Dynamics of Machines

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Module-14 Lecture-1

Vibration Measurement

We have discussed the problem of vibration of different types of mechanical systems and now we have some idea about how to analyze a system to extract information about its behavior or characteristics - that is, the mode or the frequency and so on. You also had some idea about some industrial or [engineering situation] how to either use theories we have learnt to our advantage or to design a system in a manner, so that the desired proper functioning of machine or the unit [is achieved]. Towards the end, now, I think we will just discuss briefly about how to measure vibration.

The measurement of vibration and associated instrumentation is a subject by itself. Therefore, we do not hope to do that in that extensive a manner. Our objective here will be to give a very brief introduction to vibration transducer that you pickup (01:35 min). Now, the vibratory response of a system can be expected in terms of various parameters.

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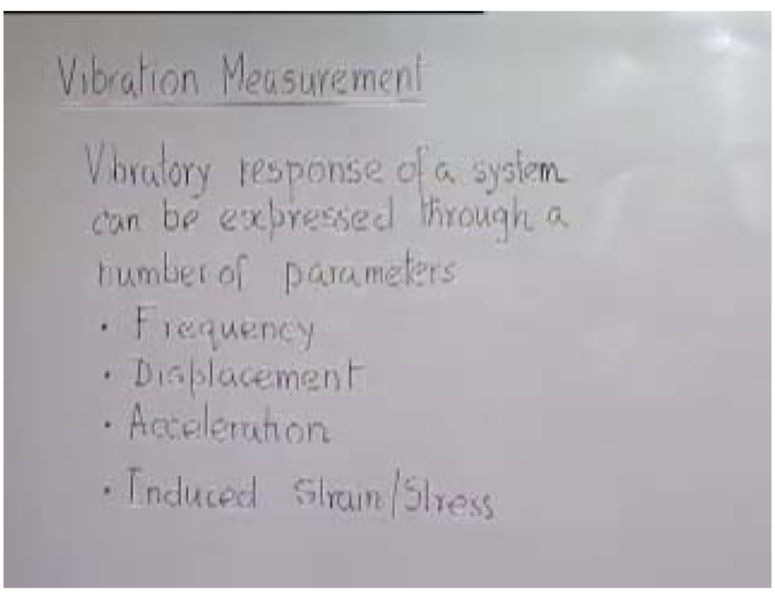

The vibration is just a phenomenon and its description or the vibratory response of a system, its expression can be done through various parameters which are associated with this phenomenon. So, these important parameters are: one is frequency of vibration; next is displacement; next is acceleration and induced strain or stress. Now, which parameter is to be used, depends on the objective of the particular experimentation and also the field of application.

For example, if we are measuring the vibration of a system with an objective to figure out what is the source of this vibration, then the best will be to find out the frequency, not amplitude or other thing; because, in a machine there are many sources of unbalanced forces and moments, there may be various rotating shafts with rotating members and if the phenomenon vibrational frequency of the machine happens to be coinciding or matching with one of the rotating members, we know there must be some substantial amount of unbalance present and that is the source of vibration. So we can rectify this equation.

Sometimes, for example, we are interested in analyzing or studying the comfort of ride in case of a vehicle mechanic problem. So, there we have to find out the acceleration to which the passenger or the driver is subjected to. So, therefore we will measure the acceleration in such cases or that will be the predominant parameter. Similarly, if we are investigating the fatigue life (05:28 min) of a structural member, then in such cases induced strain or stress will be the predominant parameter. So, like that it depends on what we want to do, what kind of usage will be of our experimental results - that will tell the predominant parameter of measurement.

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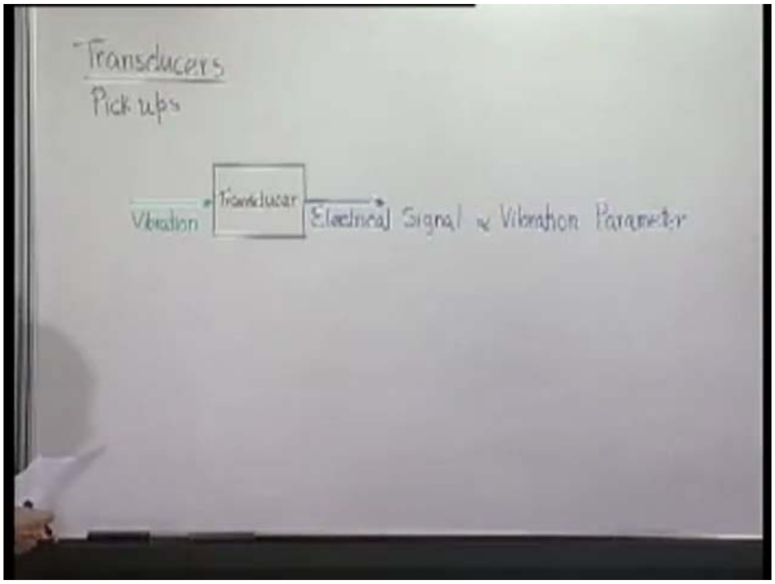

A vibration measurement is done through transducer or in common word you can say pick ups. The transducer's job is to sense the vibration, say which is a mechanical movement and to produce a proportional electrical signal, which is proportional to vibration parameters. In general, though there are very rare operations where we may not use an electrical output of the transducers, but in general, in almost all cases we get the mechanical motion sensed by the pick ups or the transducer and it transforms that parameter into a proportional electrical signal, which can be then seen or recorded or whatever you want to utilize it. Sometimes it is used even in an online feedback loop in a machine to do something. So, this is the basic function of a transducer. In this session, I think what we will do is we will discuss the design and basic principle of some of the commonly used transducers. First thing what we will take up is a very simple problem of frequency measurement.

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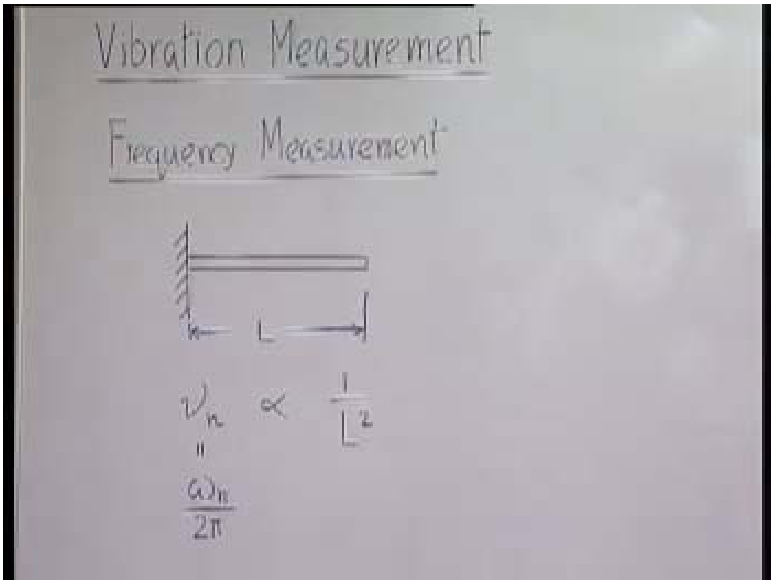

Now, frequency measurement in many situations may come out as a by-product, like say for example, if we can sense the whole signal (Refer Slide Time: 08:55) obviously frequency, amplitude, velocity all may be simultaneously same. But in simple industrial scenario, quite often a mechanical vibration transducer we can utilize which will give a frequency. The basic principle is the transverse vibration of a cantilever beam. If this is the cantilever beam, uniform beam of length L, then the natural frequency in the first mode is proportional to 1 by L square, that you have seen; this is nothing but omega$_n$ by 2 pi.

This is the circular natural frequency, this is in Hertz; this is in radians [per second]. Therefore, if we can have arrangement through which we can adjust L, then its natural frequency will change and then what we will use is the principle of resonance. If we attach this to the vibrating body, the vibration will be most severe, only when the natural frequency of this matches with the actual vibration. If we can somehow calibrate it, that means, for what length, what natural frequency we get, then, from that we can figure out what is the frequency of the object to which this has been attached or connected.

So, simply we will get natural frequency; it is not very accurate because, by visual observation it may not be always possible to say when the amplitude of vibration is maximum, but it can be easily augmented by attaching a simple electrical pickup signal which gives the vibrational amplitude and measuring the amplitude by some meter or by some oscilloscope we can do a better experiment, but in a crude way, since the objective is to get some idea about the order of magnitude of the frequency, visual observation is enough.

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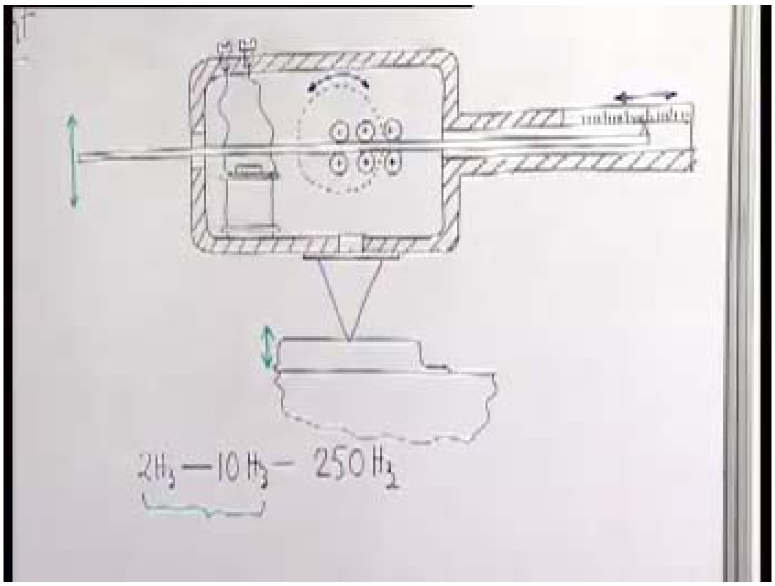

So, this vibration pick up probably (in a mechanical vibrometer) is like this. It is a very simple arrangement; this is a uniform field scale kind of thing, which is supported by a series of rollers; that means, this end may be considered to be equivalent to a thick cantilever beam because the slope and displacement both will be 0 here. Now, one of these is connected to a knob outside which can be rotated by hands and this can be made sliding in this direction. Thus the length of the over end portion of this cantilever beam could be adjusted by this and the reading can be taken here - that what will be the corresponding natural frequency when you touch it to the machine body vibrating, so it will vibrate.

So, only one thing we have to keep in mind - that in this mechanical vibrometer the range is from 2 hertz to 10 hertz to 250 hertz - this is the effective range in which it can be utilized. However, when we use this lower range, this range (Refer Slide Time: 17:35), in this range a standard mass is attached to this because, otherwise this range is so large that it will be essential to change the length of the cantilever portion to a very large value. It will make the length of the whole equipment very large; it may not be always desirable. So, what is done? This range is taken care of by this sliding and then the natural frequency of the whole beam is reduced by attaching a separate mass at the end. We know that if we attach a mass here, immediately the natural frequency will be less. So, therefore that brings…. again then the length will change; it will give us a different scale. So, attaching a known standard mass at the tip, that is all what you have to do.

Here, one disadvantage is that at higher frequency the sensitivity is less. What is meant by sensitivity is this: that here I think if by changing the frequency - if the frequency changes, how much should be the displacement of this? If it is too small, then it becomes insensitive to the frequency. Now, it can be easily seen that if we say that natural frequency is proportional to 1 by L square or we can say natural frequency is equal to some constant by L square, then a small change in natural frequency - this will be equal to minus 2C by L into dL.

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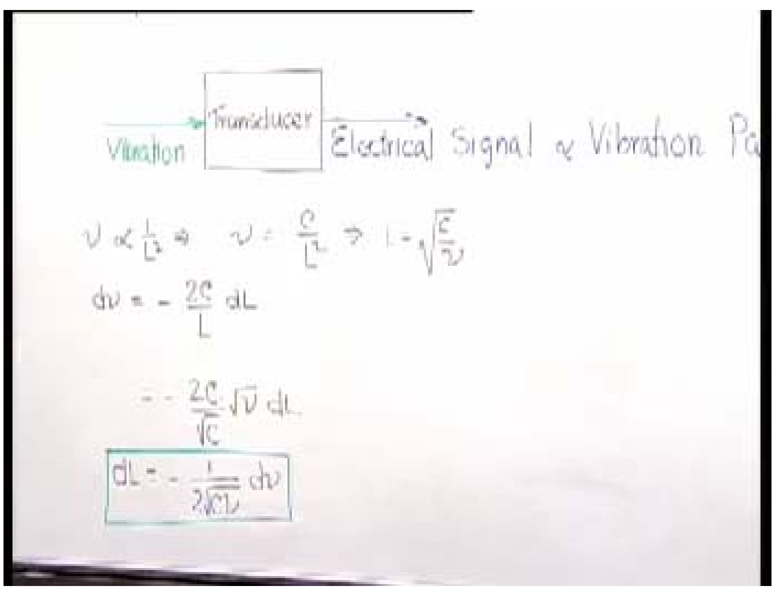

Now, from this again we know L is equal to square root of C by nu. If you substitute this here, it will become minus 2 square root of C by square root of nu into dL. So, how much change is necessary for a change of frequency, this sigma, can be written now - it will be minus 2C and L is how much? It is square root of C into square root of nu into dL. So, change in the length is given by minus 1 by 2 square root of C nu into d nu. Thus, it is very clear that when we are measuring frequency and frequency is small, then the change of length required for a corresponding change or for a change of the frequency, when this is small, this is substantial, but when nu is very large for a given change in frequency, the change required here will be very small. So it will be insensitive to this change of frequency at high speed; so this is one disadvantage. Secondly, of course, the range of frequency which can be covered is also limited.

Therefore, the modern transducer or vibration pick ups if you want to study, first let us see what are the various types or broad classifications.

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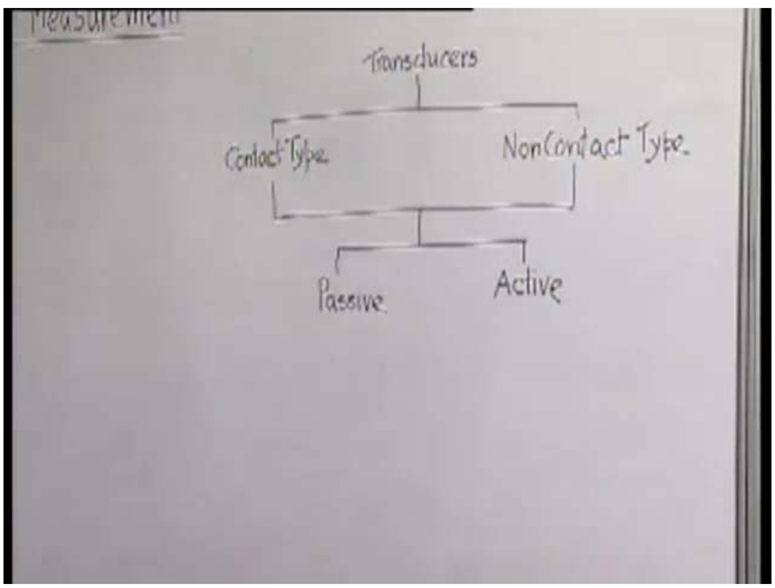

The transducers can be classified into contact type and non contact type. This means, in contact type, the transducer is put on the vibrating body or attached to it. On the other hand the non contact type is a pick up which does not touch the actual vibrating body it senses its motion from a nearby position. So, this we can do or we can use a contacting type pick up only when putting or attaching the pickup to the main body or the main system to the vibrating does not alter the characteristics of the system itself.

So, therefore the big machines like (23:45 min) rocket systems etc., you know we can use the contact type vibration pick up, because, it will not have much effect on the characteristics of the system which we are going to measure. On the other hand for very delicate systems, very light systems or flexible systems, we cannot put a pickup attached to that because the systems natural behavior is going to change. In such cases we try to sense the vibrations through a non contact type and obviously the limitations also come. So, here since the pickup is attached to the body then what is the movement - the amount - is not restricted. On the other hand, here it is placed very near the object which is vibrating; so obviously, it will be possible to measure it properly only for a small movement, because, large movement is going to cause problem for this type of vibrations.

So, both these contacting type and non contacting type can be further classified into two groups which are passive or active. In passive type pickup what happens? We do not have to have a separate electrical power source to activate the transducer or to get the results. On the other hand, in active type of vibration pick up it has to be connected to a separate electrical source or its power, then only it responds. So, both types are there and they have their advantages and disadvantages. So, what we will do now is the basic principle of certain transducers.

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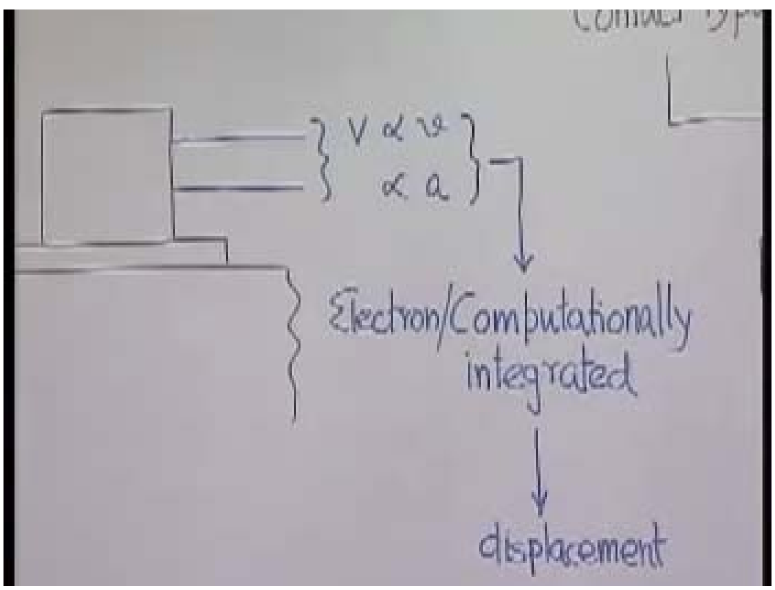

So, what the fundamental things we have to keep in mind is that, suppose we use three types of contact and non contact type of transducer pickup and we attach it to the body of machine or system which is vibrating. So, then the electrical signal what we will get out from this voltage. Generally this voltage will be proportional to velocity or proportional to acceleration; generally, these are the two cases what we will get. Then what will happen, this can be computationally integrated or electronically integrated and finally we get the information about the displacement; that is the basic principle. So, we start with the most common type of vibration pickup, that we have (27:10 min); they are called seismic pickup vibration principle. The basic principle can be understood from the analysis which we have already done or vibration with basic (27:31 min). Effectively, these pickups are nothing but the strength which is excited by the body to which it is attached, I am talking about contact type pick up and obviously it is nothing but a vibrating system with basic [scheme], it is like this.

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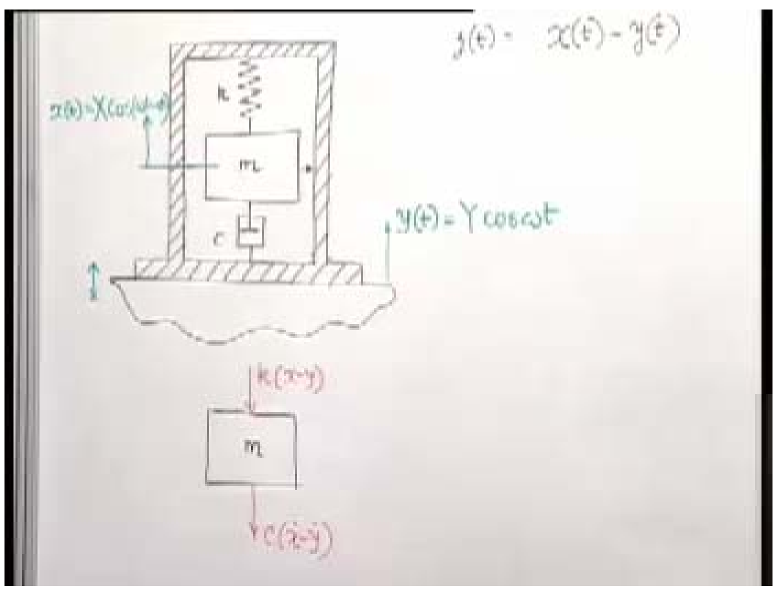

So, this is the basic scheme of the pickup, it is connected to a machine body or system body which is vibrating like this (Refer Slide Time: 30:20) and we call this vibration as a function of time. Then, we know that from equilibrium position the vibration of the mass we can describe by x, which is a function of time given by X. Now in such a situation what we can really sense is the movements of this mass inside this container, that means, the relative movement of the mass with the container (31:03 min).

Now, in a very crude way it can be sensed by putting a scale here and the point at which it is attached to the mass, what movement would be there? But really that is not the way it is done. What is done that it is this movement here is sensed by some electromagnetic effect. If we can put a coil and a permanent magnet attached to this or the other way around - the coil is attached to this and there is a permanent magnet which is fixed to this body - then the relative movement between the coil and the core that will generate a signal. So actually it is done that way, but anyhow our primary objective is as we can see is the measurement or the sensing of the relative movement between the container and this. So this relative movement what is sensed is z - the function of time, which is nothing but the relative movement between the… (Refer Slide Time: 32:18).

Now, let us see the free body diagram. This is the mass at any instant of time, the force, that means, if it as gone by x and (Refer Slide Time: 32:41) this has gone by only y then the compression is given by x minus y or if we want say it is…. So, therefore, it will be k into x minus y. Similarly, if this is going by x dot velocity and this is following by y dot velocity, then the relative velocity with which it is stretching is x dot minus y dot. The force will be c into x dot minus y dot; gravity we ignore, because, we know that if everything is considered from the equilibrium position it is already under the action of gravity then gravity will not be there, everything will get cancelled.

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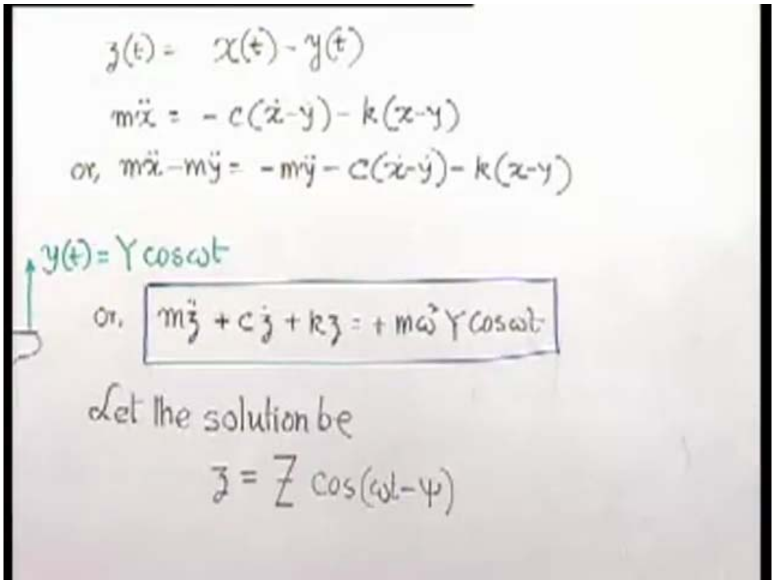

So, the equation of motion for the system will be: mx two dots is equal to minus c. Now, if we subtract my two dot from both the left and right hand side and add minus my two dot on both sides, taking m common or it will be nothing but mz two dot because z two dot is x two dot minus y two dot, we take this c into z dot this is nothing but z dot plus k into z is equal to plus m omega square y cos omega t; this is used here. So, now this being the equation of motion, what will be the solution? We know in steady state the solution we can always assume to be another harmonic function of time with the same frequency, but at a different stage, so let the solution be this (Refer Slide Time: 35:14).

If we substitute z in this and equate the coefficients of cosine omega t on both sides and sine omega t on both sides we will get two things. So, substituting in equation of motion and equating the coefficients of sin omega t and cos omega t, if we do that we will get this.

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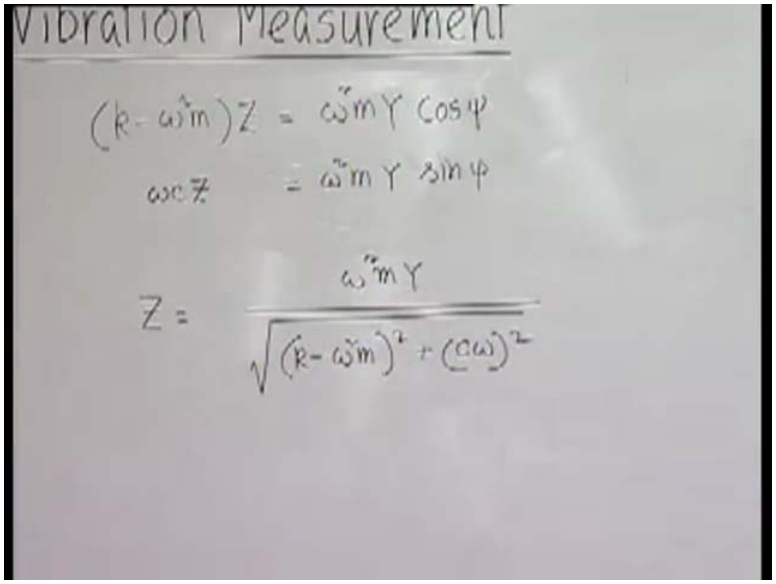

Now, dividing both sides by this quantity which is common and requiring an adding we get this (Refer Slide Time: 37:37).

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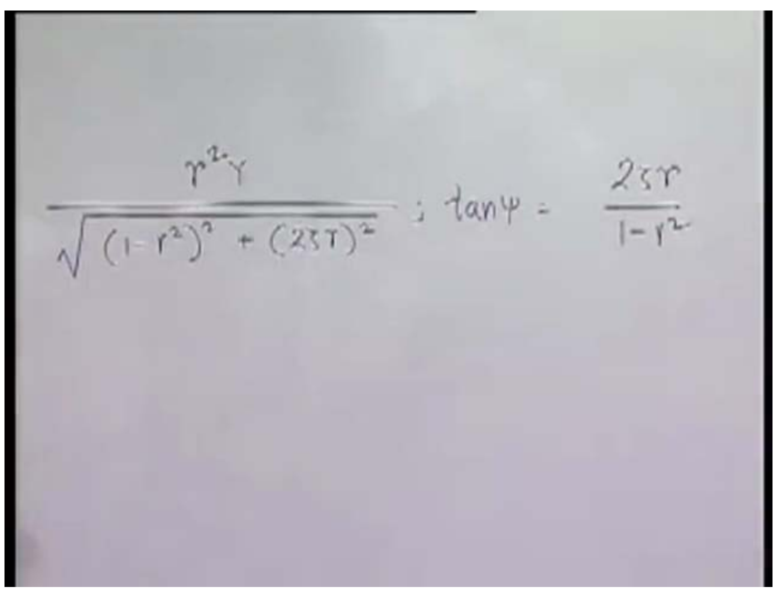

Using the same symbol which we have used so far, this will become nothing but if we divide both the numerator and denominator by k, this becomes omega square by k by n that is omega n square, that is r square; r is the frequency ratio and the denominator becomes again then, 1 minus r square whole square and tan psi, if we divide this, this by this, tan psi will be given by this (Refer Slide Time: 38:42). So, now I think, let us see how it looks when you plot the characteristics.

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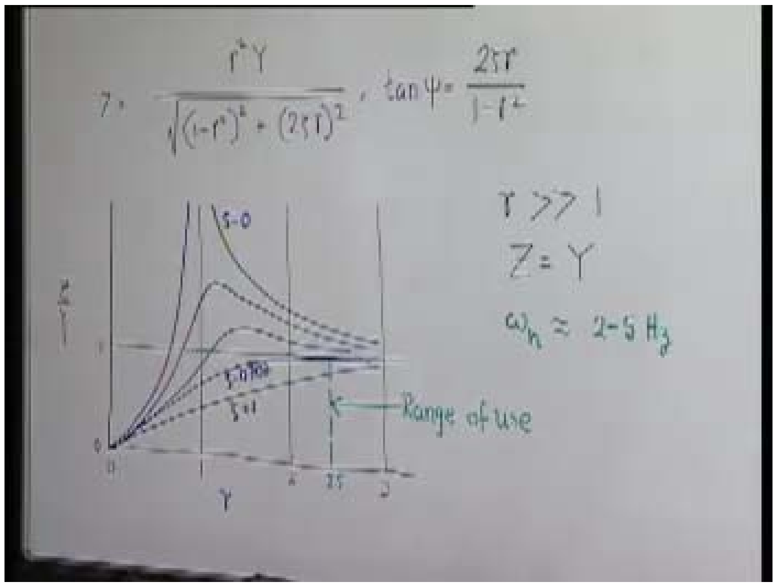

From this z by y characteristics now we can see that for zeta, that means for the various values for damping, the response curve is like this (Refer Slide Time: 39:25), but as we have seen that for in this case when r is very large compared to 1, that is the case we were discussing, then z is equal to y, as you have seen from this (Refer Slide Time: 39:54), but that really gives excellent result only beyond certain value of r.

Now, to get the widest possible range, we should try to see that for which value of damping factor we get z equal to y at the earliest possible value of r. As you can see, that for the damping which is equal to 1 by root 2 or 0.707, the z by y curve approaches 1 at the earliest and beyond 2.5 we can safely use this as the velocity meter. Therefore, in this situation the value of natural frequency of the system, of the seismograph should be very low; the frequency with which we will measure or the vibration which we will measure must be at a much higher frequency so that r is much more than 1 and beyond r is equal to 2.5 we can get excellent result, if we can match the damping factor to this value; otherwise also they are all ultimately approaching 1 as we find from the asymptotic tendencies. Now, a pick up with a low natural frequency - that means, the natural frequency of such pickups are generally 2 to 5 hertz - so, we should plan to design the seismograph in this range. One thing also must be noticed that the damping factor when you try to adjust, if you try to have as close to zeta is equal to t as possible.

Now I think let us consider the other extreme case, this extreme case was r much greater than 1. Now, let us consider the situation when r is much less than 1. What does it mean? r very less compared to 1 means, the frequency of the vibration we are measuring is very low compared to the natural frequency of the system or in such cases the natural frequency of the vibration pick up will be very high, so that is case two - r is very small compared to 1.

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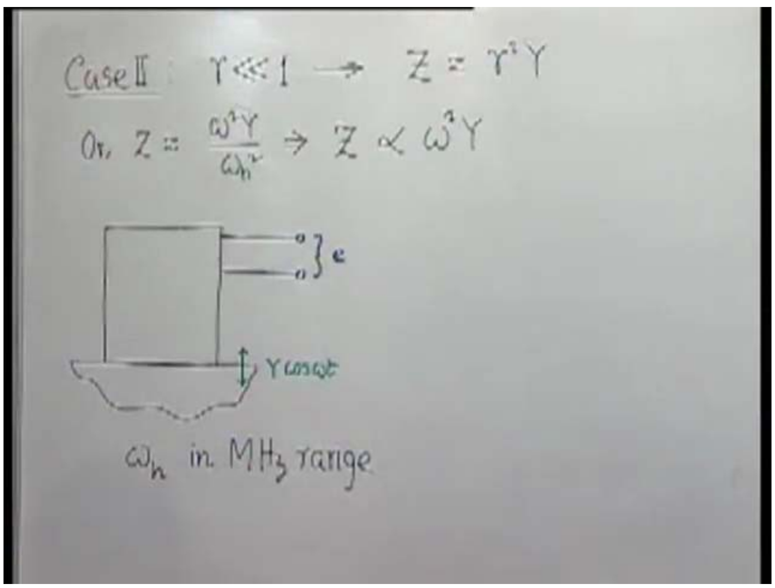

If r is very small compared to one, then from this what we find? If r is very small, it means we can consider this (Refer Slide Time: 42:56) to be 1 and we can ignore this; so therefore denominator becomes 1 and this… (Refer Slide Time: 43:05). so the whole denominator becoming 1 means z is approximately equal to r square into y. We know that or z is approximately equal to omega square y by omega$_n$ square, where omega$_n$ is the natural frequency of the pickup.

Now, natural frequency of a particular pick up is a constant quantity. So we can consider this to be constant and this means that relative movement is proportional to omega square y. omega square y is nothing but the acceleration of the vibrating body whose vibration we are going to pick up. Thus, here we treat these kinds of units as an accelerator that means output of the pickup will be an electric (Refer Slide Time: 44:10). If this is the pickup mounted on an object which is vibrating, so its acceleration is omega square y and magnitude of that. If the output of this voltage that will be proportional to acceleration and it can be considered as accelerometer, but the conditions what has to be satisfied is that r must be extremely low or omega$_n$ must be very high. So, omega$_n$ will be in megahertz range.

Another important thing has to be also kept in mind that here how do you generate this electric voltage? In the previous case, we know we used electromagnetic induction and thus the voltage was proportional to the velocity. In this case, if the output has to be proportional to acceleration, so, one is that this relation as to be approximately valid which is ensured by this. Second is that the voltage which you generate here, the mechanism of that, its transduction, should be at where the voltage is generated by displacement or proportional to the displacement of the object; that means - z - relative displacement.

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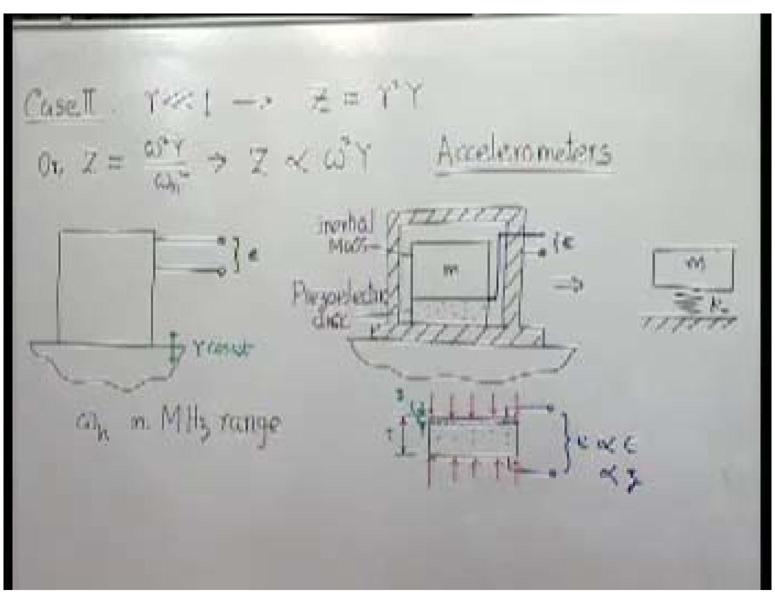

Now, both these conditions are very nicely satisfied by one type of pick up where this is the inertial mass mounted on a thin disk of a special kind of material which we call piezoelectric material.

This is the container; this is the inertial mass; this is the piezoelectric crystal. Now, the property of piezoelectric crystal is that, if we have a piezoelectric crystal (Refer Slide Time: 47:45) of thickness t, then if we subject this to a deformation by some compressive force, by developing some compressive stress, then a voltage is generated at this, that means, if we connect this, voltage will be generated which is proportional to a strain; that means, which is proportional to this displacement of this (Refer Slide Time: 48:38).

So, due to inertial force which is my two dot, mx two dot and this inertial force we will call compression or deformation of this piezoelectric crystal. That will generate a voltage and this voltage will be proportional to this strain to which this is subjected; strain means it is proportional to deformation, because, strain is nothing but this deformation divided by the original thickness t, that means which t is constant, so it will be proportional to z or the displacement. So, the voltage will be proportional to acceleration. This type of pick ups where the output voltage of the transducer is proportional to the acceleration of the object is called Accelerometer.

As mentioned here, since the stiffness of this piezoelectric plate can be very high, mass is small, therefore, effectively the natural frequency of this mass is (50:33 min) because it is equivalent to mass supported by a spring where this spring stiffness is very high. So the natural frequency will be very high in megahertz range. So, we can measure quite a few tens of kilohertz with the help of this kind of (51:00 min).

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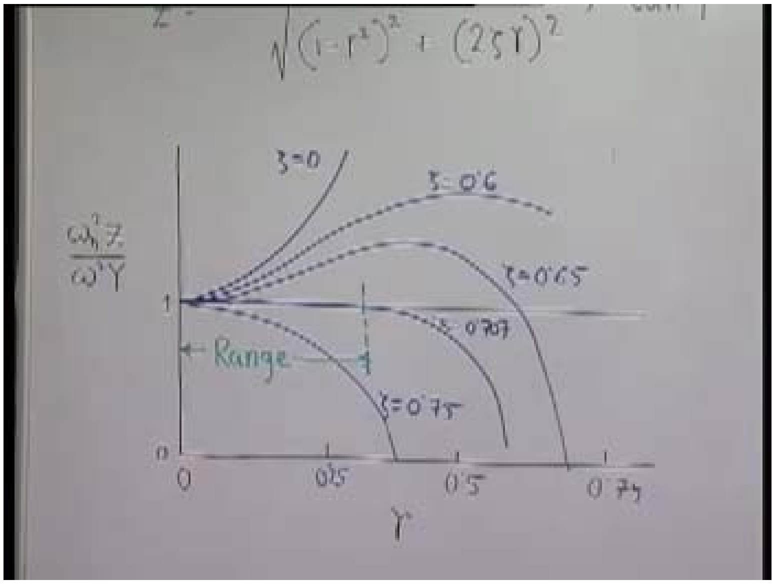

Now, if we plot the characteristics… so since this is equal to this. So when this will be satisfied, then omega$_n$ square z by omega square y will be 1; so when that happens if you want to see (Refer Slide Time: 52:38). So when you plot the details we can see now that here, of course, our condition is that r as to be very small. Again we should compare to 1, again we should try to have as much range as possible and we find that a large range, when this relationship is approximately satisfied, it is obtained when zeta is equal to 0.707 compared to all other possible values. Therefore, again zeta equal to 0.707, this gives us this range of operations which is the largest; otherwise, you have to operate very near this (Refer Slide Time: 54:01) or with 0.707 we can go r to almost 0.3 and still satisfy that (54:03) activity.

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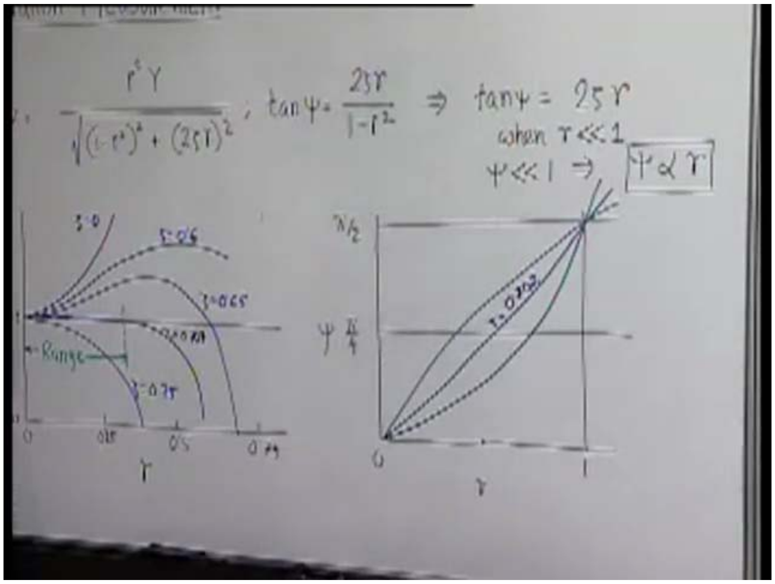

If we plot the phase (Refer Slide Time: 54:17) and this if we use some lower value of zeta like this, (Refer Slide Time: 55:01); from this we find when r is very small compared to 1 then tan psi is equal to 2 zeta r. If that be so, since zeta itself is never a very large quantity so tan psi itself is very small; that means, psi itself is very small - 1 and that tells us psi is proportional to 1. That is why at z is equal to 0.707, you will find that we will get this approximate linear relationship equation. Others also will be like this when r is very small it will be straight line, but when r is even goes up to 1 it will be approximately straight line for this particular value of zeta. This has very important application or important result that we will now see. This proportionality of psi with r helps the transducer to maintain the [phase] of the signal or the vibration what we are measuring without any distortion; that means, the voltage which we get will be a faithful reproduction of the actual vibration.

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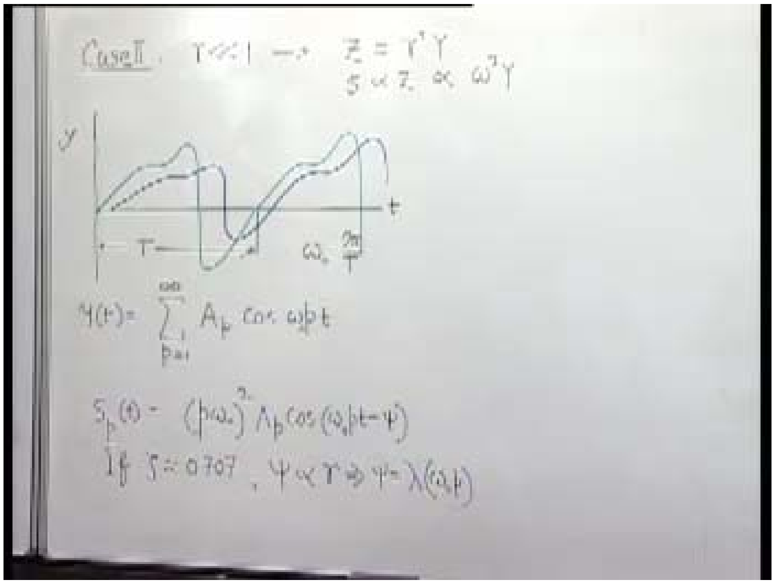

So, for example, if the original vibration is something like this, then the voltage which will be produced by faithful reproduction will be something like this (Refer Slide Time: 57:20). That means, the shape will be very similar to the shape of y; how it happens that we will see. Say, the vibration is given by y function of t; if it is periodic, but not harmonic exactly we can represent it through a Fourier series (Refer Slide Time: 57:57) where omega$_0$ is the fundamental frequency; how do you find it out? You find out the time period and omega$_0$ is equal to 2 pi by T. So, we can easily form the vibration pattern of the basic objects which we are investigating; we can find out omega$_0$. So this will be this. So, therefore, the signal for the pth harmonic will be… signal is proportional to (Refer Slide Time: 58:50) z; that means, proportional to omega square y. So, omega for the pth harmonic is p omega$_0$. So it will be p omega$_0$ square A$_p$; y is equal to this A$_p$ cosine omega$_0$ pt minus psi; that is the signal generated by the pth harmonic. Now, if zeta is approximately 0.707, then psi is proportional to r or we can say psi will be equal to some constant lambda into the frequency that is omega$_0$ p, because r is nothing but omega by omega$_n$ and omega is omega$_0$ p for the p of harmony. So, if we write it here or the signal for the pth harmonic which we get is this (Refer Slide Time: 1:00:14).

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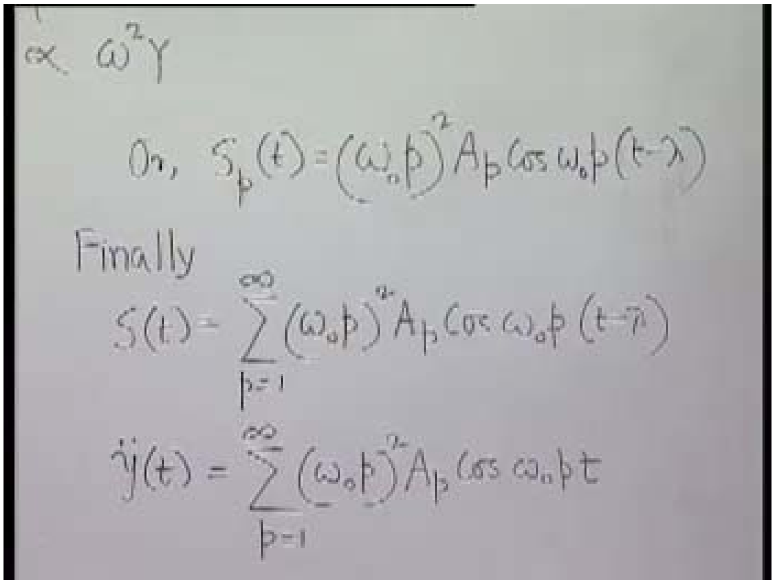

Now, if we use psi is equal to lambda omega$_0$ p and omega$_0$ p we take from outside where lambda is a constant. The total signal will be sum total of all the harmonics. If this be the vibration, what will be the acceleration? The double derivative of this, which is y two dot p, acceleration is this (Refer Slide Time: 1:01:23). So, comparing these two, we find that even if the vibration is not harmonic but periodic, the acceleration pattern and the voltage generated that pattern, they are same only there is a slight shift in the origin of the t that is all, but the shape is undistorted; that is one great advantage. Since accelerometers are used quite extensively, this property helps in the analysis.

Therefore, we find that this seismic type of pickups can be primarily of two types; they are both contact type pick ups as you can see and they are also both passive that means they do not require any outside power source. In one extreme case like when [r is] the natural frequency of the system is very low, in such cases it will be treated as (1:02:30) the velocity meter, if the velocity or voltage is generated by (1:02:30) electromagnetic induction. On the other case, when the natural frequency of the system is very high and the operational frequency generally very low compared to that, then what we generate as a voltage using piezoelectric pick up which will be proportional to the acceleration of the system and it is called accelerometers. Accelerometers by nature, because, the whole thing can be very small and concise, they are very small in the form of buttons, on the other hand the velocity seismic pickup are somewhat bulky.

In the next presentation, we will discuss other types of transducers and vibration pickups.