Dynamics of Machines
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Module No. #3
Lecture No. # 1

Unbalance in Machines and Balancing of Rotating Systems

In the previous lectures, we have discussed how to find out the forces, moments, acting on the various components of a machine when it runs. Once these forces, moments which are acting on the various components are known, we can design the dimensions or we can find out the dimensions of the various elements. So, thus, we can have a physical system with basic dimensions determined by the kinematic analysis, and the actual dimensions of the components and its various elements are determined through the determination of the forces and moments.

Next question is, once a machine is designed this way and fabricated, is it satisfactory or will it run properly, or its function will be satisfactory? That is the question now.

Now, we will now see that it is not enough to design a machine, just on the basis of its kinematic functionality and the determination of the forces and moments to find out the sections and dimensions of the components. What we need for a machine to be perfect in this design and construction is that it should run smooth. What is meant by running smoothly?

So, if you have a machine, So if you have a machine. Just (()) by the outline, and it is supported. When static condition, when the machine is not running, the support will be subjected to only the gravitational pull which does not change its time; that means, it is a static force; its magnitude and dimension, both are constant and they do not change in time. But if the machine starts running and then the support starts getting force and moment which are changing with time, or varying or fluctuating with time, then the whole system will definitely have a disturbed situation; that means it will vibrate.

We all know from our experience that having a machine and if it runs and if it is not properly designed, it will vibrate. So, therefore, the condition of proper design should ensure that, when the machine runs, the support which carries the whole system, that support is not subjected to dynamic forces or moments; that means what? In common terms, we call that the system should be balanced.

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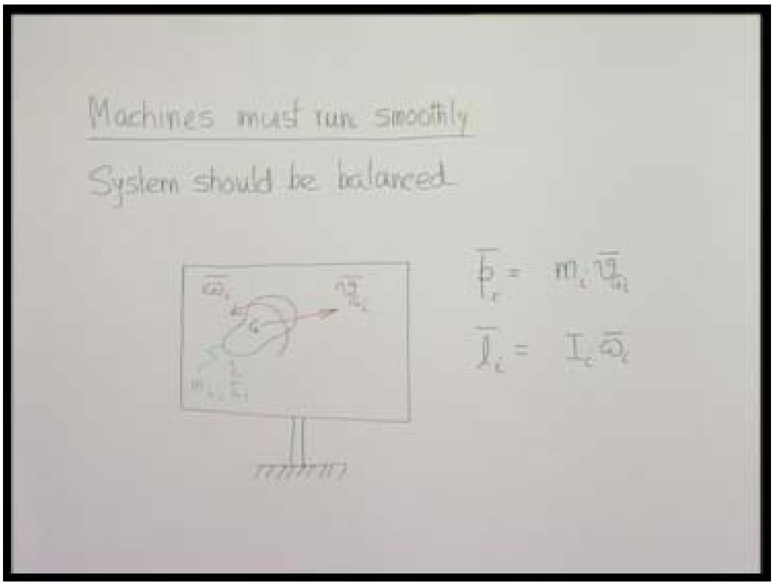

Before we take up the problem of finding out how to balance a system or when a system is balanced, let us first find out that why dynamic forces appear or they get exerted on the support, once the machine starts running.

Let us consider one of the components of the machine say I, whose velocity of the center of mass is V G I, G being the point and it also has an angular velocity omega i at this instant. Let the inertial properties of these members be denoted by m i that is mass of the object and I i that is the centroidal moment of inertia of the object. Then the angular and linear momentum will be given by (Refer Slide Time: 05:28).

So, when we consider all the bodies together consisting the or consisting the whole system or machine, then the total and linear momentum of the whole machine will be given by the sum total of all the individual linear momentum sum over all the components. If there are n number of (Refer Slide time: 06:17), and similarly, the total angular momentum of the system will be vector summation of the individual angular momentum.

Now, if this momentum both linear and angular momentum remains constant with time, then external force applied by the support and external moment by the support on the body will be zero. On the other hand, if they are not zero, then what it is… let us see. Then, what will happen? If this total linear momentum changes with time, then this linear momentum rate of change is nothing, but the instantaneous linear force acting on the whole system.

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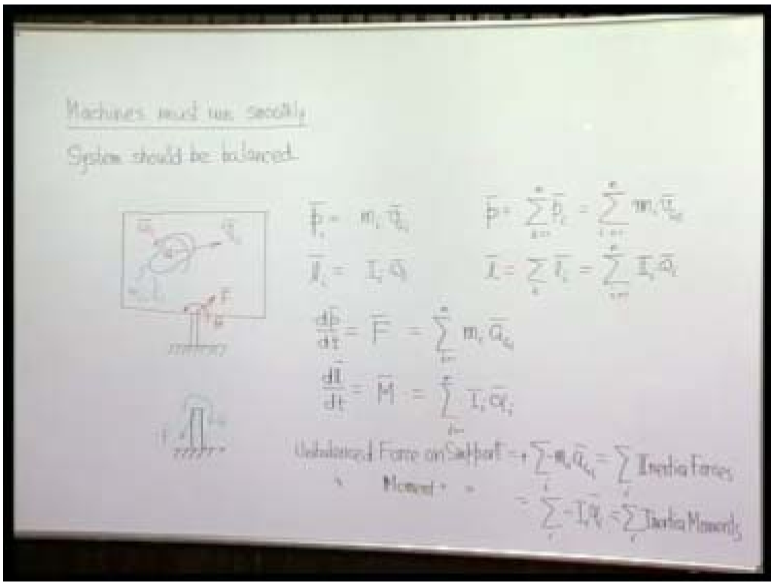

Now, a force besides the weight can act on the system is only through the support. So, this force is responsible for the rate of change of linear momentum. So, this is nothing but m is constant; so, V G i d t will be (Refer Slide Time: 07:55). Similarly, the total momentum instantaneous rate of change is nothing but the moment which is acting on the body from outside, where alpha i is the angular acceleration of the ith component a G i is the linear acceleration of the center of mass of the ith component. So, therefore, we find that the system or the support will receive a force from the machine like this (Refer Slide Time: 08:44).

If this is the support, support will receive equal and opposite forces as an effect of the machines running. So, what is the unbalanced force received by the support is nothing but minus m i a G i which is nothing but the sum of the inertia forces of all components. So, inertia force of the ith member, if you sum it over all the members i from one to n, that inertia force when you sum over the whole machine, then actually this (Refer Slide Time: 10:11), then you will get the total unbalanced force which will be experienced by the support. Similarly, unbalanced moment on the support will be nothing but (Refer Slide Time: 10:29), which is nothing but the sum of inertia moment.

Thus, the reason for support experiencing unbalanced force and moment is the non-zero inertia force of the system at an instant or non-zero inertia moment of each member at an- instant. The only way we can have total force acting on the support to be zero is that the total systems inertia forces all need to rise amongst themselves or we add some extra bodies to utilize that, and at the same time, if all the inertia moments of the various components utilize themselves or we add something extra to utilize the resultant, then only the total dynamic force and moment on the support will be zero, and the running of the system will be smooth and it will function properly; there will be no vibration; life of the whole system will be long.

So, therefore, we will now have to be further active in designing machine, not just satisfying the dimension, not being enough to produce the required motions kinematically; it is not enough for the components to have enough strength to sustain the forces and moments, but the mass distribution of the whole machine should be such that the support is not subjected to any dynamic force.

Now, this apart from these forces (Refer Slide Time: 12:25), there is of course, one static force, all the time acting; that I have not shown, but it will be always there. You should remember that; that is the weight of the (( )), but this weight is a static force. It neither changes in direction nor in magnitude, and therefore, we are not worried about that. But both these F and m will change continuously as this quantity a G i and alpha i, they fluctuate with time because the machine, remember, runs.

So, as I mentioned just now, there are two ways we can have no inertia force acting on the… or no resultant inertia unbalanced force acting on the support is that, where the individual inertia forces are balanced or it is zero or they balance or they utilize each other when there are more than one system, same with inertia moments. And therefore, there are what we will now consider is that an activity to make machines run smoothly and the whole process is called balancing.

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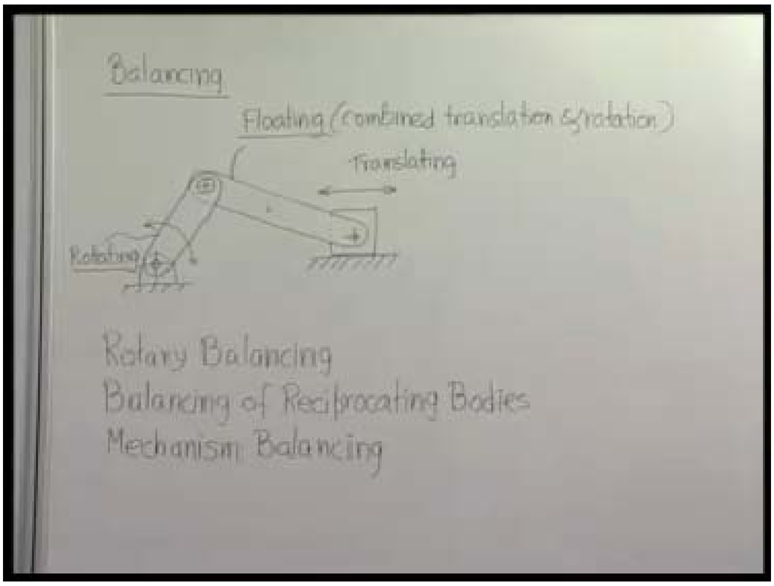

Now, in any machine, we generally have three kinds of members; say, for if you take this one, say large number of machines, you will find. Just I am taking an example.

Large number of machines consist of a mechanism what we call as slider crank mechanism. Now, here, we find there are three moving objects; one which you call the crank, already you know, the purely rotating member, it makes pure rotation about this hinge. Similarly, this one makes pure translation. So, this block also executes a motion which is also a simple type. That we call pure translation, but the problematic component is this (Refer Slide Time: 15:36) which is having both the translation and a rotation. So, it is a combined motion. This we call floating moment, and it is a combined translation and rotation.

So, a machine in general or a mechanical system in general will consist of three different kinds of objects; one type where the components are executing pure rotation; in another situation, the components execute pure translation; either rectilinear or curvilinear translation; another kind, where the objects are having combined rotation and translation. So, we will take up the subject of balancing for three different situations. So, one is rotating body or rotary balancing and balancing of translating object or reciprocating objects, and finally, mechanism balancing. So, in a (( )) what we have done so far is that, we have found it is not enough.

A machine to be designed only on the basis of its kinematic requirement and the forces and moments acting on it and strength design or rigidity design for proper functioning of a machine. It must run smoothly. And what is meant by running smoothly is that, the support on which the machine is placed, are not subjected to this support. Support is not subjected to any force or moment that fluctuates with time. That means it should not be subjected to any dynamic force or motion. There will be no vibration and it will run smoothly.

Now, what are these forces experienced by the support? dynamic forces.

This, apart from the static force weight, the dynamic force and the dynamic moment which are being experienced by the support are nothing but the sum total of the inertia forces of each of the all the components and sum total of inertia moments of all the components. We have also seen that a system or machine will have number of components; some of them can be of purely rotary type. An example of that is the crank of a slider crank mechanism. Some of them can be reciprocating body or translating objects as the slider of a slider crank mechanism or it can be a floating or complex motion or combined motion body as in case of the coupler of the slider crank mechanism.

And we will now handle the problem of balancing a machine; that means bring something so that it runs smoothly. All the forces acting either neutralize among themselves or we put some extra object to neutralize the resultant unbalanced forces and moments, and that we will take up one by one.

First, we will take up only rotating objects and how to balance those; then we will go for reciprocating bodies and their balancing; finally, we will discuss something about the mechanism balancing. This balancing problems which we will take up now, one by one, is a theoretical way of bringing it at the design stage. Sometimes, what happens, that after the machine is built, if it is very big and if it is placed on its bearings or on its support, the deformations etcetera due to its gravitational pull can make the system distorted from its original idealistic design and it may give rise to some unbalanced effects. And in such cases, it has been found that it may be necessary to do some balancing in its actual working position.

That kind of an activity is called field balancing and we will take up how to do it; the procedure and the theory of it also.

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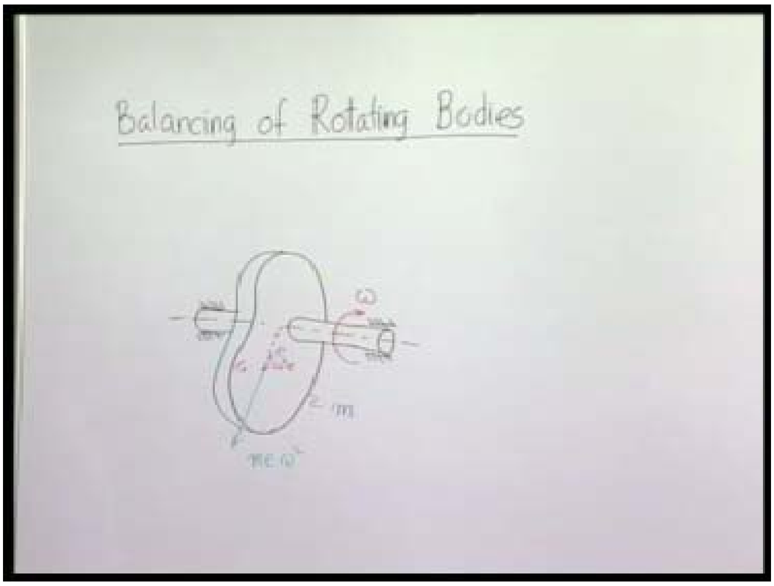

Let us take up a simple case. When an object is mounted on a shaft which is rotating, let us see what will be unbalanced force experienced by the bearing or the support. Now, we know that there can be two situations. The object itself may be of irregular shape by virtue of which the center of mass is away from the axis of rotation, and let the eccentricity of the center of mass be denoted by e, and let the mass of the whole object which is rotating be m.

There can be two situations, as I mentioned just now, one is that the shape itself is like that, that the center of mass is away from the axis of rotation. There can be another situation that it is a perfect disc, circular disc, and supposed to be symmetry body. But even then, the center of mass of that may have a small miss alignment with the axis of rotation because of assembly which is not by design, but by error.

Now, when this shaft is rotated at an angular speed omega, then we all know that the inertia force of the object is what? The acceleration of the center of mass which is towards the center, which is omega square into e into mass that is m omega square e and this force, will be Inertia force will be minus of that. So, it will be…(Refer Slide Time: 23:30)

This is the…acceleration is in this direction; I should have indicated it. And minus m into acceleration; that is, the inertia force is the centrifugal force; so, you all know that. Now, therefore, at any instant, what will happen to the support?

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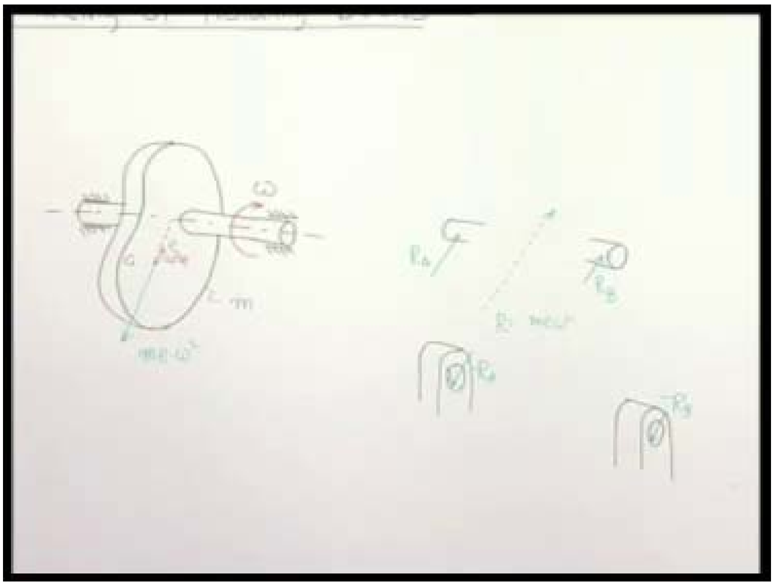

That means, this body will be then subjected to the two reaction forces, will be acting on this, whose resultant will neutralize (Refer Slide Time: 24:35) and in this direction. So, therefore, for the dynamic view of this, the two bearings or support will be subjected to two forces whose resultant is equal and opposite to this. And since the whole thing is rotating, these two forces which is also in the same plane, that means the plane constituted by R A and R B also contains the force, centrifugal force and the whole plane is rotating with a speed omega.

So, for these two forces, though their magnitudes are fixed, their direction is continuously changing. That means it is a dynamic force, and as a result, the bearings, if you see the bearings of the support. If this be the support then this will be subjected to forces. So, the supports are also subjected to the opposite of these forces and which rotate along with this shaft. So, therefore, the support is subjected to a dynamic force.

Now, this is a very complicated way of drawing all the time. So, henceforth what we will do?

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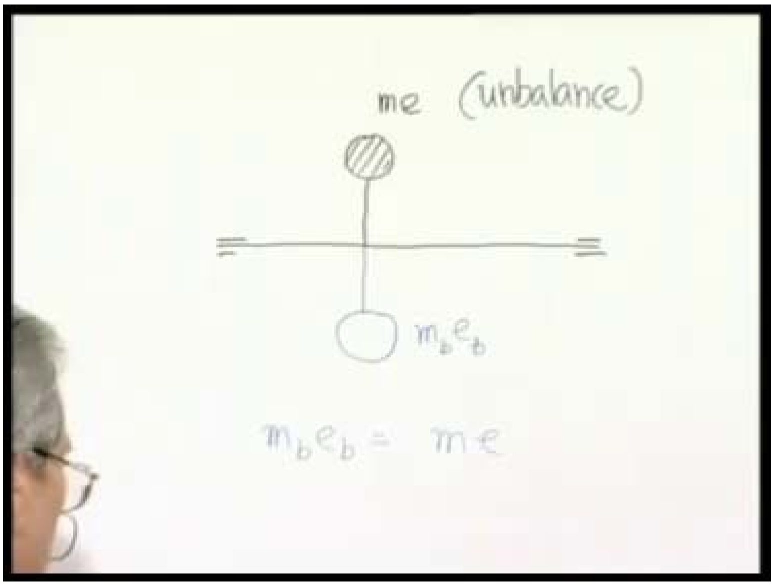

We have a simplified way of representing a dynamic body, rotating dynamic body. If this be the support axis, then let this be the eccentric mass and now as we find that it is the product of the mass and eccentricity which determines the force, this (( )) is omega. We will always represent the unbalanced as a product of mass of the object and the eccentricity of its c G from the axis of rotation.

Now, the simplest way to resolve that means, if we have to balance this, what will be the simplest way? We will say that. ok. We can add another mass here m b at an eccentricity e b so that m b e b is equal to me.

If that happens, then the centrifugal forces of the two bodies, one originally existed of the system and another one added later; so, they will neutralize each other, and therefore, there will be no resultant force on this support (Refer Slide Time: 24:47).

What we have done by doing this is, effectively, we have brought the center of mass of the total object, just on the axis of rotation. If the resultant center of mass comes on the axis of rotation, resultant inertia force will be zero that is resultant centrifugal force will be zero. Sometimes, it may happen that there is no space of adding a mass like this. So, in such cases also, there is no problem.

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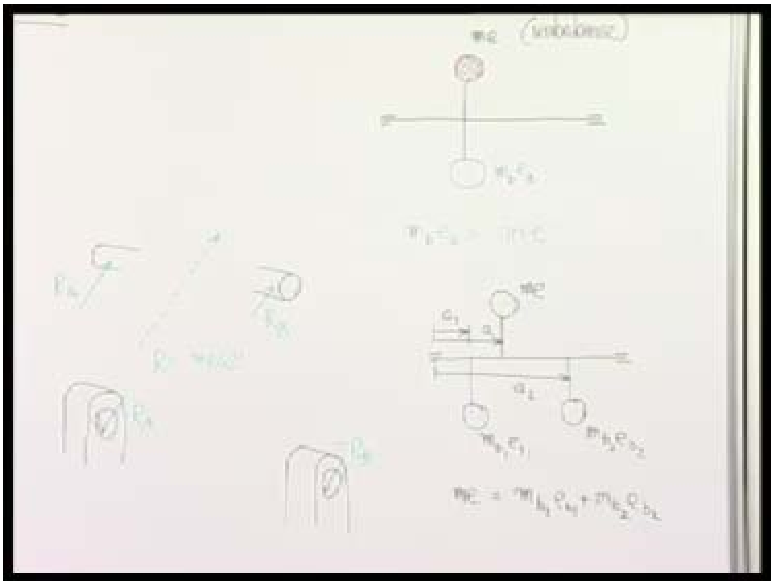

What we can do? We can add two masses instead of one. So, we will call m A b e b e A b M B b e B b, in such a way, that we call this as, or rather I will follow something which I will take up later. Also, I will represent this by 1 and 2. M b 1 e b 1; M b 2 e b 2; this is a 1; this distance is a 2 and this distance is a.

In such case, we have to satisfy two conditions, that the resultant centrifugal force by these two balancing masses produce is equal in magnitude, and of course, opposite in direction with that. So, that will tell us that since all of them are rotating at the constant angular velocity or same angular velocity omega; so, their total centrifugal force of the balancing masses will be equal to the centrifugal force of the unbalanced, if this equation if satisfied (Refer Slide Time: 30:06).

So, this will ensure that the sum total of the two centrifugal forces here is equal to the centrifugal force of the original unbalanced. So, they will neutralize, but to neutralize that we need another condition that they must also be equal and opposite, and pass through the same line.

Now, the condition which will tell that the resultant of these two centrifugal forces will pass through this point, you all know, from basic mechanics is that...(Refer Slide Time:

30:43). So, this is the condition which needs to be satisfied if the resultant of the two has to also pass through the same point, along which the original unbalanced centrifugal force is acting. So, therefore, we have two equations and we have two unknowns.

Now, as I have mentioned already, it is the product of the mass and the eccentricity which is dictating; we cannot individually find out mass and eccentricity unless other conditions are prescribed from design point of view, but we can definitely find out the this unbalance, and this unbalance (Refer Slide Time: 31:33) satisfying these two equations.

We have two unknowns' m b 1 e b 1 and m b 2 e b 2. These two unknowns can be determined following these two equations, and in this case, even if they are not along the same direction, the resultant system will be free from any unbalanced forces as experienced by the support. But there is a difference which you should not forget. In this case, the force which was acting here is neutralized by a force here (Refer Slide Time: 32:05); both were equal. In this case what happens? A force which was here is neutralized by two forces here.

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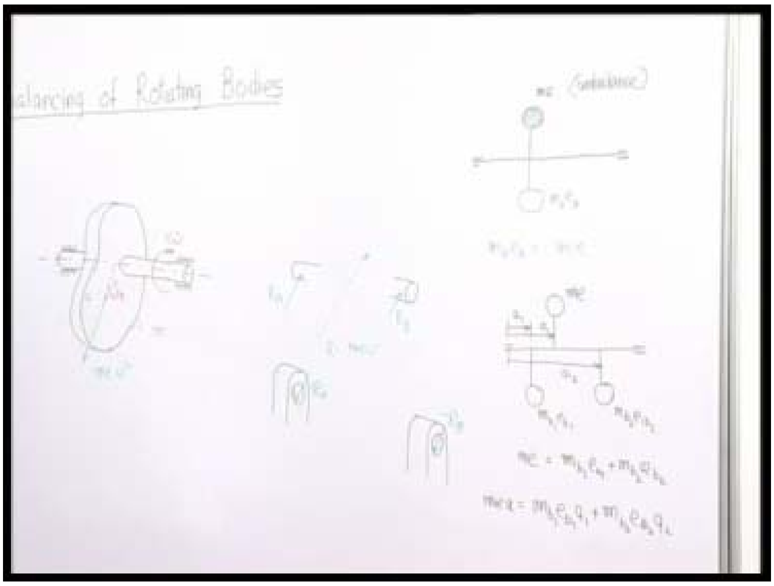

The difference is that though, so far as the supports are concerned, both will be free from dynamic forces. In this case (Refer Slide Time: 32:35 to 33:01), the shaft is completely free from any effect due to the centrifugal force; there will be no bending moment or stress to reach the shaft will be subjected to…. On the other hand, here we will find this portion of the shaft will be subjected to a bending moment and therefore definitely it will be subjected to some stress because of the rotation. And if the rotational speed is very high, this bending moment here can be, of course, important for the design of the shaft

So, finally, to say that, in this case the balancing is absolutely complete; even internally there is no effect to be seen, and that is called internal balancing or the system is internally balanced.

In this case (Refer Slide Time: 33:31), it is not completely free from stresses etcetera due to rotation and unbalance forces, but the support is free form that. So, we call it external balancing, or the system is externally balanced. There can be two different situations.

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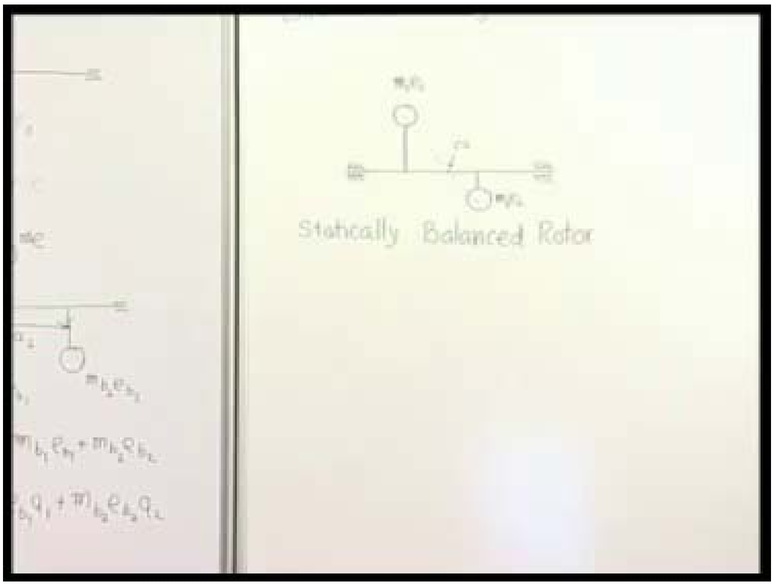

In rotary balancing, there is another situation which we have to keep in mind. What we are doing, it may appear that our primary objective is to shift the center of mass to the axis of rotation. When it happens, then the resultant centrifugal force is zero, but that may not be always enough for balancing. That means if you take a disc or a rotor and we put it on some very free bearing, if the system is completely balanced, that means the center of mass of the whole thing is exactly on the axis of rotation. Then, you can put the whole system at any position; it will remain there; that means it will be in static condition. You can put it anywhere. It will appear to be balanced. Such systems are called static balancing.

And for static balancing, what is needed is that the center of mass has to be on the axis of rotation. That means, now, if we have two unbalances, in such a way that their center of mass is here (Refer Slide Time: 35:33), then this system in static condition, it will appear that it is balanced because you can put it any angular position; on a free bearing, it will remain there. So, this is a statically balanced case. So, this rotor or rotating object is statically balanced, but what happens when it starts rotating?

Moment it starts rotating, this centrifugal forces will appear and this centrifugal force and this centrifugal force (Refer Slide Time: 36:20), they will produce a couple, and when this is rotating, in rotating condition, in rotating condition, these two forces - equal and opposite forces, they are going to produce some unbalanced force on this because now this couple which is acting on the shaft (Refer Slide Time: 37:08) will need to be neutralized by equal and opposite couple produced by the bearing reactions which are also in that case equal and opposite.

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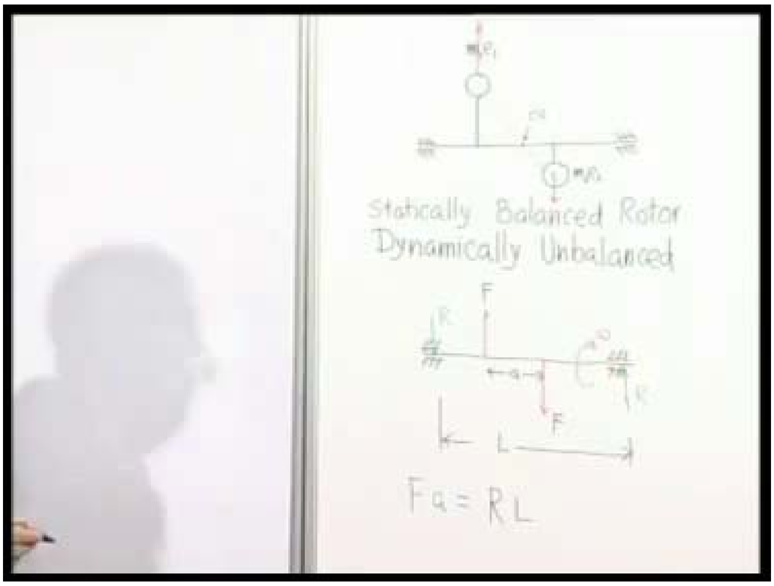

Only thing what you have to satisfy is that, if this is F, and F, this is a, and if this is L, then F into a, this couple must be balanced by the couple produced by the bearing reactions R into L, but nevertheless, we find that the bearings are subjected to forces which also rotate along with the rotor, and therefore, these are dynamic forces. So, as soon as it starts rotating, the bearing starts experiencing dynamic forces. So, therefore, the same system which appeared to be perfectly balanced in static condition becomes unbalanced in the dynamic condition. So, this system is statically balanced, but dynamically unbalanced. Dynamically unbalanced. Now, to balance this, what we have to do?

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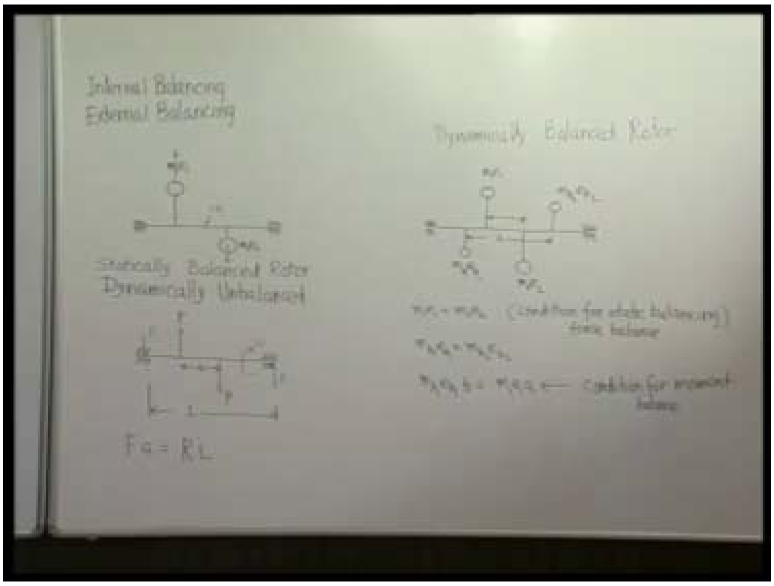

That means if we have to produce a rotating object which is both statically and dynamically unbalanced, we have to ensure that the resultant force and resultant moment produced by the objects… then what you can do?

See if you had these two objects, in such a way that it produces a couple, if this is b and that is a, then the couple produced by this when m 1 e 1 is equal to m 2 e 2. That is the condition for static balancing. Now, if we have m b 1 e b 1 which is equal to m b 2 e b 2, and m b 1 e b 1 into b is equal to m 1 e 1 into a; this will produce an equal and opposite couple, so that the unbalanced couple produced by these two masses will be neutralized by the unbalanced couple produced by the other two balancing masses. So, the resultant moment and resultant force, both should be zero. So, this is the condition for dynamic balancing. This is the condition for static balancing or simply force balancing. So, when both the conditions are satisfied, Condition for force balance and condition for moment balance, then the system is dynamically balanced.

Next, what do we want to do?

We want to show that any rotor consisting of any number of unbalanced masses can be balanced by two balancing masses at any two chosen planes. These any two chosen planes which we can take for putting the balancing masses, of course, depend on the design considerations and other features of the system, but theoretically, any two planes you can select, where you can put two balancing masses that can balance any rotor with any number of unbalances. It is extremely important, this theorem because that shows that to balance any rotor because even a continuous rotor may be treated as infinite number of thin discs which are individually unbalanced, and so, even a continuous rotor can be balanced by only two balancing masses placed at your chosen planes.

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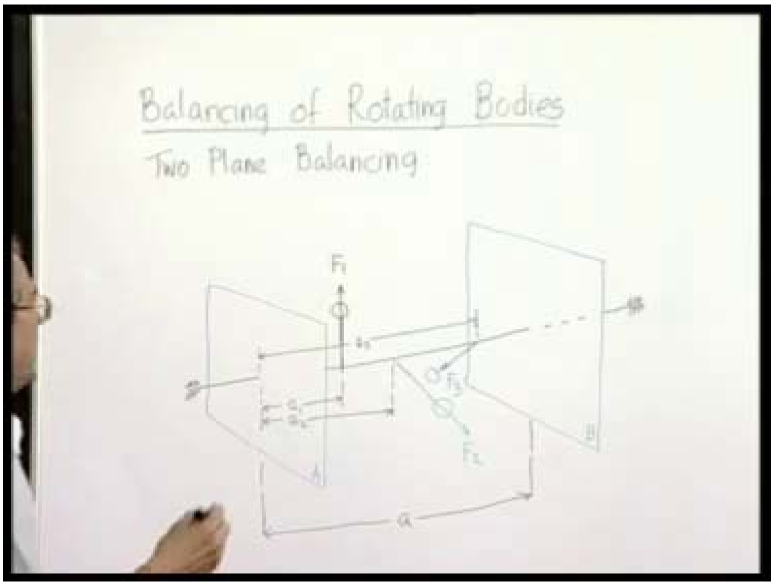

Now, let us take first two planes; let us draw it like this; so, let this be plane A; this be plane B, and this is the shaft which carries the unbalances. So, let us consider a case with three, as you will see very soon, that irrespective of the finite number I am selecting. So, let us consider only three unbalances (( )) here: m 1 b 1, m 2 b 2 e 2 and m 3 e 3.

Now, it means what? That this will produce a force F 1; this will produce an unbalanced centrifugal force F 2 and this one will produce an unbalanced centrifugal force F 3 (Refer Slide Time: 45:20).

Now, let us consider distances. So, let this be A 1; let this be A 2 and let this be A 3. The location of these unbalances and the overall distance between the two planes is say capital S or say a.

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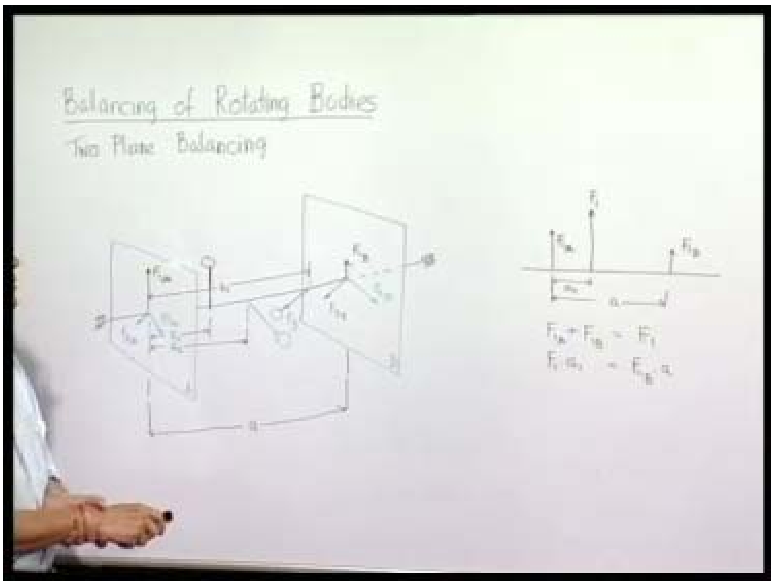

Now, any force here can be represented by two forces; that means, suppose if I have a force acting here F, I can always represent it by two forces at any two points by F 1 A and F 1 B, so that these two forces' resultant is F 1. So, I can replace the force by this. So therefore, what I can do?

I can replace this force F 1 by two forces: one is F 1 A, another one is F 1 B, and there is no more force here (Refer Slide Time: 47:14 to 48:14). Similarly, this force F 2 can be also split into two forces in these two planes and this is F 2 A and this will be F 2 B, and since I have replaced this force by these two, there is no need to have this, and finally, this force can be also replaced by two forces F 3 B and F 3 A. The magnitudes can be easily figured out.

How it is going to be?

As you can say, if this distance is a 1 and this distance is a (Refer Slide Time: 48:43), as I have chosen here, then we have F 1 A plus F 1 B must be equal to F 1 and F 1 into a 1 must be equal to F 1 B into a. This is the condition or these are the equations to be satisfied if this force system is to be equivalent to this force.

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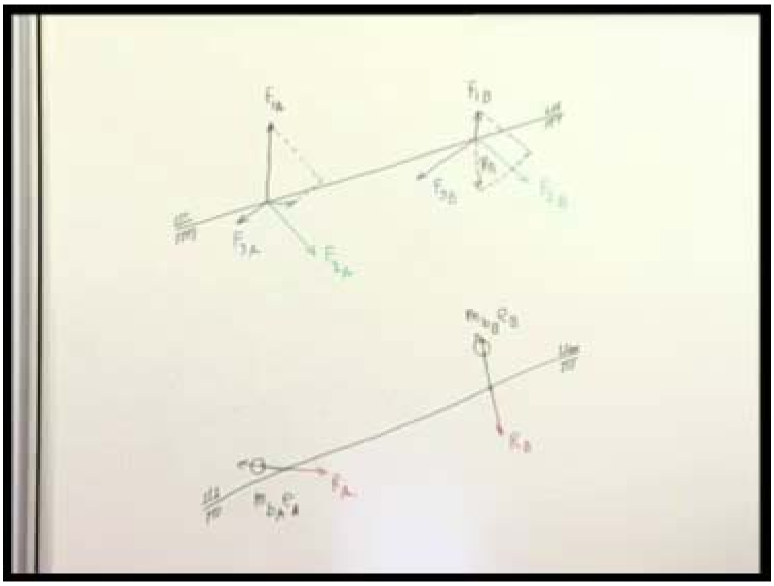

Now, you see what we have. The two planes, all the unbalanced forces are represented by an equivalent system in which, say let us draw the situation now. The situation is now like this, that we have a shaft. We have F 1 A, F 1 B, F 1 F 2 A, F 2 B and F 3 A and F 3 B. So, this force system is equivalent to the three centrifugal forces at these three locations. So, what we get, now finally, is that the resultant force acting; that means, now, this is again equivalent to… What we do? We can add these three force vectors and this becomes a resultant. So, we can say this location (Refer Slide Time: 51:35), the resultant force is R a, and the resultant here of these three vectors, if we add F 1 B, F 2 B add; then add this; so, this is R (( )). So, resultant here will be...(Refer Slide Time: 51:58).

So, finally, you would have found that the three centrifugal forces which you had can be represented by these two forces at these two locations. Now, these two locations are arbitrarily chosen, well, depending upon your convenience that where you select the two planes, and once a system you have, where you have two unbalanced forces, to balance it is very simple by putting two balancing masses M b b e b, M b a e a, so that this centrifugal force and this centrifugal force, they exactly neutralize (Refer Slide Time: 53:05).

So, therefore, with this system of three unbalanced objects, if you put two balancing masses here, M b a e a and M b b e b, then these five objects will balance each other. There will be no unbalanced force or no dynamic force experienced by the (( )).

Now, we have chosen, we have shown this whole thing with three; this number could be any because each of every force centrifugal force. We can replace by two forces in the two planes and the sum total of all these forces which you have placed here can be represented by one resultant, which can be neutralized by a centrifugal force produced by a suitably placed balancing mass there; same thing happens here.

So, if you consider even a continuous rotor which can be considered to be…(Refer Slide Time: 54:20). Then, this can be split into thin discs, infinite number of thin discs and each one of these discs may have its own unbalance. But it does not matter even if the number is infinite because it can be always balanced by only two balancing masses placed at two suitably chosen planes which depend on the designer’s convenience. So, this is a graphical description of the whole thing (Refer Slide Time: 55:08), but i think we can sometimes find it convenient to solve such problems in an analytical procedure which I will start discussing in next lecture.