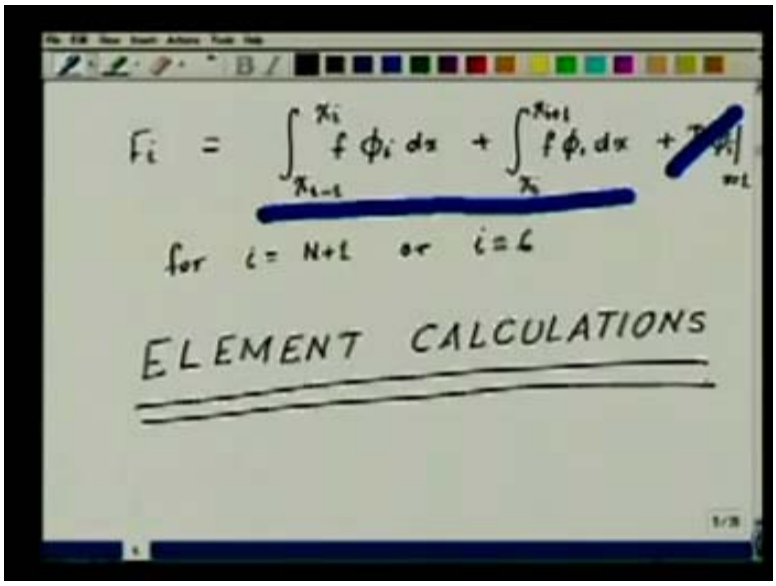

That is the load vector it will be equal to because phi_i is non zero in the element i minus 1 and the element i so $x_{i\ minus\ 1}$ x_i f phi_i dx plus integral x_i to $x_{i\ plus\ 1}$ f phi_i dx; plus we have P phi_i at x equal to L for our model problem. Let us ignore this part for the time limit. That is for the time being, let us forget this thing. We will see why. I mean you should be able to know the reason why. You note that, for all phi_i's, because we have taken a six element mesh, that is for i is equal to five element mesh plus or for the model problem, i is equal to 6 for that case only the phi_i at x equal to L is 1; all other phi_i's are zero here. So this part P into phi_i is only going to contribute to f_6 that is in the general case $f_{n\ plus\ 1}$. To all other f phi_i's it is not going to contribute that is why let us keep it out for the time being, we will handle this later. So now, we have this part of the load vector and if I go to the previous page (Refer Slide Time: 16:15), this part of the stiffness matrix to handle.

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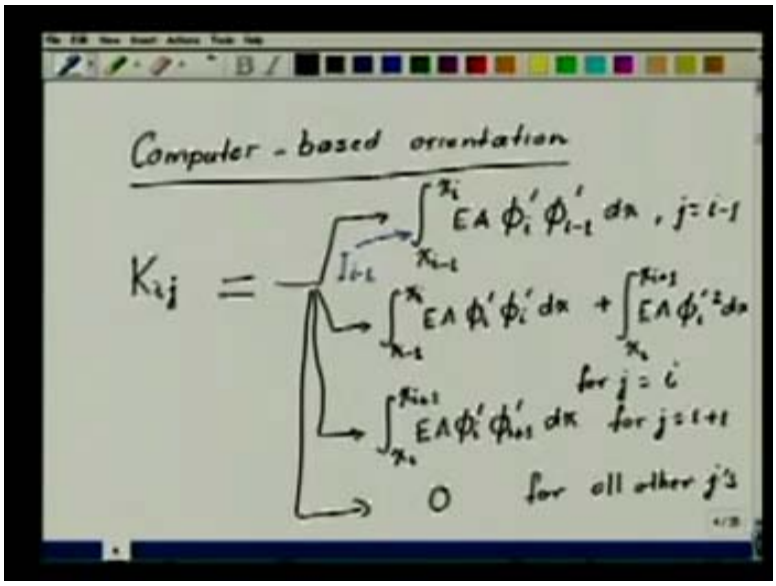

So how do we go about handling this? One thing that we will do is we will try to compute these integrals because you see that they are limited to elements i minus 1 i or i plus 1 at the most. We will try to compute them using the element level integrations. Why? Because these are summation over the elements as we did in the previous lecture also.

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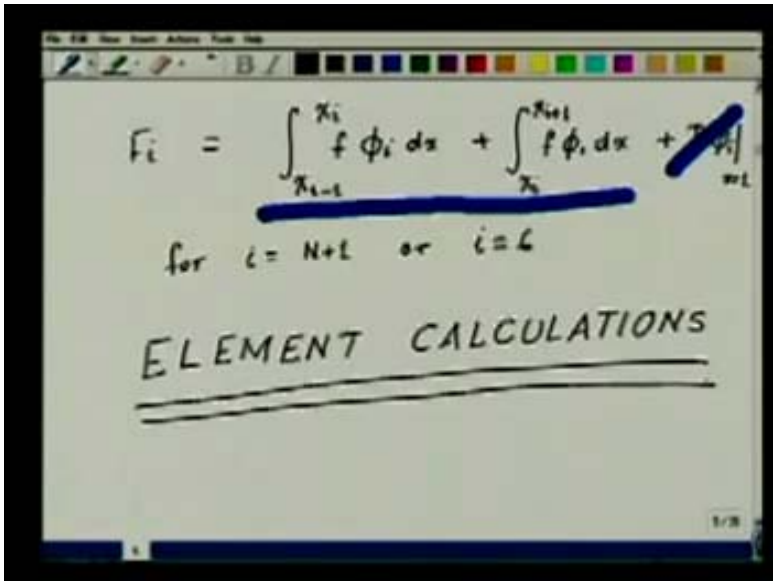

The integrals are obtained but summation of the integrals over these elements. So let us now go to a very important part of the finite element method which is called the Element Calculations.

For the element calculation what do we do? Let us draw certain pictures. In finite element, one has to draw pictures there is no way out.

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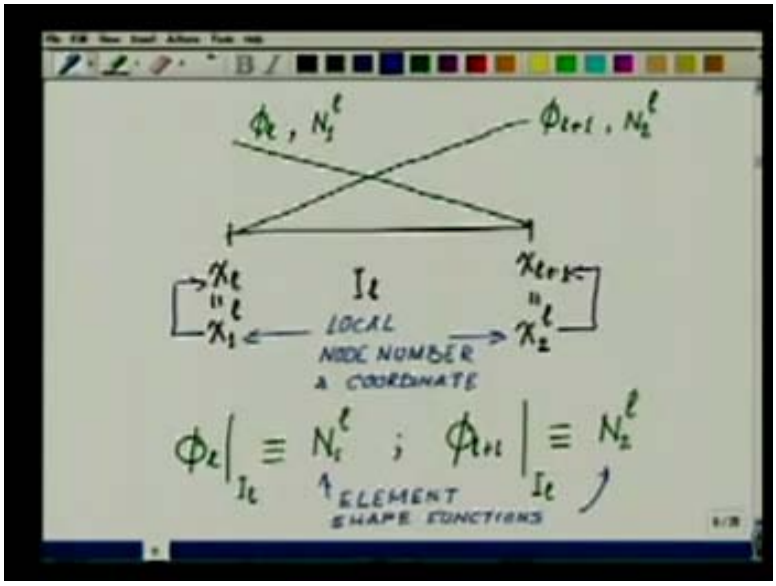

So let us look; I ask the following question. I give you element l that is, this is element I_l. For this element what are the nodes that will be the end points of the elements? They will be x_l and $x_{l \text{ plus } 1}$. First thing that I am going to do is, since this element has two nodes I am going to give them local number – a local name to these nodes. What I am going to call them? I will call them of x_1 of element l this node and this node is equal to x_2 of element l. So for each element, I have an x_1 and x_2, which are representing the two extremities of the element and x_1 for the element l will equal to the global node x_l or the coordinate and x_2 will be global node $x_{l \text{ plus } 1}$.

This you see we will give it a name; this is the local node number and coordinate. This is inherited from a global number. That is it is inherited from here (Refer Slide Time: 19:17). There is a one to one correspondence between these local nodes and the global nodes. Another thing that we will do is, let us now look at the function phi_i. Which of the phi_i's are non zero in this elements? We have given the answer many times over. It is these two functions (Refer Slide Time: 19:42). So this is going to be phi_l

and this is $phi_{l\ plus\ 1}$. These are the only two phi's which are non zero in this element. So what we are going to do is we are going to now give them again an element name and that is the name when I is at an element I_l. So what name will I give, quite natural. I am going to give it the name N_1^l and this one will be N_2^l. That is, we should remember that this phi_l in the element I_l is equal to N_1 of element l. Similarly $phi_{l\ plus\ 1}$ restricted to the N_2 element l is identically equal to N_2 of this element l. What are these N_1 and N_2 called?

This N_1 and N_2's for the element are called the Element Shape functions. This N_1^l and N_2^l are given a name called the element shape functions. What you would see in most of the books is that they start from N_1^l and N_2^l remember that N_1^l and N_2^l are nothing but phi_l's restricted to the element i. So once we have these elements shape functions N_1^l and N_2^l then you see what we are doing; we are defining everything with respect to the element. All the global quantities that we are defining them with respect to the elements. So now tell me what is N_1^l and N_2^l?

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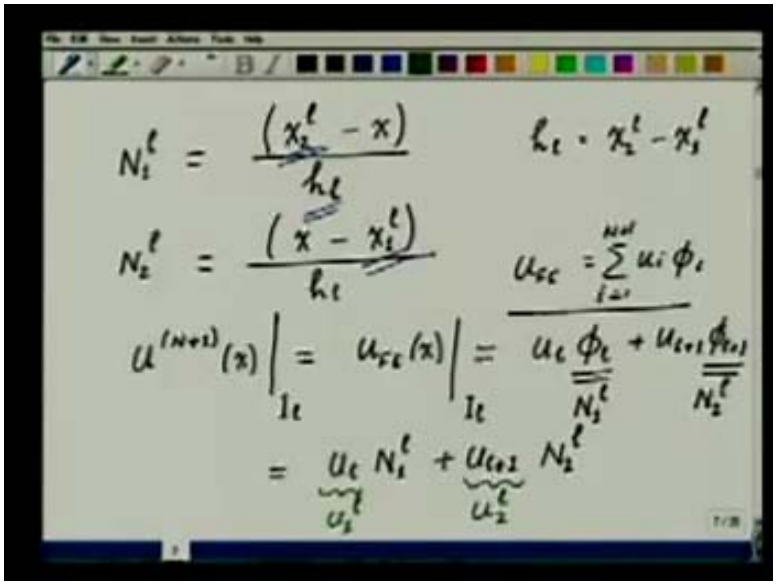

It is very easy N_1^l will be equal to what? Since it has to be 1 at the point x_1^l and zero at the point x_2^l. It will be x_2^l minus x divided by h of the element l; that is the size of the element l and remember that h_l is equal to x_2^l minus x_1^l that is the size of the element is h_l given by x_2^l minus x_1^l and note that here what happens when I put x_1^l that is at the point x is equal to x_1^l I get N_1^l is equal to l that the point x is equal to x_2^l N_1^l is equal to zero.

Similarly, N_2^l is equal to since it is zero at the point x_1^l and 1 at the point x_2^l it is equal to x minus x_1^l divided by h_l as simple as this. This is nothing but phi_l N_1^l is the phi_l and N_2^l is the phi_l plus 1 in the element I_l. So we have defined what these element shape functions are in terms of the element coordinates and in terms of the element size h_1. So once we have defined these, then we ask the other question; what is finite element solutions, what is the representation of it in this element I_l? So we have if you remember u we had in the generic case u to the power of N plus 1 x is I will call it different connotations different places this will be equal to what? Again the question is which of the phi_i if you remember that u_{fe} is equal to sigma i is equal to 1 to N plus 1 u_i phi_i. So the question is, which of phi_i will be non zero in the element. The answer is it is going to be phi_l and phi_l plus 1. So what will get? u_{fe} in the element will be equal to u_l phi_l plus $u_{l\ plus\ 1}$ $phi_{l\ plus\ 1}$. Now we know that this phi_l in the element is equal to N_1 of l and $phi_{l\ plus\ 1}$ in the element is equal to N_2 of l. This u_{fe} in the element $I_{l,}$ in the element $I_{l,}$ is equal to u_l in to N_1 of l plus $u_{l\ plus\ 1}$ into N_2 of l.

Now these u_l and $u_{l\ plus\ 1}$ are global quantities, because they are defined with respect to this series so now we will give them a local name that is an element wise name. So this we will represent as in the element this is u_1 of l and in this element this is $u_2$of l. You should remember that u_l is equal to in the element u_1 of l and $u_{l\ plus\ 1}$ is equal to in the element $u_2$of l.

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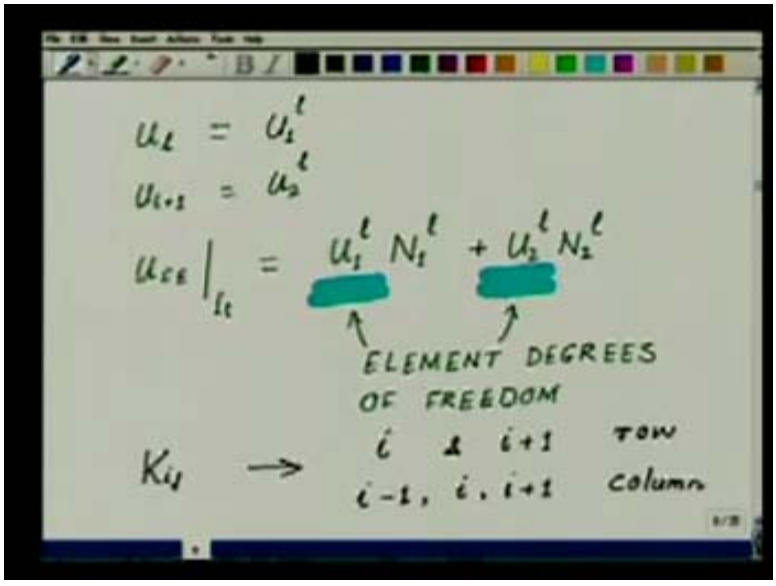

So what do we have? u_{fe} restricted to the element l is equal to $u_1^l N_1^l$ plus $u_2^l N_2^l$. So these are called, they have to be given a name; these are the Element Degrees of Freedom. The u_i's in this finite element connotations globally are called degrees of freedom while the element - they are element degrees the freedom. Now we will ask the following question as to which of the K_{ij}'s does the integral from the element contribute with the non zero entry. The answer is directly related to the phi_i's which is non zero in the element so that it comes that, for the row's i and i plus 1 row globally and for these rows the columns will be i minus 1, i and i plus 1 column. So these rows and these columns are the ones to which the element contributes. So what we see is that the element contributes to the ith and i plus 1th row of global stiffness matrix K and to the i and i plus 1 column.

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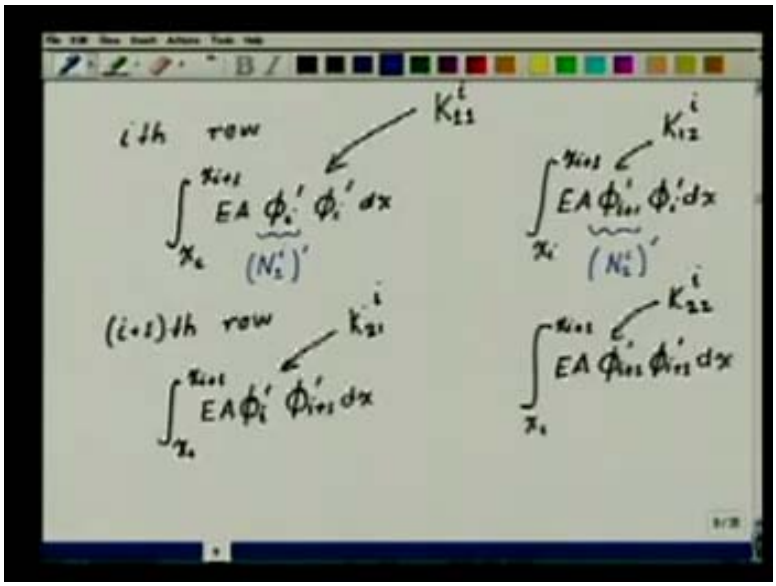

So let us now look at these contributions. What are these contributions? What we have is that for the ith row. The element contributions will be integral from, since we are talking of the ith element, so it will be x_i to $x_{i \text{ plus } 1}$ EA phi_i prime phi_i prime dx this is going to be non zero. Similarly, x_i to $x_{i \text{ plus } 1}$ EA $phi_{i \text{ plus } 1}$ prime phi_i prime dx this is going to be non zero. Similarly, for the i plus 1th row, what are going to be non zero? x_i to $x_{i \text{ plus } 1}$ EA phi_i prime $phi_{i \text{ plus } 1}$ prime dx and integral x_i to $x_{i \text{ plus } 1}$ EA $phi_{i \text{ plus } 1}$ prime $phi_{i \text{ plus } 1}$ prime dx. So you see that these are the four entries in global stiffness matrix to which the element contributes with non zero matrix. So what we are going to do is, we are going to call these entries we are going to write in the form of the matrix and the first entry will be called $K_{11}^{i.}$. Second entry will be call $K_{12}^{i.}$ The third entry will be called K_{21}^{i} i and the fourth term will be $K_{22}^{i.}$ Tell me why I

am calling in K_{11}, K_{12}, K_{21}, and $K_{22}$of element i? Because, in the element if you remember, I can replace phi_i with N_1 of the element i prime. Similarly, I can replace $phi_{i\ plus\ 1}$ with the N_2 of the element i prime.

So for a given element, you see that naturally this will be nothing but the first integral will be EA N_1 prime and N_2 prime second entry will be N_1 prime N_2 prime. The first entry will be N_1 prime N_1 prime second one will be N_1 prime N_2 prime, second row the first entry will be N_2 prime N_1 prime and the second entry of the second row will be N_2 prime N_2 prime. So naturally this comes in terms of the element convention as the matrix K for the element i is equal to K_{11} of element i, K_{12} of the element i, K_{21} of the element i, K_{22} of the element i, and this will be equal to let me rewrite it again, integral now I am rewriting the integrals all in terms of the element quantities.

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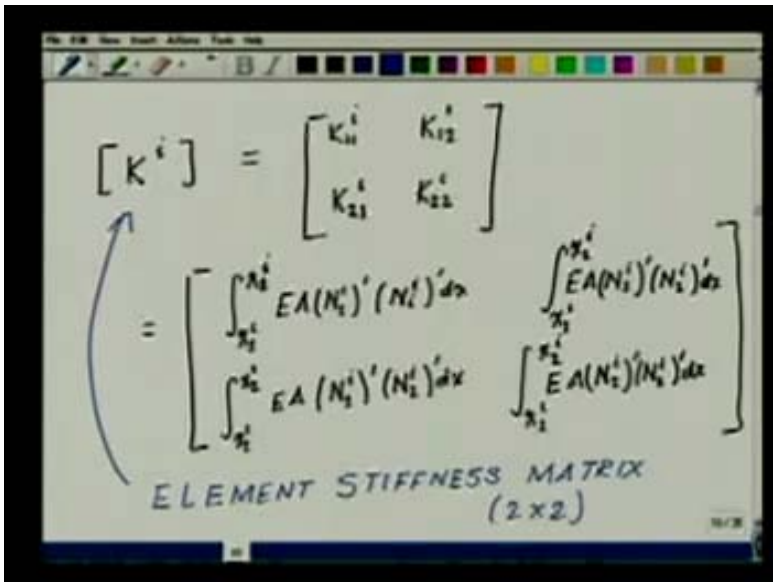

So it will be integral from x_1 of the element i to x_2 of the element i EA N_1i prime N_1i prime dx and the second one will be x_1^i to x_2^i EA N_1i prime N_2i prime dx and the first entry of the second row will be x_1^i to x_2^i EA N_1i prime N_2i prime dx and x_1^i x_2^i EA N_2i prime N_2i prime dx. This is all; this matrix if you remember, we had given it a name it is called the Element Stiffness matrix. In this case, the element stiffness matrix is a 2 by 2 matrix because only two of the phi_i's are non zero in the elements. All other phi_i's are zero. If there were more phi_i's which we will see later one which will be non zero in the elements the matrix size increases by that much. So this is the element stiffness matrix and if you see that we have computed now the entries of it. What we had done in the previous stage, we had simply

rewritten in terms of the element connotations that we are introduced. So they are given in terms of the element level entities, let us now go and use this for a particular problem.

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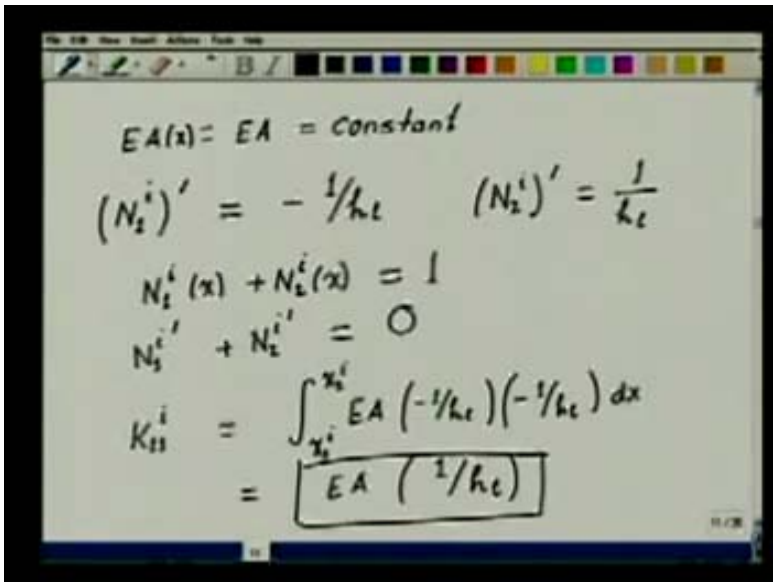

So let us take the case where EA as a function of x is equal to EA is equal to constant. So a simple thing, now what do we have? We will have, remember that we need the derivatives of N_1 and N_2. Let us take for the element of N_1^i the derivative for the element i the derivative of N_1^i this will be equal to what from the definition of N_1^i, so let us go back to the previous pages and see the definition of these functions N_1.

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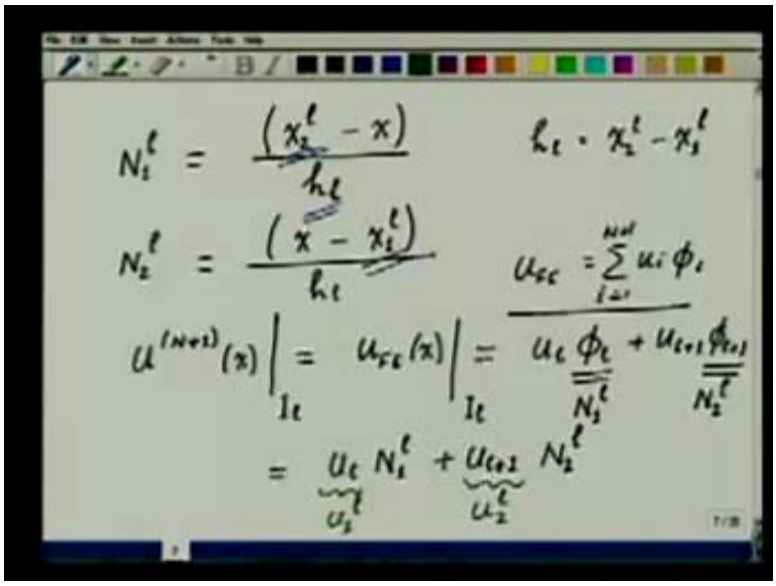

So you see that, if I take the derivative of this expression N_1 then it becomes -1 by h_l. The derivative of N_2 here it becomes plus 1 by h_l. Let us go further and introduce this becomes equal minus 1 by h_l.

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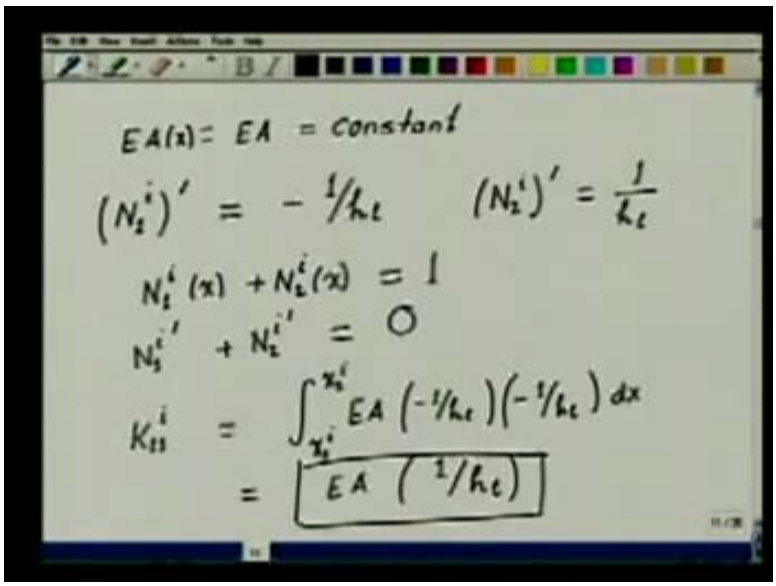

Similarly, N_2 of i prime is equal to +1 by h_l. This has some very interesting properties. You should check for yourself that, N_1^i at any point x in the element plus N_2^i at the same point in the element is equal to 1. If you look at this thing here, this expression N_1^i prime plus N_2^i prime is equal to zero. In a way this is a

check for what you are computing. No if N_1^i plus N_2^i comes out to be non zero you know there is a problem in whatever you have done and in similarly if N_1^i plus N_2^i is not equal to 1 then there is a problem; N_1 prime plus N_2 i prime is not equal to zero then there is a problem. These are some small checks that we can incorporate and whatever we do.

Using this let us now go to our element calculation. So what will K_{11} of element i be equal to? It will be equal to integral x_1^i to x_2^i EA which is now a constant for this particular case in the example that we are taking. Now N_1^i prime is equal to -1 by h_l. Again N_1^i prime is -1 by h_l dx and this is this expression, the integrant is a constant. So this will be equal to integral EA in to 1 by h_l square into x_2^i minus x_1^i. But what is x_2^i minus x_1^i? It is equal to h_i. This will be equal to EA into 1 by h_i. So K_{11}^i came out to the EA by h_i. So let us now calculate K_{12}^l.

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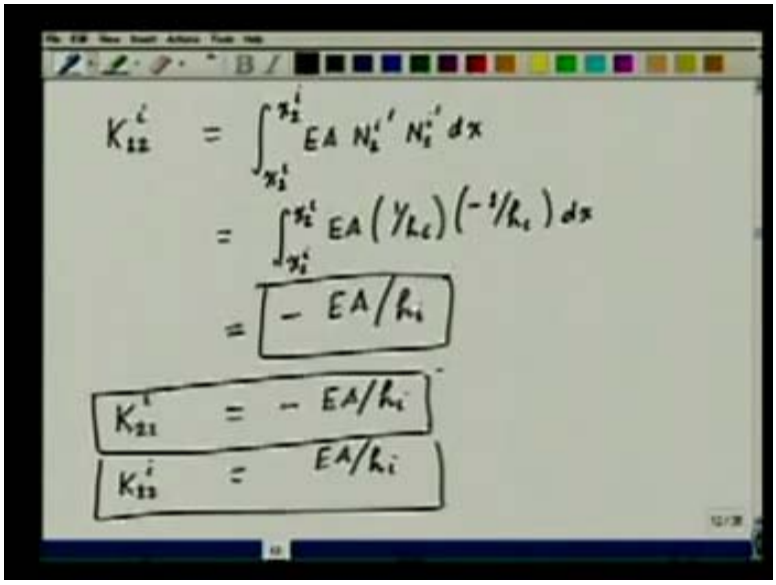

This will be equal to integral of x_1^i to x_2^i of EA N_2^i prime N_1^i prime dx. This is equal to integral over x_1^i to x_2^i EA in to 1 by h_l into h_i into -1 by h_i dx. So this will be equal to again by doing this integration it becomes minus EA by h_i. Actually for the previous expression also it is not EA by h_l, it is EA by h_i because we are talking of the ith element not the lth element, okay.

So this is what K_{12}^{i} is. So by the same token if you do the same job for the next row, K_{2}^{i} will be equal to minus EA by h_i and K_{22}^{i} will be equal to EA by h_i. These are the four entries of the element stiffness matrix we get. What does the element stiffness matrix become?

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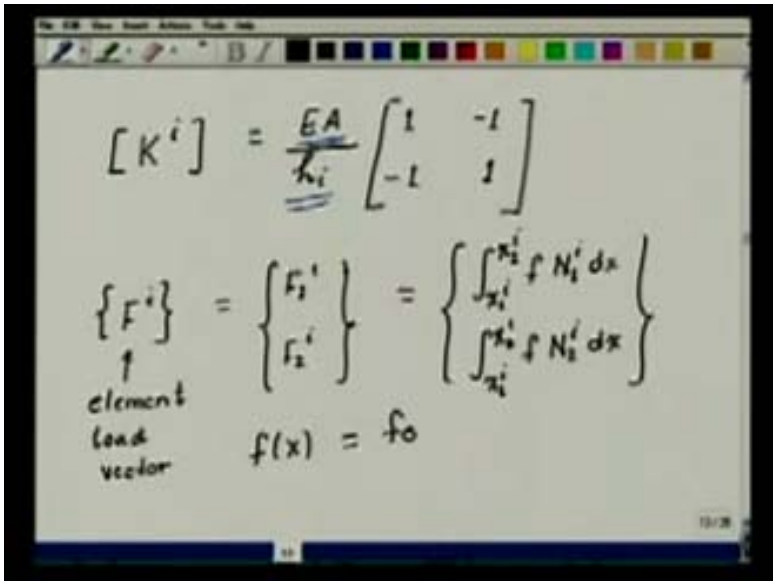

K for the ith element is equal to EA by h_i into here 1 here -1 here, -1, here, 1 here. See this also is symmetric; the element stiffness matrix is also symmetric. You see that we have formed a recipe for the ith element. So now it is very easy to do this job for any of the elements because all you need as far as the information is concerned is value of the EA and the size h_i of the element. Once you have that, then this together gives me the element stiffness matrix. I do not have to do this computation repeatedly, so I do it once and this structure can be used for all the elements. This is one of the beauties of the finite element method. Because, the shape functions are so defined that this structure remains uniform in something. Similarly, if I am looking at F^i that is the contribution to the load vector. So this is an element load vector. This will be equal to F_1^{i}, F_2^{i} and this will be equal to integral from x_1^{i} to x_2^{i} f N_1^{i} dx and here also integral over these elements f N_2^{i} dx. Let us take the case that fx is the constant. There is an example, let us say it is equal to f_0. So then what does this become?

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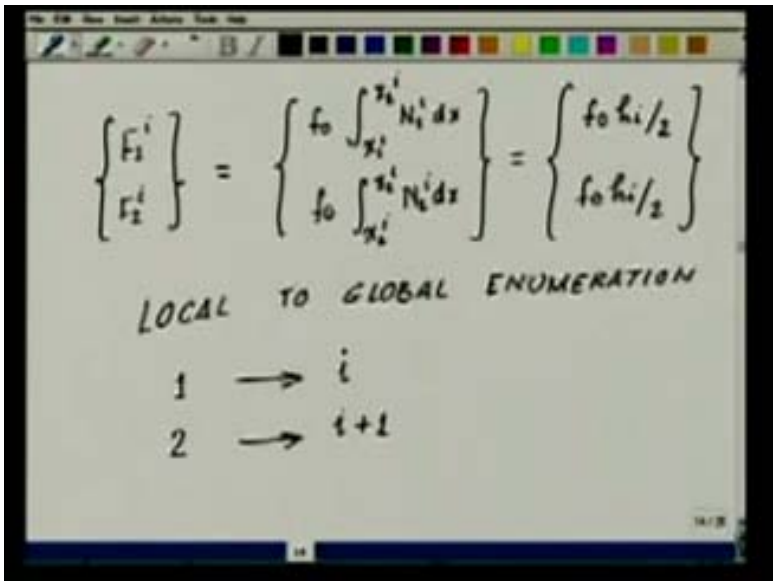

So here we will get $F_1^i F_2^i$ is equal to f_0 into integral over the element and this after doing the integration, it is quite easy to show is equal to f_0 into h_i by 2 and f_0 into h_i by 2. Now we have obtained all the entries of the element stiffness matrix and the element load vector. The question is now which of the global equation it is going to go to? So imagine that I started from this element calculation. That is, I define the element shape functions, found the element stiffness matrix, and element load vector; I would like to know which of the global entries they are to be added to. For that we need so-called local to global enumeration for numbering and which we have already stated that the local 1 for the element maps to what? Globally it maps to I; local 2 of element maps to global i plus 1. So the job is very simple. We know which of these global entries I have to add these local entries to. What do we do in terms of, remember our job is to form the global stiffness matrix K and the global load vector f.

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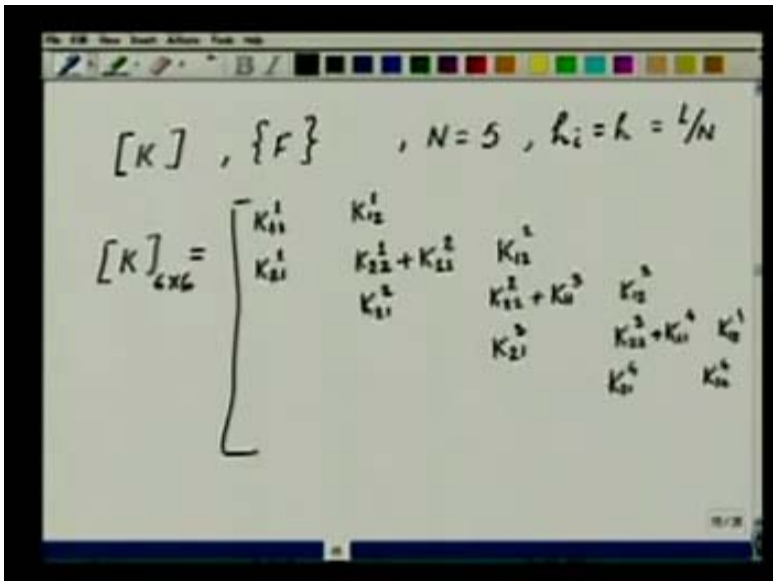

What we have done is, we have done the integrations over these elements and now we have to add them together to form the global entries. Tell me for the 5 elements mesh, let us say N is equal to 5 and also let us assume that all the elements of the same size that is the uniform mesh that is h_i is equal to h is equal to l by N. Let us take this particular case. So for this particular case for the choice of the EA and f that we have taken can we form the global stiffness matrix? So what do we do? We first make all the entries so the global stiffness matrix will be size of 6 by 6 or N plus 1 by N plus 1. So we are first going to make all the entries of the global stiffness matrix 0. This process is call initialization. That is the set the values to zero. Then I start looping over the elements. See I am talking in terms of the computer program. So I start looping over the elements and adding the stiffness entries and the load vector entries from the element to this global K and global f. I am also going to initialize all the entries of f to 0. So let us do that. Here I have the big K matrix, so from the first element the first and the second entries of the element go where they have to go to the first row and the second row and the first and second column because for the element 1 the global phi_i's which are non zero or the phi_1 and phi_2. Here will come K_{11}^{1},here will come K_{12}^{1}. This is all that these elements contribute to the first row that is the first equation.

Similarly, in the second equation it will have K_{21}^{1},K_{22}^{1}. So the element 1 entries go to the first row first column and second column - second row first column and second column. Let us go to the element 2. Where will it add up? Element 2 if you see the phi_2 and phi_3 are non zero in element 2. So where will the K_{11}^{2} add to? It is going to add to this K_{11}^{2}. Similarly, you will have K_{12}^{2} here K_{21}^{2}, K_{22}^{2}. Then I go

further look at element 3, for element 3 phi_3 and phi_4 are non zero, so it only contributes to the third and fourth rows and the third and fourth columns. So third row and third column will have K_{11}^3 and similarly, here I will have $K_{12}^3$and then here I will have in the 4th row I will have K_{21}^3 and here I will have K_{22}^3

This way I can keep on adding. Then I add K_{11}^4, K_{12}^4, K_{21}^4 and K_{22}^4 and so on. If I now write this 6 by 6 matrix after doing all this, the element, the global matrix K becomes equal to EA by h into what do we get?

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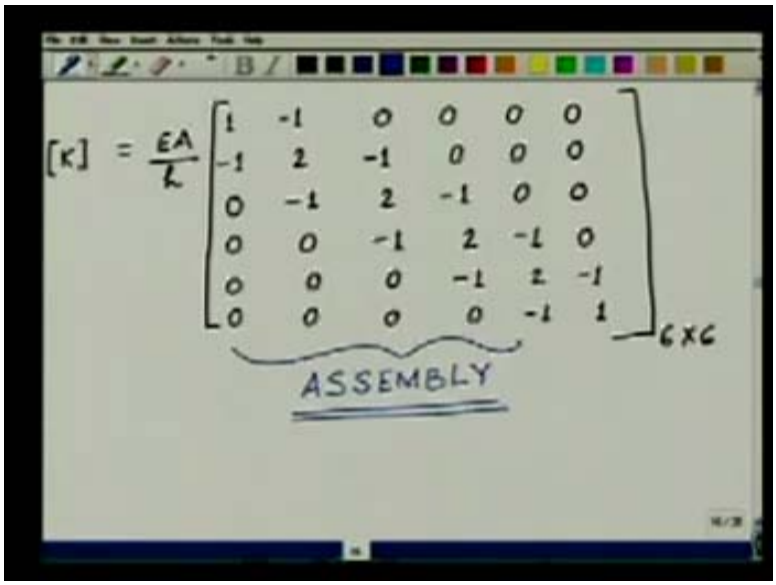

Here I will have the first entry will be K_{11}^1 which is 1. Next one will be -1. Then what else? Is there anything else which is going to be there? The answer is no. All other entries 3, 4, 5, 6, will be 0.

Similarly, in the next row I will get, -1 here by substituting the K_{ij}'s from the elements. Here I will get 1 plus 1 is 2, here I will get -1 and everything else is going to be zero. Here the first one in the third equation you see that phi_3 and phi_1 are… there is no point in the domain where they both are non zero. That is phi_3 is 0 in element 1 and phi_1 is 0 in all other elements. So by the token, the first entry will be 0 and as we have done it in the previous page also that is what it comes out to be. Second entry will be -1 third will be -2 -1 0 0. Continue the process, 0 0 -1 2 -1 0 0 0 0 -1 2 -1 and the final equation 0 0 0 -1. So

this is the global stiffness matrix that we were after and you see very nicely by doing the computation at the element level and adding them up in the global matrix I form the global matrix.

This whole process of doing the computation at the element level and then adding up these entries in the global matrix at the right location in the global matrix using the local to global enumeration is called Assembly. In the computer program you can see very clearly essentially you have to write it to loop. You will start looping over all the elements get the element entries add them up in the write location in the global matrix and that is the whole assembly procedure. I have assembled the global K matrix; let us now assemble the global F matrix.

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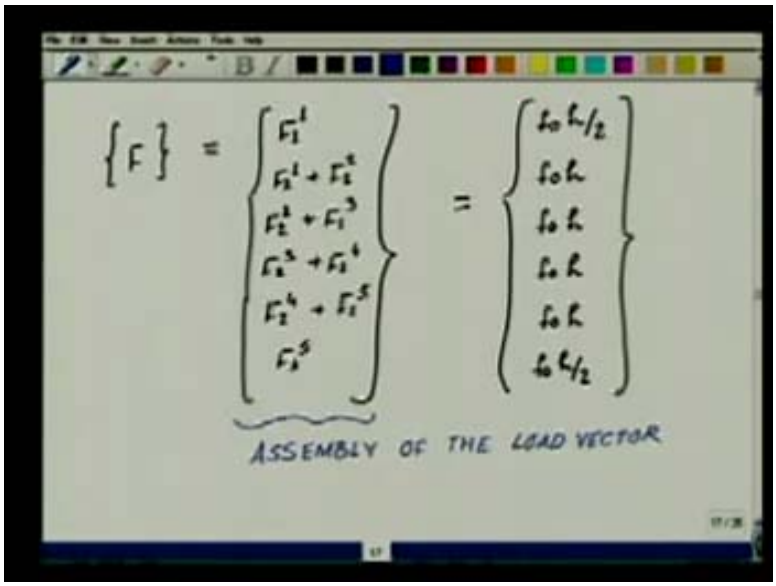

So the global F will be equal to what? You see the phi_1 will only be the non-zero in element 1. So it will be F_1 of element 1, here phi_2, so F_2^1plus $F_1^{2,}$ here I will get, F_2^2plus F_1^3, F_2^3plus F_1^4. Let me erase this part then F_2^4plus F_1^5 and then I will get, F_1^5. This is my global assembly of the load vector and we have already done computations at the element level, so it is very easy to do it here. What do we get? F_1^i is equal to what do? We have f_0 into to h by 2. Here I will have f_0h by 2 plus f_0h by 2 because all elements are the same size this becomes f_0h. The third one becomes again f_0h, fourth one becomes f_0h, fifth becomes f_0h and sixth one becomes f_0h plus 2, because it is only getting contribution from one element. So this is nothing but the assembly of the load vector. Now you should see something that I have

deliberately not touched the boundary condition here. How do we now go and apply the boundary conditions for the problem that we have?

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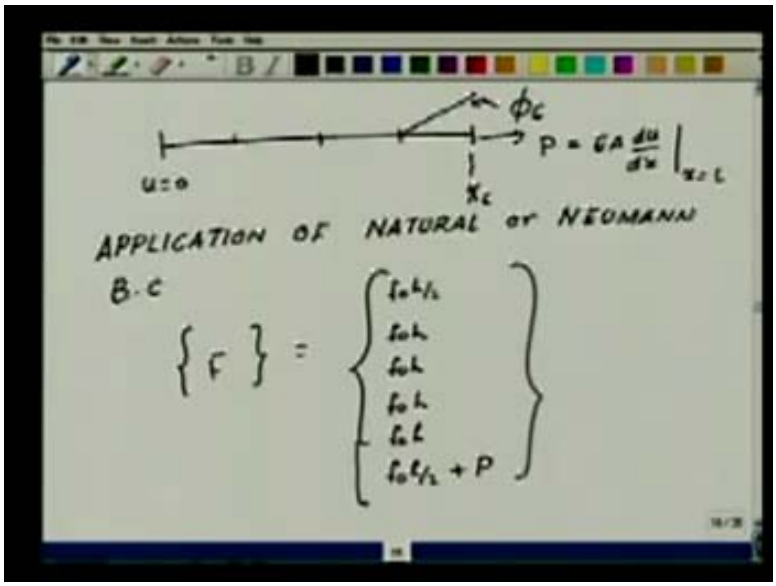

So again let us take the particular problem that u is equal to 0 at x is equal to 0 and here P is equal to EA du by dx at x is equal to L. So now you see that first thing that will do is will apply the natural or Neumann boundary conditions. How do we apply it?

So you see here, that this also corresponds to global node 6 and remember that if I have all these things only phi_6 is non zero at the point x_6. So if you remember back what we have done, we had to add p phi_i evaluated at x equal to six only for phi_6 we will get that expression equal to P into 1, because phi_6 is equal to 1 for all other phi_i's this expression is going to be 0. What do we do? We find out, see in any problem you may have load condition applied at any of the two ends. So first of all we find out which end this load condition is applied at or at which node. We may have concentrated load at this center and as we have said that where ever the concentrated load is applied that has to be node. Which node we have? Here we are going to modify F to be f_0 h by 2, f_0 h, f_0 h, f_0 h, f_0 h and the last 1 f_0 h by 2 plus I will have P. Here, what we do is this Neumann boundary condition applied by simply adding this term P to the corresponding a row of the load vector. So this is how the Neumann boundary condition is applied. In the next class, next lecture we are going to talk how to apply the Dirichlet boundary condition.

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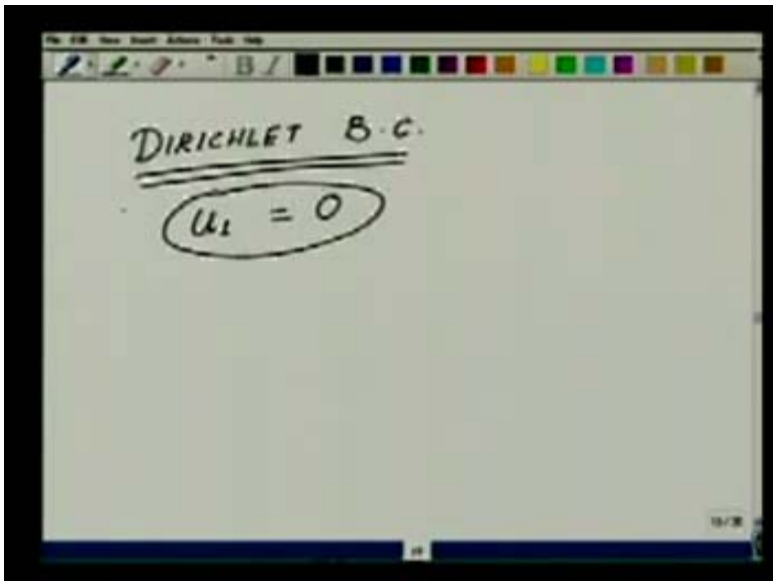

This is little bit more involved and see we have not yet get applied the problem, we know that out of the Dirichlet condition u_1 should have been 0 so how do we impose u_1 is equal to 0. So once we have done this we have essentially given the outline of how to go about doing a finite element computation. I have given you hints as to how we should progress with the programming of all this in a computer program. It does not matter which language, but the various blocks or the building blocks or the subroutines or the functions that will be involved in the computer program should come out of this kind of a discussion. Later on we will see that how does this help. That is if I have to improve the accuracy of the solution what do I have to do? How can I do easily and how can I increase the order of approximation which we are not touched? It touched here to get a better approximation solution.