Finite Element Method
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Module – 10 Lecture - 02

In the last lecture, we discussed a little bit about how to handle curved boundaries and curved elements and what we have said is we can handle them through appropriate mapping of the curve domain.

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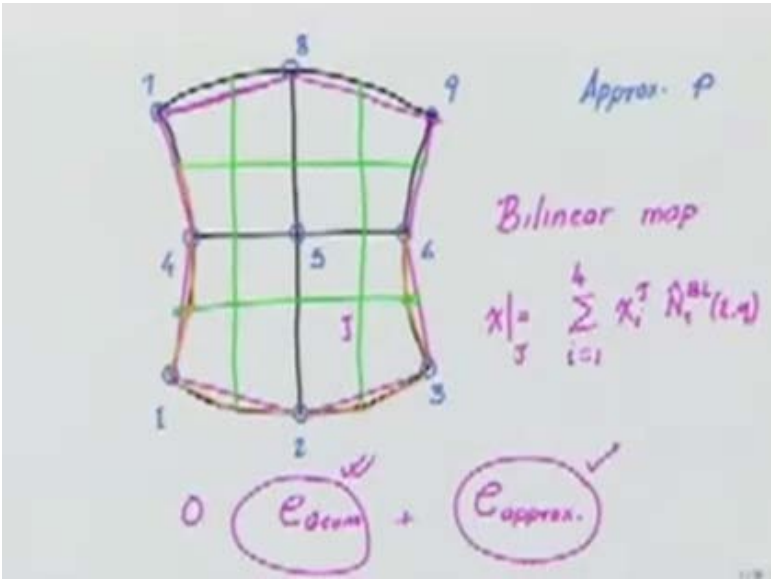

Let me re-enforce this point through a simple example. Let's say I have a domain which is like this. It has all this curved edges and I want to make a mesh of 4 quadrilateral elements on this domain. What will I do? If I make the mesh as a figure I will make of a mesh of straight edges, let me do the following I will take this and this. What are my nodes, nodes will be this, this, this, this, this. If we have given bilinear map here, then bilinear map lets say this is my node 1, node 2, node 3, node 4, 5, 6, 7, 8, 9. Remember let my approximation be of order P, the mapping of the geometry from the physical to the master is through a bilinear map. In that case if I take the bilinear map, then I will get edges will be joined by these straight lines.

You see that the computer sees a domain, because why this way is because we have taken a bilinear map that is in each element I am giving the x is equal to sum of in the element tau. Let it be any generic element tau, i is equal to 1 to 4, x_i tau and N_i. I would say bilinear hat as a function of sine eta. Similarly for y term, the x and y is given in terms of only these 4 corner coordinates of the 4 corner nodes of the elements. The domain that the computer sees is not the actual curve domain, but this domain. What we have

obtained by the mesh and you see that there is a tremendous discrepancy in the representation of the domain itself.

The geometry is wrongly represented and this will lead to errors in the finite element solution that you obtain, because you are solving for a different domain. We would like to minimize the error in representing the geometry of the domain in order to control the error in the finite element solution. The error in the finite element solution will have 2 parts. One is due to the geometry, error due to the representation of the geometry and you will have the error due to the approximation. Remember that. In most of the problem that we have done till now in fact all the problems for us the geometry that we have taken was polygon. It consisted of straight edges. The error in the representation of the geometry was 0, we didn't bother about this. While this error was what we bothered about and we discussed how to do mesh refinement and so on in order to control this error. What will happen in this case this error will be dominant, will be significant need not be dominant, but it will be significant. How do you control this error? One way of controlling this error is to say fine let me refine this mesh. Can I take a finer mesh?

What we are doing? In taking a finer mesh we are not going to take these points of the finer mesh out of the previous mesh; we will have to move these points to this term. We will be taking the finer mesh in such a way that I have these as my edges. We see that the error in the representation of the geometry also has gone down, as well as the approximation error because as we had said that when the approximation is of order P. We refine the mesh the error goes as h to the power of p, the energy norm of the error or the square root of the strain energy of the error goes as h to the power of p, but here also we have improved the representation of the geometry and so the representation of the geometry is better and we can get better solutions. The question is do we have to refine the mesh, because when you refine the mesh, the cost of the computation goes up number of unknowns goes up. Why not control the error in the representation of the geometry by doing a better representation of the geometry. That's what we had been talking towards the end of the last lecture.

In doing that what did we do? We said that we will use a higher order representation not the bilinear map, but something more and we said okay let us take a serendipity quadratic map. If you take the serendipity quadratic map for the same domain, then how will I make the serendipity quadratic map for the same domain?

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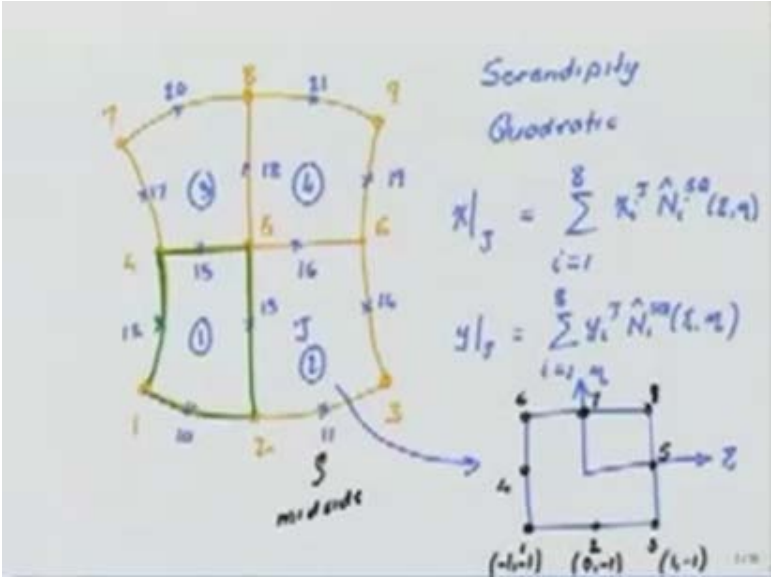

I will have my domain is like this. We had been made this mesh to start of 1, 2, 3, 4, 5, 6, 7, 8, 9 these are the nodes used to represent the vertices of the elements, not for approximation. We said that let us now go to serendipity, serendipity quadratic. In this case if I take a generic element tau here. What we are going to do, we will write our x in the element tau as sum of I serendipity quadratic as 8 nodes x_i tau N_i serendipity quadratic hat as a function of psi eta. Similarly y term, in doing this what do we have to do? We will have to put additional nodes in the element. What we will do? We will put these nodes somewhere in the middle of the edges. You can put it anywhere, but if you put too close to any of these nodes, one of these existing nodes then the mapping has a problem.

Let's decide that we will put it somewhere close to the middle of the edge. Middle of this edge, middle of this edge similarly here, we will do it for all the elements, will create these nodes which are in the middle of the respective edges. I will call this nodes by remember that these are only for representation of the geometry here 13, 14, 15. We have 21 such nodes. If I am talking of this element in which this is my element 1, this is my element 2, 3, 4, then actually in element tau is element 2 the way I have shown, we'll have the x of 2, x of 11, x of 3, x of 13, x of 14, x of 5, x of 16, x of 6. Let's give a numbering pattern and this goes gets mapped to the master square. How will it map? This will be my in the master square 1, 2, 3. We will call it 4, numbering is something that is up to us 6, 7, 8.

You see that the 2 here in the physical element becomes a 1 in the master 11 here becomes the 2, 3 becomes the 3, 13 becomes the 4, 14 becomes the 5. 5 becomes the 6, 16 becomes the 7 and 6 becomes the 8 and these nodes are now in the middle of the edges. This has co ordinates -1, -1. This will have co ordinate 0 -1, this will have co ordinate 1 -1. You see that this mapping takes us back to the same master square that we had earlier for the linear map of the bilinear mapping. Remember this that we have this

kind of a feature for the mapping and we will say mid side. If I do this kind of a fit where I am taking more point's lying on the curve in order to represent the geometry of the element, then we should do better than what we have done earlier and you we will see that you will get a very good fit as far as these edges are concerned.

There can be error, there will be errors if this curve is not a perfect quadratic curve with something else then there will be some errors, but that error will be small. So our representation of the geometry that is the geometry that the computer sees is much more accurate then the geometry that we had earlier with the bilinear map. That's why we should use a higher order map, need not be always isoparametric for example If I am using a^{th} order approximation I need not use an a^{th} order map, but we have to use some sufficiently higher order map. It could be which is a parametric or it could be isoparametric if my P equal to 2 for the approximation, P=2 here will do the job.

That is the idea. That is each geometry has to be controlled and you see here something curious that in doing what we have done, we have ensured that the mapping is continuous. What does it mean that if I look at these edges, the common edges between 2 elements there after the mapping I do not see a gap between the 2 elements? That is it should not end of giving me a gap that is a spurious space in between in the 2 elements. And how is it ensured? You see on this common edge the mapping is the same from both sides, it uses the same information it gives me the same line on the common edges, same curve on the common edge. This is one feature that we have to maintain the mapping from both sides on this common edge, so naturally there will be no gaps. This mapping is invertible also, but finding the inverse that is sine terms of sine eta in terms of x and y is not so easy when it comes to higher order mapping but one can do it.

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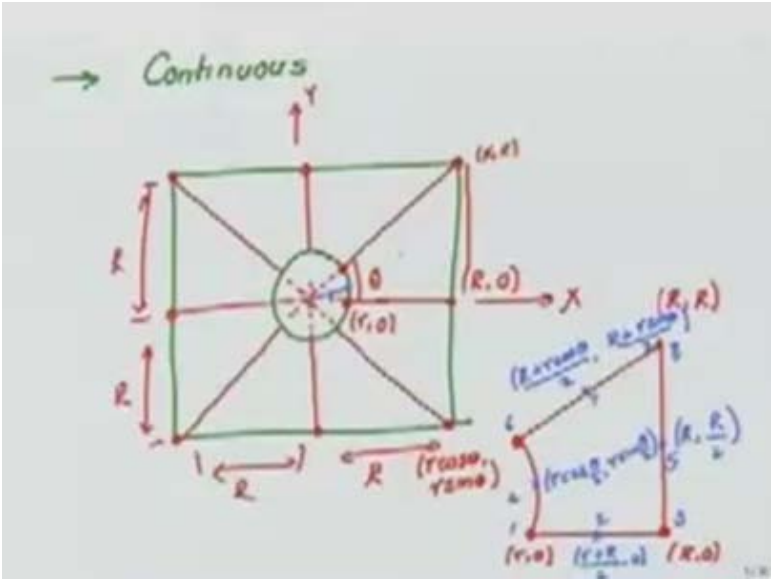

These are some of the features of the mapping, let me take an example to demonstrate this, how am I going to do it. Let say this is my domain with a centrally located circular

hole, in the mesh I am going to make is like this. This is the centre this, this. This is the kind of a mesh, we can make mesh of a quadrilaterals. Let us say this inner one, the co ordinate system is fixed like this, x and y. This point is located at r (, 0), this point is located at big R, 0. If I see that these are our initial nodes that we have created in the basic mesh. Let's look at the mapping of this element. If I take that element out so let's first fix the corners, then we will bother about the rest.

This angle is some angle theta, which can be obtained from this geometry. This is point r, 0, this is point big R, 0, this is point r cos theta, r sin theta. This one is point R and here it will be the other co-ordinate, we can easily obtain as if it is a square, let say it is a square so this distance is R we will keep this distance is also R. This is point (R, R). Let say this is point R, R. The height has to be obtained has to be given. Let say this is distance R, this is also distance R and this is distance R and this is distance R. We have these coordinates, now we have to put these middle points. The middle point here is nothing but r plus big R by 2 zero, middle point here is R and R by 2, middle point here is R plus r cos theta divided by 2, R plus r sin theta divided by 2 and the middle point here. What will the middle point here be? If this angle is theta, let we take this angle is this will be theta by two, this angle will be theta by two.

This will be nothing but r cos theta by 2, r sin theta by 2. This is how we can create the local co ordinates, I will call this local 1, local 2, local 3, local 4, local 5, local 6, 7 and 8 and using these co ordinates we can go and construct the mapping.

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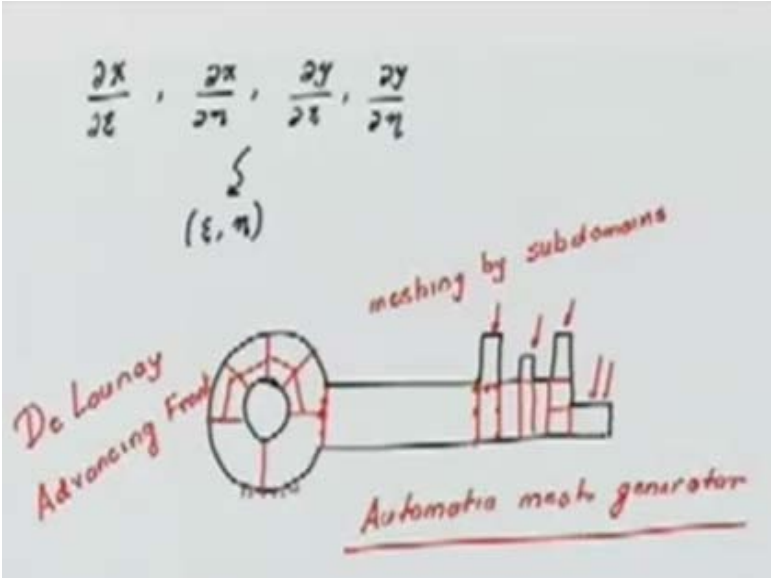

Then after we have constructed the mapping then the rest is the same that is we need from our computation point of view, we need to obtain del x del psi, del x del eta for the matrix of the transformation del y del psi and del y del eta and in order to compute the jacobian. We compute this from the expressions for the x and the y in terms of those sin eta that is in terms of the shape functions that have been used to represent the geometry

and once we have that then we can obtain these expressions as functions of sin eta and we are in business. We can find the Jacobian, we can do everything else that we need. This is how we are going to handle a curved geometry.

Let me (18:20) let me harp on another issue lets say that I have this kind of a domain. Lets say this is what under some loading, let say I have some loading acting on the structure, some constraints somewhere lets say I am fixing it and I would like to solve this problem, find the state of stress everywhere in this domain under this loading and this constraint. How do you do it? And let say this is planar problem. In terms of the meshing its better to break the domain up into smaller pieces or sub domains and then do the meshing. One thing you can do is break it like this. We can right away break this domain into smaller pieces and do the meshing by pieces or meshing by sub domains. Why do you do that? Because this will give you a quitter nice looking uniform mesh and it is not always possible to make the mesh by hand, you use something called an automatic mesh generator which takes the domain, the controls of the domain given by you and gives you a mesh.

In these we have the feature of finding sub domains and most of the standard automatic mesh generators and there we define a sub domains and then do the meshing by sub domains to get a uniform mesh with a good looking uniform mesh such that the continuity is maintained that is the nodes from the 2 sides should overlap. On the common edges the nodes should overlap. Like here on these edges common edges, these nodes should overlap and in doing this we can construct a fine mesh where we need the mesh to be fine. We can construct a course mesh in the region where not much is happening. We can play around with the meshing and create a non uniform grid or mesh which is in somewhere economical and also accurate in terms of getting to the solution.

Automatic mesh generator, writing is very difficult. It is not an easy job there are various procedures by which one can write an automatic mesh generator there are methods like yank piving and your advancing front method and so on all those things. There are course which are available on the net, free of cost which can be downloaded which can do a very good job of automatic mesh generation in 2d. For two dimensional domains it can be done. You can go and download there are some course, which is based on de launay triangulation. The concept of de launay triangulation, some which are based on the advancing front method and so on.

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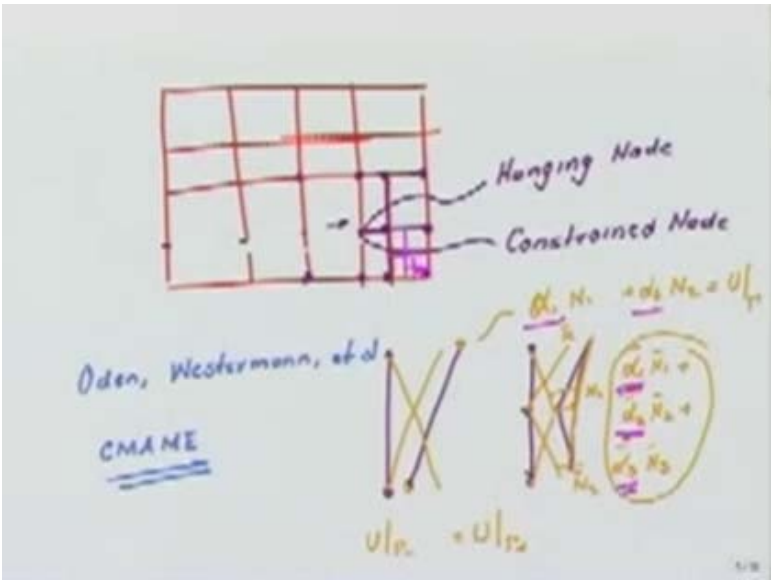

You can get these course, download them use it for your problem and the documentation also for this is available some of those course I can take the name one is by jonattan sourchak and the other is called anganer. These courses are quite nice they can be used. Lets now shift gears and go from this problem to a little bit of a different issue, that is till now we have been talking about approximation or meshes where the nodes are overlapping. That is we had ruled out such situations this kind of a situation was ruled out. Why? Because you see that here if I simply take bilinear here you have a node here which does not have counterpart from this side. This is called a hanging node.

This was ruled out in what we did. We said that well this has to be taken care of either you do this and keep on doing it to fill up the thing or you just cant do it. It terms out that it is not very difficult to have this kind of a situation and still I am sure that your approximation is continuous. How do you that? You do that by constraining these node, it is called what does it mean? I will tell you that is the approximation form this edge looks like this. If it is a bilinear, the approximation from this side would look like this, because of this node being present.

What will you do? You force this approximation to exactly match this one that is you overlap, this guy here you overlap this guy here and force this one to get its value from here. That is on this side the approximation is given in terms of this nodal values that is alpha_1 N_1 plus alpha_2 N_2. These N_1 N_2 are the shape function from this side. It will be like this. From this side the approximation is given in terms of you will have alpha_1 bar, N_1 bar plus alpha_2 bar I will call it N_2 bar plus alpha_3 bar N_3 bar (24.50). This could be your N_1 bar this is your N_2 bar and this is your N_3 bar. You force this is going to be u on the edge, the force the u on this edge from the negative side to be equal to u on this edge from the positive side. This gives me a constraint on these degrees of freedom alpha_1 bar, alpha_2 bar, alpha_3 bar, in terms of alpha_1 and alpha_2 so this is called a constrained approximation.

What is the advantage of it? Advantage of it is let say I have a crack here or a singular vertex here, then I can zoom in the approximation towards this without propagating this refine meshing elsewhere, the cost of computation goes down. It is much more economical because I know here the solution is unsmooth, I will have to take care of it locally so I keep on doing this kind of local refinement to capture that unsmooth behavior and I am done. While the mesh elsewhere remains continuous. You can see this in the literature there is a very nice paper I think published in 1989 by Oden, Westernmann and Otal the computer methods in applied mechanics and engineering this journal. You can see this paper, it is very nicely describes how to do this job.

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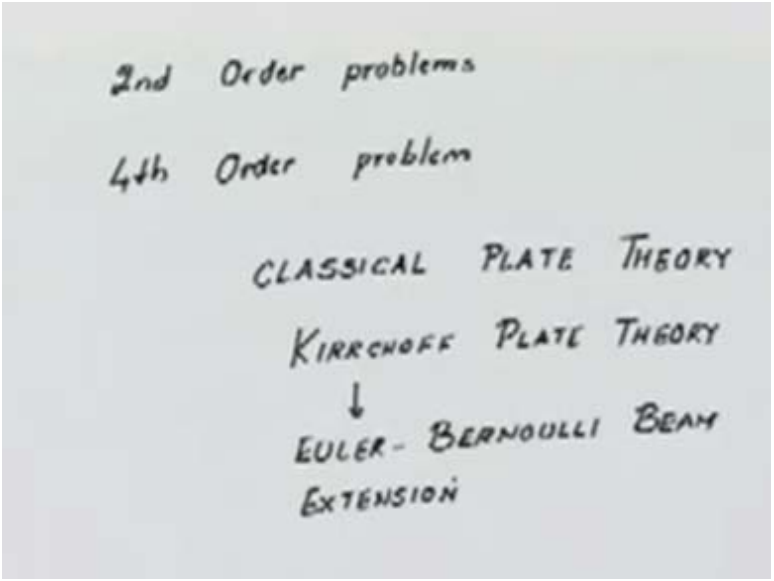

Let us get down to again more classical things. Till now we have been discussing second order problems, where the highest derivative occurring in differential equation was the second derivative, partial derivatives. We talked of problems in terms of one variable which was an example was the steady state heat conduction problem, the temperature was the unknown or the torsion problem where the (27.08) function was the unknown. While we also talked of planar elasticity problem, where we had a system of unknown that is two unknowns which are the in plane displacement u and v.

Let us look at a problem which is the fourth order problem, this is what we know as the one example of it is the classical plate theory. It is an attributed to Mr. Kirrchoff this is called the Kirchhoff plate theory also, sometimes the Kirrchoff (28.00) in name is also added. This is a direct extension of the Euler Bernoulli beam theory in one dimension.

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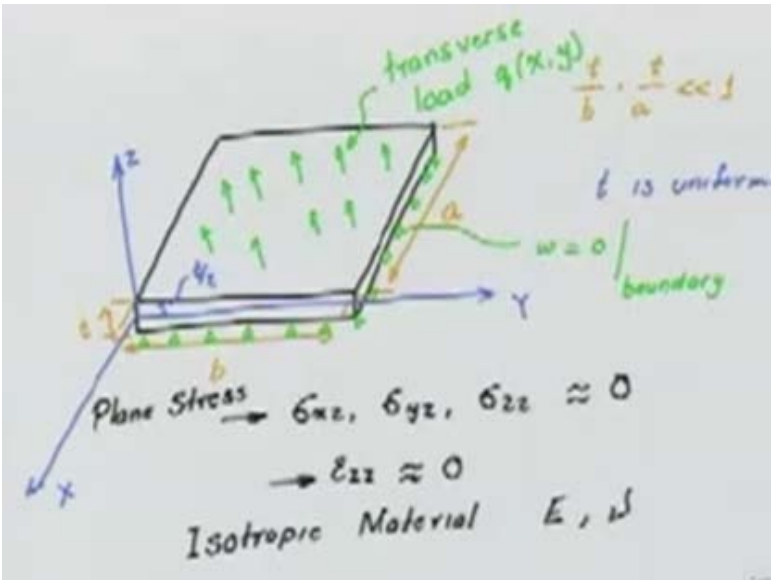

How do we take care of what do we do here? The idea is simple let us take a very simple problem that is I have a rectangular plate, plate means the depth direction is much smaller, depth dimension is much smaller as compared to the in plane dimensions. That is here if this is my width b, this is my length, I will put my co ordinate system accordingly so you will put this as a, this is b. Then this one, this dimension here is t, we will say that t by b, t by a is much smaller than 1. Lets now as you saw that there is a difficulty in writing things let's fix a coordinate system. A coordinate system is such that I have this is my x direction, this is my y direction and this is my z direction. We assume the thickness is uniform, t is uniform everywhere that is it does not change with the x and y location, and it is a rectangular plate.

If you see this co ordinate system is such that this height is t by 2. This is in the middle of the thickness of the plate. On top you have t by 2, on bottom you have t by 2. What we will assume further is that this plate is loaded by only a distributed transverse load. Under the action of this transverse load and suitable boundary conditions, what are we going to do as far as the boundary condition is concerned, we are going to constrain the transverse movement of the plate on the edges that is we are going to force the transverse displacement on this edge to be zero. It is an extension of the pin support in one dimension. You will have the w, the transverse displacement w is equal to zero on the boundary, that's it. This is what we have has our and there are no other loads applied on the structure, the transverse load we have w is zero on the boundary this is what we are going to take. For this problem, let us see how are we going to get the solution? There is a standard way what we call as a semi inverse method or in the strength of material way you try to figure out what is going to be the deformation pattern.

For this plate it can be shown when it is thin, that sigma_{xz}, sigma_{yz}, sigma_{zz} are essentially zero. That is they can be ignored or neglected with respect to sigma_{xx} sigma_{yy} and sigma_{xy} that is the magnitude is much smaller as compared to those. We further have

an assumption here that the transverse straining is much smaller as compared to the in plane strain, that is E_{zz} is on top of this assumed to be zero. This gives us the plane stress assumption and this is the additional assumption that we make for the classical plate model.

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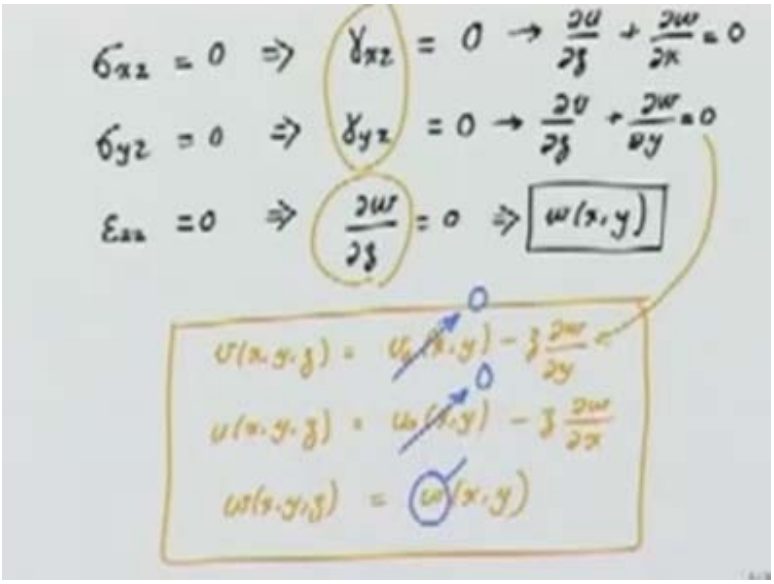

Let us assume that we have an isotropic material with young's modulus e and poisson's ratio mew. If I go ahead and expand what we have made as our assumptions out $sigma_{xz}$ in this case equal to zero implies the engineering strain $gamma_{xz}$ is also equal to zero, $sigma_{yz}$ is equal to zero this implies the engineering strain $gamma_{yz}$ is equal to zero, E_{zz} is equal to zero, this will imply that del transverse deflection, del w del z is equal to zero, which is what we are going to make, the w is only the function of x and y. This is the first outcome of this assumption.

This will imply that del v del z plus del w del y equal to 0. This will imply del u del z plus del w del x is equal to zero. This assumption will lead to v as a function of x y z is x y minus z del w del y and u as a function of x y z is u_0 x y minus z del w del x. Where did these things come from? These assumption say that a plane perpendicular to the middle plane remains perpendicular to it after deformation just like we have done in the beam that the line perpendicular to the middle line remains perpendicular to it after deformation. And that line does not stretch in length. These assumptions give us the representation of the displacement field.

It can be further shown that in this case for the isotropic plate the values of v_0, the functions u_0 and v_0 will be 0 everywhere, because they will be governed by a the planar elasticity problem. It terms out because of these being a bending dominated problem. That the only displacement you are going to have is w and due to the w, the u's and the v's will come. In this case for this kind of loading we will have only w being non zero, u_0 and v_0 will be 0. If I had an in plane loading along with this transverse loading then yes

we would have u_0 v_0 also. If I had anisotropic material then also u_0 v_0 will be present. So remember in this very special case u_0 and v_0 become 0 functions.

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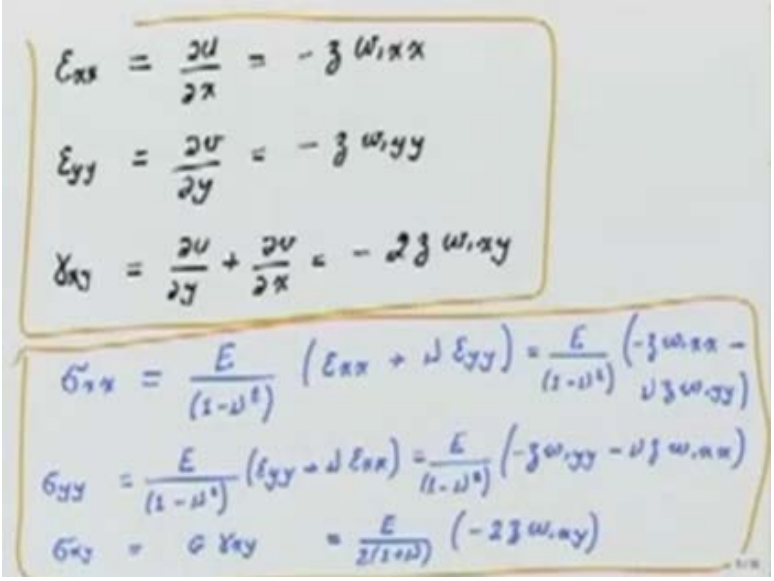

We can write what is the strain, E_{xx} will be del u del x, which is equal to minus z. I am going to write everything now in short form w, xx which is del to w del x squared E_{yy} will be equal to del v del y which is going to be equal to minus z w, $_{yy}$ and $gamma_{xy}$. These are the only strains which are non-zero in this model, this is equal to del u del y plus del v del x this is equal to minus 2 z w, xy, because you see from here. We will get minus z w, xy from here you will get minus z w, yx; both y x and x y are the same. This is the state of strain.

What about the state of stress? State of stress will be $sigma_{xx}$, because it is plane stress problem, because of the assumed state of stress you will get e by one minus mew squared into E_{xx} plus mew E_{yy}. This I can write as e by one minus mew squared into E_{xx} will be minus z w, xx minus mew z w, yy. Similarly I have $sigma_{yy}$ is equal to e by one minus mew squared E_{yy} plus mew E_{xx} which is equal to e by one minus mew squared into minus z w, yy minus mew z w comma xx and $sigma_{xy}$ which is sometime written as tau x y is equal to g into $gamma_{xy}$ this is equal to E by two into one plus mew into minus two z w comma xy. So we have the state of strain and the state of stress for this problem known to us.

We are going to use the total potential energy we need to write the total potential energy expression in terms of the state of stress and the displacement given to us, w is not known but we will write it in terms of w and then how are we going to obtain the equations of w by saying the solution w is the minimizer of the total potential energy. We are using the approach of the variational formadation. We could have used the principle of virtual work or we could have used the differential equation to get to the v form.

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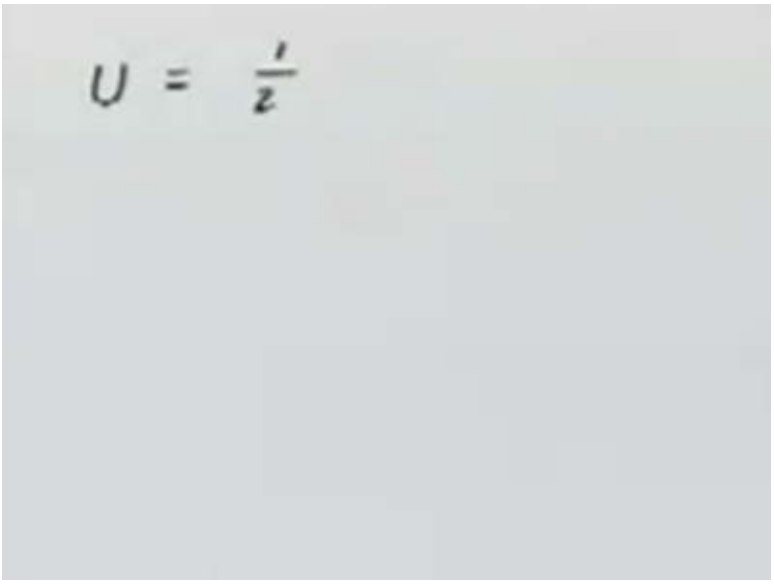

We will write the strain energy u equal to 1 by 2 integral, let's go back to our figure that we had earlier we will call this whole plate is said to have a volume v and this face if you look at the projection of it, this area will be set to have an area A.

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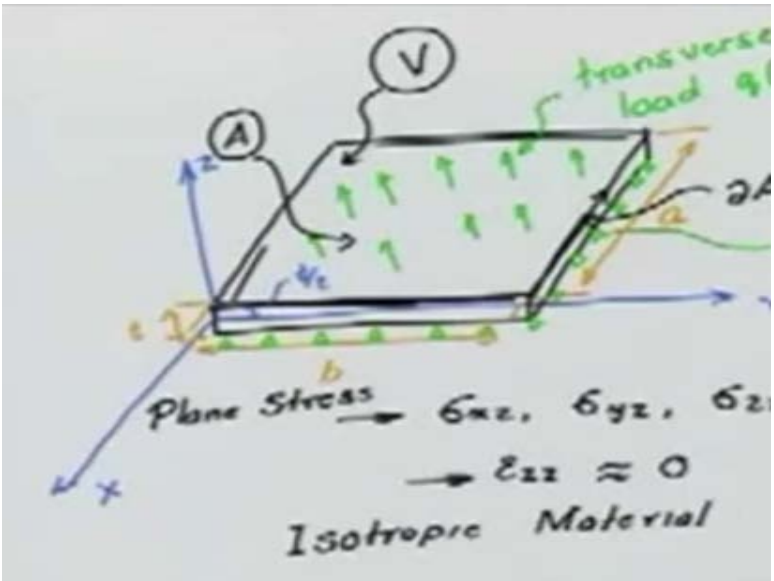

For the area we will have the contour, the bounding edges which will be given by delta A, such that this bounding edges are arranged in a counter clockwise manner. This is the naming conversion we are going to follow, the whole volume of the plate is v the projected area of it is A, this is the same through the depth, and the bounding contour of the area is called delta A.

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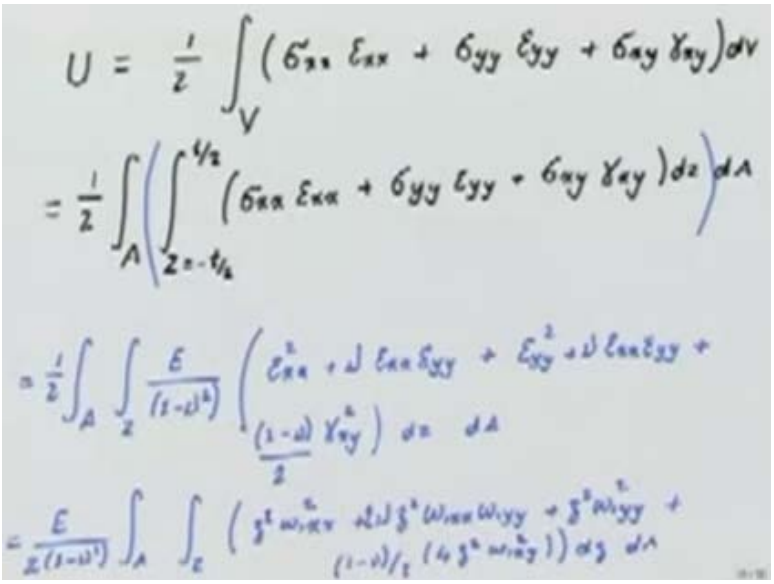

In this case u becomes integral over the volume of $sigma_{xx}$ E_{xx} plus $sigma_{yy}$ E_{yy} plus $sigma_{xy}$ $gamma_{xy}$ d v. This if you write will be equal to 1 by 2, we can write it has this integral over the area, you see that the volume becomes essentially integral over the area into integral for z going from minus t by 2 to plus t by 2 of $sigma_{xx}$ E_{xx} plus $sigma_{yy}$ E_{yy} plus $sigma_{xy}$ $gamma_{xy}$ into dz dA. We can say we will put those within the brackets, because why are you doing this because we know the expansion of the u v w in terms of z explicitly.

You would like to eliminate that dimension, which in terms of a mechanics means that we are looking at equilibrium or the responses or the forces not in terms of stresses, but as in terms of its stress resultants which are the moments, the shear forces and so on. This integral here we will get as this is equal to 1 by 2 integral over the area integral over z, 1 by 2 we will have E by one minus Mu squared into $sigma_{xx}$ E_{xx}. From the previous one we will have E_{xx} squared plus Mu E_{xx} E_{yy} plus from here E_{yy} squared plus Mu E_{xx} E_{yy} plus G $gamma_{xy}$ squared.

It will not be G here, if you go back to what we have done this will become one minus Mu divided by two into this because e into one minus Mu by whole squared I have taken it out as common. This will be E by 2 into one minus Mu squared integral over the area integral over z. Lets expand this thing, it will be z square w, xx whole squared plus Mu in fact two Mu z square w, xx w, yy plus z squared w, yy whole squared plus one minus Mu by two into two. If I go back to the expression for my $gamma_{xy}$ it is actually equal to two z w comma xy. We will have by two into four z squared w comma xy whole squared d z dA.

You see that here through all these things we have the z square sitting in each one of these expressions, the integral of z square and the rest of them are all functions of only x

and y. The w, xx, w, yy w, xy are all functions of x and y so they are functions in terms of the area only.

We can explicitly integrate out the part corresponding to the z so z square integral from minus t by 2 to plus t by 2 will be z cube by 3 evaluated from minus t by 2 to plus t by 2 which will be equal to t cube by what will you have t cube by 12.

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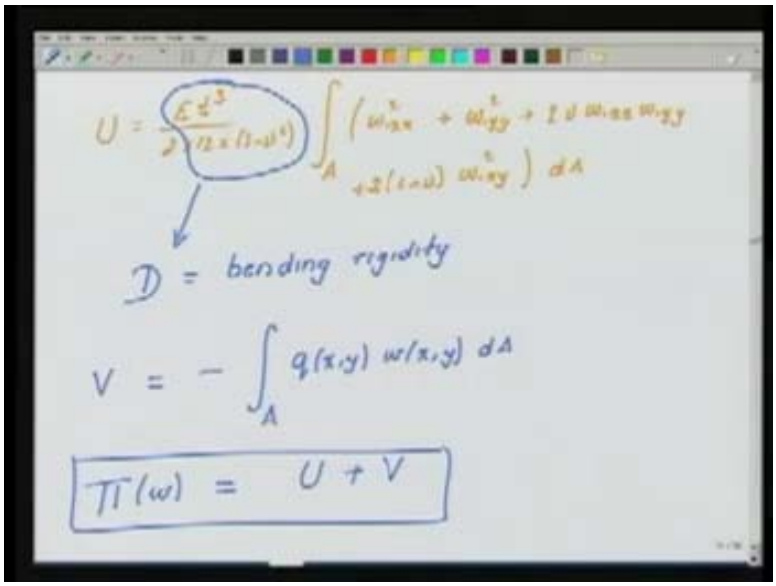

T cube by 12, u will be equal to actually E into t cube by 2 into 12 into one minus Mu square integral over the area w, xx whole squared plus w, yy whole squared plus two Mu w, xx w, yy two into one minus Mu, because of this last expression four divided by two is two, two into one minus Mu into w, xy whole squared dA. You see now we have integrated out the effect of the part of corresponding to z to get this expression over the area. In terms of the function w which is the function of x and y only. If you look at this expression here, this is given a name D which is called bending rigidity. Essentially it tells us how rigid is the plate in bending? How much it resist bending action?

We have this expression for u, similarly v is equal to the minus of the work done by the external forces so here the work done by the external forces is integral over the top area which is nothing but the projected area itself, because we have said that in the depth direction the cross section or the profile does not change. This will be equal to q (x, y) into w (x, y) w is constant with z, it doesn't change from the top to the bottom, Then our pi as a function of w this is a function null will be u + v.

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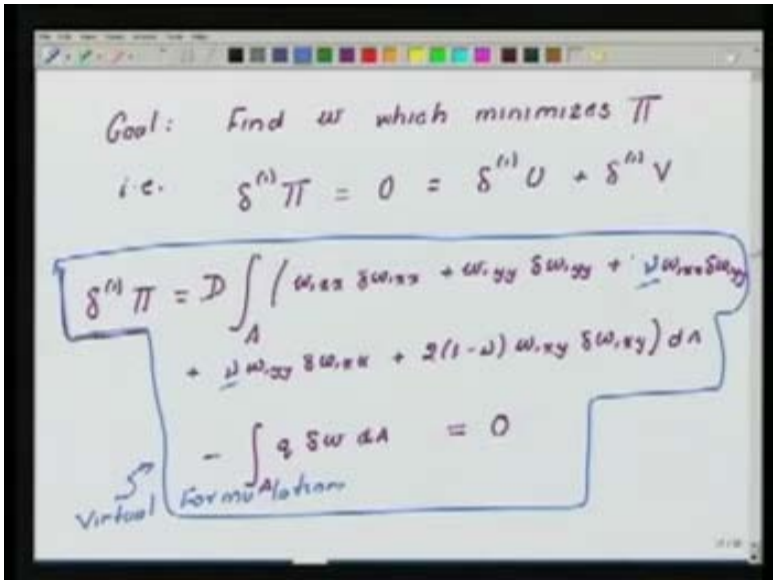

And the solution to the problem so the goal is find w which minimizes pi, that is has we have done earlier the first variation of pi is equal to zero, which is then equal to the first variation of u plus the first variation of v. This if you work out you will be quiet easy. How do you work out the variation? Essentially is like taking the differential or derivative of these quantities, we will end up getting delta one pi is equal to integral over the area D into w, xx del w, xx plus w, yy del w, yy plus two nu w, xx del w, yy plus two nu w, yy del w, xx plus 2 into one minus Mu w, xy del w, xy dA minus integral over the area q del w dA, this is equal to zero.

This is our virtual formulation. This is the virtual formulation that we are going to get out of what we are obtained here. I think the 2 will go away yes I have made a mistake here. Actually it will be 2 by 2. This two will go away, it will not be present there. This two will not be present there. I will only have the Mu is here. This one should also work out to see but I have made a mistake there. Once we have these formulation, then now we are in business we can solve the problem. In order to solve the problem we have not yet done any approximation here. We have simply posed what is the definition of the displacement or representation of the displacement from there, we went to the state of strain to the state of stress obtain the total potential energy. That is we wrote the functional from there we did minimization of the functional to get the virtual formulation, and this is the approach one has to follow in all the problems where we can easily obtain the energy expression. You need not always go from the weak formulation, you can go form the virtual formulation also whichever is convenient we use that.

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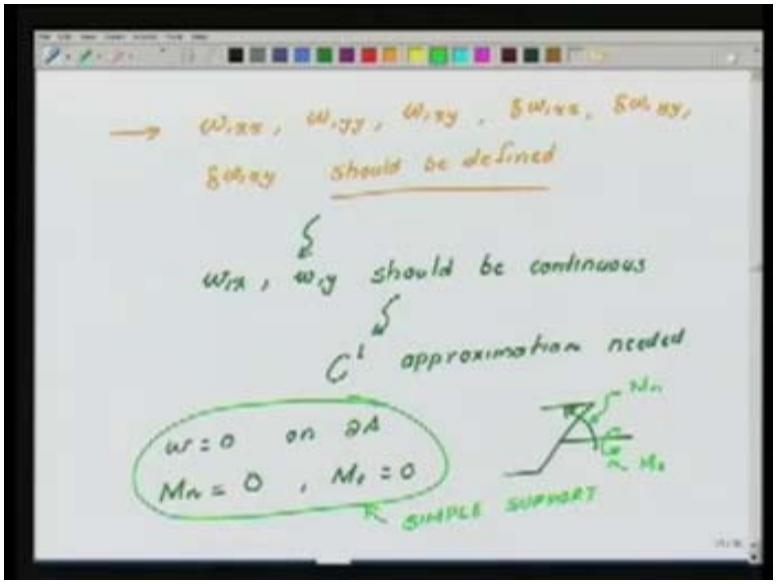

You see the problem that we have an interesting thing is here w, xx w, yy w, xy is setting. What we have here, in this problem that we want w, xx w comma yy and so far del also comma xy should be defined. If they are not defined then the energy becomes meaningless, because it becomes infinite and the right hand side in the virtual formulation I would say this is the left hand side will also become infinite. This should be defined, which implies that we want that w, x w, y and so with del w should be continuous. That is we need C 1 approximation minimum for the representation of this w. Minimum smoothness required is C 1 that is the first derivative of the w with respect to x and y should be continuous.

Let me reiterate, if the place where w is defined what we know about del w at those places del w has to be constrained to be zero. Remember that in the places where w is defined del w has to be constrained to be zero. In this problem just like Euler Bernoulli beam, in the Euler Bernoulli beam we also had a rotation that is del w del x they could also be defined on the boundary. Similarly here we could have on the boundary, the slopes or the rotation del w del x del w del y defined in that case the delta del of delta w del x and or delta of del of delta w del y should be zero correspondingly, that is in terms of the slopes also, we will have geometry constraints just like we did in the Euler Bernoulli case.

However for the problem we have taken, it is simply supported that is w is zero on delta A and on this edge when we go and do things properly this means, that we also have the normal component of the moment is zero and the tangential component of the moment is zero on the edge. This is the edge here this component of the moment is zero as well as this component of the moment is zero. This is m normal, this is M tangential. You see on the edge a bending moment and the twisting moment both are zero in this case. This is one of the so called simple supports. This is in fact also called soft simple support you would have hard simple support also and so on.

What if this edge was free, if an edge was free on that edge your M_n M_t resultant moment bending and the twisting as well as the normal shear force v will also be zero. That would be a free edge, but the bottom line is in any of these all these cases the work done by the forces on the edge is zero. That is why, it didn't come in our definition of the total potential energy. We could have moments and shear forces applied on the edge, and then we will have to account for it in the expression for the total potential energy.

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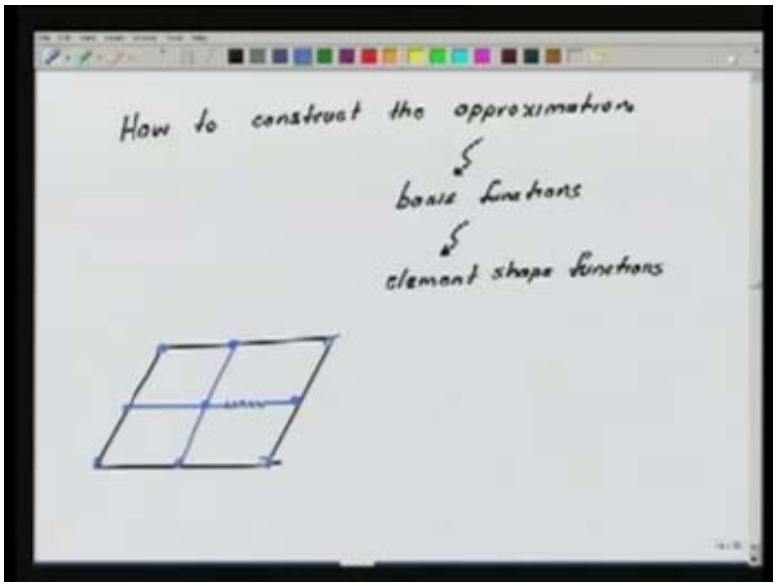

In the next time onwards, in the last class what we are going to look at is how to construct the approximation. That is what are these basis functions are globally, which means what are the element shape functions. We will see that we can construct one family of the basis functions globally which will not be confirming in the sense that they will not ensure continuity of del w del x del w del y everywhere along the edges. That is if I take the plate and mesh it with rectangles, then along these edges del w del x del w del y will not be continuous they will only be continuous at the node points.

This is actually violating our requirement, but it is shown that this does a fairly good job. We could however at least in this case construct a strictly confirming approximation that is use a extension of Hermite cubic polynomials from the 1d to the 2d, but for the rectangular mesh and then we will have the confirming approximation. We would look at both the cases not strictly confirming and the confirming in the next class.