Dynamics of Machines

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Module - 12 Lecture - 6

Forced Vibration of Multiple Degrees of Freedom Systems

Once the free vibration problem of a multi degree freedom system is solved, it is possible to solve forced vibration problem of the same system. Of course, earlier we have solved a 2 degree freedom systems forced vibration, but that was with a very specific objective to its application vibration absorber. Now in this presentation, we would like to discuss the matter of forced vibration of a multi degree freedom system. There are a couple of approaches, one approach of course is a transfer matrix method which is applied to - very often we apply to - torsional oscillation problem with a number of discs and typically the method is known as Holzer's method and its standard text books can be consulted to find that. What I would like to present here is a very generalized approach through the post-vibration problem of multi degree freedom system.

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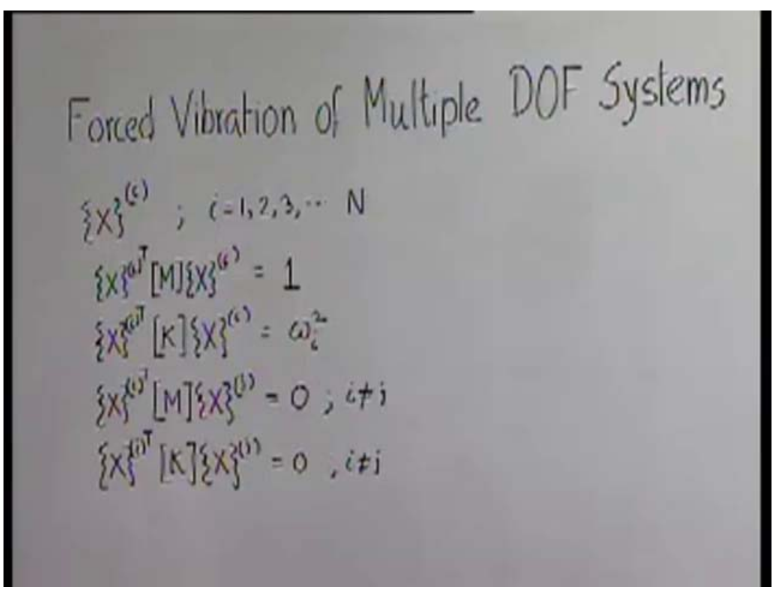

So, once we solve the free vibration of a multi degree freedom system, we know the natural mode where i is equal to 1, 2, 3, up to N, which is the number of degrees of freedom. We also normalize the natural mode in a manner (Refer Slide Time: 02:55) otherwise, for free vibration amplitudes we do not have any definite value, so to get some kind of normalized approach, we use this generalized mass equal to 1 and obviously in that case (Refer Slide Time: 03:24) it will be equal to $omega_i$ to the power 2. Of course we know that (Refer Slide Time: 03:38). Therefore, after you normalize you get some unique values of the normal modes; of course, you also know the natural frequency. With the approach I present it is possible to convert the forced vibration of an n degree freedom system into n single degree freedom systems forced vibration problem? Since single degree freedom systems have been solved and you know this solution, we can use that and then retransform the problem to get the actual desired value.

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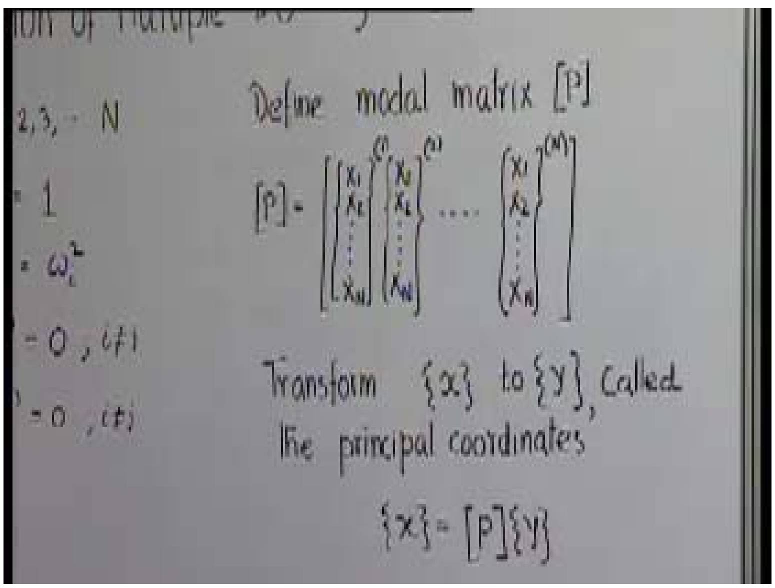

Let us first define the modal matrix, where the columns of this (Refer Slide Time: 05:18) are nothing but the columns of this square matrix; they represent the natural mode. This is the first natural mode, this is the second natural mode and for an n degree freedom system, there will be n natural modes. So, there will be n columns and obviously $x_{1,}$ x_2 and x_3, it will go up to x_n there will be n rows. This represents a square matrix of order n. This we call modal matrix P. Next transform (Refer Slide Time: 6:36), how we transform x P.

So, this is the way we transform and get another set of coordinates in case of x this transformation rule and this y is called generally the principle coordinate.

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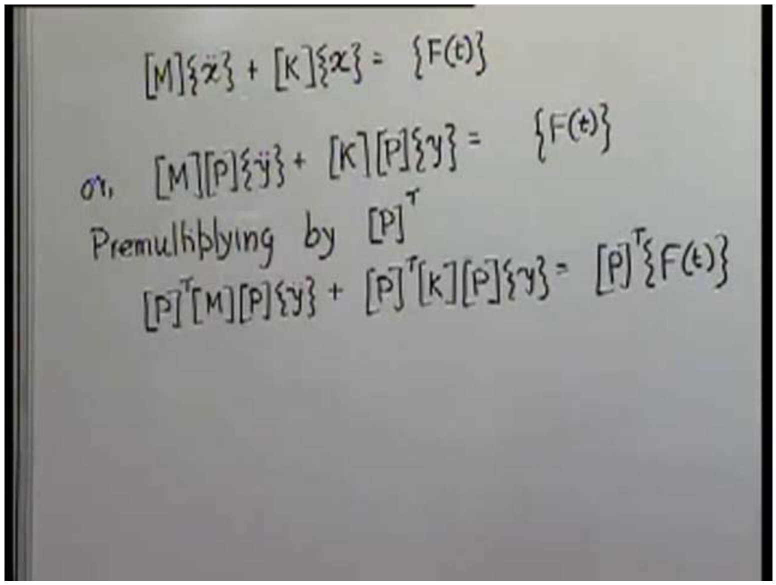

Now, we know our equation of motion, it is (Refer Slide time: 07:50). This is the forcing function and of course, there are n such equations and they are all coupled, we know m and k matrix. Therefore the second derivative in P matrices is independent of time; P being a constant, P constituting constant elements; so a time derivative that means the second derivative will be this. In terms of principle coordinates, we get this equation (Refer Slide Time: 09:10). Replace x 2 dot by Py dot x by Py then pre-multiply both sides by P transpose. (Refer Slide time: 09:57) Now, what will be this? We will find that P matrix is this. Therefore, when we transpose it, this first column becomes the first row and so on and here you will find because of the orthogonality condition, all the cross terms will go; only the terms representing this and this will remain. So, the results of this will be (Refer Slide time: 11:31).

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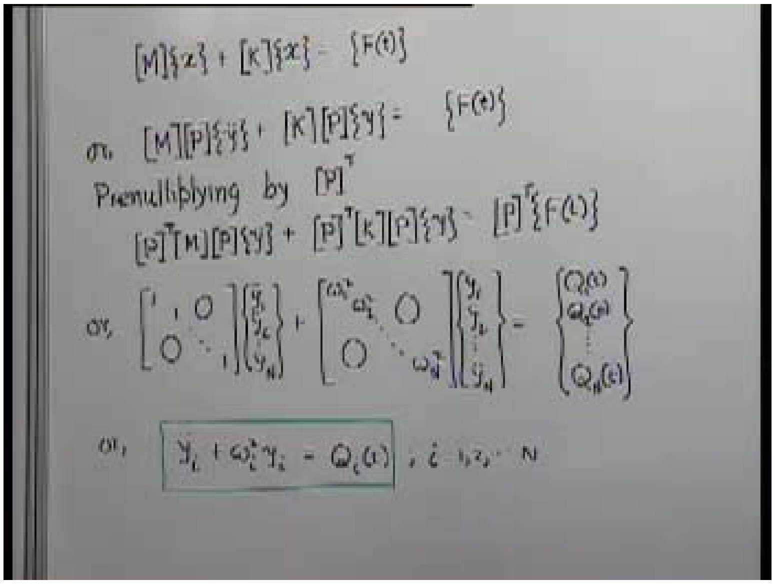

All the terms will be gone except the diagonal terms; this will be a square matrix, only the diagonal terms will be there and because of normalization all the diagonal terms will be one. It will be a unit matrix (11:58) and here again, it will be a diagonal matrix where the diagonal terms will be this and the rest will be 0. Here, it will be a column matrix. These are the transformed (12:55). Therefore, what we are getting is nothing but some decoupled equations, the first equation will be or what we will get y_i two dot plus $omega_i$ squared y_i equal to Q_it and i varying from 1, 2 and so on up to N. So, you get a simple equation which is for the single degree freedom system; we are getting n number of such equations.

This constitutes only y_1 two dot and $omega_1$ squared and y_1 and Q_1, next one is only in terms of y_2, next one is y_3. So, they all will be decoupled equations and therefore each one representing the solution for a single degree freedom system. Therefore, you know the solution to this equation and we can find it out and once we find out and Q also we have to find out by this multiplication. Once we find out y, (Refer Slide Time: 14:17) we pre-multiply that by P matrix and then we get the actual solution wanted in terms of x. So, this is the general procedure and I will explain now the procedure in detail with the help of an example.

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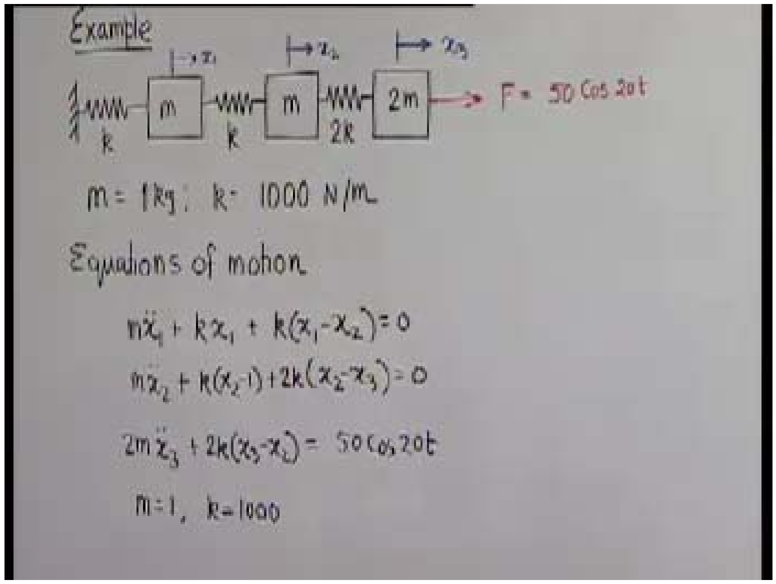

This is the problem; let us solve it. So, there are three masses connected by 3 springs: this is 1 kilogram; this is 1 kilogram; this is 2 kilogram; this is 1000 Newton per meter; this is 1000 Newton per meter; this is 1000 Newton per meter and mass 3 using acted upon by an external force which is 50 into cosine 20 t. The forcing circular frequency is twenty radians per second and the magnitude of the exciting force is 50 Newton per meter. Now, the equations of motion of the three masses can be written as this (Refer Slide Time: 18:35). m is 1; k is equal to 1000. Now, in matrx form equations can be represented (Refer Slide Time: 19:40). So it is now clear that we are writing the equation of motion for a normal mode of vibration.

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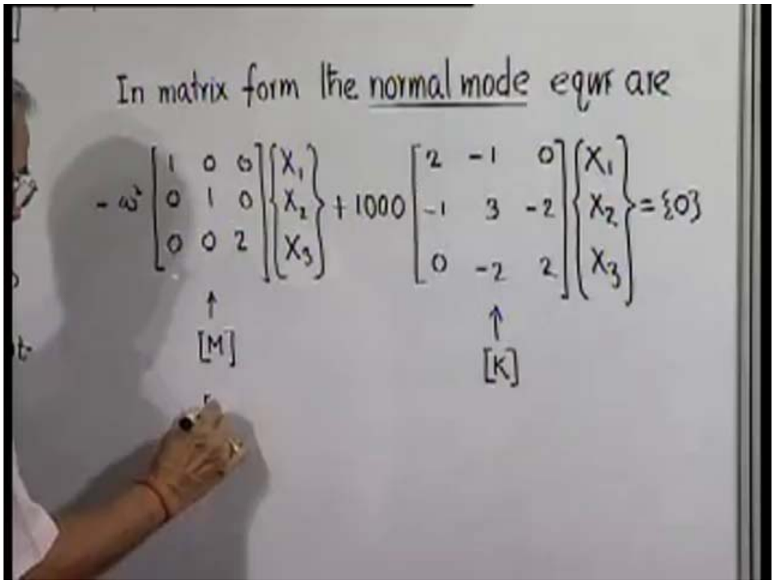

Once it is normal mode vibration, we know that each one will oscillate with the same frequency and x can be written as x_1 cosine omega t and so on. (Refer Slide Time: 21:00) Since it is normal mode oscillation, it is free oscillation that you can see; 0 means this is the vibration for free oscillation. Our objective is as I mentioned that before we solve the-forced vibration problem, we have to find out the natural mode and the natural frequencies. So, first we have to solve the free vibration problem; that is what precisely we are doing. The characteristic equation now, as this can be written as minus omega squared, this is the m matrix and this is the k matrix, then m inverse will also be - you can easily see even by inspection - inverse matrix will also be a diagonal matrix. Multiplying by m inverse, the equation becomes (Refer Slide Time: 23:08) minus omega squared, m inverse into k will be A matrix and A matrix in this particular case will be 1000, we can take common outside, this unit will be second square as we know, because we are now solving a numerical problem. We have to be careful about the units.

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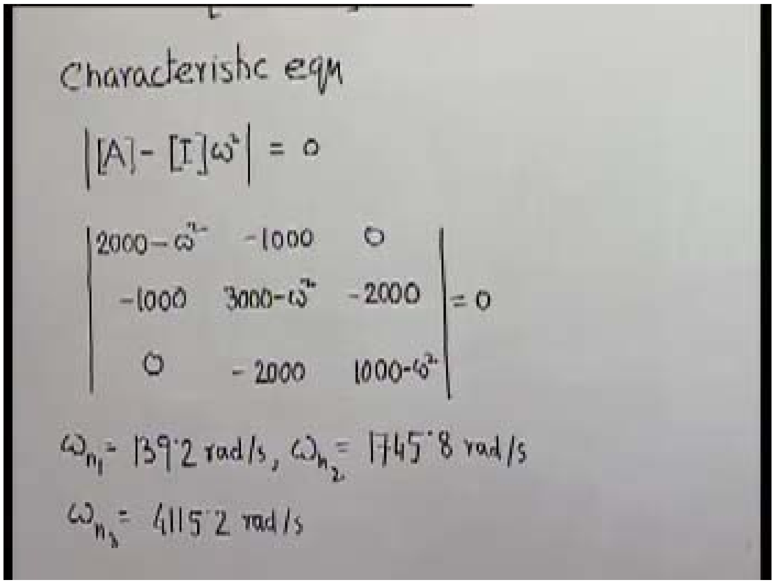

So, the characteristic equation will be - because these are set of three homogenous equations so the determinant must be 0, that means (Refer Slide Time: 24:54), when we write in a standard form, becomes 2000 minus omega squared minus 1000 0 minus 1000, (Refer Slide Time: 25:35) (poor audio) omega squared. Solving this, we will get three roots or three values and we can tell you that the three roots of natural frequency will be (Refer Slide Time: 25:59), second natural frequency will be and third natural frequency will be.

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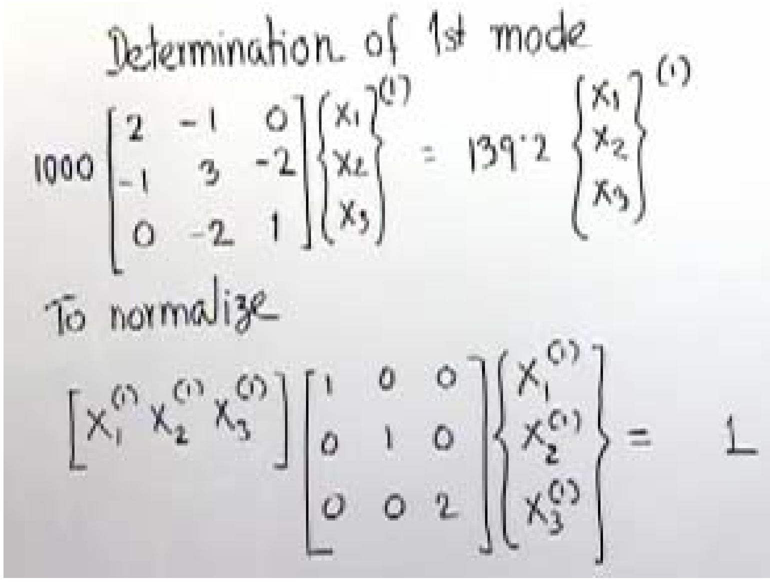

Three natural frequencies after solving this equation will be (Refer Slide Time: 26:41). Once we know the natural frequencies, now to find out the modes. First let us find out the first mode. This procedure I will show and the same procedure will lead to other modes. We have seen the equation, this equation A is this 1000 into 2 minus 1 0, minus 1 3 minus 2 0 minus 2 1. For first mode now, this is equal to, these are all… is equal to, when you take the other side, minus becomes plus omega$_{n1}$ squared for the first mode is 139.2 (Refer Slide Time: 28:39). This is the equation for the first mode. We have to find out x_1, x_2 and x_3 for this first mode. We have already normalized x_1, x_2, x_3 - otherwise has no meaning - now you have to normalize it using that condition you remember x transpose m x equal to 1.

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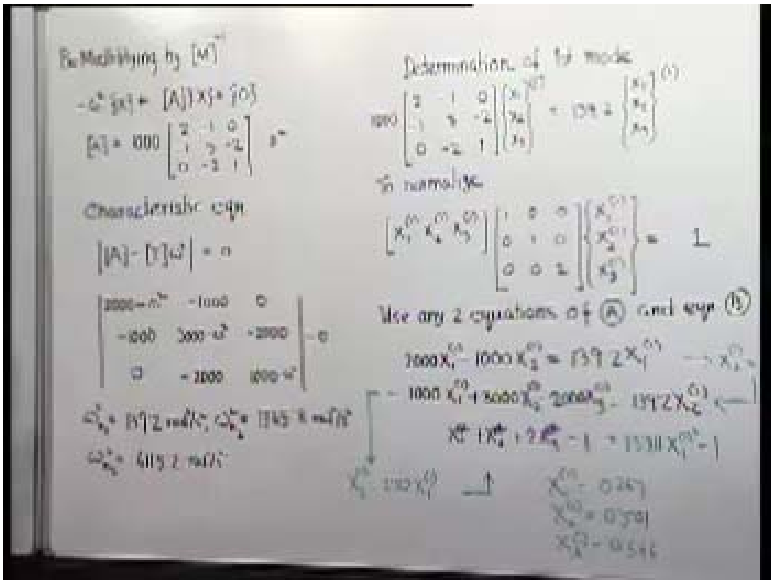

What we have done is X_1 one X_2 one X_3 one, that is the transpose of X one, m matrix was 1 0 0, 0 1 0, 0 0 2, that was the m matrix you must remember and this was X_1 one X_2 one X_3 one, the first mode and that is equal to 1. That is normalization. Now, there are three equations here and one equation here, in total four - we do not need four - so what we can do is use any two equations from here and this equation from here of the state A and equation B. Let us solve the first two; $2000X_1$ minus $1000X_2$ equal to $139.2X_1$. That was the first equation, the second equation will be minus $1000X_1$ plus $3000X_2$ minus $2000X_3$ equal to 139.2 X_2, that is the second equation, we do not need the third, we will have to do this equation X_1 squared plus X_2 squared plus $2X_3$ squared equal to 1, so these three equations we can solve to find out X_1 one X_2 one.

Since, it represents a particular mode, it should be, it is not a general equation, so, from this one it has an equation of only X_1 X_2 that will give us X_2 in terms of X_1 this will give us X_2 one equal to 1.861 X_1 one. If we use this here: X_2 will be in terms of X_1, and X_3 will be there. So here this will give X_3 in terms of X_1 straight forward. Then using both X_2 and X_3 in terms of this, we can get an equation in terms of X_1 only and this will give 13.811 X_1 one squared equal to 1 and the result will be X_1 one equal to 0.269, X_2 one is just 1.861 (33:25) this is 0.501, X_3 one is equal to 0.586.

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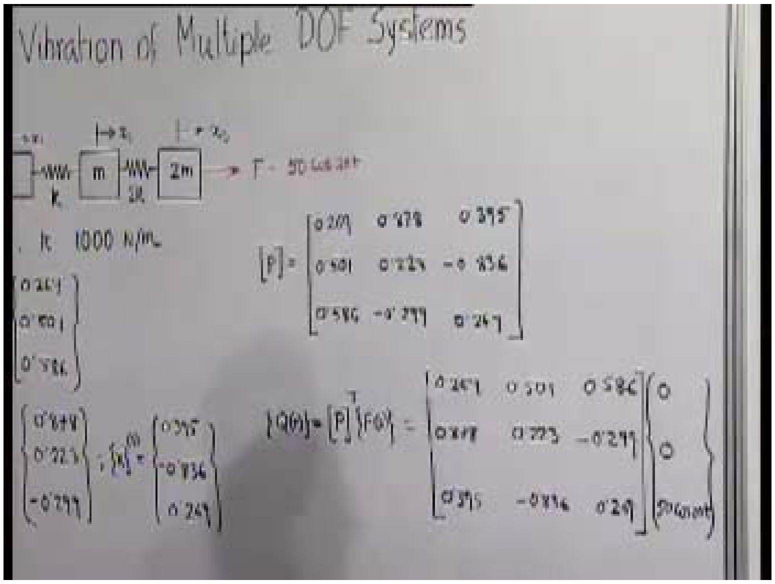

So, we have found out X_1 the first column of the modal matrix. (Refer Slide Time: 34:06) All the time we have to remember, we are getting these unique numerical values; because you have normalized the natural mode, otherwise there is no meaning of talking about amplitude in a free vibration problem. Similarly, X_2 will be following the same procedure; only thing we have to do is we have to use for the second mode and this is going to be the square of second mode that is 1745.8. So, the equations will be different and solving that set of equations we will get (Refer Slide Time: 35:08). Again after normalization only we will get numerical values like this. The third mode again we will find out by using the third natural frequency 4115 in place of 139.2 that and this equation will remain the same. Then we will get the third mode the numerical value and our modal column matrix, this is our important matrix (Refer Slide Time: 36:39) which will transform x into the principle coordinate y. Therefore Q(t) will be also found out by pre-multiplying the forcing matrix which is nothing but this (Refer Slide Time: 37:11).

This will be 0.269 0.501 0.586 will be first row in place of first column, second column will be the second row and third column will be the third row. This is P transpose and f t matrix if you remember force acting on station 1 is 0 force acting on station 2 is 0 force acting on station 3 is 50 cosine 20t so this is f t matrix.

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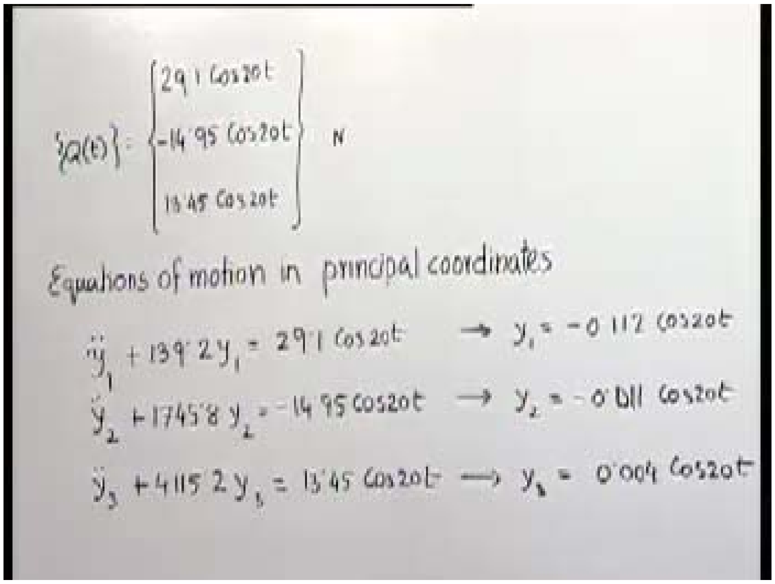

Now after, we do this Q (t) becomes (39:33). This will be neutral. So, the equations of motion in principle coordinate will become free decoupled equations like this.(Refer Slide Time: 40:10) We know the solution of this steady state(41:10) - of course you have to consider - steady state solution for each one (41:10) and so we get from here y_1 equal to (Refer Slide Time: 41:22). This we bring in from our memory steady state solution (41:58). We know that, this will be 0 by you know that 1 by r.

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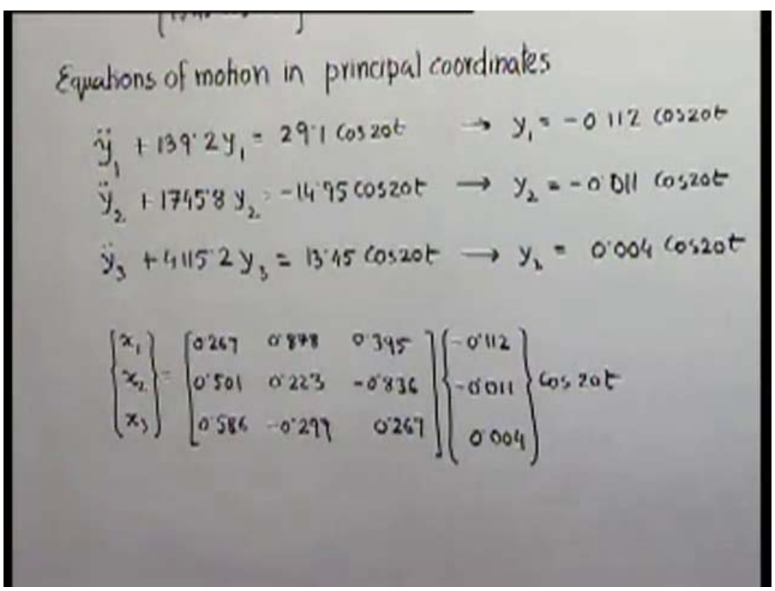

Once we get this y, x will become f_2 pre-multiply y by t; so, it will be .269, .501 (Refer Slide Time: 42:53) and y we have found out and the question is now what is the dimensions we are following the si unit, so this dimension will be in meters .This finally will become (Refer Slide Time: 43:52). So, this is the solution to find out x_1 x_2 x_3 as functions of time of course in steady state solution for a post vibration problem, we always know that the (44:34) will go and steady state oscillation will have to be in the same frequency which is the frequency of… This is the general procedure for which I think, now the question is that if there are two forces then what we can do, we can separately solve and super impose because the system is linear. That way we will get the solution by super-imposing the solutions separately obtained in this form.

Thus, we come to the end of our discussion on systems with multi degree freedom system. Multi degree freedom systems are obviously - as we have been mentioning - some modeling of the real system; that means, a beam with various cross section or a machine or a structure. We can modulate in the form of a lump parameter system, which will be approximately representing the system, the number of lump parameters or the blocks we use, more number of mass to be utilized, obviously, we can have a better approximation. Therefore, better accuracy or better results will demand larger number of masses for a better approximation I heard there are situations where it is possible not to break into lumps but to split the actual system as it is. Therefore, now we will look into the phases where, we do not lump the parameters like this we treat the system as it is.

Say, for example, a cantilever beam, a bar or a shaft in torsional mode - if we hit it, it will vibrate, you know that. So, they are all systems which are very often, rather the actual systems are composed of such elements and therefore, our next attempt should be how to handle cases where the system is not lumped and we have been doing so far what the system is treated as it is. We will take up obviously the cases like the simple elements like a uniform bar uniform beam (47:12) and how to solve the problem. So, from next session we will take up the vibration problem of a continuous system and at least a few cases (47:28). There will be some new concepts as you will understand and those new concepts also, we will explain in the class.