Dynamics of Machines

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Module-2 Lecture-4

Gyroscopic Action in Machines

In the last lecture we have derived the equation relating the components of the external moment acting on a rigid body, with the various components of its angular velocities, angular accelerations and inertial properties. What we derived in the last class is that, if small x, small y and small z be a set of axis along the directions of the principal axes of this rigid body.

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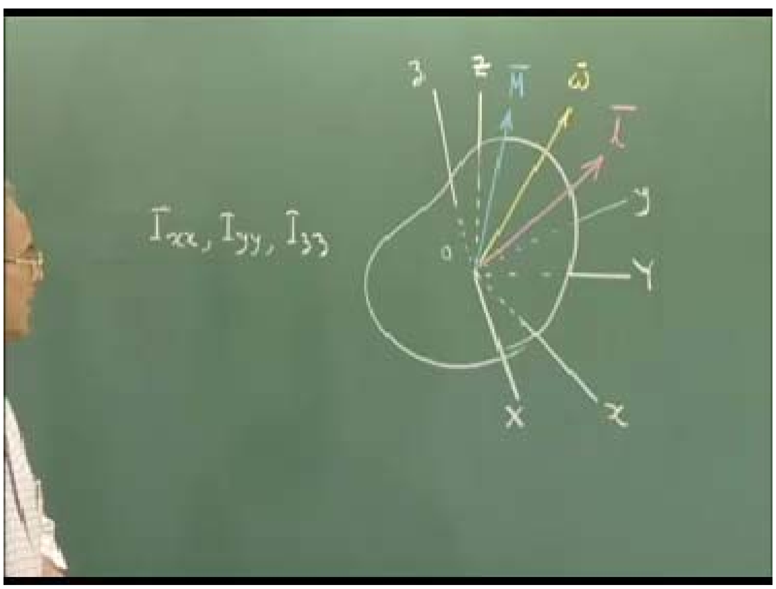

So that the products of inertia terms are zero, only the moments of terms of inertia are present. Then the moments of inertia of this in the various directions are I_{xx}, I_{yy} and I_{zz}.

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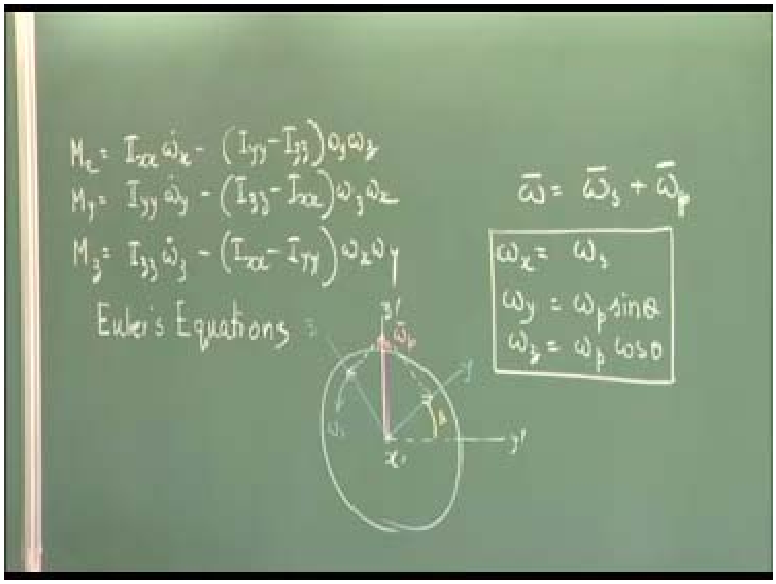

If this object is having an angular momentum l, angular velocity omega and then a moment M is applied, the relating equations will be….. (Refer Slide Time: 02:45) where M_x, M_y and M_z are the three components of the externally applied moment on the body omega$_x$, omega$_y$ and omega$_z$ are the three components of the instantaneous angular velocity of the body, and x, y, z are along the principal axes of the rigid body, which are rigidly embedded in the body. What I mean to say that the frame, small x, y, z has the same angular velocity as the rigid body itself. We derive this equation for that.

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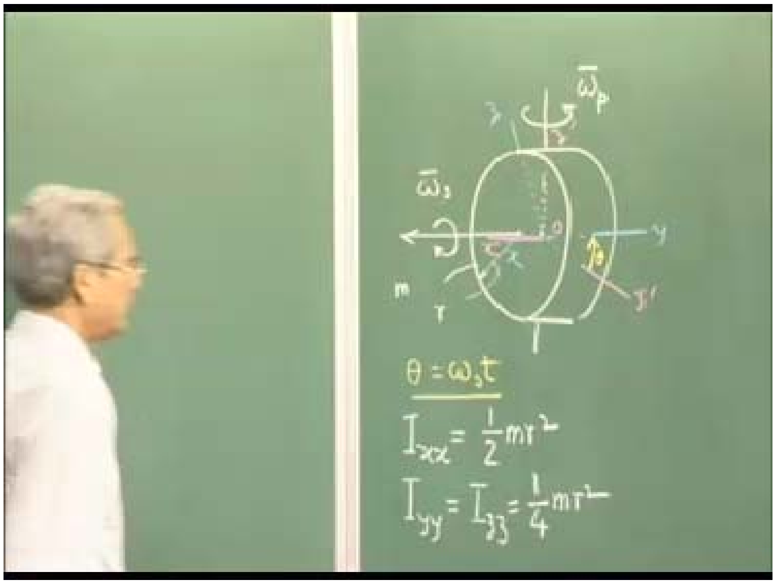

If we now apply this to a simple problem, which we encounter in engineering quite often where a disk that is rotating with an angular velocity omega$_s$ about its central axis, which has a mass m and radius r. If this body is given a rotation about an axis which is perpendicular to this angular spin, angular velocity omega$_s$, when this is 90 degrees, for this motion, if this is the motion, then what are the externally applied moments on this?

Let us find it out using these Euler's equations. What we will do first, we will attach a frame of reference small x, small y and small z, which is rigidly connected to the body.

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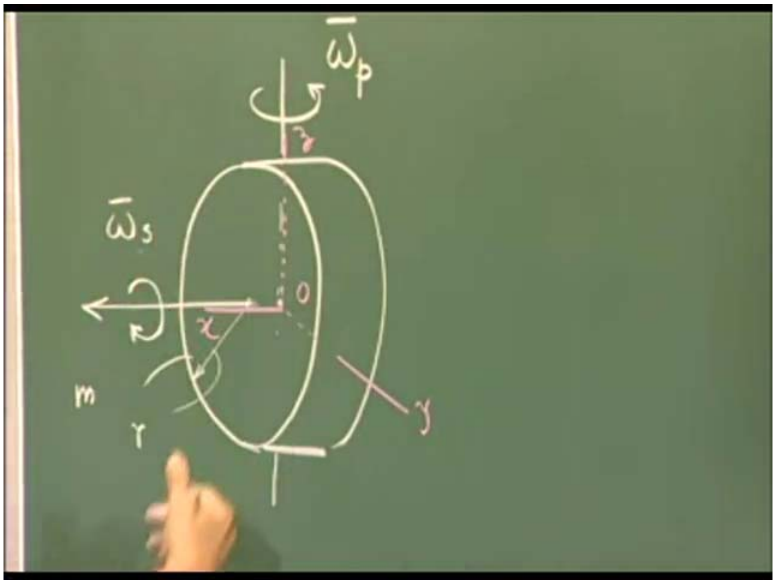

Now we should remember, since the rigid body is rotating continuously about the x-axis, the location of y and z-axis continuously changes within the space. What we can say, that these are not the instantaneous position of the x, y and z, rather this x, y and z will be the instantaneous axis, which is rigidly attached to the body and rotating along with it.

At any particular instant, we may assume that this angle is theta which is changing. Obviously theta is nothing but if you assume t to be zero at this instant, when y coincides with y prime, then it is nothing but…. Now in the inertial properties of the symmetry we understand that, this is one of the principal axes, and any diametrical axis will be also a principal axes. So I_{xx} we call the polar moment of inertia, and for a disk it is something like this I_{yy} and I_{zz} are same, equal to half of this.

Let us use these equations and try to find out. We find that M_{xx} or M_x is equal to I_{xx} omega$_x$ dot. The angular velocity of the rigid body we have to find out. You see that the rigid body has two angular velocity vectors, one is omega$_s$ another is omega$_p$. So the resultant angular velocity of the body is nothing but the vector addition of this. If we now consider i, j and k to be unit vectors along the x, y, z axis then you can directly find out the components.

What will be omega$_x$? It will be x component of omega$_s$, which is omega$_s$ itself plus x component of omega$_p$. Now since omega$_p$ is always at right angles to this, so it will be zero.

$Omega_x$ is $omega_s$, $omega_y$ if you see the frontal view of the disk, this is the x-axis, this is the z prime, this is the y prime, this is the x prime and y and z are the angular axis, rigidly connected to the body, this is theta (Refer Slide Time: 09:08). We know here that $omega_p$ is in this direction, so $omega_y$ will have a component of $omega_p$ this much. The component of $omega_s$ is zero, because it is at right angles. This is nothing but $omega_p$ sine theta and the component along the z-axis of the angular velocity. Now $omega_s$ will have no component along this and $omega_p$ will have this much, which is (Refer Slide Time: 09:52), so these are the angular velocity components along the x, y, z of the rigid body. Of course small x, y, z frame is rigidly attached to that, so now $omega_x$ dot is this minus I_{yy} minus I_{zz} , $omega_y$ is $omega_p$ sine theta and $omega_z$ is $omega_p$ cos theta. That is the first equation.

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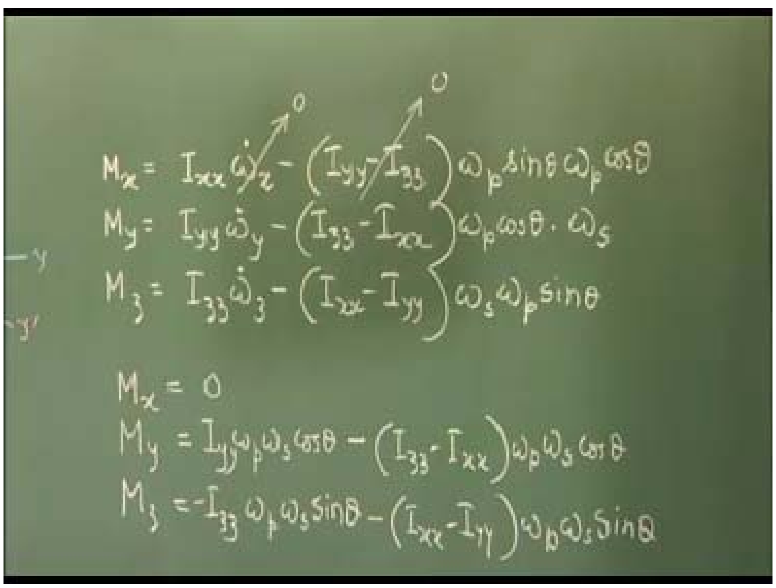

Second equation, if we use M_y is I_{yy} $omega_y$ dot. Now here $omega_z$ is $omega_p$ cosine theta and $omega_x$ is $omega_s$ and M_z is $omega_x$ is $omega_s$ and $omega_y$ is $omega_p$ sine theta. This is the set of equations that will relate the moment or applied moment components along with the acceleration and other quantities. Now $omega_x$ is $omega_s$, which is constant. We have to keep that in mind. Both magnitudes are constant, not the vector, because this angular velocity vector is changing direction. Magnitude will be constant. Since these are magnitudes only, therefore this will be zero.

We also see I_{yy} and I_{zz} are equal. So this also becomes zero (Refer Slide Time: 12:40). M_x is simply zero. Next, let us see how much we have for the y component, M_y is ….. now omega$_y$ dot omega$_p$ is constant. So, it will remain omega$_p$. Differentiate sine theta with time, it will be cosine theta into theta dot and that is nothing but omega$_p$. Theta dot from here is equal to omega$_s$. It will be omega$_p$ omega$_s$ cosine theta. This we can write as I_{yy} omega$_p$ omega$_s$ cosine theta minus I_{zz} minus I_{xx} omega$_p$ omega$_s$ cosine theta.

Similarly the z component will be I_{zz}. From this, we get omega$_z$ dot is equal to minus omega$_p$ omega$_s$ sine theta. In a similar way, differentiate that, this is a constant cosine if we differentiate this minus sine theta and theta dot is equal to omega$_s$.

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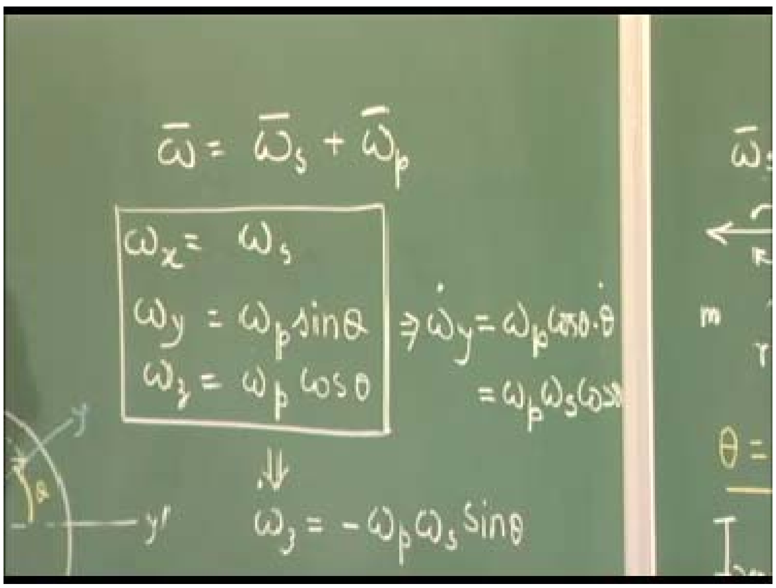

This will be minus. Further simplification will lead to this M_x is equal to zero, and M_y is equal to this. Now, I_{yy} and I_{zz} are same and equal to one-fourth m$_r$ square. The first term will be one-fourth m$_r$ square omega$_p$ omega$_s$ cosine theta minus. Now I_{zz} minus I_{xx}, if you see it, it will be simply minus one-fourth m$_r$ square. This minus coming there makes the whole thing again plus, and the first and second term become equal. You can easily see that and similarly, M_z is…. I_{zz} is one-fourth m$_r$ square. The first term is minus one-fourth m$_r$ square omega$_p$ omega$_s$ sine theta and second term also becomes minus one-fourth m$_r$ square omega$_p$ omega$_s$ sine theta. So the whole thing becomes… (Refer Slide Time: 16:00)

Interesting thing is that M_y and M_z the magnitude is same, one is cosine theta, one is sine theta, and we know that such a thing represents a vector. That means it is total. These two can lead to a vector, which is along this direction. This is half m_r square $omega_p$ $omega_s$. Its component along y is half m_r square $omega_p$ $omega_s$ cosine theta. Its component along this is obviously a negative quantity because it is opposite to the positive direction of z. That is half m_r square $omega_p$ $omega_s$. Therefore, we find that the torque which must be acting on this has to be along y prime axis.

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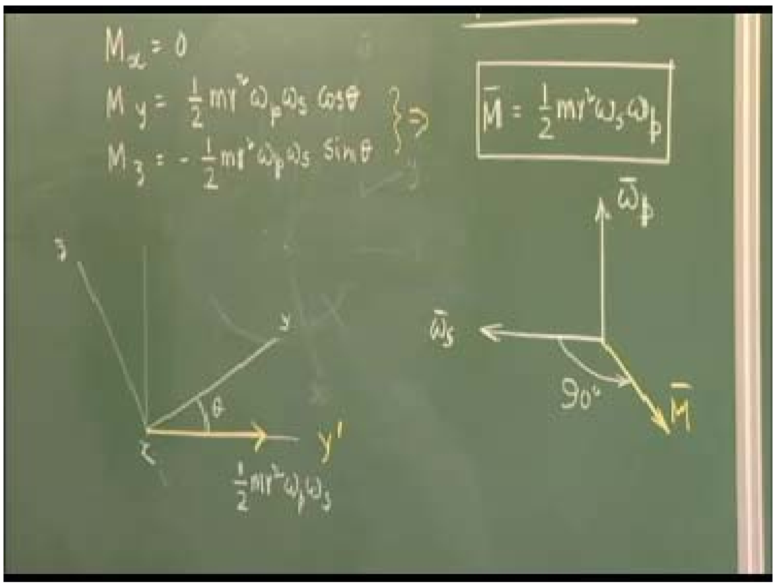

So this is the moment vector, whose magnitude is half m_r square $omega_s$ $omega_{p,}$ and this acts along the y prime axis. So, you see which is something counter-intuitive when you consider our dynamics of planar system, where you apply a moment the thing tries to accelerate in the same direction. Here what is happening, the body is spinning like this. You are trying to rotate about this. You do not have to apply a moment in this direction. Moment which you will have to apply will be in this direction. So if it is spinning like this, and I am trying to rotate the disk like this, I have to apply moment in a direction which is normal to the direction of rotation.

There is a standard simplistic situation to find out the position of these three. This is called the spin-axis or spin-vector. This is a precession vector because of such motions where the spinning axis of a rigid body also moves or rotates that rotation is called precision that is why we call it precessional motion. So this is the precessional velocity vector, then if you rotate the spin vector by 90 degrees in the direction of precession that is this it will coincide with the moment vector, this moment is applied to produce this motion.

We have to keep in mind, so this is something very strange, when a body is spinning or rotating, if I apply a moment in this direction, it will rotate in a right direction, which is at right angles. We can get a clear picture of this in a simplistic approach, and we can explain this in this manner. This is the fly wheel or disc, whatever we may say, which is spinning in this direction. If that is so, take a plane view, we can show this as the disc and this is the spin vector. When this is a large spin, we can also say that this happens to be the angular momentum vector also, which is equal to moment of inertia. In this direction that is half m_r square into $\text{omega}_{s.}$ That is the angular momentum vector of this.

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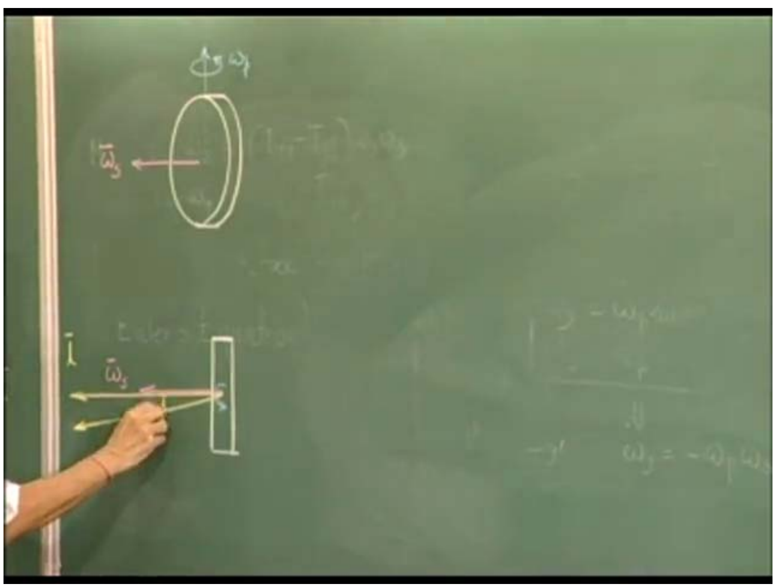

I am giving a rotation about this axis, so that means I am giving a rotation about this with a velocity omega_p. In time delta t the whole thing will come somewhere here, this is the new angular momentum vector. How much will the angular velocity be? It is omega_p into the time, which is delta t. This is nothing but omega_p into delta t. This angle is the change in angular momentum because, original angular momentum plus the change makes the new angular momentum. This is the change in angular momentum and how much is this, the magnitude of this? This is this length into this angle, this length is l and this angle is omega$_p$ delta t. The quantity delta l by delta t, delta t tending to 0 becomes l into magnitude. I mean to say omega$_p$ and omega$_l$ is nothing but the moment of inertia of the disc into omega$_s$ is the angular momentum. That is equal to half m$_r$ square omega$_s$ omega$_p$ and which direction now you know that angular momentum vector rate of change, of that is same as this.

This being the change in angular momentum, the rate of change of angular momentum vector will be in this direction, and that tells us that this will also be moment vector matching with this. So this is the moment vector and its magnitude is this.

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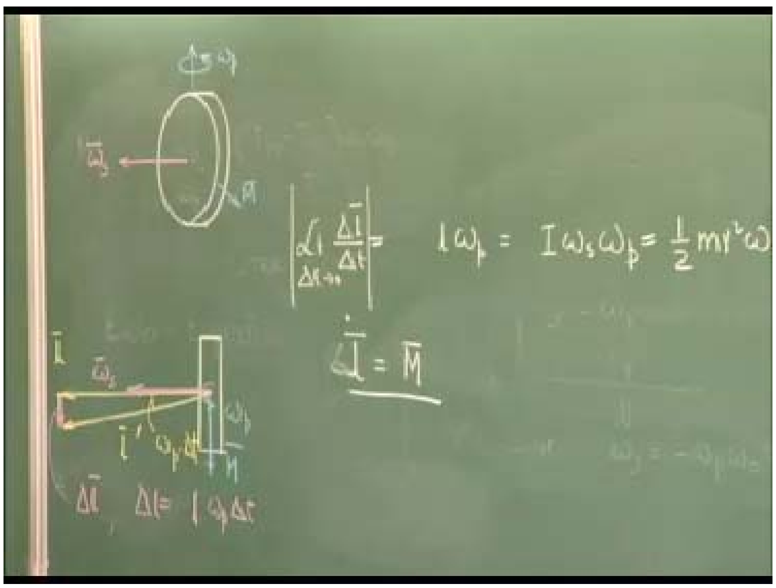

We found this out from the exact analysis of the whole thing or direct analysis of this using Euler's equation. This has an important consequence for engineering system because, we have many cases where a spinning or high speed rotating disc or rotor or a body, and which again is forced to turn its axis or its orientation because of the motion. For example, the turbine in the jet engine, there I think the body of the plane contains the turbine rotor which is simply represented like this.

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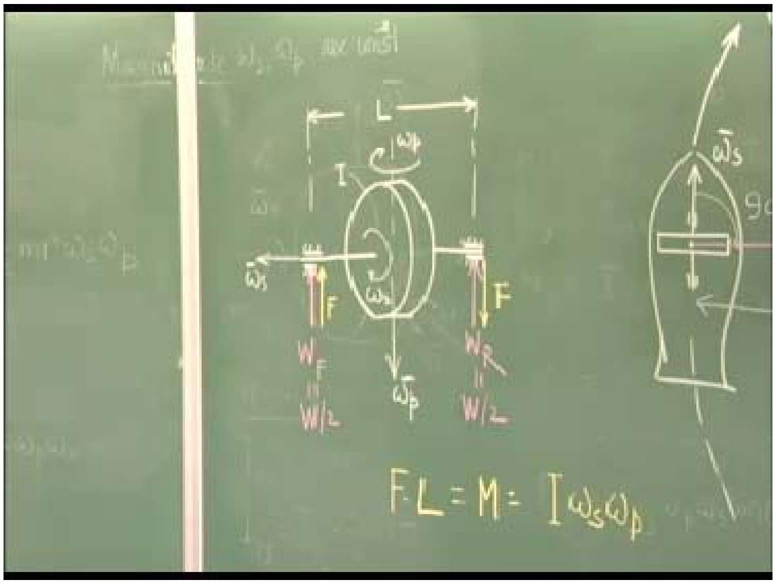

This is the turbine rotor supported by bearings. Let the distance between the bearings be the moment of inertia about the central axis is I or we call it polar moment of inertia. Its speed is omega$_s$. Suppose, now the whole thing which is taking a turn the velocity V and say radius of curvature R, the whole thing is taking a curve, a turn towards right or towards left. What will be the bearing reactions? When it is going straight, the only thing the bearing will have to support is the two parts of the weight. If it is exactly at the center, then this is going to be total weight by 2, this will be also total weight by 2.

Since it is taking a turn like this, the whole axis of this rotational axis of the rigid rotor is taking a turn in this direction. How much will be the processional angular velocity? That means, what will be the rate at which it is rotating towards this? It will be same thing as the speed at which it is the distance divided by R. So omega$_p$ in this case, that means the amount of rotation. Suppose if it is here, this is the axis of rotor-spin, and after sometime delta t the axis of rotation is this, because it has to be always at 90 degrees. This amount of rotation will be same as this. Now in time delta t if it as rotated this much, this will be delta t into delta t into omega$_p$.

If omega$_p$ be the rate at which it is changing its orientation, and we know that this is nothing but this length, which is velocity into delta t. When delta t is small, divided by this radius, that is delta phi into R is this arc length and small limiting case. This is nothing but V into delta t. So delta phi by delta t is nothing but V by R, and delta t by delta phi is actually, you have seen omega$_p$, so omega$_p$ is nothing but the velocity of this plane or this movement of the vehicle divided by the radius of its. Since it is rotating towards right in this view, if this is omega$_s$, omega$_p$ will be up or down? It will be down and this is omega$_s$.

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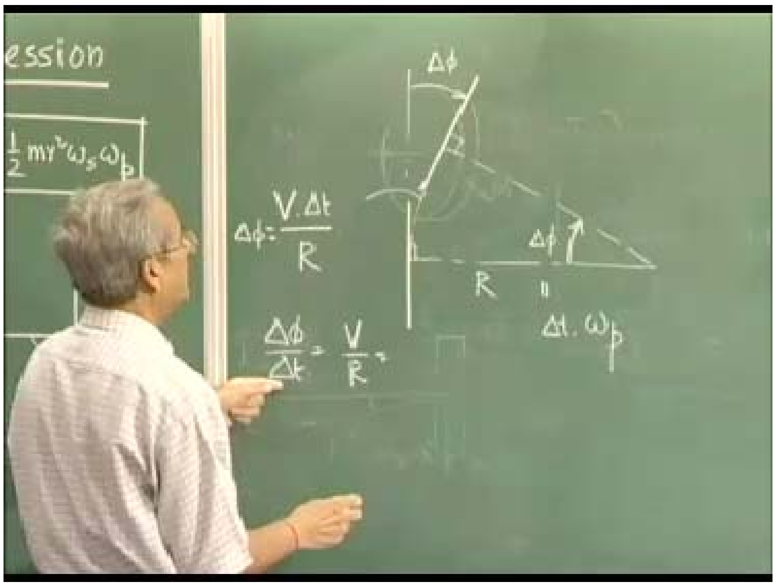

We have seen that if you rotate the spin axis in the direction of precession by 90 degrees, you get the top direction. If you rotate the spin axis by 90 degrees in the direction of precession, you get the moment vector. Moment vector will be this, the moment which must be applied on the body to produce this motion. Now, how a body can be subjected to outside moment in this particular case, the only place where it can get some external force or moment is from the bearings, so on the shaft of this rotor. There must be forces whose resultant effect on this body will be a moment in this direction. Therefore a force has to act here and an equal and opposite force has to act here, in such a way that the moment of this force which is F into L is equal to M.

The magnitude of this moment also we know from our analysis. It is moment of inertia of this rotor I into omega$_s$ into omega$_p$ and the direction of the two forces, which are equal and opposite on the shaft, will be depending on the direction of rotation. Therefore, the resultant reaction on the rotor shaft is going to be now different in different location. Here it is going to be W by 2, if it is symmetrical placed plus F. F is equal to I $omega_s$ $omega_p$ by L. Here you can see that the force is going to cause a reduction in this. So, resultant force here will be W by 2 minus I $omega_s$ $omega_p$ by L.

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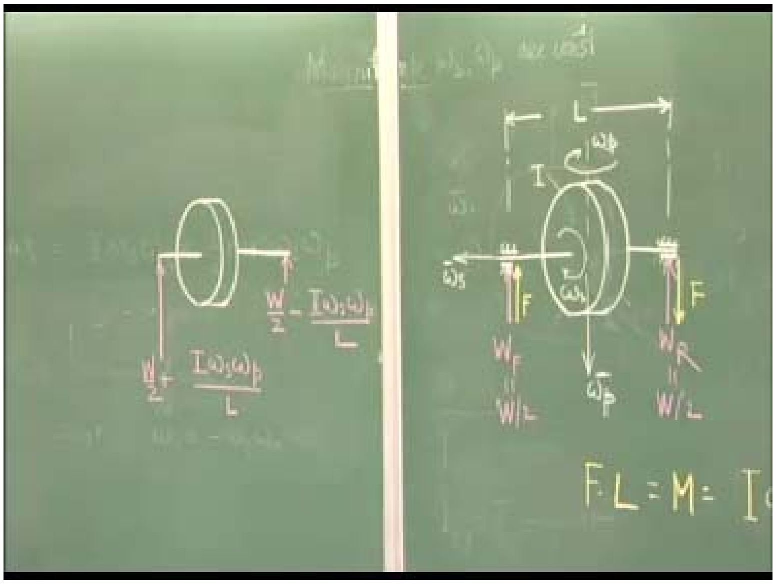

Therefore the bearing reactions will be depending now on this particular effect, and this effect that is rotating or spinning body when its axis of rotation is given a precessional motion. Moment has to be applied in a direction which is at mutually perpendicular direction, is called a Gyroscopic action or Gyroscopic effect. As a simple case, we have been seen here Gyroscopic action leads to alteration of the bearing reactions, and this must be taken into account while designing the system. If we consider that the bearings are subjected to only W by 2 that is the way it will be wrong because, one of the bearings is going to subject it to a more load than what simple way it will result in, this can be also used effectively sometimes.

One very popular usage of this is the crasher, where this effect is effectively utilized. See, if we take a crushing machine, this is the crasher this is a simple configuration I am taking where the radius of the crasher is r, distance of the crushing disc is capital R and this particular case, this be 90 degrees mass of this crasher be m. What we expect is and this is here, what we expect here is that the total force here acting is on this will be simply the weight of this, if we ignore the weight of the arms etc.

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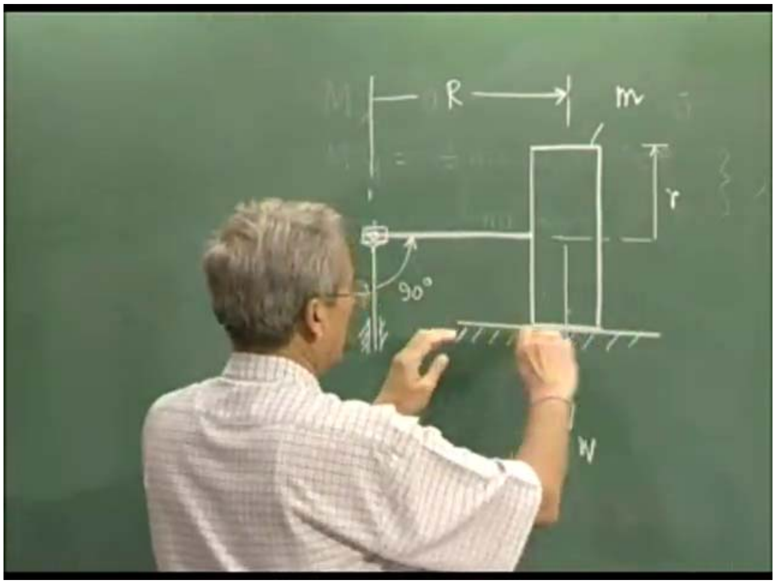

If the weight of the disc is W, then the reaction between the floor of the crusher, and the crushing roller will be W. This weight will be just balanced by this and the crushing force will be W. If you call a 10 ton crusher, it means that the roller has a 10 ton weight and it can produce a total crushing force at the contact point below W. That is what happens if whole thing is rotating in this direction, in angular velocity omega. If we assume that there is no slip which is quite practical then what will happen, this will rotate in this direction. That means it will have an angular velocity or spin velocity in this direction without considering any slip here. This can be found out from simple relation, that is, this point is moving with a velocity omega into R. If this point's velocity is zero, because it is in contact with the floor and then again to produce the same velocity here, it must rotate with an angular velocity omega_s and this distance is r. So omega_s, the spin velocity will be this, which will generate some angular momentum of this heavy roller.

At the same time we find that its orientation is changing with the same angular velocity omega. See if it is now here, after sometime it will be like this. It will be like this after sometime, it will be like this. So this whole orient plane of the disc or the axis about which it is spinning is rotating with this angular velocity. omega_p of precessional velocity will be simply omega. Now if we assume this to be a disc, then its moment of inertia will be W by g, is the mass half r square is the moment of inertia.

We know now that this is omega$_s$, this is omega$_p$ then we know that, torque must be acting in this direction of the system. That means, there must be a torque which is acting on this body. According to our thing, if we rotate the spin vector by 90 degrees, it will be coming out of this, let us see how this can be given a moment in this direction. Moment in this direction means the extra force in this direction, in such a way that this distance being R, F into R is the moment which is omega into omega$_s$ is the moment. That is equal to F into R. So the moment which total moment which must be acting on this is W in the downward direction due to gravity, R minus reaction F into R. Now this F that means the force it is receiving from the contact at the floor is the crossing force.

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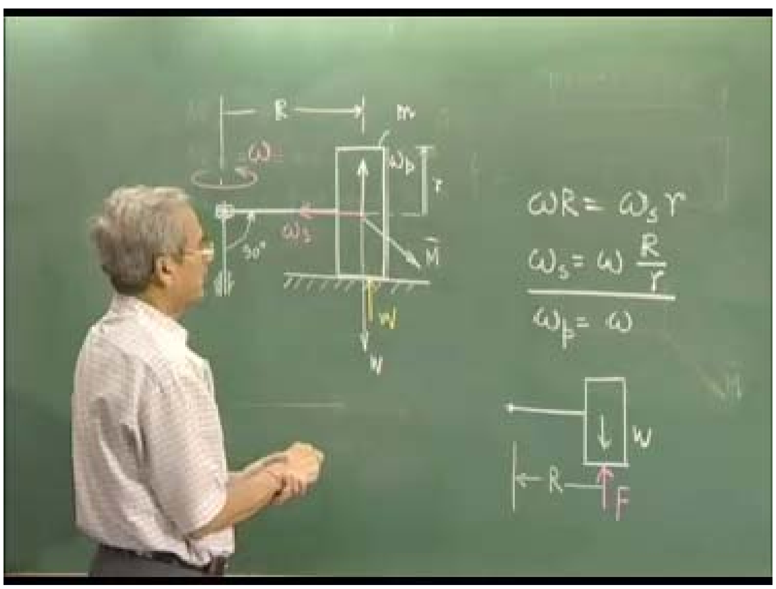

This is the crossing force, what I have been telling just now is that. This is the magnitude of the gyroscopic moment which must act on this to have this motion angular speed in this direction. A precessional motion in this direction and following the condition, as you have mentioned earlier, the moment must be acting along this. That means about an axis like this, now to produce that, so therefore this force multiplied by R can produce a moment in this direction F into R. But the gravity will produce an opposite moment minus W into R. This will be the total moment acting on this roller system and F is nothing but the resultant force of contact between the floor of the crasher and the roller of the crasher or effectively this is the crashing force.

The crashing force which you will get from this is equal to, this relation gives us W plus half r square omega omega$_s$ by gR into W, I divide both sides by R and take W to this side, or this can be written as W into one plus now, omega$_s$ is simply this so omega square r square R by gRr. So, omega$_s$ we have written as this, so this gets cancelled and finally we get W into one plus omega square r by g. We find that resultant crushing force is more than the weight that is the gyroscopic effect here, has helped to produce a crushing force which is more than the weight of the roller, which normally we expect to be the crushing force. The magnitude of the increase, it depends on the speed for higher angular speed. We find that we can generate a higher crushing force without increasing the weight of the roller.

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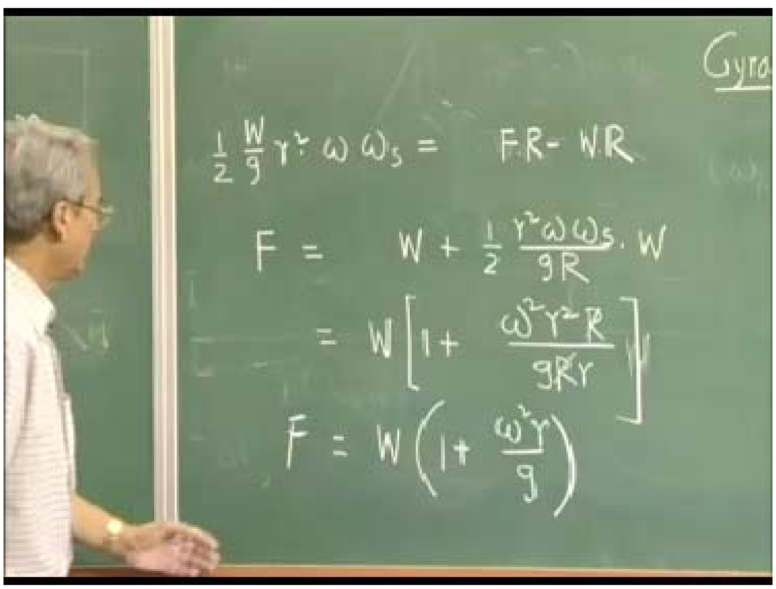

May be a 10 ton roller can produce a 15 ton crushing force. This is an advantage we get, so this kind of an effect that brings the forces or bearing reactions or moments, which all get generated because of the Gyroscopic action, must be taken into account. Whenever a rotating body changes its axis of rotation, there the gyroscopic action will come into action. There are many examples where you will find that the there are like a fly wheel of a car it is quite heavy and it reaches rotating at a high speed. When it is the car is taking a turn, the gyroscopic action will change the bearing reactions, and that excess force which gets generated due to the gyroscopic action needs to be taken into account while designing the system.

Sometimes a situation is bit more complicated. Now for very symmetric bodies like discs rotors, it is fine. But sometimes we have not perfectly symmetric bodies like say of a plane. It is not perfectly symmetric though it has an axis of symmetry. In such cases, it can be shown that the gyroscopic action is not a static force but it is the dynamic force. That frequency of that variation of the gyroscopic action is related to the spin or the rotational speed of the propeller, which is normally very high. In such cases what will happen, that bearing reaction will no longer be a static force like this, but will be subjected to an extra component or extra Gyroscopic action. Which is again vibrating or changing with time, that means it is changing, fluctuating between a maximum and minimum with this frequency, with which the unsymmetrical body is rotating. The problem is more serious because it can generate very dangerous vibration of the system and lead to failure. I think there are many examples in machines, where whenever we find that the rotating body is changing its axis of rotations direction, we have to be careful. We have to consider the gyroscopic action into picture and calculate the various forces which are generated due to this.