Dynamics of Machines

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Module No. # 3

Lecture No. # 2

Rotary Balancing: Graphical and Analytical Approach

In the last class, we have seen that any rotating system or any rotor can be completely balanced with the help of only two balancing masses placed at any two chosen balancing planes.

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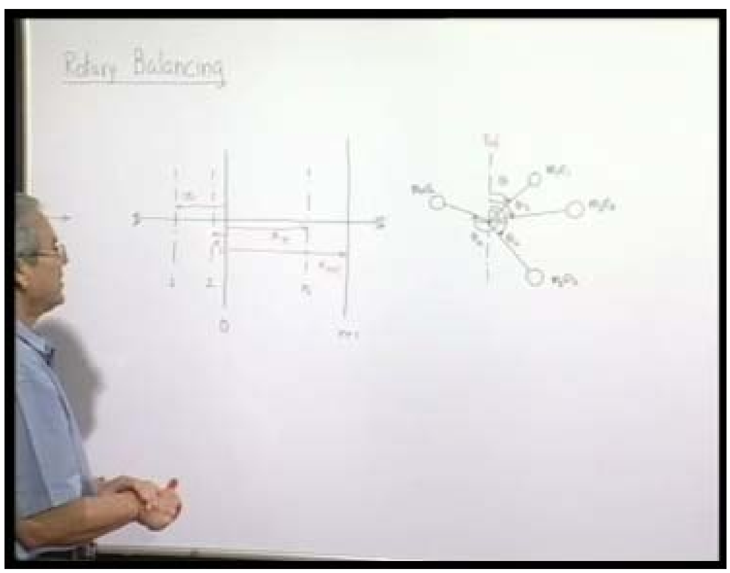

Today, we will take up analysis of such rotating systems and find out the masses which are needed to balance such a rotating system at the design stage. So, let us take a rotating shaft that carries n number of unbalanced masses; let the plane of this unbalances be 1, 2 and so on up to n. When we view from this side the angular position of this unbalances are as shown in this slide (Refer Slide Time: 02:00).

For example, $theta_1$ form a difference which we can take as the vertical direction and this is m_1e_1, say this is m_2e_2 making an angle $theta_2$, let this be m_3e_3 making an angle $theta_3$ and so on let this be m_ne_n making an angle of $theta_n$ (Refer Slide Time: 03:11). So, the angular position and the magnitude of unbalance at these positions are all known. Now, say for example, we have to place two balancing masses at two chosen planes depending on the convenience of the designer; that where such balancing masses can be placed physically.

Let this be one of the balancing mass; we call 0 and let this be another balancing plane which we can call plane m plus 1. One other thing which needs to be taken care of is the distances of these planes, where unbalances are present from the one balancing plane. For example, in this case it is this one (Refer Slide Time: 04:03). Let these be the distance along the axis of the rotor from one of the balancing planes, the 0th balancing plane. Let this and this are the planes, where you have to place the balancing masses.

Now, we have a complete idea about the configuration of the system. We have two balancing planes and we have the planes of the system where the unbalances are present whose magnitudes and the angular orientation with respect to some reference. In this case, we have chosen this as the reference (Refer Slide Time: 04:54) where all are known.

Our problem is to find out m_0e_0 and $m_{n\ plus\ 1}\ e_{n\ plus\ 1}$ placed at these two balancing planes and the values of $theta_0$ and $theta_{n\ plus\ 1}$, so that the system is completely balanced. When you say completely balanced? What we mean is that there will be no unbalanced resultant inertia force and there will be no resultant couple due to this inertia forces about any point. Now, the problem can be handled in two ways. We can solve the problem by an analytical approach or we can discuss or we can solve the problem in a graphical manner.

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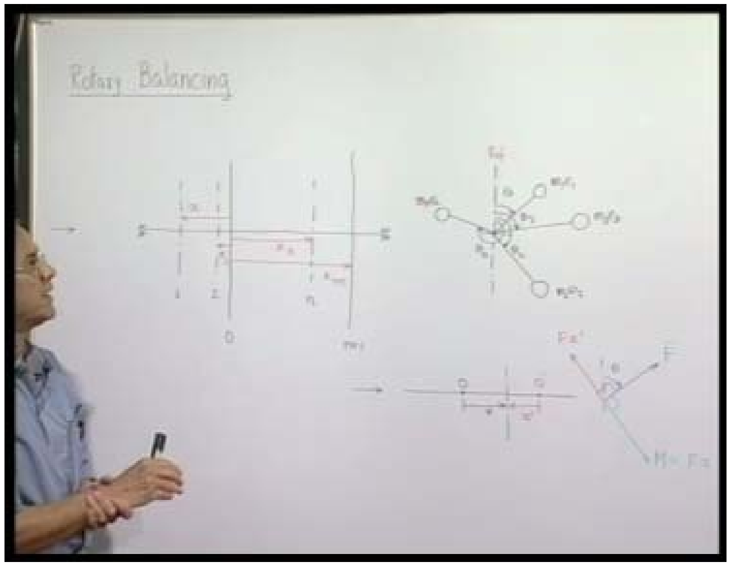

The graphical manner gives a better physical understanding. First, let us take the case where we discuss or solve the problem by graphical analysis. Before we take up the problem let us consider, one important task; say if this be the shaft and this be one plane where the mass is here; that means, its centrifugal force will be here at an angle theta (Refer Slide Time: 06:56).

So, what will be the moment of this about a point? Say here, where this distance is x. Since, we are plotting this force or drawing this force as viewed from this, we should also draw the moment vector diagram from the same view.

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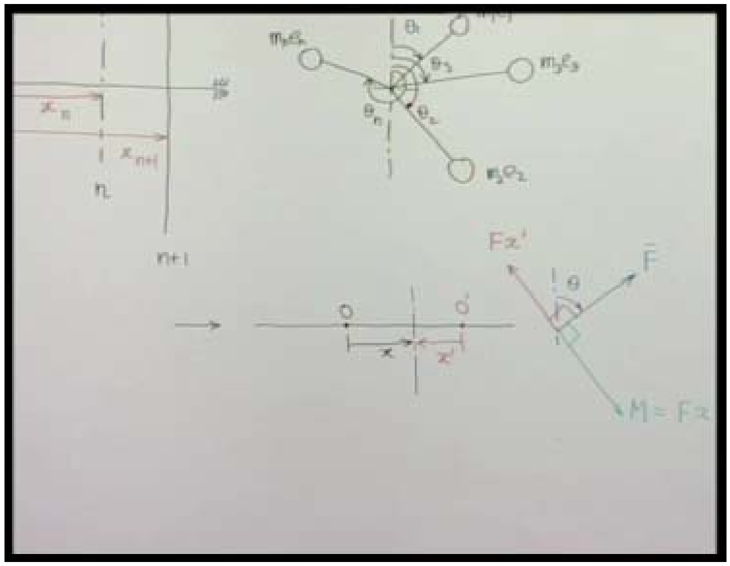

That means, the moment due to this force F at point O, how it is going to look? You know that we will generally follow right hand screw rule; following that rule you know in this view, from this direction - the moment vector will be if it is in this direction, then moment vector will be you have to stretch your imagination little bit. This will be the moment vector whose magnitude is nothing but F into x, but it will be at right angles, in this sense, in the-clock-wise sense leading the force vector (Refer Slide Time: 08:17).

On the other hand, if the point is here about which you are taking the moment O prime. This is the force of distance x, then the moment of this force as seen from this direction will appear as a vector, which is lagging behind this force vector. Therefore, what we can say is that, we can assign a sign to this quantity x when the force is towards left, we can consider such x value as negative and when it is towards right, we can give this quantity a positive value.

The moments will be all leading the force vector by 90 degrees in the positive sense. If the moment is negative because x is negative, then obviously it will be diametrically opposite to that. That means, it will be lagging the force vector by 90 degrees as shown here (Refer Slide Time: 09:38). So, these things are to be kept in mind when you solve the problem in a graphical manner.

Next, let us go to the solution of this problem. Now, you see how many are known quantities - If you see, we have two forces or two m unbalances $m_{n \text{ plus } 1}$ $e_{n \text{ plus } 1}$, m_0 and

e_0 to be determined; that means two quantities, but it is not only the magnitudes; it is their angular positions $theta_0$ here and $theta_{n\ plus\ 1}$ have to be also determined. So, when you draw a vector diagram the resultant is 0, if it leads to a closed polygon.

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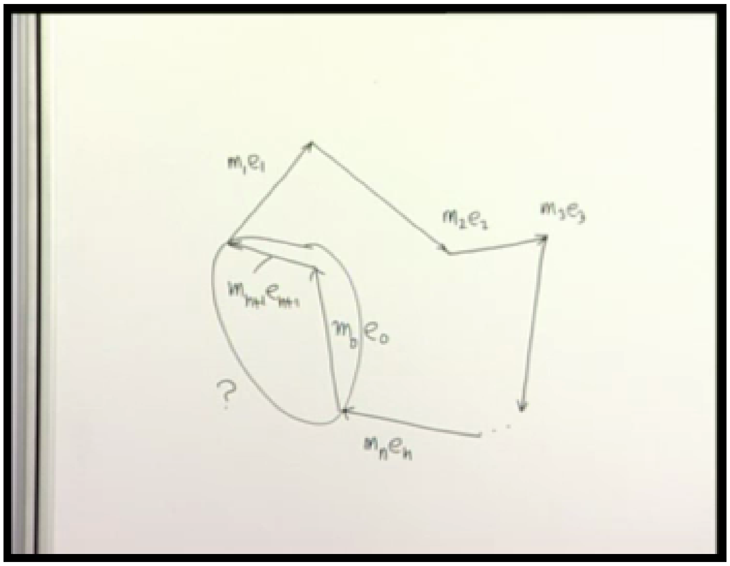

If we draw the forces, then you will find we can draw the side view of the forces, force vector will be something like this. All these terms will be multiplied by omega square and it is going to be same for all what we supposed to do.

We omit this term, omega square from all these vectors. Simply, m_1e_1 can be presently unbalanced force then $m_2e_{2,}$ next m_3e_3 and so on. There may be many and finally, we may have m_n e_n when we add the two forces generated by the balancing masses at this plane 0 and n plus 1; that means, there will be two more vectors which we do not know neither the magnitude nor the direction say, m_0e_0 and $m_{n\ plus\ 1}$ $e_{n\ plus\ 1}$ both are unknown. So, we cannot draw this force vector diagram.

On the other hand, if we consider the moment vectors what happens, moment always depends on the point about which moment is taken and that point is at our exposure, we can choose it. We also know that moment of a force about a point through which the force is passing is 0. We can always eliminate one unknown by choosing one of the balancing planes. For example, in this particular case the 0th plane as the reference about which the moments have to be taken.

So, when you draw or take moment about this point, then the moment of all the unbalanced forces except the balancing force or balancing mass produces in the plane n plus 1; rest all are known only one unknown is involved and we can draw the moment vector diagram. First what we will do, we will draw the moment vector diagram.

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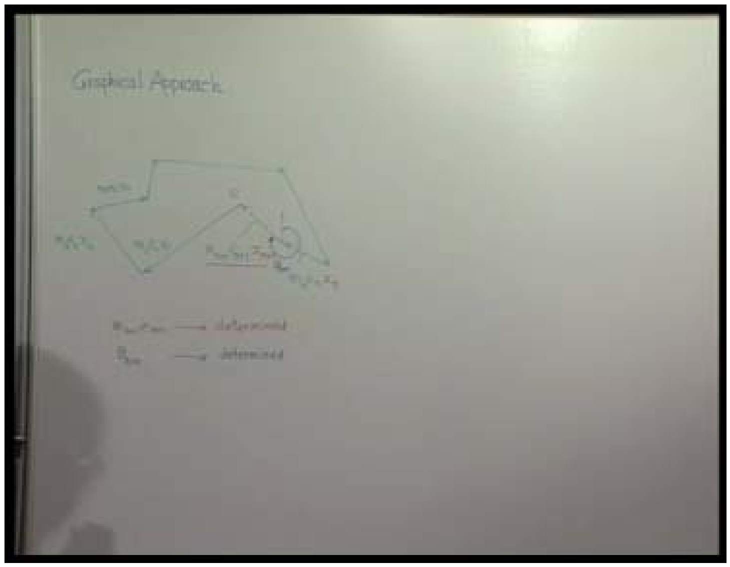

You can see one thing that all the moment vectors are going to be either leading the force vector by 90 degrees, the corresponding force vector by 90 degrees. All again the corresponding force vector by 90 degrees depending on whether the plane, where the force is being generated is on the right hand side, that is x is positive or on the left hand side, that is x is negative.

Since this is the case, what strategy we can take to make our life easier that we draw the moment vectors also along the force vectors, if the planes are on the right hand side and opposite to the force vector, if the plane is on the left hand side. Thus what we will do? We will draw the moment vector also in this and the whole thing, actual moment vector diagram will be obtained by rotating the whole system as a rigid body by 90 degrees. This will be the moment due to the unbalance (Refer Slide Time: 14:30). Let us, start from one.

Since, plane one on the left hand side, x is negative we will have a first moment opposite to this because x_1 is negative. Similarly, m_2 e_2 x_2 will be also opposite to this because x_2 is negative, but I think, if we go to three then you will find it will be in this direction, if the balancing plane or plane of unbalance be on the right hand side. Let this becomes m_3 e_3 x_3 and similarly, we keep on going finally, we come to this as m_n e_n.

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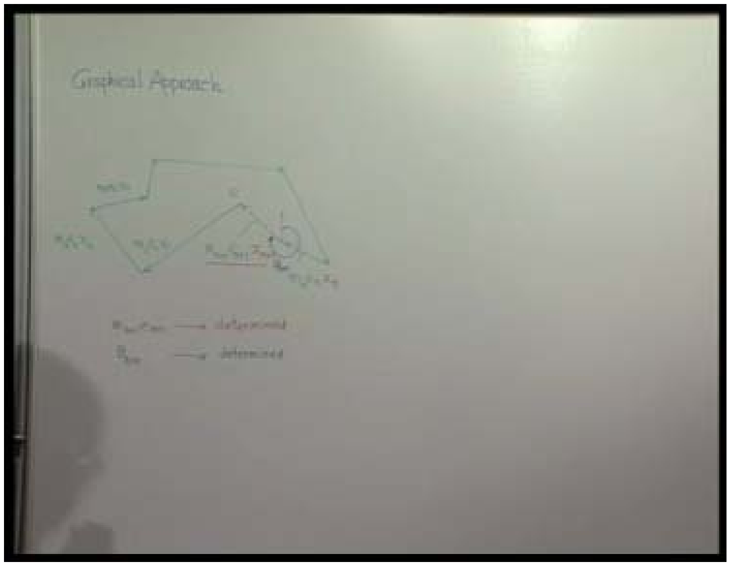

If we couple-balance or couple as to be balanced then this must be $m_{n \text{ plus } 1}$ $e_{n \text{ plus } 1}$. So, what we have got from this? The magnitude and direction of the couple, which needs to be generated by the balancing mass placed at n plus 1.

Since, magnitude is known $m_{n \text{ plus } 1}e_{n \text{ plus } 1}$ is determined; again this plane view on the right hand side that is $x_{n \text{ plus } 1}$ is positive, then the force vector that means, the $\text{theta}_{n \text{ plus } 1}$ will be also obtained from this as this (Refer Slide Time: 17:43). Thus, one of the unbalance that means, one of the balancing mass to be placed, both the magnitude and angular orientation is determined. We know that it has to be. Once, this is known we can then proceed to draw the force vector diagram because now only one force is unknown that is the balancing mass placed at plane 0. So, to do that we have to draw the force vector diagram now, this is the moment.

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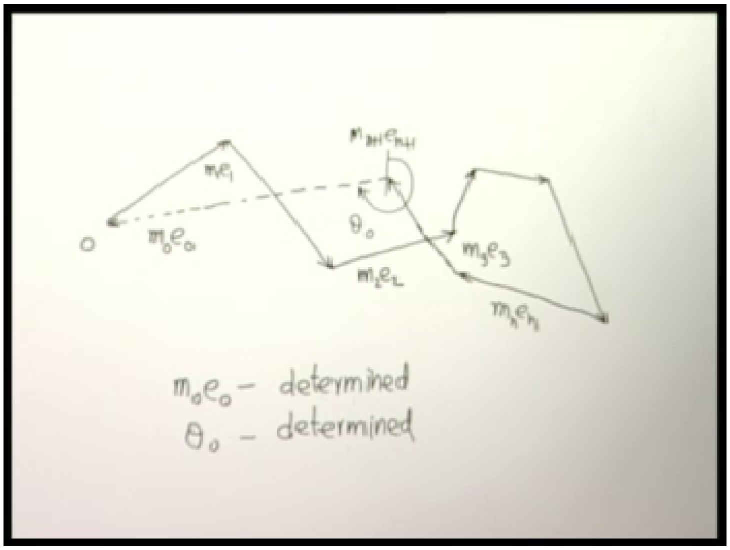

The force vector diagram will be again, will start from 0. This is m_1e_1, m_2e_2 will be in this direction, m_3e_3 will be in this direction and then we have two more as placed here. This is m_ne_n and even this one is known to close the polygon, your m_0e_0 which is the only other one force left must be this. Again, we find out its angular position theta$_0$ and its magnitude m_0e_0. Thus, m_0e_0 is determined and of course theta$_0$.

So, we have solved the problem though we have taken a particular case but it is fairly general and any ((rotate)) rotary balancing problem can be handled in this way. Of course, it has given us better physical insight and sometimes the solution also is ((vague)). As you know, that graphical procedure is having some inherent disadvantages that means, accuracy of the results what we get will be always subjected to your measurements, drawings and such things. If a problem is repeated again and again on trial and error basis, then again graphical procedure is not a very convenient one. So, in these days of computers you should go for an analytical approach.

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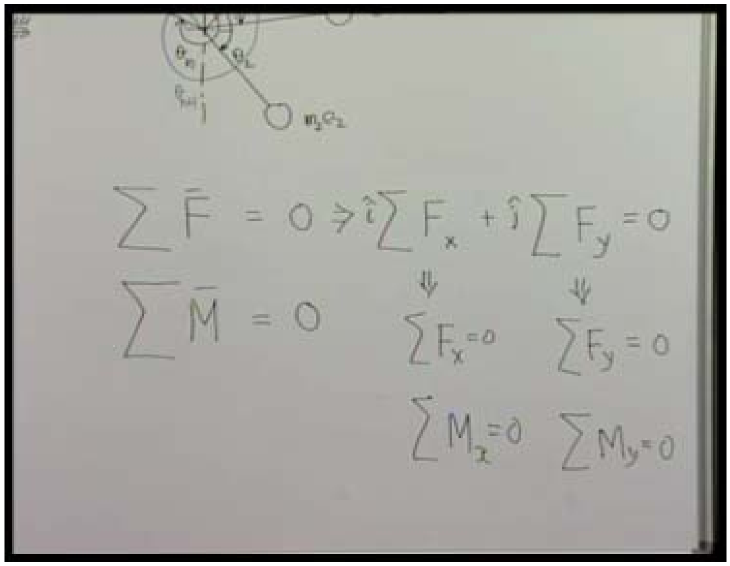

Next, what we do? We discuss the approach by using an analytical technique to find out these four unknown quantities, the problem is the same. Again, in the analytical approach, we should keep in mind that our primary objective is to place balancing masses in these two planes of such magnitude and of such angular orientation; that the total force generated due to rotation or the centrifugal force that is, the inertia force is 0. That means, vector sum of total inertia force generated will be 0 and total moment of all these inertia forces is also 0. This is the requirement for any dynamical analysis, this we have to keep in mind.

Once, we know that if the vector is 0 that means, resultant vector is 0. Then we should also keep in mind that if we resolve this resultant in two directions, the components will be also 0. In other words that if we sum up the horizontal or say x component, vector sum of x components say i and vector sum of j component is 0. This will be 0 if and only if both the sum of the x component is 0 and sum of the y component is 0 individually.

Same if the case here also; that if we take the x components of this moment that sum will be also 0, y components of the moments will be also 0 by the same logic. So, we will get four equations and we have seen that we have four unknowns m_0e_0, $theta_0$, $m_{n\ plus\ 1}e_{n\ plus\ 1}$, and $theta_{n\ plus\ 1}$. These four quantities are unknown quantities can be determined by solving these four equations. Normally, we take a tabular approach, to make the whole process more systematic and that is what I just explained now.

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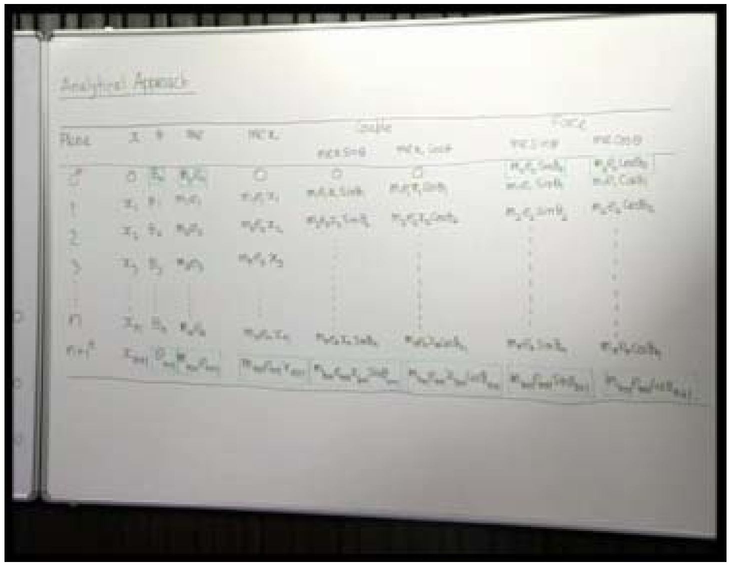

So, the columns are- first column is indicating the plane in which the mass is there, x is the distance from the reference plane, which in most places we select as the 0th plane that is one of the balancing planes. So, to indicate that, we should put some kind of a star that these are the balancing planes where you have to decide the masses.

On the other hand, the other ones are given x for these of course, x will be 0. In this case, it will be $x_{1,}$ $x_{2,}$ $x_{3,}$ x_n and $x_{n \text{ plus } 1}$ all these quantities are known, no problem. Here it is $m_0e_{0,}$ $m_1e_{1,}$ m_2e_2 and m_3e_3 and what you have to determine is this and this (Refer Slide Time: 27:45). Here it will be obviously 0, I have not put here I should put somewhere here; theta also will be $\text{theta}_{0,}$ $\text{theta}_{1,}$ $\text{theta}_{2,}$ $\text{theta}_{3,}$ theta_n and $\text{theta}_{n \text{ plus } 1.}$ Here theta_0 and $\text{theta}_{n \text{ plus } 1}$ are not known.

Therefore, the unknown quantities what I marked is: this is the unknown quantity, this is the unknown quantity, this is the unknown quantity and this is the unknown quantity (Refer Slide Time: 29:08). These four quantities are to be determined.

Here of course, couple it will be 0, this component will be 0, this component is also 0 because m e x itself is 0 (Refer Slide Time: 29:23); here it will be m_1 e_1 x_1 sine theta_1, here it will be m_1 e_1 x_1 cos $\text{theta}_{1.}$ Here of course, it will be m_0 e_0 sine theta_0, here it will be m_0 e_0 cosine theta_0. Next one will be m_2 e_2 x_2 sine theta_2, it will be m_2 e_2 x_2 cos theta_2, this is m_1 e_1 sine theta_1, m_1 e_1 cos theta_1, m_2 e_2 and this will go on, it will be m_n e_n x_n sine

θ_n, this will be $m_n e_n x_n$ cos θ_n, this will be $m_n e_n$ sine θ_n, this one will be $m_{n+1} e_{n+1} x_{n+1}$ sine θ_{n+1}. So, our table is complete.

As I have always done earlier, let us mark the unknown quantities: this is an unknown, this is an unknown, this is an unknown quantity, this is an unknown quantity, this is an unknown quantity and this is an unknown quantity ok (Refer Slide Time: 32:35).

So, as you can see that from these four equations what it means that this sum should be equal to 0 (Refer Slide Time: 32:47). This is the couple x component, this is one sum cosine component of the couple, it would be also 0. This is the sine component of the force, it should be 0; the cosine component of the force should be 0. Here of course, we have taken this as the x and this as the y components if we sum up this and make it 0.

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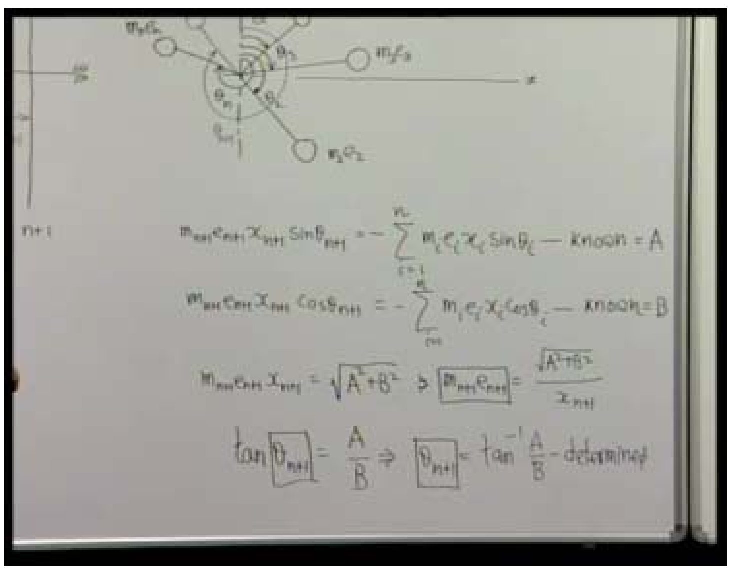
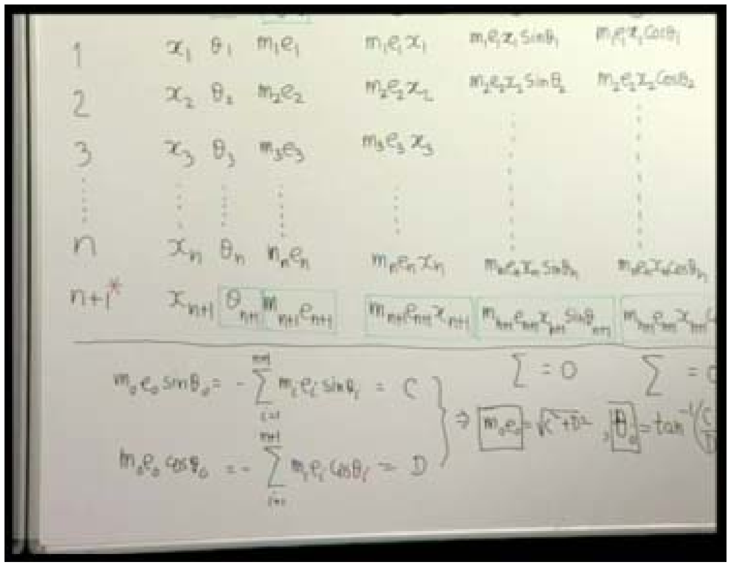


So, you can find that we get an equation with only one unknown term. So, this equation will give us that $m_{n+1} e_{n+1} x_{n+1}$ sine θ_{n+1} is equal to minus $m_i e_i x_i$ sine θ_i, i is equal to 1 to n; all of these are known quantities, so this is determined. Similarly, because the sum is 0, you get from that equation the other components of the moment you also know… say this is equal to A and say this is equal to B.

Hence, we can say that $m_{n+1} e_{n+1} x_{n+1}$ is equal to square root of a square plus b square; squaring the two terms and adding and this will give us a known quantity. Since, x_{n+1} is known or given or chosen, we know the actual unbalance or balancing amount to be placed is $e_{n \text{ plus } 1}$. So far as the magnitude is concerned, we have found it out. We can also find out, the angular position from this dividing the two. So, from this theta_n also n plus 1 it determines.(Refer Slide Time: 37:02)



Thus, this is also determined. So, from the column or two columns showing the components of the couples they give us one unbalanced mass or balancing mass and its angular position. Once, that is done then from the force columns, then this becomes already known because $m_{n \text{ plus } 1}$ $e_{n \text{ plus } 1}$ and $\text{theta}_{n \text{ plus } 1}$, they are all known quantities.

So, this is known and this is known (Refer Slide Time: 37:17). From this equation and this equation what remains only one unknown is there and we can write is equal to minus sum total of i equal to 1 to n plus 1. Let us consider this is c and the cosine component let this be d. So, these are now known, these two will give us m_0e_0 equal to square root of c square plus this square and tan or rather theta_0 equal to tan inverse c. So, both this and this gets determined.

So, you can see that following an analytical procedure explained in the form of a table. We can find out the four unknown quantities; that are the magnitudes of the balancing amount to be placed in the two balancing planes and their relative angular positions; these are found out in this manner.

Here obviously, answers will be more accurate because we have followed an analytical procedure. This is the way at the design stage when the designer knows the kinds of various objects are coming and the centers of mass of the various components coming; it can be possible by the designer to identify suitable balancing planes and to put suitable balancing amount. So, the whole system is completely free from any unbalanced inertia force.

Another kind of problem which needs to be discussed is that when we are given a rotating body; if a rotating body is given to us sometimes, it may be necessary to take care of its unbalances, which are produced by some errors in manufacturing.

Some misalignment during an assembly and such problems; in such cases, we have to take these bodies either to a machine where we can put the rotor or rotating system and do experiment to balance it or sometimes we may take the approach, that going to the location where the rotating object is present and try to do some experiment in balance. So, in the first case we take care of the balancing procedure with the help of some machines which you call balancing machines.

In the second case where you do not use any machine to put the rotor on that but, you go to the field to do the balancing, there itself we call that procedure as field balancing. In the next lecture we will take up these (( )).