Dynamics of Machines

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Module - 11 Lecture - 4

So far we have been discussing the effect of excitation, either in the form of the dynamic force or dynamic moment or some periodic motion to the support of a system. Primary objectives were, during these studies, what is the response of the system which we measured or which we defined through the displacement of the main body, which represents the systems inertia. We will extend our studies little bit in the direction, which will be extremely useful while designing a system. What we will do now is that try to analyse, that if we keep a system which is supported by kind of or maybe a combination of springs and dashpots. If the ground has some motion, what will be the motion transmitted to the main body?

It is something similar, what we have been doing in our vehicle example, where the road was wavy and the wheel was moving up and down. The driver along with the car was also moving up and down, so that is one thing, that means the motion of the support or the ground gets transmitted to the main body of the system. Similarly, we should also investigate the situation where a mechanical system is generating some dynamic force, due to some unbalanced present. Then, how much of that unbalanced force which is produced in this system, gets transmitted through the combination of springs and dashpots which we put into the ground. Therefore, what we will try to do? We will try to design our support, with the help of a combination of springs and viscous dashpots, in such a way that the system is isolated from the environment. That means, either the forces or moments generated in the machine when it runs, gets transmitted to the floor and that way it may affect other precision setups, or if a precision setup is being mounted, it should be run is such a way, that all the disturbance which are there in the ground, they do not get transmitted through this. This topic is generally termed as transmissibility and isolation.

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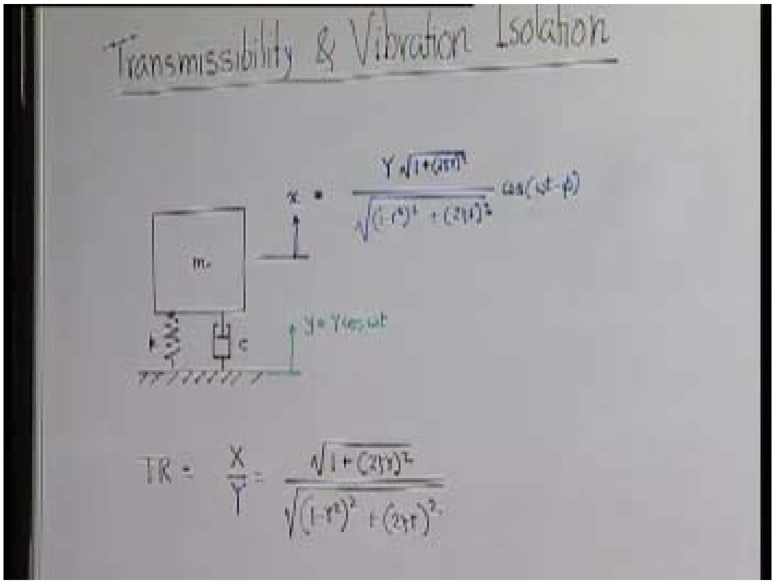

The main objective of this study will be, how to decide the parameters like stiffness and dashpot, viscous dashpot, damping coefficient in a manner, so that the system can be isolated from the environment.

This is the system we have been studying so far, a simple spring mass dashpot system. Now in this case, we have already solved the problem that when the ground has a motion like Y cosine omega t, then x is given by (Refer Slide Time: 05:08). This also oscillates with a magnitude of this. Therefore, we define transmissibility by this quantity, this is the displacement transmissibility that is X by Y, which is nothing but (Refer Slide Time: 06:06). We have already seen this, we have also plotted this earlier. Same thing it is actually defined as the transmissibility and it is like this.

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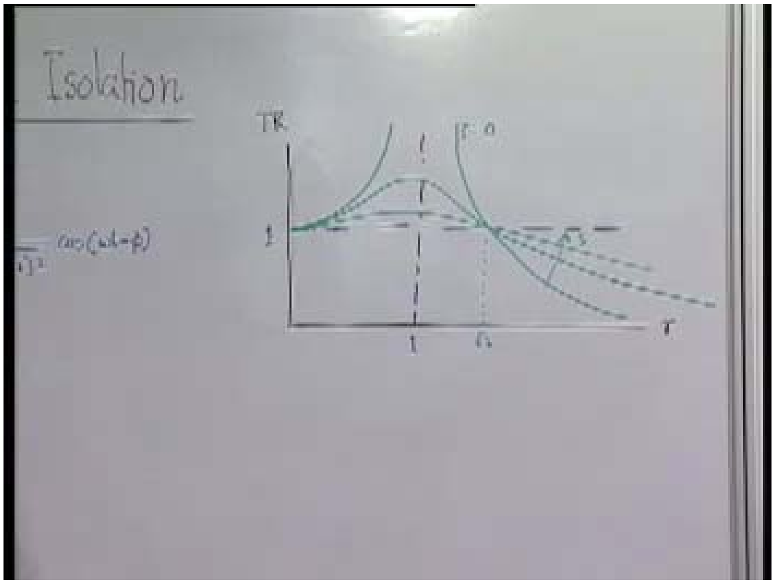

We have already seen transmissibility, if we plot against frequency ratio, zeta increasing and both at r is equal to 0 and r is equal to 0.2, transmissibility is always 1. That means, whatever disturbance I am giving, the same amount is being produced here. But if we can design the system in such a way that r is large enough, we are operating somewhere here. Then maybe transmissibility will be a small fraction, maybe only 10% of this motion, which is being given to the ground, is being transmitted.

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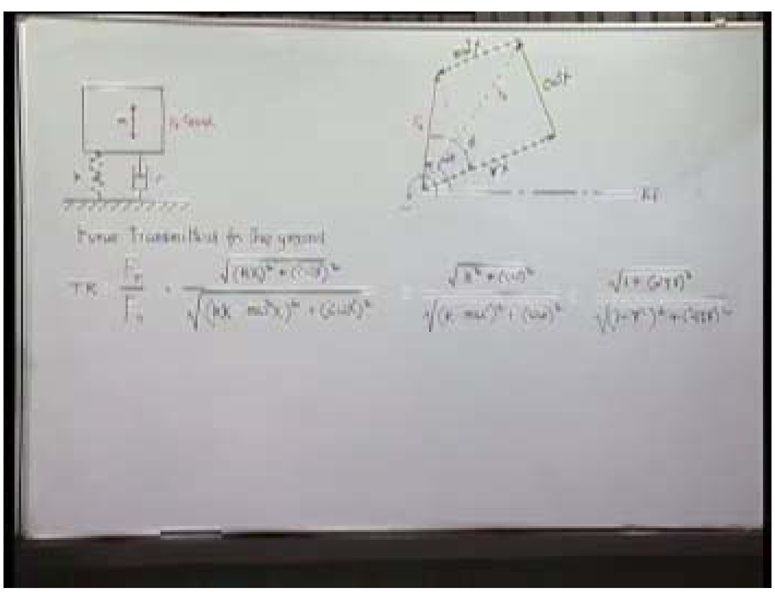

The other concept of transmissibility is as I mentioned little while ago, that is system or a machine when it runs and it generates some unbalance force. The question is how much force is transmitted to the ground or base, whatever it maybe, say it is F_T. Of course a harmonic function of time, but magnitude is F_T. Then, F_T by F_0 is called the transmission. That is another way of looking at the transmissibility, when a force is generated in a machine, how much of that force goes to the ground through the support or through whatever arrangement we have made for supporting the system.

Let us find out how much force is being transmitted. Force is being transmitted only through the element like spring and dashpot. So the vector sum of the force at any instant on the dashpot and the spring is the force, which is transmitted through the ground. We will again use our convenient rotating force. The spring force is represented by this rotating vector, the damper force is represented by this rotating vector leading the spring force by 90 degrees. Of course, inertia force is represented by this rotating vector and all these three, we know that the whole thing is rotating with fixed rpm omega and the real values at any instant, as we have been mentioning again and again are actually projection on this some reference line. If this is omega t, then this will be phi. That means, the displacement x lags the exciting force F_0 y in angle phi. Now, what is the force which is being transmitted to the ground F_T?

We have seen that F_T is the vector sum of the spring force and the dashpot force, that means it will be represented by this rotating vector and at any instant, this projection will be the instantaneous value. Therefore, let us see how much is the transmissibility? F_T is nothing but, square root of kX square plus the dashpot force divided by F_0. F_0 is again this square plus this squared, square root. How much is this? This is kX minus m omega square X, this square, plus this is again (Refer Slide Time: 13:07). This is the ratio of the magnitude of the transmitted force, with magnitude of the generated force. If we take X common outside, it goes out and it becomes (Refer Slide Time: 13:24) and you definitely have guessed by now, that we are going to get the same, if we divide by k square or take k square common outside, then it becomes 1 plus c omega by k is 2 zeta r. This is 1 minus m omega square by k is omega square by omega m square that is r square and this is like this. (Refer Slide Time: 14:15). The expression for transmissibility, both from the point of view of transmission of motion, vibration, or transmission of dynamic force which gives the same expression we find.

Therefore, it is a very important result of the study of vibration is that, we can isolate systems and machines from the environment, so that neither this unwanted disturbance come to our system, when we are a handling very precision experiments, or the ground does not get too much of disturbance from the machine on which I am working, suppose a very heavy forcing machine or machine tool, which is generating a large force, then a smaller amount is really going to transmitted to the ground. So, the ground will have lesser vibration produced by this transmitted force.

This technique of isolating a system from the environment is called isolation. The quantity which dictates to what degree one system has been isolated is called the transmissibility and for simple systems like this, transmissibility behavior is like this. Expression is like that, best will be perhaps if we solve a problem.

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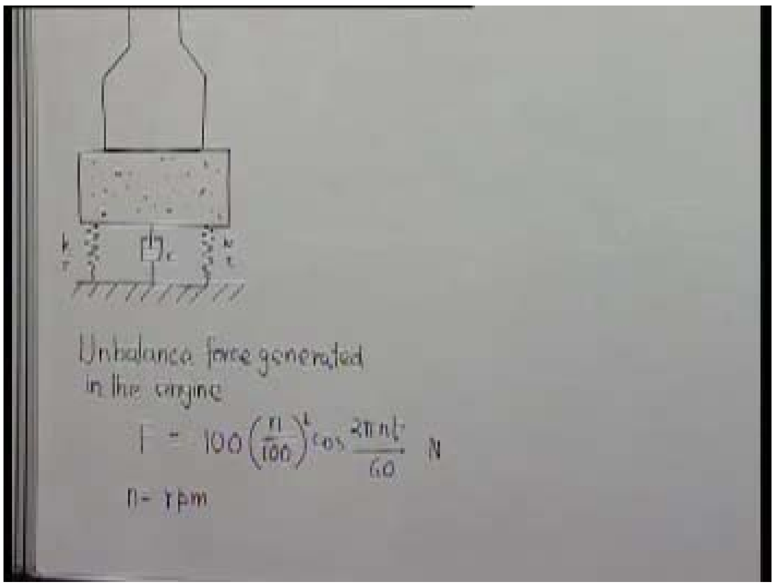

This is a vertical engine, is supported on a concrete block. This concrete block again in turn is resting on a system of springs, with total resultant stiffness of k and system of dashpot, total resultant damping coefficient c.

The unbalanced force generated in the engine is given by this expression (Refer Slide Time: 17:14), where n is the rpm of the engine. This is a simple harmonic force, we are assuming that is a primary unbalanced force. That is why it is 2 pi n by 60 t. That means it is omega into t. That means, it is connected with the rotational speed of the engine and therefore it is clear that it is the primary key. The magnitude of the force is also increasing with force, obviously because the centrifugal force or unbalanced forces will always be increasing in magnitude, not only in frequency when the rpm increases. This tells us the approximate representation of the generated force.

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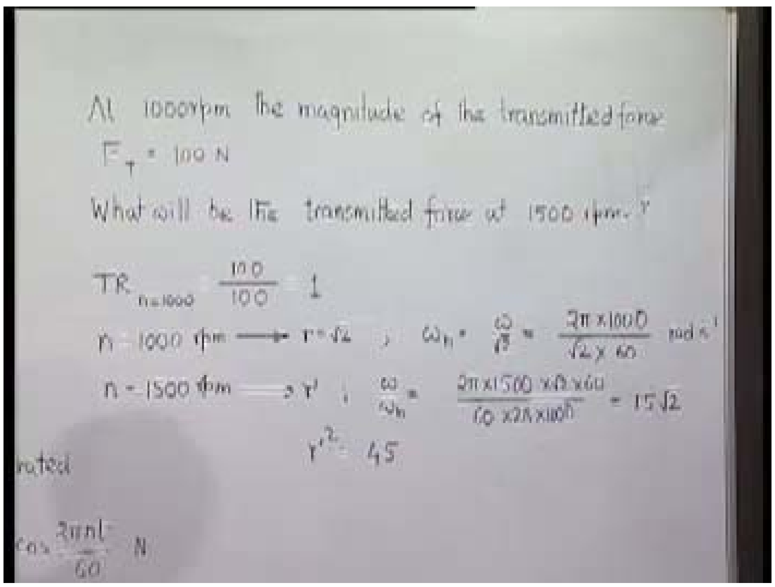

Now, we say that at 1000 rpm, the magnitude of the transmitted force, which is F_T is equal to this 100 Newton. What will be the transmitted force at 1500 rpm? If the question is this, that given the information is how in rpm of the engine, the magnitude of the transmitted force is 100 Newton. Then, if the engine runs at 1500 rpm, what will be the transmitted force? Now, we have been given the transmission ratio or transmissibility at n is equal to 1000 is equal to 100 is the transmitted. How much is generated at 1000 rpm? At 1000 rpm the magnitude of the generated force will be 100, so 100, equal to 1. Now, what does it mean from this characteristics? The transmissibility can be 1 only at r is equal to 0. That means the frequency 0, which is not possible because the engine is running at 1000 rpm. The only other possibility is that when r is equal to root 2, transmissibility will be n equal to 1000 rpm corresponds to r is equal to root 2.

We can find out the r, that means, $omega_n$ is equal to actual operational frequency omega by root 2 is the natural frequency. This we know how much is the omega. It is 2 pi into 1000 rpm divided by root 2 into 60. This gives me the radian per second is the natural frequency.

Therefore, let us find out r prime. r prime will be omega by $omega_n$ is equal to, how much is omega at 1500 rpm? This will be 2 pi into 1500 divided by 60, divided by $omega_n$. Omega $_n$ is this much (Refer Slide Time: 23:04). It will be 15 root 2 or r prime square equal to 4.5. Now, one point which was missed, when we mentioned this as such, if we just tell r prime, we will not be able to solve the problem, because c is not known. Therefore, the only possibility will be in this case, we can find out the transmitted forces with 100 rpm if the damper is disconnected.

In the first experiment, it was not necessary to disconnect the damper, because at r is equal to root 2, the effect of damping coefficient or damping factor is not present. It is irrespective of that the value of zeta which is 1. But when you deviate from this, then the transmission which depends on the damping coefficient information on which is needed, but in this case we have not given that information. Therefore, the problem is actually, what we can find out is that, when the damper is disconnected?

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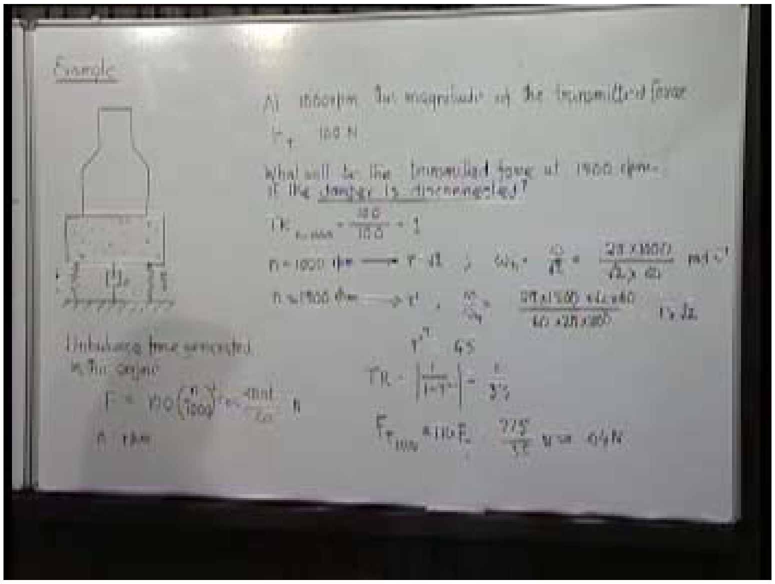

In this situation, the transmissibility is given simply by 1 by 1 minus r squared (Refer Slide Time: 25:05). Transmitted force at 1500 rpm will be F_0 by TR. How much will be the force generated F_0 at 1500 rpm? It will be simply 1.5 squared into 100, that is 225 and transmissibility will be 1 by 3.5, so how much of Newton approximately equal to (Refer Slide Time: 26:16). Therefore, we can see only approximately 30% or even less than 30% of the force which is being generated here, is being transmitted by suitable choice of this stiffness extra, we could make it still less. Therefore for simple situations, a designer should keep in mind not only the design of the machine, but also the support system. In the market, actually there are standard isolators available which are small systems that can be directly placed below a heavy machine, which is placed in a shop, so that the machine isolated not to disturb the ground.

Similarly, for mounting very accurate equipment or instrument and such devices, when they are also isolated, so that disturbances present in the ground do not come to the instrument. This brings more or less to the end of our discussion on single degree freedom damped systems. Only thing what we have not done, but as mentioned earlier that handling problem, where a system is subjected to non harmonic but periodic forces or disturbances. We can split the non harmonic, but periodic disturbance into a series of harmonic disturbance. As the systems which we are discussing are always linear, we find out the response of the system, because of each harmonic. The result will be the sum total of the F_T.

What is really happening? It is a mathematical exercise, other than any physical or mechanical activity. We will avoid that, because it will be simply a technique, which you are familiar with about how to split a periodic or non harmonic function of time into a series of infinite number of harmonics, which are pure harmonic quantities sines and cosines. The magnitude of each harmonic, you can easily derive it from the characteristics of the system, can find out the response and the total response will be sum total of all the response. We will not take up those discussions separately, as except mathematics of or the technique of Fourier series, except that, nothing new is present there. What we should pickup now is one or two cases or one or two topics, which maybe very relevant, though always not directly a part of a single degree freedom system like this.

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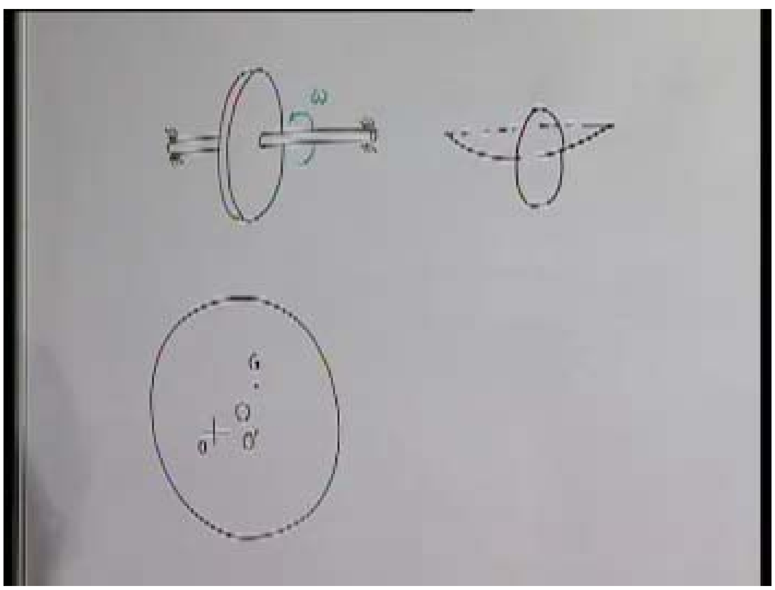

One, very common problem faced by mechanical engineering is the shafts carrying discs. A shaft like this is rotating with a speed omega. Now, whether there is any potential problem, the dynamics of rotating bodies itself is a subject. We will not be able to discuss it there, but the simplest possible case we would like to take up, because it has lot of similarities in all kinds of analysis we have been doing.

What are the possibilities, when we mount the disc on the shaft, the center of the disc of the CG will not be exactly at the axis of the shaft. If this is the shaft, the CG maybe somewhere here, center of the shaft we call it as o prime. Again because of the centrifugal force which is generated due to these rotated body and which is unbalanced, the shaft will also get bent and the axis that means, real axis rotation if we join this. That means shaft will be bent, this is the point about which the whole thing will rotate, we will take up the simplest possible case where the deflection of the shaft, then the location of the G and the center of the disc, so that the relative position remains same. That means, the bent shaft and the disc, they all rotate like a rigid body. This particular type of situation is called forward synchronise precision. It occurs very often, let us see what we can do about them.

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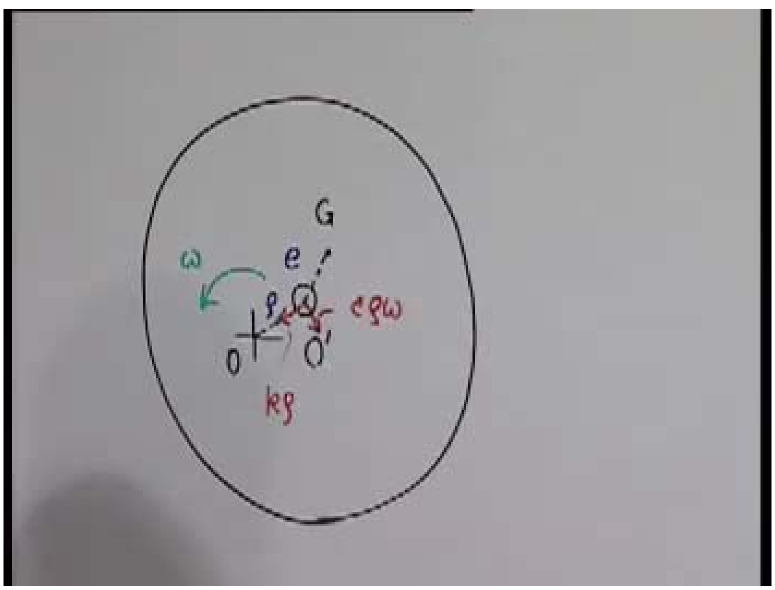

Therefore, if distance is e eccentricity which is inherently present, because of wrong manufacture or fabrication and this distance is rho. Rho represents the deflection of the shaft due to the unbalanced force produced. Now, what are the forces acting? External forces acting on the disc let us see that. The shaft has been bent, when you bend a shaft, effectively it is just like a spring. That means, a force will be acting in this direction and the shaft will act as a spring with an effective stiffness of k. This will be k and the magnitude will be proportional to the deflection of the shaft with force. Then this point is rotating like a circle, so this point velocity is in this direction and the fluid which is there, that is air or whatever media, I mean which it is immersed, that will produce a drag force whose resultant drag force on the whole disc we can assume logically, passing through the geometric center of the disc. That is the drag or the friction by the fluid present in this. This magnitude will be equal to the velocity of this point, which is rho into omega and the coefficient, which we may put c, representing the viscous friction of the surrounding field. Therefore, these are the two external forces acting and let us look into the dynamic system.

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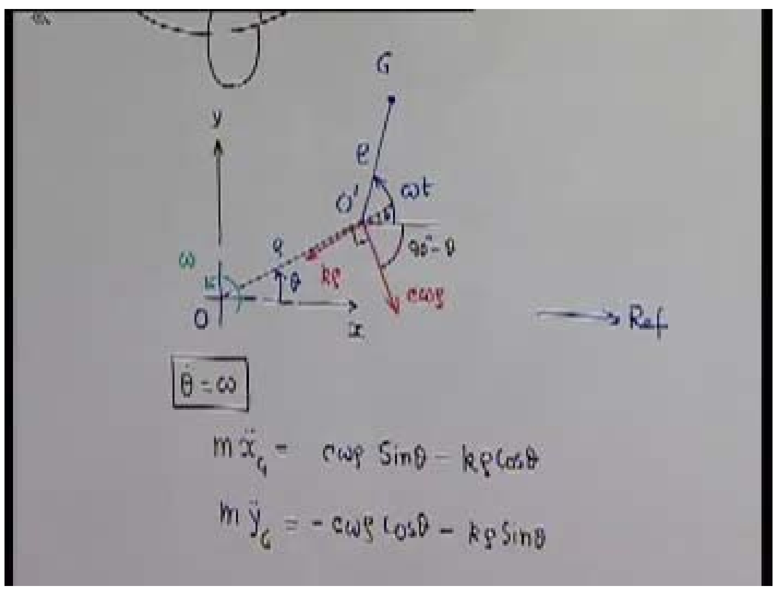

Enlarging this figure, this is o, this is rho, this is e, this is G and this point is o prime. Forces are k rho, c omega, this angle is theta and these also let us consider if this is the rotation from time t is equal to 0, that means when o prime G is parallel to this direction, that is the reference line. So if t is 0 there, this angle which makes now with that direction is omega t. Remember, theta and omega, though they are not same, but the whole thing is moving like a rigid body. Therefore, we should keep in mind that theta dot is also equal to omega. If we split everything into two components, x and y, the coordinate of the center of mass of the disc is the x_G and y_G and its accelerations will be x_G two dot and y_G two dot this component of acceleration.

Therefore, equation of motion we can write that mass of the disc x_G two dot must be equal to the total force in the x direction acting on it. How much is that? One is c omega rho. What will be the component in this direction? This angle is theta, this is omega t so omega t minus theta, therefore this is 90 degrees minus theta, because this is 90 degrees. Since this is theta, so it is 90 degrees minus theta. Therefore, it will be sine theta minus the component k rho cosine theta. Similarly, in the acceleration component in the y direction is equal to minus c omega rho (Refer Slide Time: 38:24).

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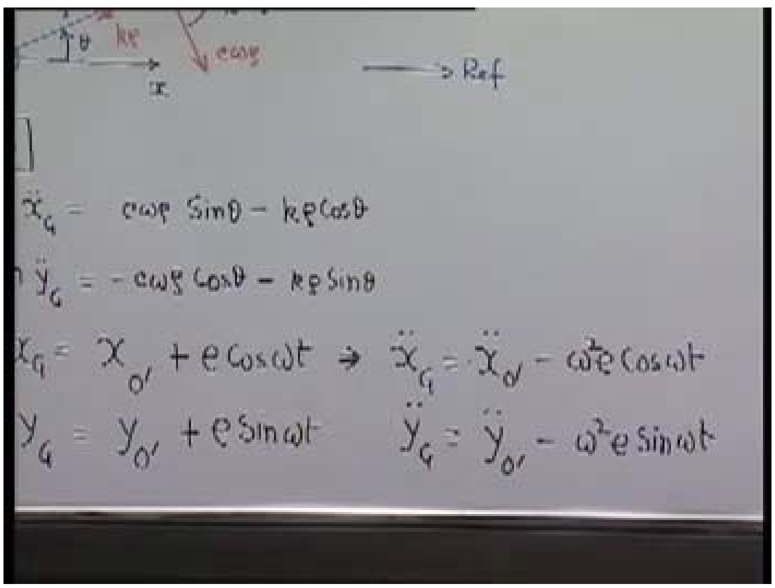

We can also represent x_G and y_G. This x_G is equal to x_{oprime} plus e cosine omega t and y_G y_{oprime} plus e. This will give me x_G two dot is equal to x_{oprime} two dot minus omega square cosine omega t and y_G two dot equal to y_{oprime} two dot minus omega square e sine into omega t.

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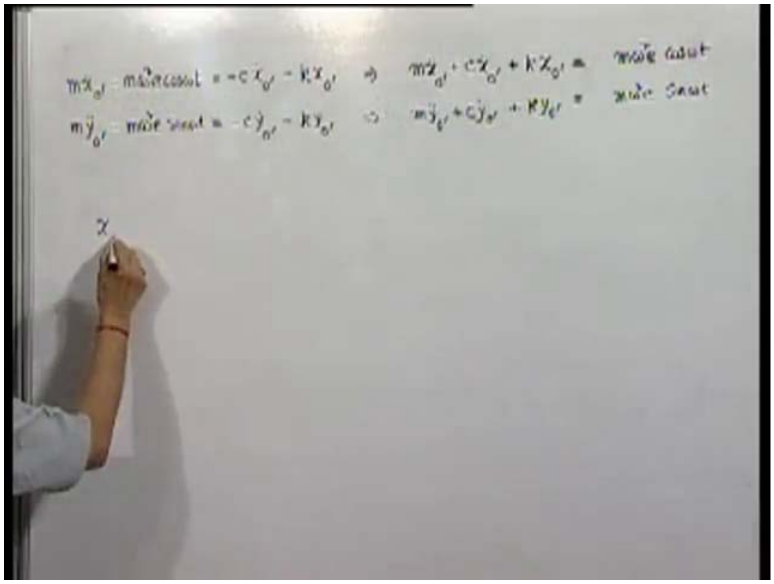

Substituting in these equations we get (Refer Slide Time: 40:00), c omega rho is nothing but velocity of this point. If we multiply by sine theta, what happens, it is the x component of velocity of o prime point. It is nothing but c into x_{oprime} dot.

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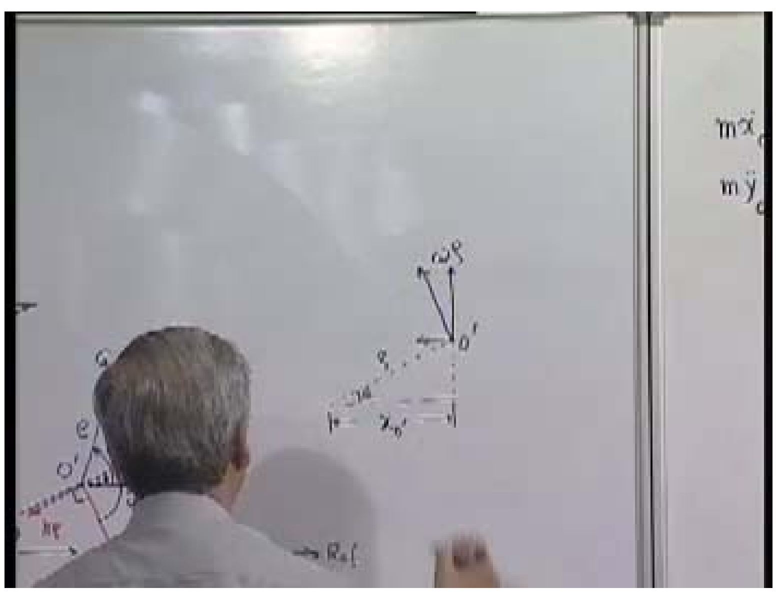

Its velocity is in this direction, velocity is omega into rho. It has two components, this component is nothing but x_{oprime} dot and c comes there. Similarly, the other one will be y_o

$_{prime}$ dot in this direction, when it is omega rho e, c omega rho cosine. So, this is this (Refer Slide Time: 41:40) minus k and rho cosine theta, this is rho and this is theta, rho cosine theta is nothing but x component of this point o prime (Refer Slide Time: 41:59). The other equation becomes, I am substituting y_G two dot by y_{oprime} two dot minus omega square e sine omega t and that is equal to omega rho cosine theta. Here we have found it is omega rho, omega rho cosine theta is y component. The positive direction is still, so minus sign will remain and rho sine theta is y_{oprime}, which is minus k y_{oprime}. This can be written like this $mx_{o\ prime}$ two dot plus $cx_{o\ prime}$ dot plus kx_{oprime} is equal to, I take this to the other side (Refer Slide Time: 43:26). This equation becomes (Refer Slide Time: 43:34). Now, we remember our solutions to this form of equation.

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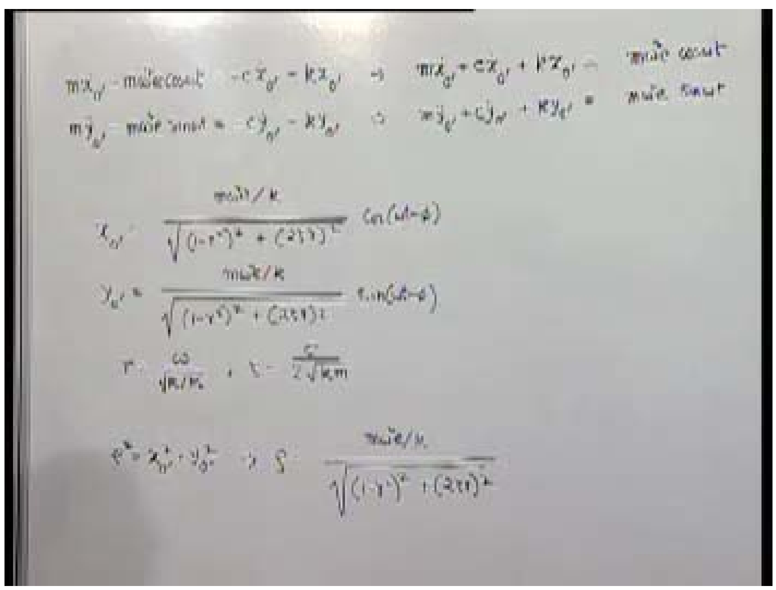

We can always write x $_{oprime}$ is equal to, now F_0 we have solved this problem mx two dot plus cx dot plus kx is equal to F_0 cosine omega t. Now F $_0$ is being replaced by this, so what we will get here, the final solution F_0 by k, I think that was the numerator (Refer Slide Time: 44:26), where zeta of course is here, this cosine. We have to keep in mind r is equal to omega, that is the rotational speed by omega $_n$, omega $_n$ is the circular frequency of natural lateral vibration of this, which we can keep it as (Refer Slide Time: 45:30) and zeta is again, we know zeta is c by 2 square root of km.

One thing is very clear, if we want to find out rho square, this is x_o prime, this is y_{oprime}, rho square is equal to x_{oprime} square plus y_{oprime} square. This will give us, rho equal to (Refer Slide Time: 46:15).

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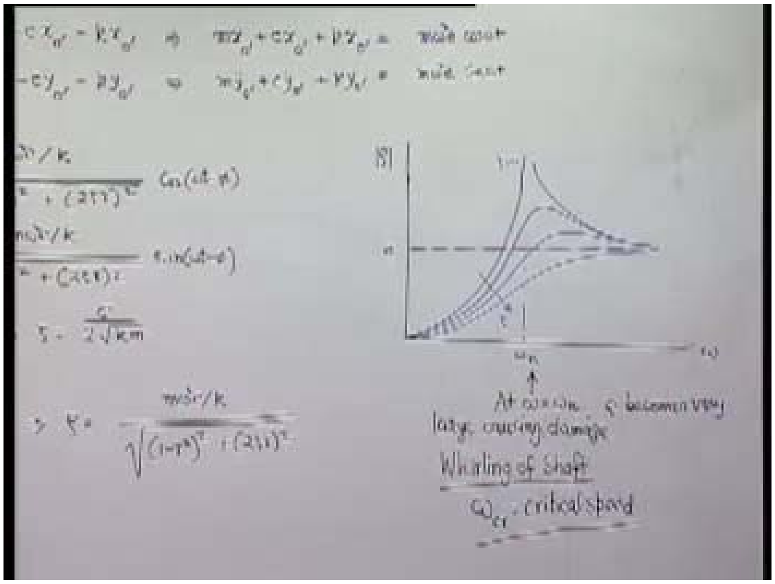

So, we find that the characteristic let us see how it is going to be. For omega is equal to 0, this rho will be equal to 0, because numerator is 0. As it increases when omega becomes very large, then r is very large you can ignore this and since this is r to the power f and this is r to the power two, so we can ignore this. After square root it becomes r square, that means omega square by $omega_n$ square that is, k by n. so it will be e.

If we plot an e rho, when r is equal to 1, suppose zeta is extremely small we can ignore this. So, denominator is 1 by r square approximately. When r is 0, omega r is equal to omega by $omega_n$, so omega is 0, means r is 0. It starts here, because rho is 0 at omega is equal to 0, and then it shoots up when r tends to 1 and then comes down, becomes e. Of course it is the magnitude I am talking about, if zeta is present, if zeta is increasing; at a very high speed, the deflection of the shaft will be equal to the deflection of the area of the center of mass from the center of the actual shaft or access of the shaft. However, when omega is nearer to $omega_n$, there is a possibility that the vibration that means the deflection of the shaft will be very large, which is something like resonance what we had.

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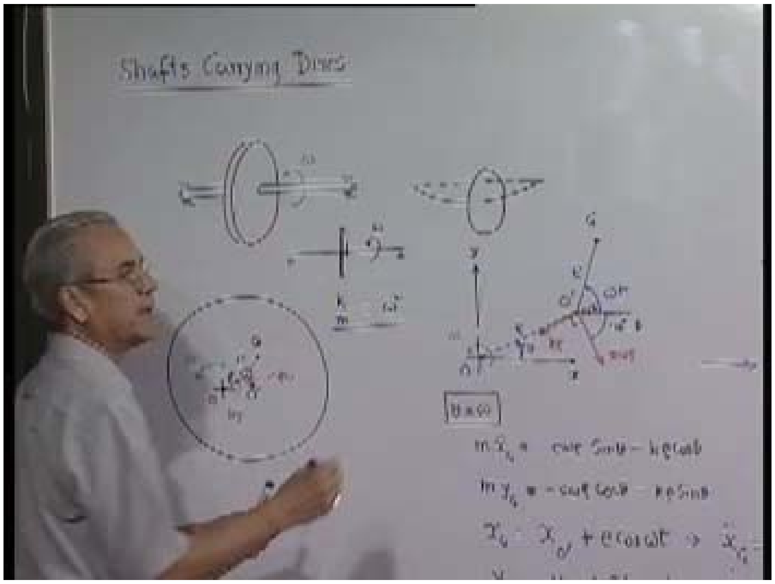

That sometimes is so large that the shaft maybe permanently damaged. This particular phenomenon, when it happens that means at omega is equal to omega $_n$. That means if the shaft is like this, then the equivalent stiffness of the shaft here is k and mass of the disc is km. If this is equal to omega square or omega is the rotational speed of the shaft, then again something like resonance takes place. This phenomenon is called whirling of shaft.

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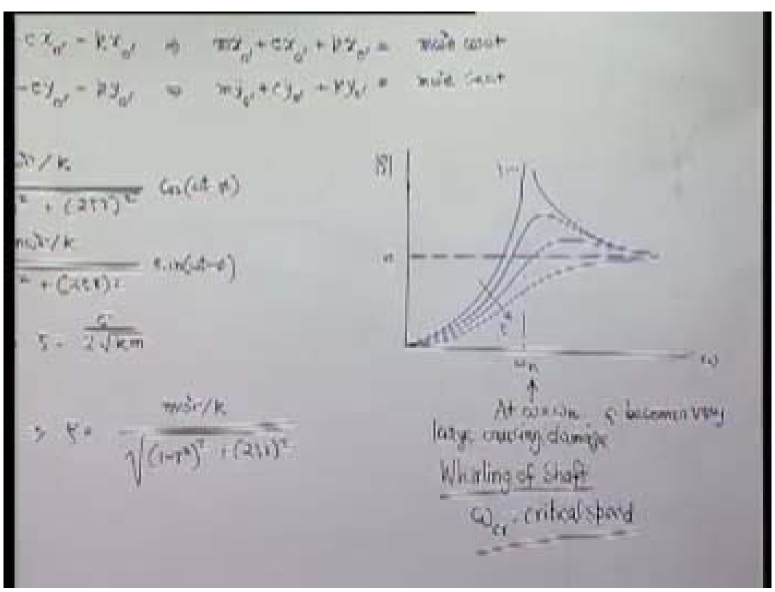

This is the simplest possible case of forward synchronise position and the speed at which it happens is called critical speed. Therefore while designing it, it is essential for the designer to keep in mind, if he is designing a rotating system, that the rotational speed of the system as designed should never go anywhere near the first critical speed or the critical speed of the system.

Sometimes when it is found that the critical speed is low and the operational speed is high. For example, like the alternators in thermal power plants. The shaft rotates at a higher speed than the critical speed. In such cases to avoid whirling and damage what is done, the shaft is quickly accelerated though the critical speed, so that it does not get enough time to build up large vibration and large deformation of the shaft. This is the simplest situation is very similar to what we get in case of a simple spring mass system behavior, when some damping is there. We have not gone into the details of finding the phase angle and all those things. It can be easily found out without any problem.

But the main thing what one has to keep in mind is that the shaft has certain space at which, the system resonance causing very large deformation of the shaft from its nominal state position and these speeds are called critical speeds. The design should be such that they should be avoided. At very large rpm, when you operate the deformation of the shaft, will be the amount of error which was there while fabricating the system, that is the e or eccentricity of the center of mass.

I think with this we more or less complete most of the things, which you are supposed to take care of in a simple system, which is spring mass and dashpot parameters. It is now important because a very large number, in most of the systems case, it is not possible to express or represent the system with the help of a simple mass. Therefore it is absolutely essential for us to now investigate, what happens when the numbers of masses are more or the degree of freedom of the systems become more or the system becomes more complicated. Subsequently you will devote your attention to systems which higher degree of complexity, in the sense that the number of degrees of freedom will be more.