Dynamics of Machines

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Module - 11 Lecture - 3

Equivalent Viscous Damping System with Base Excitation

We have solved the problem of a single degree freedom system that is viscously damped when subjected to a harmonic excitation. In practice, of course we know already that, there are many other forms of energy dissipation and not necessarily they will lead to linear equation, which can be readily solved like the (00:45) viscously damped case. In such situations, what can be done? At least approximate solutions can be obtained, if we can represent the damping by some equivalent viscous damping and then we can use the results of the viscously damped system. Now what we will do, we will find out the ways and means to determine an equivalent viscous damping for a case which is not viscously damped.

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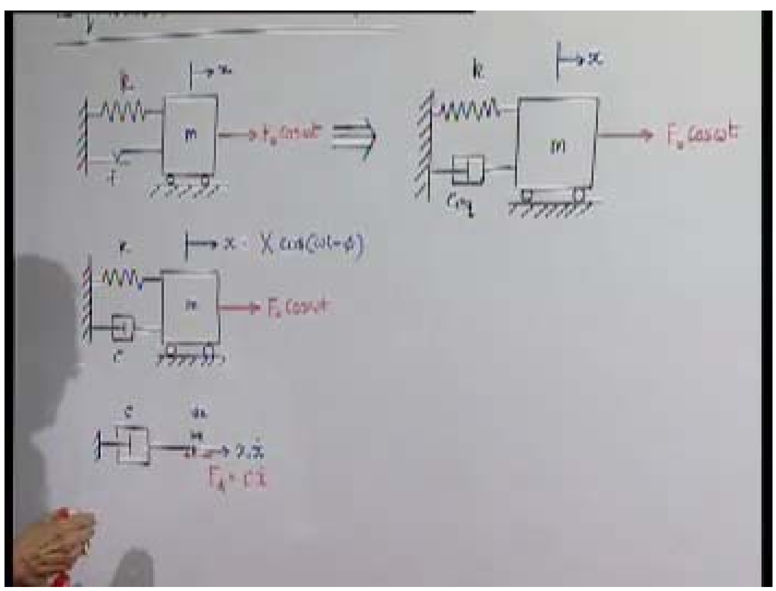

Say, let us take a particular case, it can be anything like a Coulomb damped system. Here, the energy dissipation mechanism is different, it is not viscous and the equations which we get obviously, there is not a linear equation that immediately solved. We have solved the free vibration problem by using various techniques. Suppose, if such a technique is subjected to a harmonic excitation, what will be the response which we have to find out. One approximate way of managing this kind of a problem is that, to convert this system to an equivalent system, where the stiffness mass etc., are same as the original one. Only thing what we have done, the Coulomb damping has been represented by a viscous damping here (Refer Slide Time: 04:13) with a coefficient of damping c equivalent.

Now, what will be the condition for this equivalent? One obvious thing that comes to our mind is that the job of this damping mechanism is to dissipate energy. For the same amplitude, whatever may be the dissipation of energy per cycle, if that we maintained in the two cases as equal, we can give some logistic support to our consideration that, this is an equivalent damping mechanism. To do that, let us first find out, that in case of a viscously damped system (Refer Slide Time: 04:58), what is the energy dissipated per cycle. Say it is amplitude of its displacement x is given as x (Refer Slide Time: 05:32), when it is subjected to a harmonic force F_0 cosine omega t.

What is the amount of energy dissipated per cycle? Here the only place where energy is being dissipated is this dashpot neither here nor here. (Refer Slide Time: 06:06) Therefore, let us see how much energy is being dissipated in this dashpot. (Refer Slide Time: 06:06) Now at any instant, this position also has a displacement x, velocity x dot etc., and the force which is acting here we may call damping force F_d, which is equal to c into x dot; where, c is the coefficient of damping in this diagram. At this instant, if it goes by an elemental distance dx and this force that means, actually the force which is acting on this point is in this direction and the force applied on this is this one (Refer Slide Time: 07:07). Since this point is moving in this direction which is subjected to a drag, which can be represented as this (Refer Slide Time: 07:18). The energy loss due to this drag force because of this displacement creates will be dE_d will be F_d into dx.

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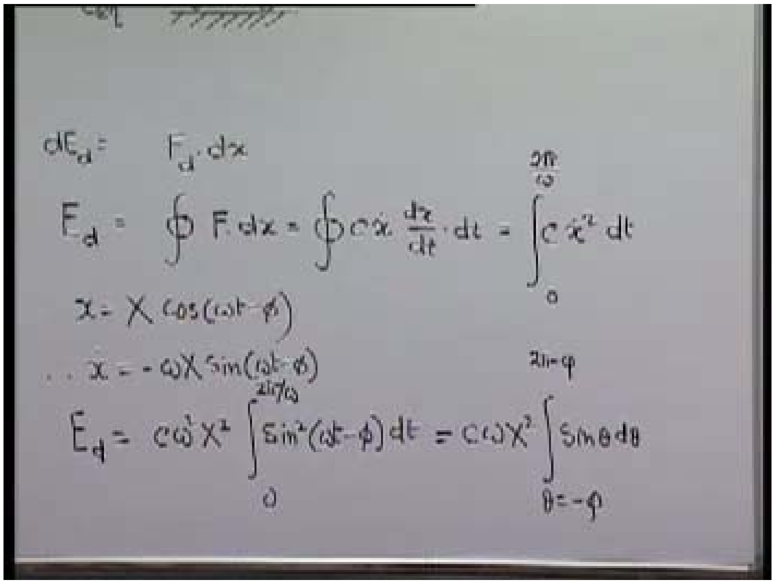

So, per cycle, energy dissipated will be just integrated form of this for a whole cycle and F_d into dx, which we can write as F_d is nothing but c into x dot and dx can be written as (Refer Slide Time: 08:02) dx by dt into dt and therefore this can be written as cx dot square dt, t varying from 0 to a time period is nothing but 2 pi by the circular frequency, that is omega, that is the time period.

Now, we know that x is X cosine omega t minus phi, so x dot is nothing but minus omega x (Refer Slide Time: 08:45). So substituting it here, we get E_d is equal to c omega square x square sine square (Refer Slide Time: 09:18), so this is integrated over a full cycle. We know that this (Refer Slide Time: 09:32) can be easily written as c omega x square sine theta d theta, theta is equal to this. This is again for a full cycle from 0 to 2 pi, we can make another transformation and we can easily make sine say phi d phi 0 2 pi square. This is nothing but pi. I am not going to the detail calculations; it is a standard result in trigonometry.

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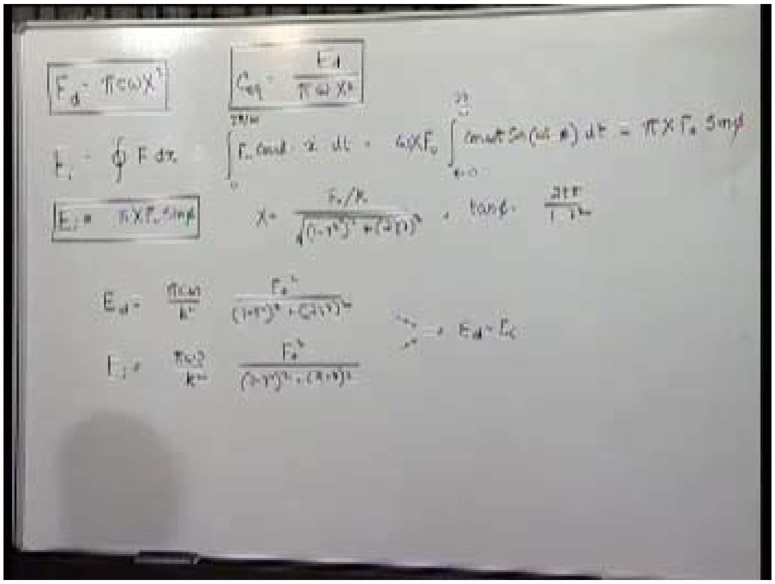

The energy dissipated per cycle is (Refer Slide Time: 10:23) pi c omega x square. So, therefore, the energy dissipated per cycle will be pi c equivalent omega x square. Actual energy dissipated in the real system will be E_d. Therefore, we can always write at (Refer Slide Time: 10:58) equivalent damping coefficient will be that, which will be the energy dissipated per cycle in the real system divided by pi omega x square. If we use this expression for the damping coefficient and use the results of this standard viscously damped single degree freedom system under harmonic excitation, we can get the result. Maybe in this particular case we do it but before we apply it to this, let us also find out that whether it matches the energy input.

This is the amount of energy dissipated per cycle. But since the system is in steady state, that means its amplitude of oscillation does not change. The amount of energy dissipated per cycle must be replenished by the amount of energy input by the exciting force, which is doing work into the system. That can be easily can be found out, that amount of energy input or cycle is nothing but (Refer Slide Time: 12:12) F into dx, F is the applied force. This is again noting but (Refer Slide Time: 12:23) and finally we know x. Therefore, x dot is already here, this will be (Refer Slide Time: 12:53) now, this minus sign will be (13:27) here that is energy input. If we solve this integration, we will get the result by standard result again, which will be one time we can do it (Refer Slide Time: 13:46) pi x

F_0 sin phi We get two different expressions, one for energy dissipation as that for energy input per cycle. Your obvious question comes to the mind is as we seen the same it must be same in the steady state.

Therefore, one has to substitute the expression for x and phi (Refer Slide Time: 14:29), using these two it can be proved easily that E_d and E_i are same and trigonometry I am avoiding it. You can show that E_d is equal to (Refer Slide Time: 15:10) when you substitute the expression for x from here (Refer Slide Time: 15:30) this will be the expression for E_d. Similarly, when you substitute sine phi, sine phi can be found out from tan phi, which will be square root of 1 plus r plus phi, the inverse of that. Therefore, using that one can also again show that this will be (Refer Slide Time: 15:54) same thing. The algebra one can him can satisfy himself. Therefore, E_d is equal to E_i in the steady state, which is a very obvious thing because otherwise it will not be in steady state.

Therefore, the technique for solving a problem, where the dissipation mechanism is not wasted and such a system is subjected to harmonic excitation and we have to find out the response. The first data to find out the energy dissipated per cycle in the real actual system then, find out the equivalent damping coefficient for a system where the energy dissipation will be the same with this viscous damping. Then, assume that system will have similar response though the result will not be exact, it will give us only some approximate idea about the response of the system. If we apply, for example, this particular phase with Coulomb damping, what will be the response? Let us solve this problem and find out what will be the oscillation of the response. Now, first is to the find out c equivalent. How did you find out c equivalent, that total energy dissipated per cycle in the real system divided by pi omega (17:35).

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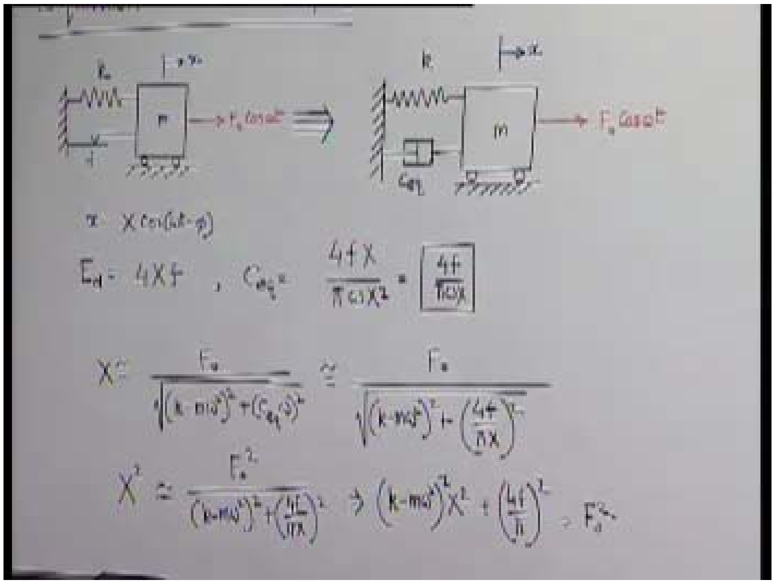

How much is this, if suppose x is x cosine, that kind of a thing if amplitude of oscillation is X, then for each half cycle, the displacement of this coulomb damper is 2x. so energy loss will be simply 2 effects because friction force is constant. Similarly on the other half cycle, it will again have another displacement and which is 2x again the energy dissipated will be $2f_x$. So, in total energy dissipated per cycle in case of this Coulomb damping, is nothing but (Refer Slide Time: 18:20) 4x into f. This is the total displacement and this is the constant force friction force, which is opposing the relative motion and so 4x into f is the energy dissipation, so that will tell us equivalent damping is going to be (Refer Slide Time: 18:39).

We find that equivalent damping coefficient, which will result in same energy dissipation per cycle depends on it is not something like a system property; it depends on the result of the motion. Anyhow once this is shown, we know for the viscously damped system x is nothing but F_0 by k (Refer Slide Time: 19:27) will be slightly different form of the result. This is the amplitude of oscillation; this difference we are not interested immediately. This is where we have applied this equivalent (Refer Slide Time: 20:02) damping coefficient. Now, substituting this (Refer Slide Time: 20:10) c omega into c equivalent into omega will be simply this (Refer Slide Time: 20:26). What we get is actually an equation in X but remember we should always use this (Refer Slide Time:

20:42) approximation symbol. Squaring both sides, we get X square is approximately equal to (Refer Slide Time: 20:53) and from this we get x will be approximately equal (Refer Slide Time: 21:49).

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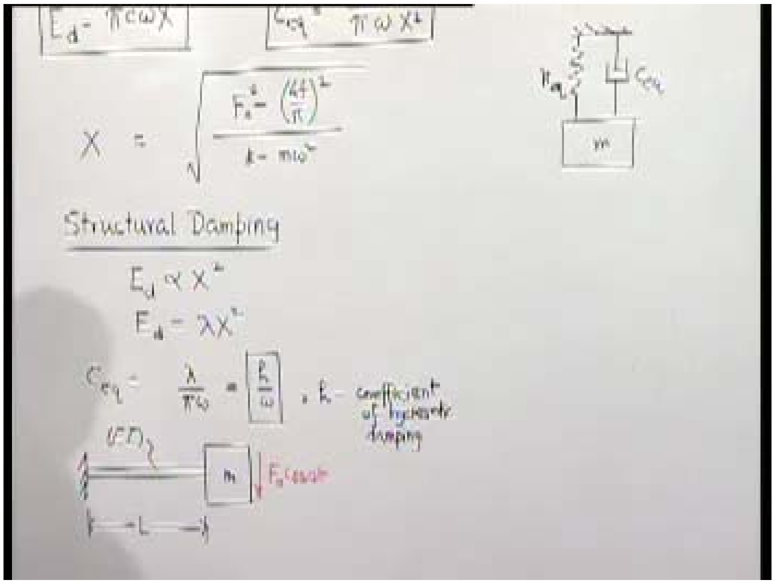

This is the expected amplitude of oscillation of the Coulomb damped system when subjected to a harmonic excitation. But, we have to always keep in mind that the result is very approximate and all the characteristics which we find, for example here, it is found that 0 being less than 4f by pi that will have no amplitude because it is going to be imaginary. But in reality, if we apply a force a Coulomb damped system whose magnitude is 4f by pi, which is more than f because 4 is more than pi, so it is definitely going to oscillate with some positive and real amplitude.

Similarly, we also notice that when the applied frequency is equal to the undamped natural frequency root over k by m, then denominator is 0 means it will never approach any steady state, that means, the characteristics will be similar, so that of a undamped system with single degree freedom. This is a very founder characteristic but when we go, that means F_0 is much larger than f etc., then the value of x will be approximately equal to the value of x in the real system but you should not try to make a deeper understanding using this expression for the system because this does not satisfy the whole thing exactly.

Anyhow there may be other kinds of damping like a structural damping. In structural damping in most cases energy dissipated per cycle is proportional to the square of the amplitude (Refer Slide Time: 24:14) or you can write that E_d is equal to some coefficient. I do not know may be lambda x square. In such cases, equivalent damping will be (Refer Slide Time: 24:32) where h is nothing but lambda by pi and h is the coefficient of hysteretic damping. Here, of course we find equivalent damping coefficient for a structural damping system is independent of x, say for example, therefore if you have a beam (Refer Slide Time: 25:18), say cantilever at the end there is a block m here. Now, if you want to solve this problem when this system is subjected to a dynamic force (Refer Slide Time: 25:42), this can be replaced by an equivalent system (Refer Slide Time: 25:51) this is k equivalent. The stiffness of the spring and this cantilever beam (Refer Slide Time: 26:00) when a force is applied at the end is same and the structural damping is replaced by (Refer Slide Time: 26:12) the rigidity modules of this the EI and length of this be L. Then, we know that, when a unique load is applied here, the deflection of the system is L cube by EI if the deflection force divided by EI into L cube.

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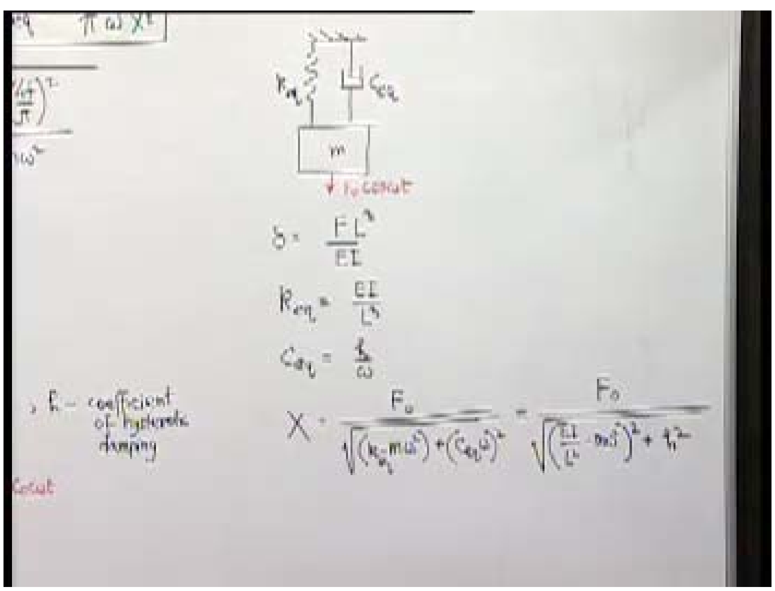

So its equivalent rigidity is going to be simply then EI by L cube. Similarly c equivalent is going to be E_d, which is c equivalent is going to be h by omega. So when it is subjected to a harmonic force (Refer Slide Time: 27:28), the displacement which we will get x will be F_0 divided by (Refer Slide Time: 27:42), where you have to just c equivalent omega is simply h (Refer Slide Time: 28:10) k equivalent is this. Therefore, we get some; again remember it will be an approximate because this equivalent is not exact. Therefore, this gives us some method of approximately solving the cases where the dissipation is not viscous and the equations lead or the resultant equations of motion of the system will not be linear.

We convert the system into an equivalent viscously damped system, find out the equivalent damping coefficient and feed the system as viscously damping system. So far we have been discussing the case where a dynamic system was being excited with the help of either a force or a moment applied at a suitable location. But it has been mentioned at the very beginning that there can be other method of excitation. One very common and frequently found that situation is where the support of a system is oscillating. For example, if there is a pendulum, I can oscillate the pendulum by applying a force to the ball or the pendulum bob but I can also oscillate it by oscillating or moving the support of the pendulum. These kinds of systems are called systems with base excitation.

Even this has lot of applications because vehicles that are where the whole vehicle is supported on springs and the wheels which touch the ground, the ground may be wavy. When it goes, the wheel gives an up and down motion, so what will be the motion of the vehicle? Similarly there can be many other situations where the excitation is not with the help of the force but from a disturbance to which support. Therefore, to represent the pa lump parameter model.

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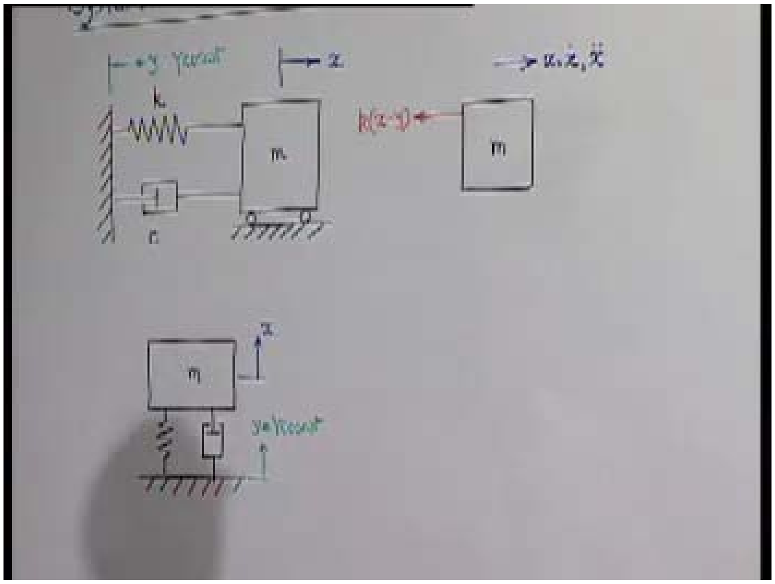

Now, we are exciting this (Refer Slide Time: 31:50) mass, not by applying a dynamic force here but by providing a harmonic oscillation (Refer Slide Time: 32:01) to the base or the foundation. It could be the other type also (Refer Slide Time: 32:14), this is very common. Something is resting and a ground is vibrating or here this is vibrating or a pendulum, the point of support or the hinge is oscillating. These are all cases of excitation given through a displacement of the support. Now the equation of motion we will try to solve this problem. (Refer Slide Time: 33:13) Its displacement velocity acceleration, we have all this, the forces acting will depend on the amount of stress of the screen at any instant, which obviously will be then x minus y because the amount of stretch will be displacement here, because this minus displacement. Similarly the damping force will be (Refer Slide Time: 33:57) the relative velocity which when the two ends of the dashpot.

The equation of motion therefore mx two dot is nothing but total force in this direction, which is minus c into x dot minus y dot minus k into x minus y or we can rewrite this equation in this form (Refer Slide Time: 34:27)

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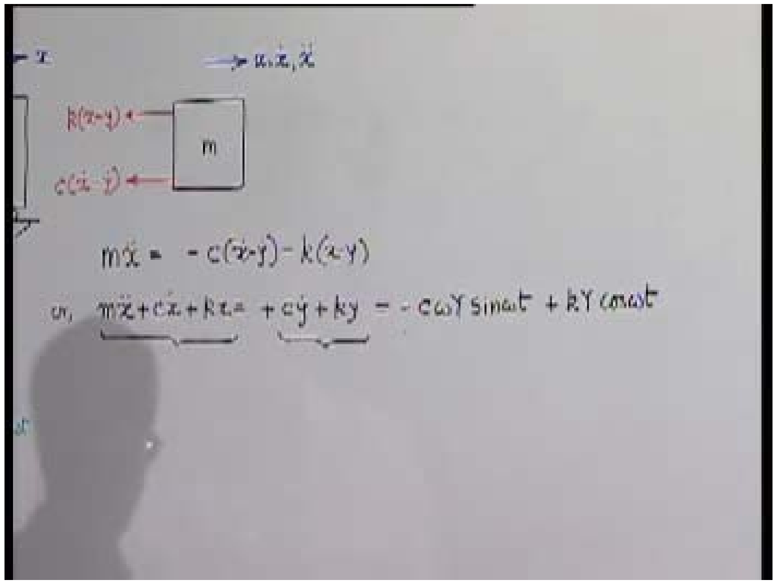

Now y is already known as Y cosine omega t. This can be written as minus c omega y sine omega t plus kY cosine omega t. Therefore, we find that right hand side again is a harmonic function of time (Refer Slide Time: 35:17), so the case if the equation becomes identical to what we have solved for a force excitation. Only thing that x_0 cosine omega t is being replaced by these, so we can solve the equation, as I have mentioned to you, that sometimes it is much easier to solve a problem by using the technique of rotating (35:45) let us apply that.

So, this is the left hand side (Refer Slide Time: 35:53) and this is the right hand side. These two must be equal, so how much is kX which is represented by (Refer Slide Time: 36:09) a rotating vector kX, cx dot is represented by vector leading (Refer Slide Time: 36:22), this by 90 degrees with the magnitude c omega X, m omega square x is this, that means, this result must be same as cy dot plus ky, ky can be something like this and cy dot will be another rotating vector leading the y vector by 90 degrees. This must be equal to some of these from this differential equation and the whole thing is rotating like a rigid body as we know.

Actually the real quantities are always projections of these on to one plane, in this case may be horizontal plane. One thing we find here that the phase difference between the excitation that is y and (Refer Slide Time: 37:47) the response that is this. But more importantly let us find out the expression for x in terms of y another quantity.

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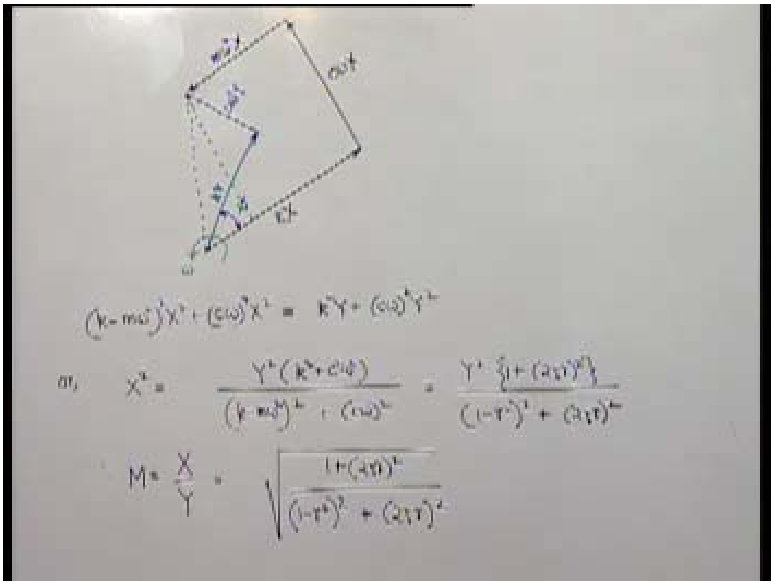

We do one thing, we find out this length (Refer Slide Time: 38:01), how much is this? It can be square plus and this is square equal to (Refer Slide Time: 38:15) k minus m omega square x square, that is this square and this is nothing but c omega x and this must be ky square plus c omega y square or easily we get x square is equal to y square into (Refer Slide Time: 38:47), now if we divide both the numerator and denominator by k square we get (Refer Slide Time: 39:15) now c omega by k is nothing but 2 zeta r in terms of those parameters we should replace now (Refer Slide Time: 39:30) and here it is again 1 minus r square whole square again c omega by k whole square. Therefore, the magnification factor e by x by y is given by square root of this (Refer Slide Time: 40:00).

So, this is the ratio of the response magnitude with the excitation magnitude. If you want to see the nature of this how it varies (Refer Slide Time: 40:31). When zeta is 0, no damping, it will be 1 by 1 minus r square. It is r that is for zeta is equal to 0; of course it should always mean that it should be magnitude. Now when zeta is equal to infinity extremely larger then, this 1 is negligible, we can keep only 2 zeta whole square. Again this becomes negligible because zeta is very large to this, so numerator and denominator is same irrespective of the value of r. m becomes independent of r and is equal to 1 that case it is also easy to imagine that zeta infinity means which is a rigid connection.

Whatever will be x, same will be y or whatever will be y same will be x, very easy. In between, when zeta is increasing from 0 and so one thing, we have noticed (Refer Slide Time: 43:11), obviously that all these points, all these curves they meet at one point here (Refer Slide Time: 43:17) that means the magnification factor or this kind of a system is equal to 1 when r is 0 because r is 0 means numerator is 1 denominator is 1.

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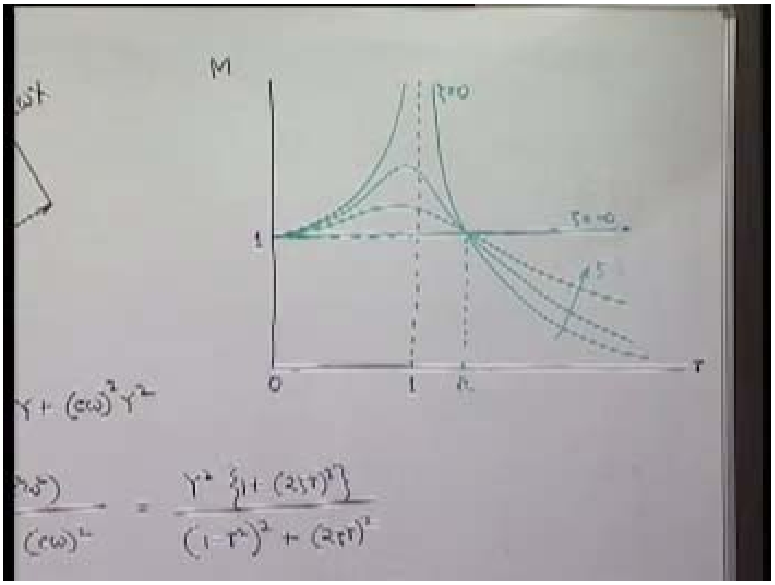

There is another value of r for which it is again always 0, that is r square is equal to 2. So it becomes 1 plus 2 zeta r whole square, here it becomes again 1 because 1 minus 2 square is 1 plus 2 zeta. So, numerator and denominator is again same irrespective of the value of zeta. So, all the curves they pass through again a common point where r is equal to root 2. Therefore, here also we find that in such a case the ratio of the response to the excitation depends on the damping coefficient or damping factor and the frequency ratio.

For example, if we operate in this range (Refer Slide Time: 44:24) what happens, we get a substantial response it will provide some excitation but if our natural frequency to be designed in such a way that, the exciting frequency is much larger and then obviously you are operating somewhere here, r is very large and then the response is much smaller than the excitation, though the ground may be vibrating substantially the actual system on which we are interested its vibration maybe small. We will come back to this point again but before we may close this particular discussion. Let us solve a problem

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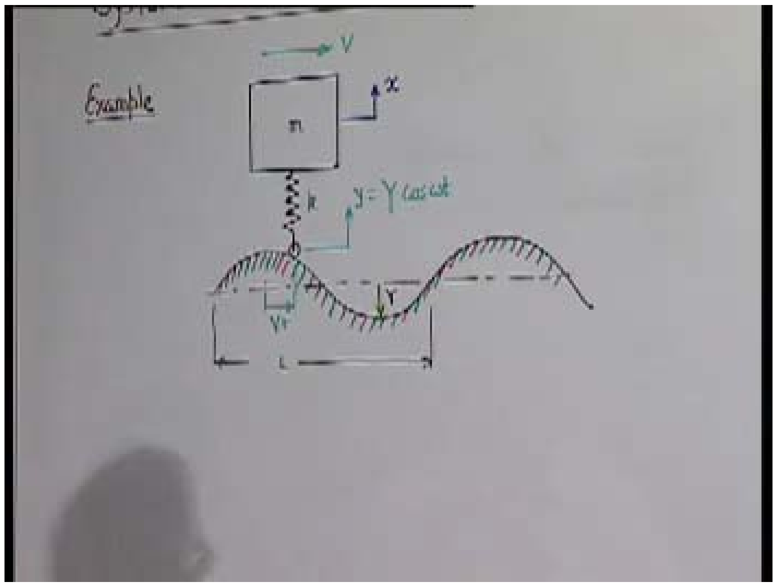

We represent a vehicle with extremely simple form, it is a block of mass m supported by just a spring with stiffness k is on the ground, is a sine wave with periodic length or with the length of one cycle is L that is wavelength and the maximum variation from the mean position is Y. Now we will give some numerical values here of course (Refer Slide Time: 46:37) this is x from its mean position. The vehicle of course is moving in this direction (Refer Slide Time: 46:49) with a velocity V. Now from vehicles point of view, we can consider a vehicle is stationary and the road is moving in the opposite direction so what will happen, (Refer Slide Time: 47:05) this point will be subjected to displacement y, which is nothing but y cosine omega t. If we consider t is here (Refer Slide Time: 47:23) or simply we can say not omega t, it is vt.

We give the problem like this that for a particular case m is given to be 450 kg, the speed is given as 96 kilometer per hour and wavelength of the sinusoidal road is approximately 6.1 meter, the variation the amplitude of variation is 0.025 meter that is 2.5 centimeter or 1 inch. The problem has three parts. First is how much will be the k, so that the system resonates. Number 2 will be if x has to be 0.6 centimeter, what will be the value of k? Third part is, does the driver leaves his seat, that is, if the seat and driver on which he is sitting gets disconnected.

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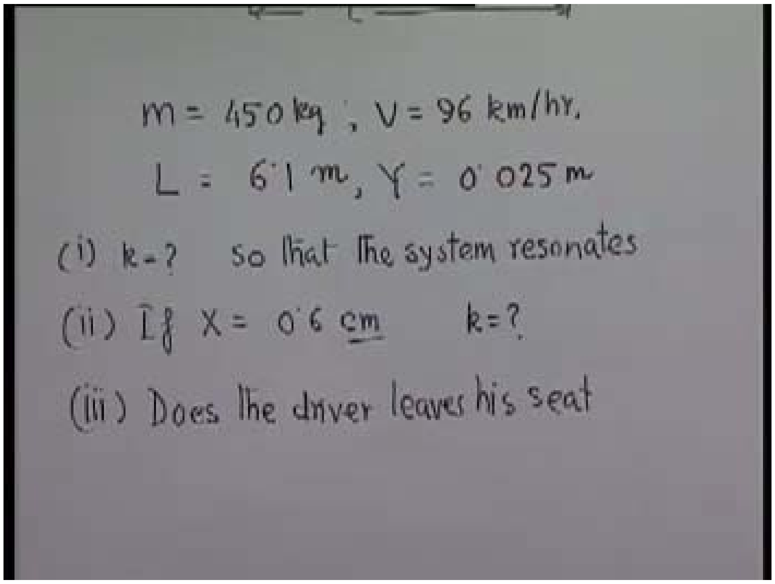

In this case the equation of motion is simply mx two dot is equal to minus k into x minus y, no damping or mx two dot plus kx is equal to ky cosine omega t (Refer Slide Time: 50:19), where omega you should keep in mind, omega is what it is completing, when it is moving, by moving at a speed V then how many cycles it will complete in unit time, it will be V by L. V is the distance moved in unit time. L is the distance equal to complete one cycle. So number of cycles completed in unit time will be V by L. Each complete cycle represents 2 pi of rotation of the vector, which represents this harmonic quantity. It is going to be 2 pi numbers of times (Refer Slide Time: 51:03), so we can find out the k for which the natural frequency will be again same.

Therefore, we can say for part 1 to resonate, if the system to resonate is 2 pi. Now, V will be an equivalent to 96 kilo meter per hour, which is equivalent to 27 meters per second. We have to find out these into 27 by 6 dash 1(Refer Slide Time: 51:51) must be equal to square root of k by 450 and this will give us k has to be equal to 34 into 10 raised to 4 Newton per meter and then the system is resonated. Next, we have to find out if it goes at the speed and the driver or this block mass has amplitude of 0.6 centimeter (Refer Slide Time: 52:35) that means point 0.006 meter, what will be the stiffness necessary?

Second part, we know that (Refer Slide Time: 52:49) x by y is given by square root of 1 plus 2 zeta r whole square but zeta is 0 in this case. Therefore this will be 1 by 1 minus r square and this is equal to point 0.006 divided by 0.025.

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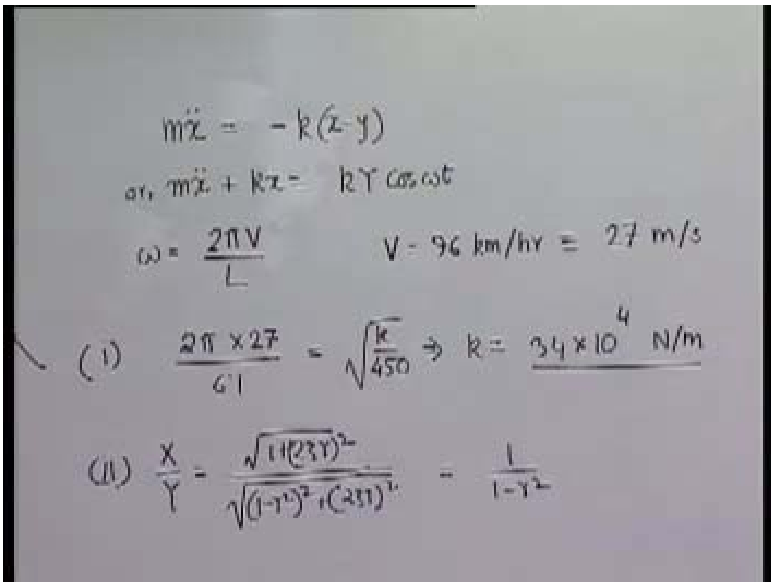

This will lead to r is equal to 2.275 that means this is equal to omega by omega_n.

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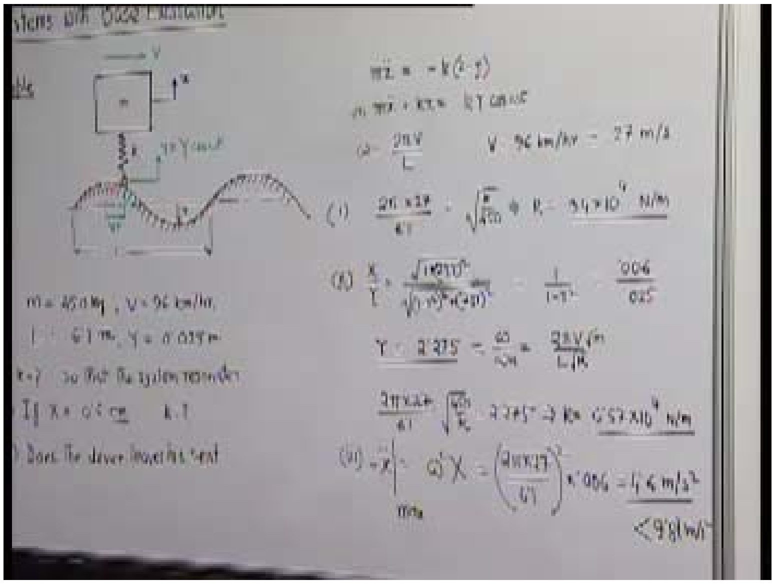

Omega we have already found out with the same speed, that means 2 pi V by L square root of k square root of m (Refer Slide Time: 53:57). 2 pi into 27 by 6.1 square root of k and (Refer Slide Time: 54:20) this m is 450 we can apply it here. This is a simple equation for k and that will give us (Refer Slide Time: 54:43) k is equal to 6.57 into 10 rises to 4 Newton per meter. That is the answer for the second part. Third part wants us to check (55:01) during this motion, whether the driver will leave the seat.

If we do that, we have to keep in mind that if this platform (Refer Slide Time: 55:10) or the vehicle is moving up and down, during the downward motion if the acceleration with which the seat of the driver is falling down is more than g, just above g, then only driver because he is falling under the action of gravity. If the seat accelerating downward is more than the value of g, then you will leave the seat to the floor. We have to find out the acceleration of the maximum acceleration of the seat or the vehicle in the downward direction. So maximum acceleration in the downward direction is equal to maximum (Refer Slide Time: 55:51) will be omega square X. Now under this condition, omega is known 2 pi into 27 by 6.1 square and x is equal to 0.006.

This we find is 4.6 meter per second square and this is less than 9.8 (Refer Slide Time: 56:36) which is g. Therefore at any time, the acceleration of the seats on which the driver is sitting is not going to fall or go down with acceleration more than 4.6 and that brings the maximum, which is power less than the acceleration due to gravity the driver will be always on the seat. Therefore he is not going to leave the seat under these conditions. This is a simple problem but it illustrates the basic principle. We will take up further discussion on this.