Finite Element Method

Prof. C. S. Upadhyay

Department of Mathematics, IIT Kanpur

Module- 11 Lecture - 1

In the previous lecture, we have talked about the classical plate theory.

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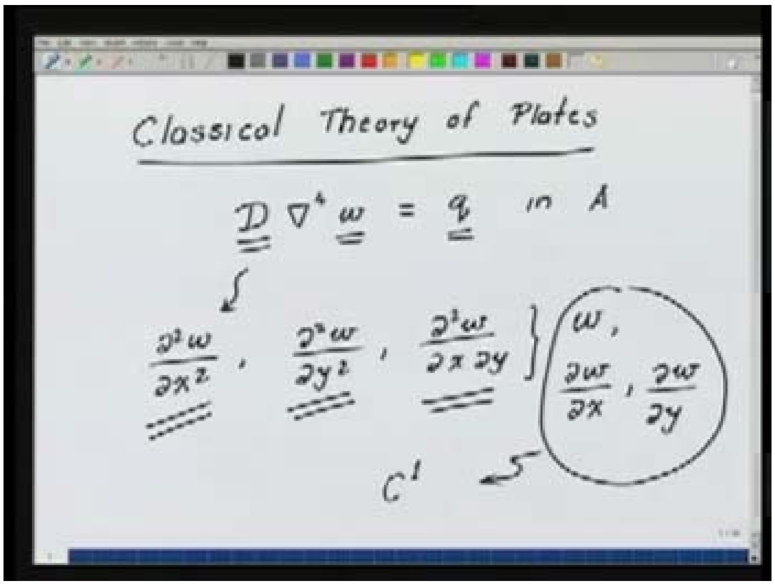

As an example of a problem which leads to a fourth order partial differential equation which we had said that if we work things out it will be this is in the area A where w as the transverse deflection, D is the flexure rigidity constant and q is the applied distributed load on the top or bottom surfaces of the body. This led to a variational form which was in terms of the second derivatives of w. Since, it is in terms of variational form, in terms of the second derivatives of w, we require these derivatives of w to be defined. We need continuity of w del w del x and del w del y. So, this is what we had required as far as continuity and we said this would lead to just analogous to what happened in the Euler Bernoulli beam theory in the 1D case to C one continuity requirement in two dimensions. We had said that we will look at certain ways of constructing these C one continuous elements.

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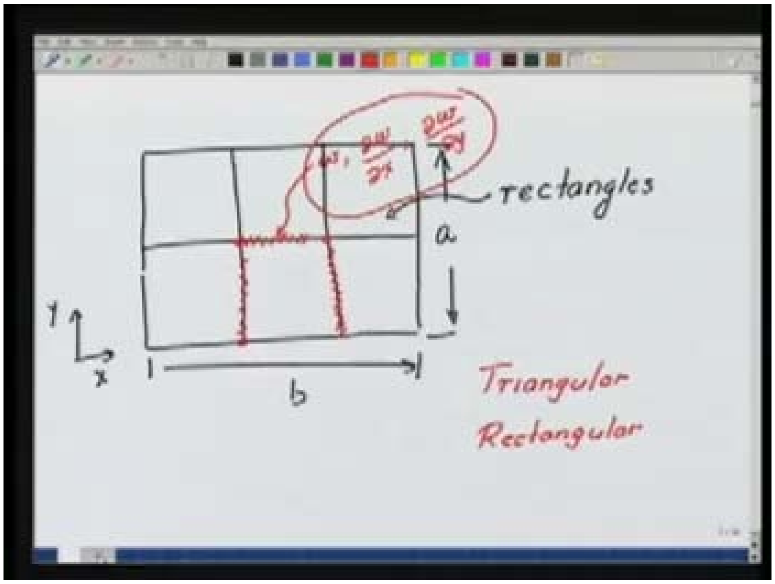

One way of doing it is let us say I have a rectangular domain, I will take only a few elements; so, this is size a, this is size b, let this be our x and y direction, so these are all rectangles. So what we want is at the interfaces of these rectangles with each other including these end points, I should have continuity of del w del x, w and del w del y. If I can construct basis functions which can satisfy continuity of these quantities then we are in good shape. There were several attempts to make such elements and various versions of triangular, here we are not considering triangular; triangular and rectangular elements have been reported in the literature, various types of such elements. The major problem here is to satisfy these constraints exactly in the general case.

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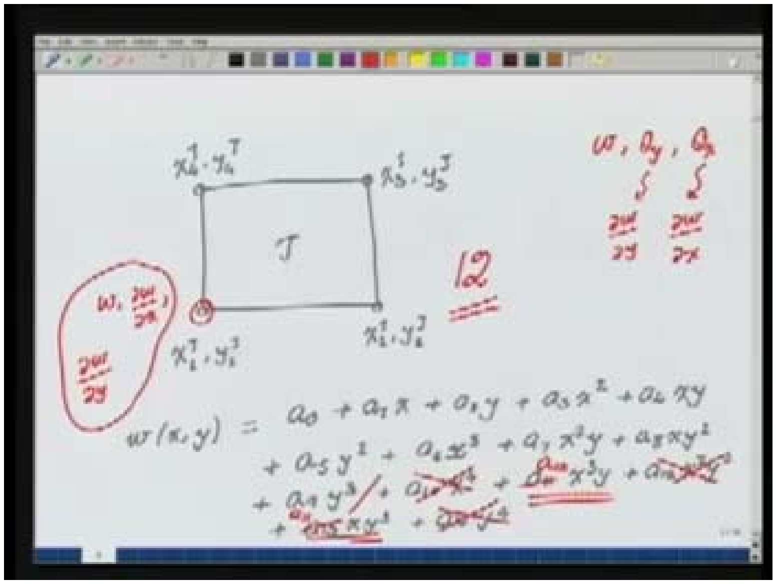

Let us take the simple case and look at an example that let us say this is my element, rectangular element. This is my rectangular element with the nodes x_1 tau, y_1 tau, here x_2 y_2 and here is x_4 tau and y_4 tau (Refer Slide Time: 04:42). What is the basic idea in constructing such an element? We say that w as a function of x and y is equal to, a_0 plus a_1x plus a_2y plus a_3x square plus $a_4$xy plus a_5y square plus a_6x cube plus a_7x square y plus $a_8$xy square plus a_9y cube plus a_{10}x to the power of 4 plus a_{11}x cube y plus a_{12}x square y square plus a_{13}xy cube plus a_{14}y to the power of 4. Why did I take this whole polynomial? Because if I would like to have continuity of w, del w del x, del w del y at these 4 points of these 4 nodes of the element. I will have to have w, del w del x, del w del y as the variables at each of these nodes.

We will call them w, this is essentially theta y, theta y is the rotation about the y axis which is given by del w del x and theta x is rotation about the x axis. I am not bothered, I am not taking here the positive or negative signs that one can find out; this is equal to del w del x. This is del w del y and this is del w del x. So these 3 quantities are specified at the each of these nodes. There are 12 quantities which are specified at all the 4 nodes combined for this element. I need to have a polynomial which has at least 12 coefficients. But, here we said if we look up to the cubic in 2D, it has 10 coefficients - unknown coefficients. We have to do more than that; so, we go to the fourth order. When we go to the full fourth order there are total of 15 unknowns. We need only 12 coefficients, so what do we do? Out of the fourth order we retain only the so called symmetric terms, this term and this term (Refer Slide Time: 07:43). While, we drop off from the fourth order polynomial these terms. What we are left with is the whole cubic plus now I will rename this, I will call it a_{10} here and I will call it as a_{11}. This is the polynomial that we are going to fit to these values.

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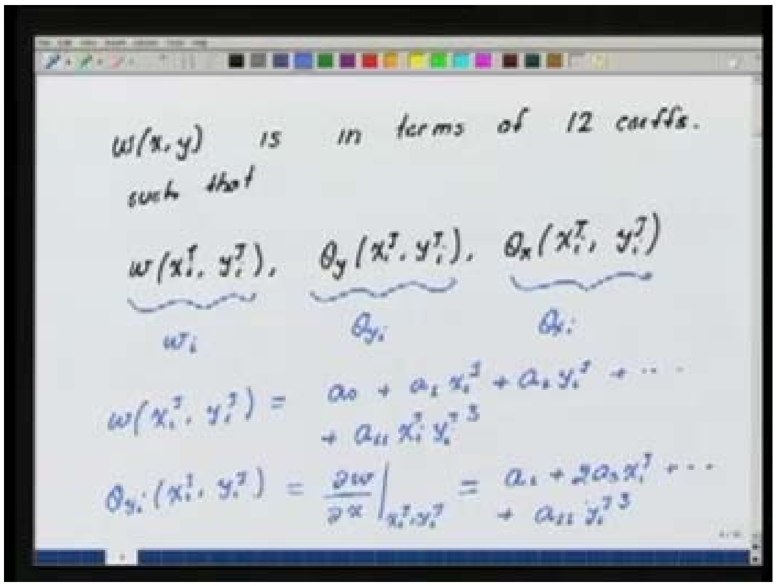

But how do we do it? Again the idea is simple that our function w (x, y) in the element is in terms of 12 coefficients such that the value of this function w at the nodes x_i tau, y_i tau then theta_y at the node x_i tau, y_i tau, theta_x at the node x_i tau, y_i tau which we are going to call by a new name; this I am going to call as w_i, this is $\text{theta}_{yi,}$ this is theta_{xi}. The value of this function equals the coefficients at these nodes. So we have w at x_i tau, y_i tau is equal to a_0 plus a_1x_i tau plus a_2y_i tau plus up to $a_{11}x_i$ tau y_i tau cube. Similarly, theta_{yi} at x_i tau, y_i tau is equal to del w del x at the node x_i tau y_i tau. This would be equal to a_1 plus, a_2 would not be there, $2a_3x_i$ tau plus up to $a_{11}y_i$ tau cube.

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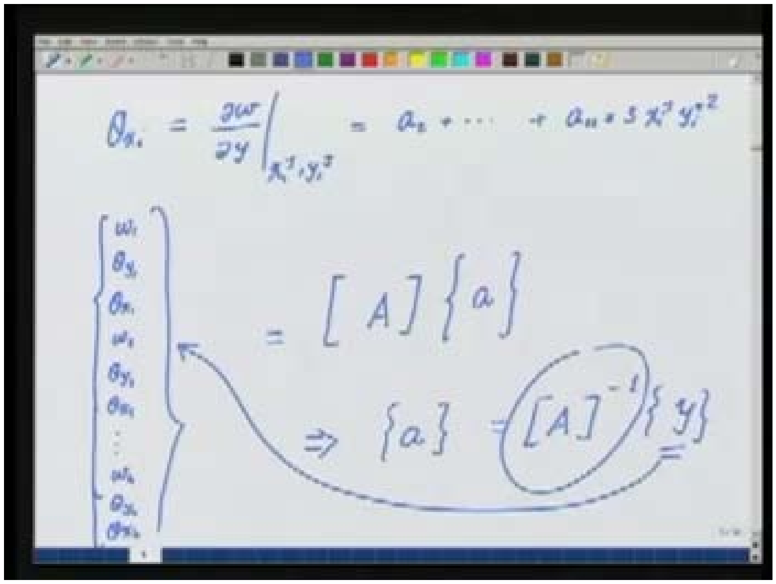

Similarly, we can give theta$_{xi}$ is equal to del w del y at the node x_i tau, y_i tau; so this would be equal to a_2 plus up to a_{11} into $3x_i$ tau y_i tau squared. We can write the theta$_{xi}$, theta$_{yi}$, and w_i at each of these 4 nodes which have coordinates x_i tau, y_i tau in this expanded form. How many values we have? We have 12 values; it is a twelfth order, there are terms in this expression. We can write it as, this whole thing w_i, w_1 theta$_{y1}$, theta$_{x1}$, then w_2, theta$_{y2}$, theta$_{x2}$ up to all the way down to w_4, theta$_{y4}$, theta$_{x4}$ this is equal to this matrix A into the vector of this coefficients a_0 to a_{11}. What are the entries of this matrix A? It is essentially one x_i, y_i depending on the expression for theta$_{xi}$, theta$_{yi}$ and w_i in terms of the x_i's and y_i's, so then implies we can get a is equal to A inverse into y where this y is this vector. I can find the coefficients of this polynomial expression for w in terms of the values of w, theta$_x$, and theta$_y$ and the nodes and all we can do is essentially, define the finite element solution in terms of these nodal values of w theta$_{xi}$ and theta$_{yi}$. This expression can be easily obtained once it is obtained then this is known; now we go ahead and do the same job for all the elements and we have the representation of the solution in the element and also globally.

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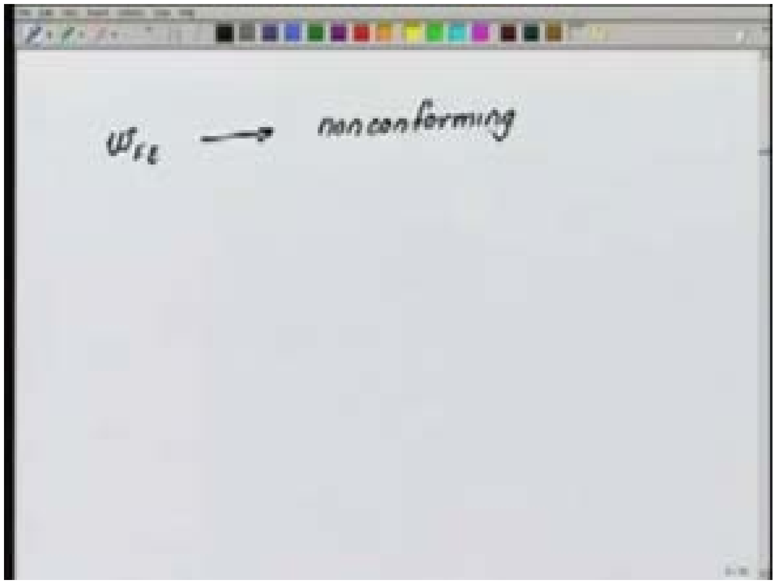

This way we can construct w_{FE}. What will happen is w_{FE} is nonconforming in this case.

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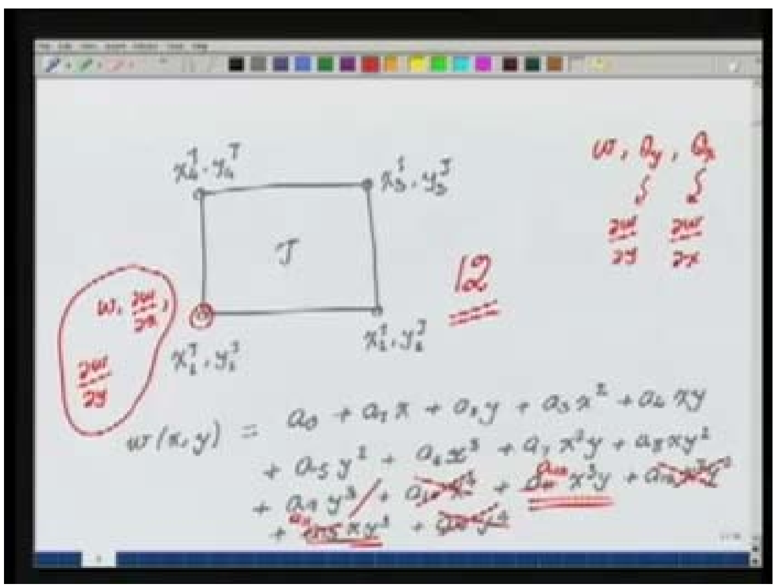

What do we mean by nonconforming is that, it does not strictly satisfy the constraint of continuity of value and derivative on each of this faces on each of this edges of the element from this side and from this side (Refer Slide Time: 14:00). The two elements sharing this edge will have on this edge it can be shown continuity of w and the tangential derivative of w. Not the normal derivative, the normal derivative here from the two sides is not continuous. For example, on this face on this edge del w del y will not be continuous but del w del x will be and w will be. This is the problem because, we required a weak solution to have a minimum continuity condition of all these derivatives and w but we are violating it, because of this violation finite element solution suffers. In many cases we get pretty decent solutions and this can be used. Can we improve it? Yes we can improve it and where does the improvement come from?

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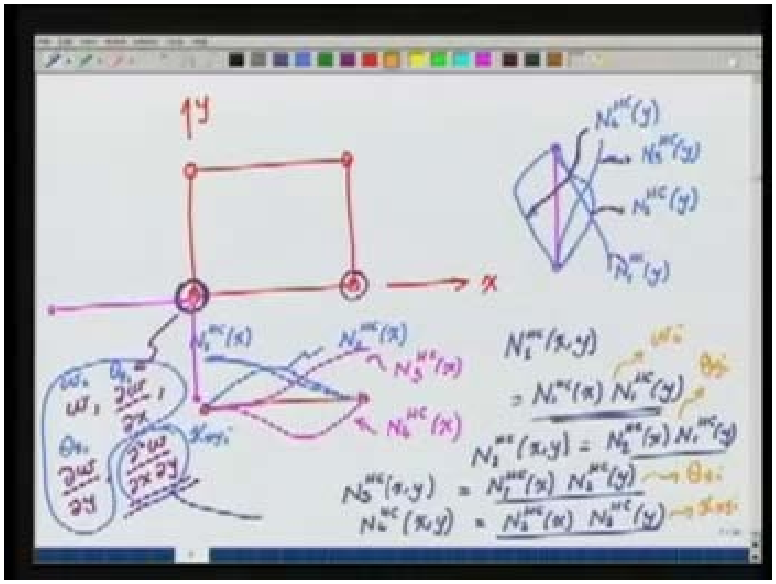

This is the rectangular element, so here this essentially the x direction, this is the y direction of the element. In this direction in the x direction, what if I took this hermite cubic function as we had created in the case of the beam analysis? If I take this one this is what we had as N_1 hermite cubic as a function of x. This one (Refer Slide Time: 16:20) would be N_2 hermite cubic as a function of x. Similarly, I can create the other hermite cubics; this will be N_3 and N_4, this will be N_3 hermite cubic as a function of x and this will be N_4 hermite cubic as a function of x.

What does the hermite cubic do? If I have another element here and I have the same hermite cubic here also same definition, it ensures continuity of the derivative also in this direction which is essentially the x derivative del w del x. Similarly, I will define now the hermite cubic with respect to y in this direction. so the hermite cubic with respect to y would be this function (Refer Slide Time: 17:20). These functions, so I will call this N_1 hermite cubic with respect to y, this is N_2 hermite cubic with respect to y, this is N_3 hermite cubic with respect to y and this N_4 hermite cubic with respect to y. These are our hermite cubics, so this is N_4, N_3, N_2, and this is N1 so we have this hermite cubic in the x and the y direction so in tensor product family, for the quadrilaterals same thing we can do here. How many functions we can create now by taking products of these hermite cubics in the x and y direction? We will create 16 functions because they are 4 in the x direction. There are 4 in the y direction and these functions if we remember, will now be given in terms of nodal values of some quantities.

It turns out that it will be in terms of nodal values of w del w del x, del w del y and del 2w del x del y. These 4 nodal values in terms of these 4 nodal values I can define the hermite cubic. One thing has happened; this also enforces an extra constraint that we are asking for this cross derivative of w that del two w del x del y to be also continuous in this case. Nevertheless our basic requirement is satisfied of this w del w del x, del w del y being continuous; so how can I construct this hermite cubics? At every node we have w_i this we had said theta y_i, this was $theta_{xi}$ and this will call it has kappa x y_i. Essentially, we will get the values the representation of w in terms of these 4 values at the node that is total of 16 values.

It is very easy to construct this functions, so I will call my N_1 hermite cubic as a function of x and y will be equal to the first the hermite cubic corresponding to this line and this node it is N_1 hermite cubic due to x and hermite cubic which is one here, value one corresponding to this line so it will be N_1 hermite cubic y. Similarly, N_2 hermite cubic xy will be equal to, let us say I will take the same value one here and the derivative one here. It will be N_1 hermite cubic in this direction x. Let me take derivative in this direction, value in this direction so I will take N_2 hermite cubic with x, N_1 hermite cubic with y similarly, N_3 hermite cubic with respect to x y is equal to I will have N_2 N_1 hermite cubic with respect to x and N_2 hermite cubic with respect to y. N_4 hermite cubic with respect to xy becomes N_2 hermite cubic with respect to x and N_2 hermite cubic with respect to y. We have essentially the 2 hermite cubic in each of these directions which are non zero which have some meaning here which are non zero at this point. Either in the value or in the derivative with respect to those hermite cubics, products of those hermite cubics we construct the 4 functions here. N_2x into N_2y will correspond to this cross term that is product of the 2 derivatives. While, the value here will correspond to N_{1h} hermite cubic will correspond to the value w_i and the others will correspond to the slopes in one direction or the other direction.

I can say that this will correspond to w_i, this will correspond to $theta_{yi}$, this will correspond to $theta_{xi}$, this will correspond to kappa x y_i. At every node so I can define this hermite cubics with respect to the non zero hermite cubics . I will have the N_1, N_2, N_3, N_4 here then I will have N_5, N_6, N_7, N_8 in terms of the hermite cubic which are active here and so on. This way I can create all the 16 hermite cubics and now I have so called conforming approximation over the rectangular elements. Whether, for this kind of a domain if I make a mesh of quadrilaterals, I make the mesh of the quadrilaterals something I will do here. In this mesh of quadrilaterals whether, this hermite cubics do satisfy the continuity of x and y derivative on these faces. If they do then they are actually conforming in all cases. If they do not, then they are not but for the rectangular domain they will satisfy conforming condition; we check here this is an assignment problem which you can work out and check.

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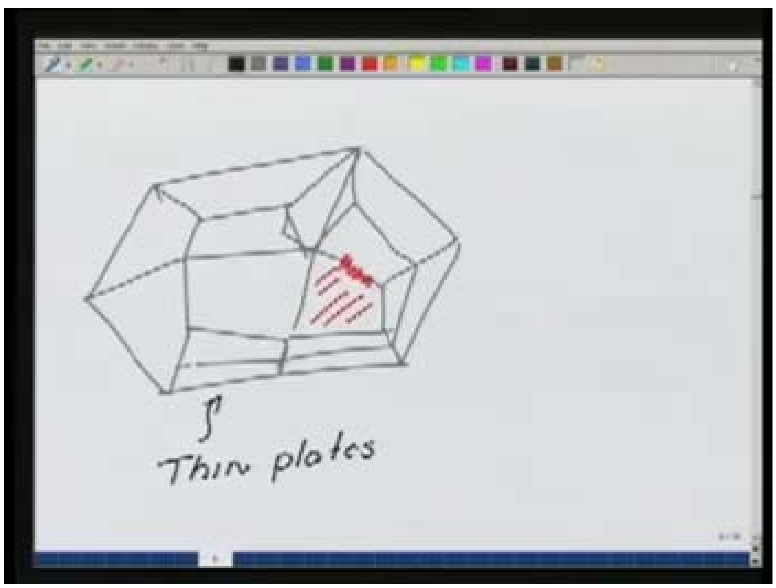

Plate theory: since we are in the topic of plate theory is that, we have dealt with now the Kirchhoff plate theory was for very thin plates where the shear effects could be neglected.

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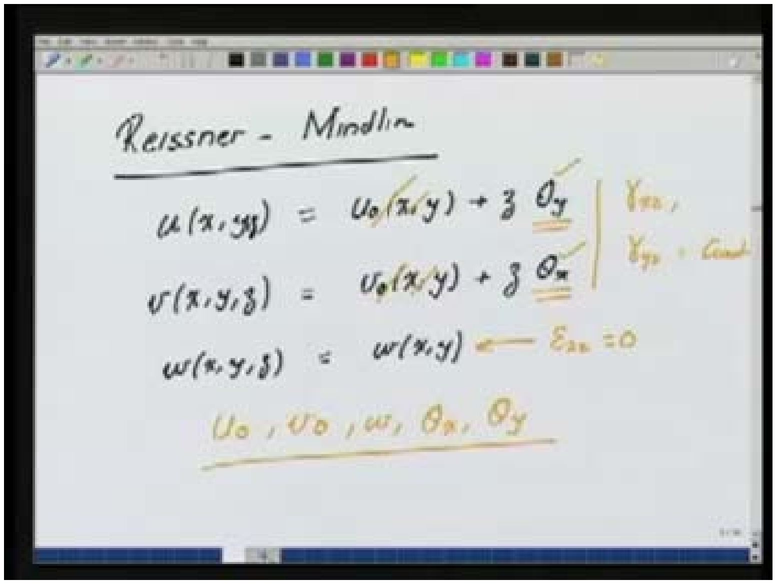

Let us go little further, so we can have a plate theory which is called Reissner-Mindlin plate theory which is an improvement of over the plate theory that we have discussed which led to the fourth order differential equation. The idea behind this plate theory is this kind of a representation of the displacement field in terms of the transverse coordinate. Here I will have this has theta y, v (x, y) should be a function of z also, z is equal to v_0(x y) plus z theta x and w (x, y, z) remains w (x, y). What this theory does? It assumes that e_{zz} is still equal to 0. This means gamma xz, gamma yz are constant While, from our standard strength of materials we know the gamma xz and gamma yz is actually parabolic across the in the in terms of z. Nevertheless, this is an approximation which is made. I am not going to dealt too much into this plate theory as such but let us bring out an important aspect of it.

What happened in the case of Kirchhoff plate theory that we said these things were 0? Because these things were 0, this theta y and theta xx came out as minus del w del x and del w del x and del w del y. These are essentially free or independent variables so in this problem we solve for u_0, where is u_0, v_0 coming from, if I have in plane loading? So u_0 v_0, if I do not have in plane loading then we can ignore these two parts where z is with respect to again the centre line of the plate. So u_0, v_0, w, theta x and theta y these are the 5 unknown functions that have to be solved for in order to construct the full solution.

This is one feature that we have further Reissner-Mindlin plate theory as compared to the Kirchhoff's theory where we only needed to solve for u_0 v_0 and w_3 functions.

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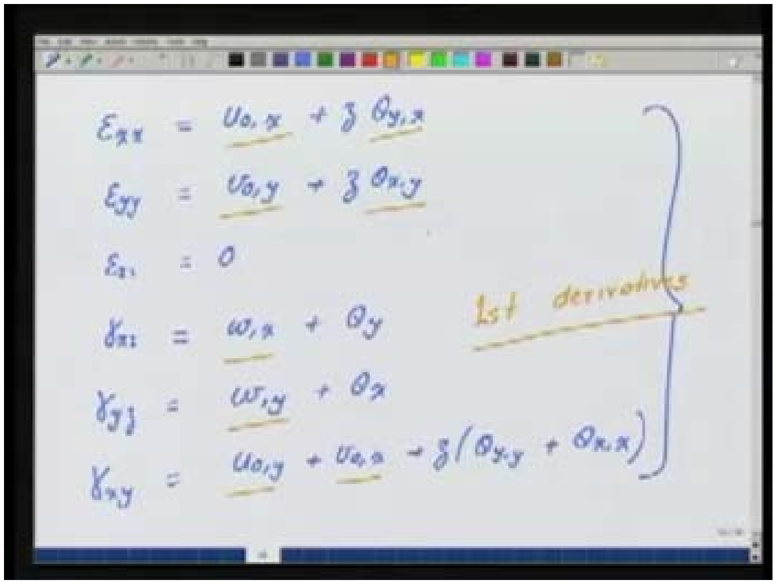

If I do this, then let us see now we should able to tell me that e_{xx} is actually equal to u_0 comma x plus z $\text{theta}_{y,x}$, e_{yy} is equal to $v_{0,}$y plus z $\text{theta}_{x,y}$, e_{zz} is equal to 0 gamma_{xz} is equal to w comma x plus theta_y gamma_{yz} is equal to $w_{,y}$ plus theta_x and gamma_{xy} is equal to u_0 comma y plus $v_{0,x}$ plus z theta y comma y plus $\text{theta}_{x,\, x}$. This is the state of strain at any point given in terms of u_0 v_0 theta_y and theta_x and w. In this strain terms the first derivatives of w u_0 v_0 and theta_x theta_y are sitting. Only first derivatives, so if I now write the strain energy that is I write the stress in terms of the strain so the stress will also are in terms of the first derivatives of all these quantities. The product stress into the strain sigma xx into e_{xx} will be in terms of the first derivatives of u_0 v_0 and so on.

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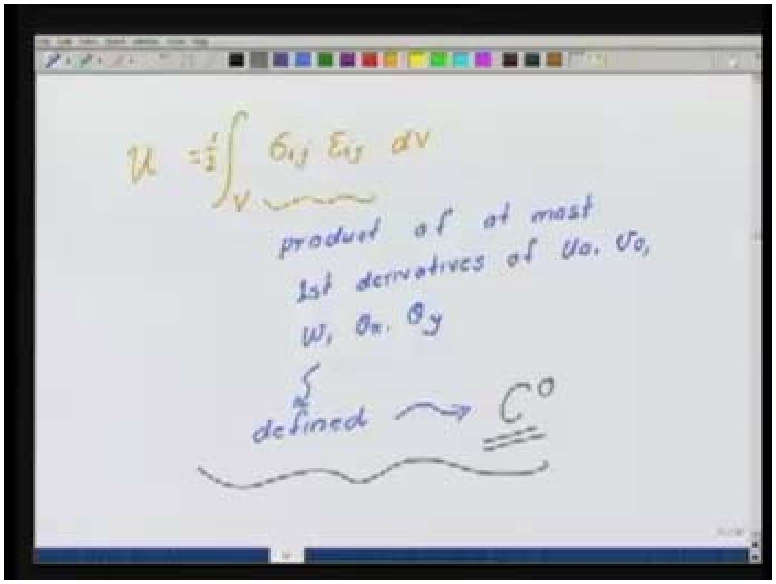

The strain energy definition as we had written strain energy is integral over the volume of half of sigma ij e $_{ij}$ dv, ij going from 1 to 3. This expression would be product of at most first derivatives of u_0 v_0 w $theta_x$ and $theta_y$. It is quadratic in terms of the first derivatives of all these unknown functions. If I look at the strain energy for the strain energy to be finite we only want these to be defined. We only want this first derivative of u_0 v_0 with respect to x and y $theta_x$, $theta_y$ and w all with respect to x and y to be defined which means that here if I want to now construct a basis function to do the approximation we need to use only C zero. I can use the standard C zero elements that we had talked about in a very detailed way earlier when we started the 2 dimension problems. Those elements those basis functions like we had the linear quadratic, cubic, triangles the tensor product, quadrilaterals, the serendipity quadrilaterals all those things can be used for the approximation of u_0 v_0 $theta_x$ $theta_y$ and w in terms of x and y. This problem can be solved very easily using the machinery of the shape function generation and integration and all those things that we had developed earlier. This way we can essentially use the tools available to us in a judicious manner. This model it is not to say that this model will do better than the Kirchhoff's plate model or it will do verse. This model has its inherent problems which can be corrected to take care of things. This one should read the books and enough of material is available.

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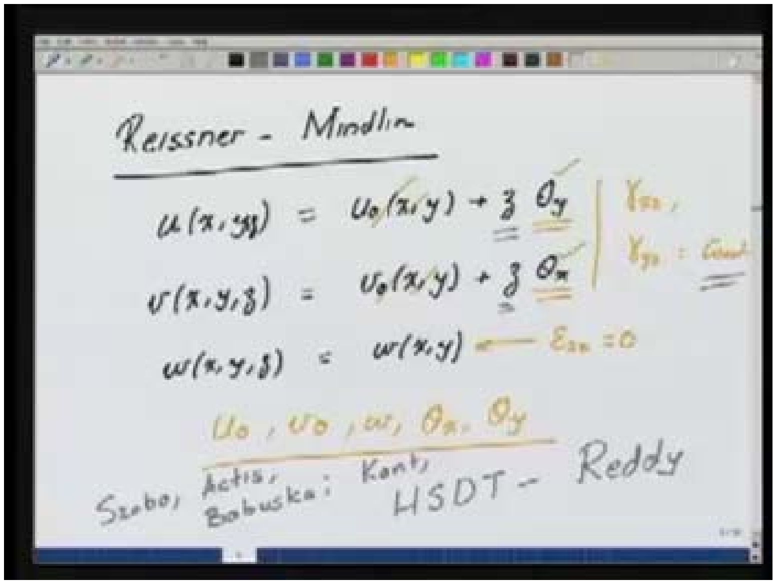

There are also higher order plate models which go beyond this expression of u in terms of z theta y z theta x and we add more terms in terms of higher powers of z. We would have z square, we will have some other quantity which is a function of x and y plus here also z square more. This way we can construct so called higher order shear deformable theories. In those theories shear stress or strain will be non zero and the higher ones will also give a non constant shear stress. We will use sufficient number of terms to get a parabolic variation of shear which is when we take cubic terms that is celebrated higher order shear deformable theory. What is known as, HSDT due to Reddy and lots of other people have done this. Avery good reference is on this plate theory is papers by g n Reddy papers by Tarun khan from IIT Bombay and from Szobo, Actis and Babushka on a different approach to creating this plate models.

I will not deal with those things in detail. I think we are in a position to deal with all these plate models and the refinements, if we need them and how to handle them? Let us go little ahead with what we are doing? Let us now talk of a more interesting problem in common practice now a day, because computational tools have improved tremendously.

We can get dual processor machines with very high speeds 3GHz -3.4 GHz sitting on a desktop, at a very affordable price even here.

In that case, the capabilities have improved and because of that our desire to do more refines modeling of the physical problems. If we see the beams, the plates, the bars these are all idealizations of a 3D situation, we do 3 dimensional analysis.

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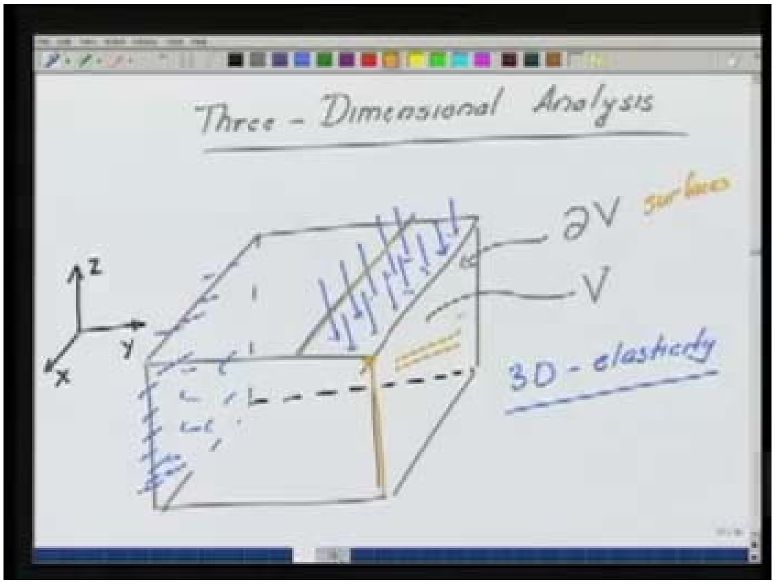

I will briefly touch upon 3 dimensional analysis and how it is done? The language the procedures are direct analogous extension of what we had in the 1 and 2 dimensional cases. Let us say that here we have a 3 dimensional domain, I will take a cubical domain for simplicity let us say the volume of the domain is v and the surface is given as delta v. Over this domain, I would like to find the response of this structure when let us say on this phase, I am going to fix my displacement completely fix this phase and on this phase partially I want to apply a transverse load. This looks like a bending problem; now if it is a long slender member we will see that what we get out of the beam analysis will be closed. If it is flat and both dimensions are similar plate analysis will do a good job but that is for us to see. Here we would like to do an honest 3 dimension analysis with the load is applied here.

How do we solve this problem? First of all we are talking of 3D elasticity as an example of a 3D problem and for 3D elasticity we would like to develop the variational formulation and then suggest how to go and create the basic functions or the shape functions which can be used to solve the problem. Some things that we should keep in mind before we proceed, we see that here I deliberately applied the load in only part of the domain. This we should keep in mind and the boundaries now of this 3 dimensional domain are surfaces that also we should remember, we will have surfaces, surfaces will have edges and the corners. These are phases, these are edges and these are corners.

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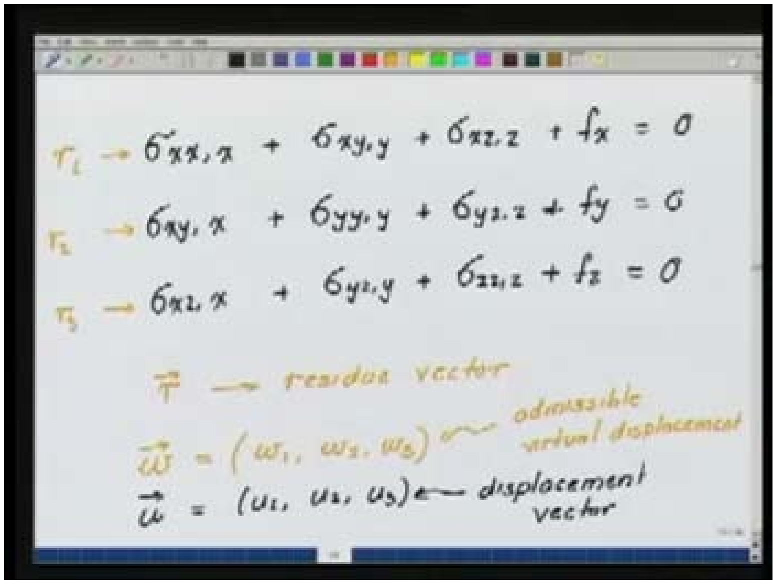

Let us do to write the 3D equation of equilibrium for the theory of elasticity sigma $_{xx,x}$ plus sigma$_{xy,y}$ plus sigma$_{xz,z}$ plus f_x is equal 0. Similarly, I have sigma$_{xy,x}$ plus sigma$_{yy,y}$ plus sigma$_{yz,z}$ plus f_y is equal to 0. Here I am assuming that the stress tensor is symmetric that is sigma$_{xz}$ is equal to sigma$_{yz}$, sigma$_{xy}$ is equal to sigma$_{yx}$ so on and sigma$_{xz,x}$ plus sigma$_{yz,y}$ plus sigma $_{zz,z}$ plus f_z is equal to 0. This quantity I am going to call as r_1, this is r_2, and this is r_3 that is these are components of the residue vector. As we have done in the 2 dimensional case, let us take w with components w_1, w_2, w_3 as an admissible displacement, this is an admissible displacement, virtual displacement. While, we will have the vector u with components u_1, u_2, u_3 when each of these components u_1, u_2, u_3, w_1, w_2, w_3 are functions of x y z; this is the unknown displacement field.

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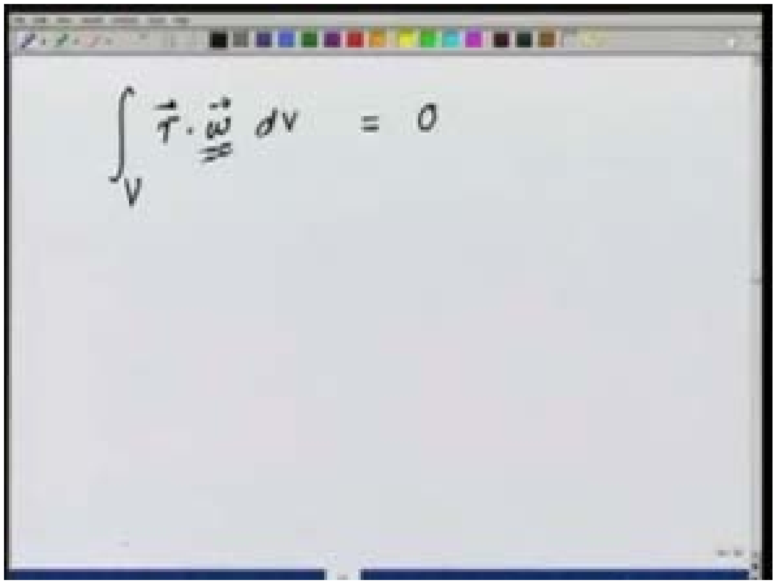

What we had done earlier? We had said that we will take the weighted residual formulation that is we will take the volume take r dotted with w and integrated over the volume because r was zero this will come out to be zero. Against any w is any admissible virtual displacement, admissible means it should satisfy the geometric constraints wherever u is specified. From our, let me jump the gun and say what we want our w to do in the standard situation.

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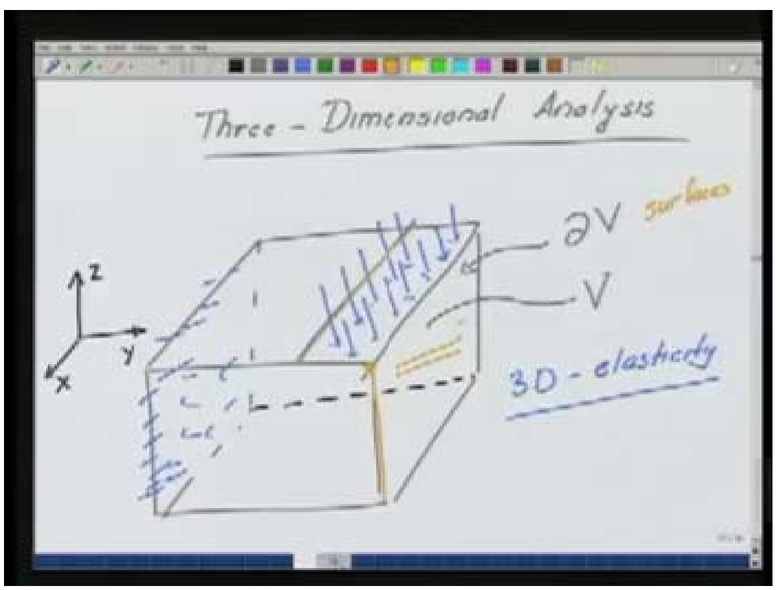

We have not yet done integration by parts; we will do it is the w_2 should satisfy the constraints this one and this one.

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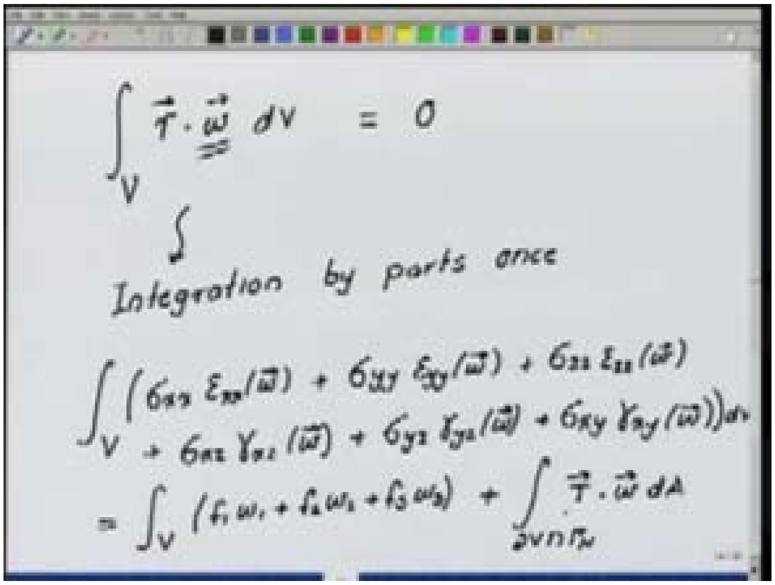

Out of this I am not going to write the long expression, I will do integration by parts once. By doing integration by parts an end of getting this expression sigma $_{xx}$ into e_{xx} due to w plus sigma$_{yy}$ into e_{yy} due to w plus sigma $_{zz}$ into e_{zz} due to w plus sigma $_{xz}$ gamma $_{xz}$ due to w plus sigma $_{yz}$ gamma $_{yz}$ due to w plus sigma $_{xy}$ gamma $_{xy}$ due to w this whole thing dv will be equal to here f_1w_1 plus f_2w_2 plus f_3w_3 plus integral over delta v, I will explain what is this traction vector t dotted with w dA.

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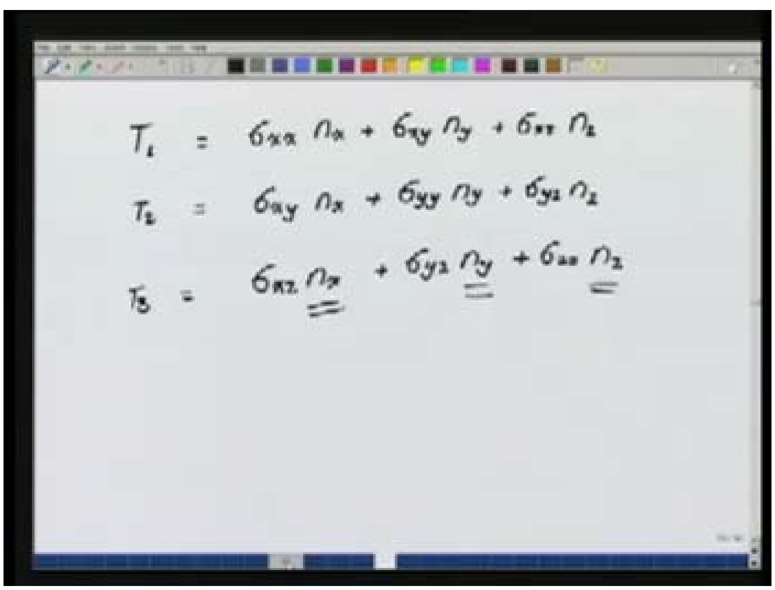

This is that traction t what is traction t given as, if we remember t_1 is equal to sigma $_{xx}$ n_x plus sigma $_{xy}$ n_y plus sigma $_{xz}$ n_z t_2 that is the component of a traction vector in the x direction t_2 is the component of the traction vector in the y direction this will be n_x sigma $_{yy}$ n_y plus sigma $_{yz}$ n_z and t_3 is equal to sigma $_{xz}$ n_x plus sigma $_{yz}$ n_y plus sigma $_{zz}$ n_z this is standard, if we do this integration by parts and use the green divergence gauss, green divergence theorem we will get this in terms of that expression. We will get this, now we see what are these n_x, n_y, n_z these are the components of the unit outward normal on the area; so here lets come back to our problem.

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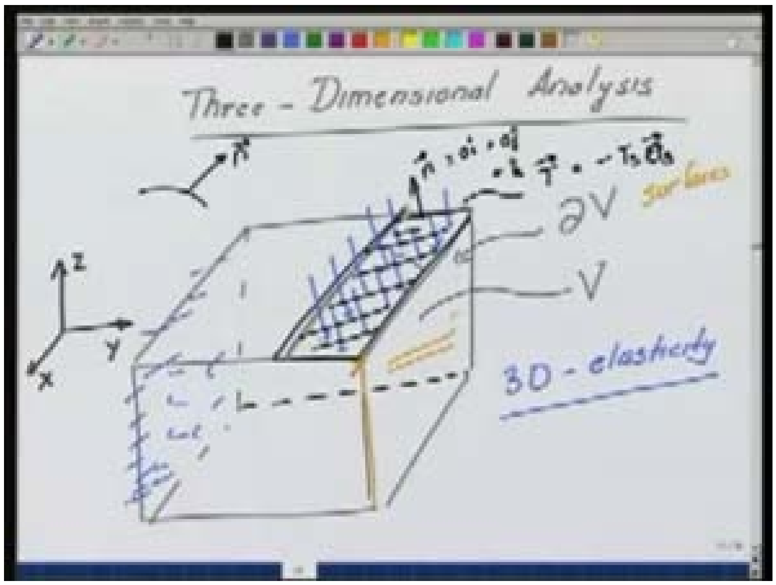

The unit outward normal is the vector n here, in this case since these are all parallel n will be equal to 0_i plus 0_j plus k so it has only the component n_z which is equal to one on this phase. But in the general align phase we will have all the 3 components n_x, n_y and n_z .we will have all the components of n. If we see that where does the t become non zero only on this part of the boundary area and in this case we see that t_1 is 0 because I have drawn these as vertical forces t_2 is 0, here t is equal to vector t is equal to minus t_3 e_3 unit vector in the third direction.

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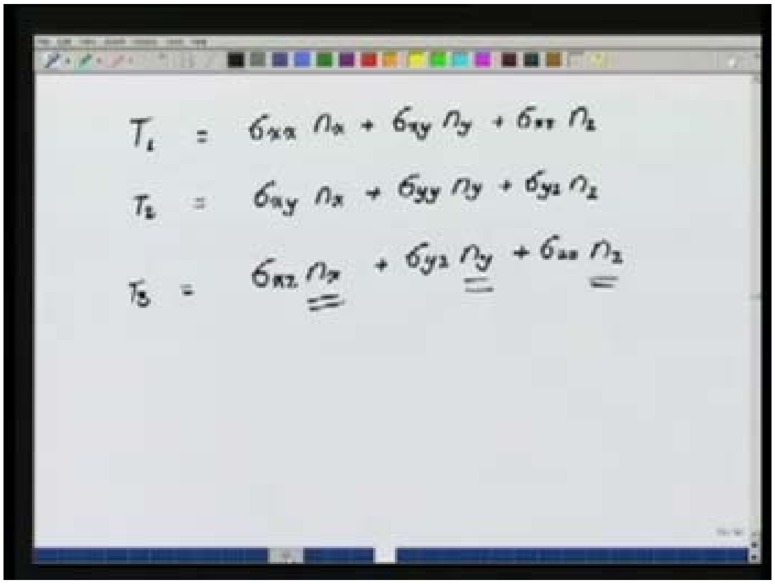

I will put this expression for the t in the expression here and I will essentially go ahead from here now again coming back to what we drawn there.

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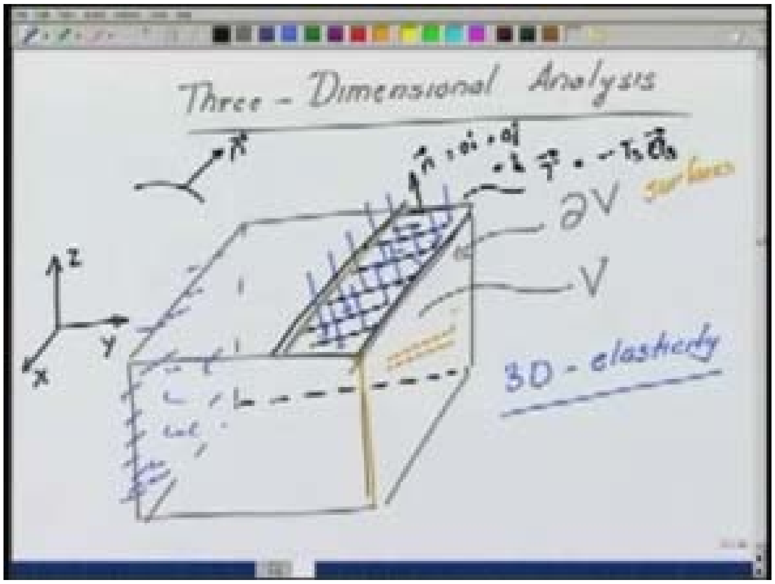

Now I am done the weak formulation the one I had written in the last expression is the weak formulation, the weak formulation tells us that sigma $_{xx}$ due to the u into e_{xx} due the w so on, have to be defined in order for this integral on the left hand side to be finite which means sigma $_{xx}$ is in terms of the first derivative of u and first derivative of v and so on. Similarly, for e_x, e_y and so on is all in terms of the first derivative of u, v and w with respect to x, y, z.

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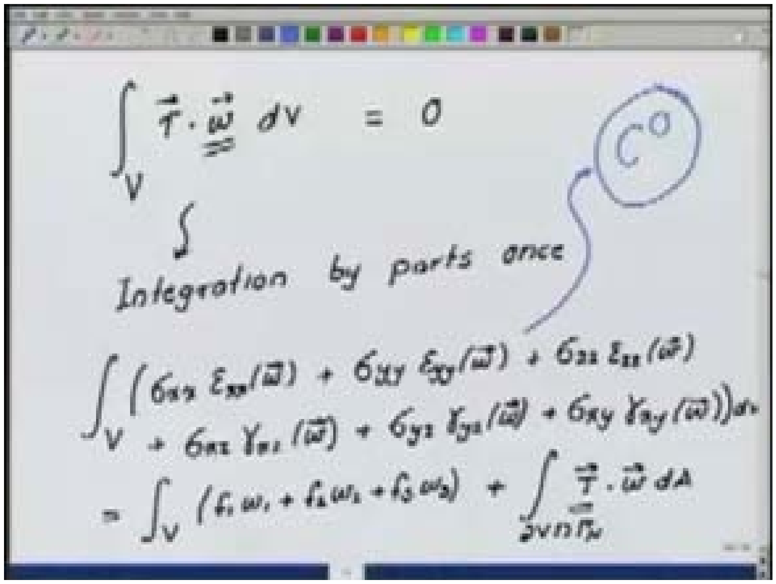

Our weak form now requires only the first derivatives of the vector u and vector v and vector w to be defined which means that in this case, we need with respect to what we have done as far as the approximation is concerned, we need to construct a C zero approximation in three dimensions that is we only want the value of this functions u_1, u_2, u_3 and similarly for the test function w_1, w_2, w_3 which is the virtual displacement to be continuous, derivatives need only to be defined. We now define continuous shape functions first of all, how do we discretize the domain? Let us take our domain of the previous figure and let us make a mesh here.

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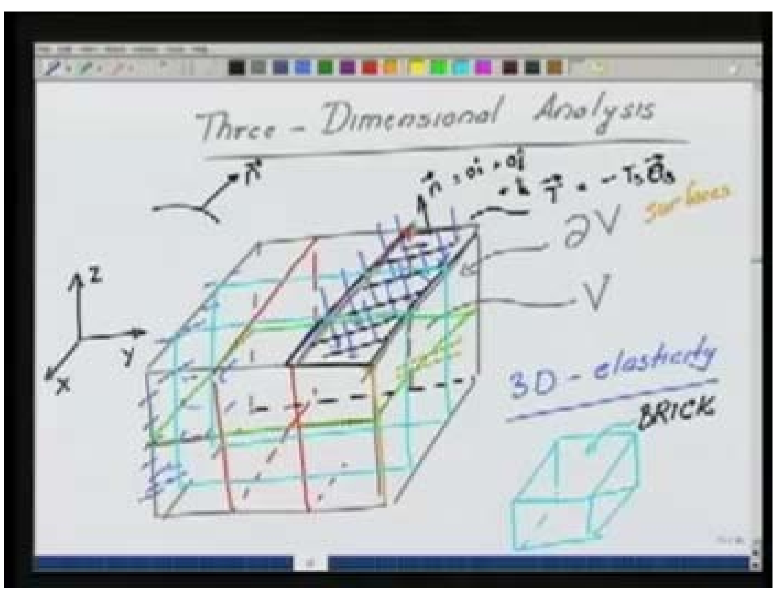

How will I make the mesh? For the mesh I have to honor the boundaries of the load profile boundaries of material differences, so wherever my material has a change that boundary will have the nodes and the edges and the vertices nodes and phases of elements sitting there. What kind of element should I need? In this case, life is a little simpler so I will make this kind of a partition, very simple partition we see here I will make with the different colour so that we can see the effect. Similarly, in the third direction I will make the partitions like this. If we see, what we have done? We have constructed elements which are actually cubical in shape okay elements which are cubical in shape. Well the sides could have different lengths so these are called in the language of finite elements brick elements. There are other types of elements we can make but I am not going to discuss them here.

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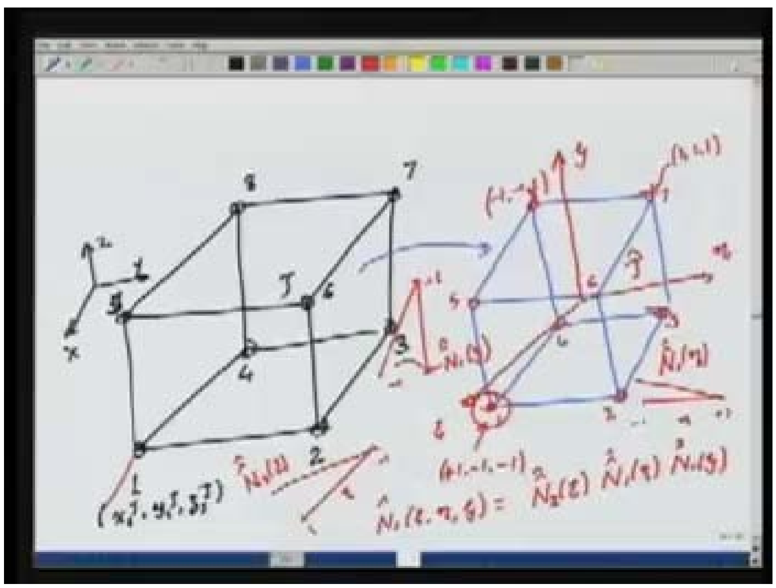

This is the brick element, so let us do a very simple job of constructing a finite element basis functions or here we will concentrate only on the element so the shape functions which are C zero continuous now for the brick element. Let us say co ordinate system is this is x, this is y, this is z, x y and z. I will call this for the element, let us say this is my generic element, this is my generic element tau, this is the node 1 of the element tau node 2, node 3, node 4, node 5, 6, 7 and 8 so this will have co ordinates x_1 tau, y_1 tau and z_1 tau similarly all the other nodes so the mapping now we will take this and I am going to go straight away to the master element this will map to the master element which is the master cube.

We make a master cube like this; this will be my psi direction, this will be my eta direction and this will be my zeta direction such that this point is at psi eta zeta minus 1. This point is at psi zeta eta 1 just like we had the 1D element master element then the 2D master element; now we have the 3D master element. So, this point will have essentially psi your zeta will be 1 psi will be plus1 zeta will be minus1 eta will be minus 1, zeta will be minus1 so this will be plus1. So here my psi will be minus1 zeta will be psi will be minus1, zeta will be minus1, eta will be minus1 and zeta will be plus1. So this way we can construct the 4 nodes. We want to define the basic functions or the shape functions with respect to these master nodes such that they are such that the shape function is defined with respect to this node is 1 here, 0 at all other nodes, 0 at all other master nodes. How do we define? Very simple; we now take tensor of product of the 1D shape functions that we had created C zero shape functions in the direction. N_1 here, N_1 hat as a function of psi eta zeta will be equal to N_2 with respect to psi N_1 double hat with respect to eta N_1 double hat respect to zeta and that is all so I will take the tense of product as if this is my psi direction minus minus1 to plus1, this is the eta direction from minus minus1 to plus1 and this is my zeta direction from minus1 to plus1 so my confusion was because my psi direction is positive in x direction. Here this shape function was non zero with respect to psi with respect to eta, with respect to eta this term will be non zero with respect to zeta, this term will be non zero, so for this zeta, this is the first one this is N_1 double hat eta this is N_2 double hat psi this is N_1 double hat zeta.

In terms of this 1D shape function we can construct these 3D shape function and we see there will be 8 of them. They will satisfy completeness linear independence and all those properties in this way. Putting them together we can construct the global basis functions remember that the basic functions will be piecing together these shape functions from all the elements which share this node and then we can construct the 3 dimensional finite element solution for each of these components u_1, u_2, and u_3 solve using the weak formulation that we have done and we are now in a position to solve the 3 dimension problem, but remember that here applying the boundary condition is a tricky.

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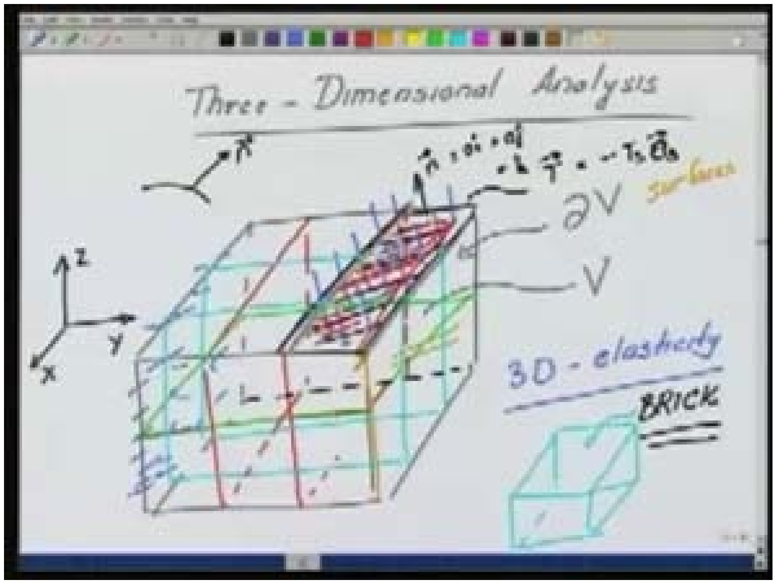

Because we have to do integral of the given loads over the area of the phases, in general for the general 3 dimensional domains defining the domain meshing the surface of the domain, meshing the interior of domain is not such an easy job. For that we need sophisticated mesh generators which have to be used to construct the mesh on the surface of the outer surface and the internal volume. With this I will stop my brief foray into the 3 dimension problem, before which we have discussed the plate problem which was essentially in between the 2D problem and the 3D problem. Before that we talked of the honest 2D problem which is the planar stress problem and planar strain problem as well as the 1 dimensional heat conduction problem.

Next we are going to develop methods for a different class of problems. Problems which relate to Eigen value problems at the continued level. For example, the free vibration analysis or the buckling analysis, how do we solve those problems using the finite element method?