Dynamics of Machines

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Module No. # 01

Lecture No. # 03

Rigid body motion: Dynamic Force Analysis of Mechanisms (Analytical Approach)

In the previous lecture, we have discussed the method of solving dynamics force analysis problems of plane mechanisms. We have seen, the dynamic force analysis problems, which means, when the complete motion of a system is known, we have to find out the unknown joint forces in the mechanism and also the externally applied forces or moments, which cause the system to move in the manner prescribed.

In this of course, we have to keep in mind that for a mechanism or a machine with single degree of freedom, only one component of externally applied forces can be independently found out. On the other hand, if we have more degrees of freedom then the number of unknown forces which is externally applied can be more. In the previous lecture, we adopted a technique of using graphical procedures, though we get answers which are often adequately accurate and precise for designing purpose, it becomes very difficult if we have to repeat the analysis again and again for a large number of situations.

For example, if a mechanism is moving that is continuously rotating, say a crank rocker mechanism, if you have to find out the forces for the complete cycle of the system, we have to repeat the force analysis problem for each and every position of the crank. Maybe, we have to take the crank at every 2 degrees, then we have to solve the problems for 180 times. Apart from that whatever may be the procedure or the equipmental use, the graphical analysis will always have some limitation from the point of view of accuracy of the results.

Third, if we have to optimize a system or if we have to achieve an optimal design then by doing or solving the problem using graphical approach, it becomes difficult. So, in today's lecture what we will do? We will solve the same problem of dynamic force analysis by adopting an analytical approach.

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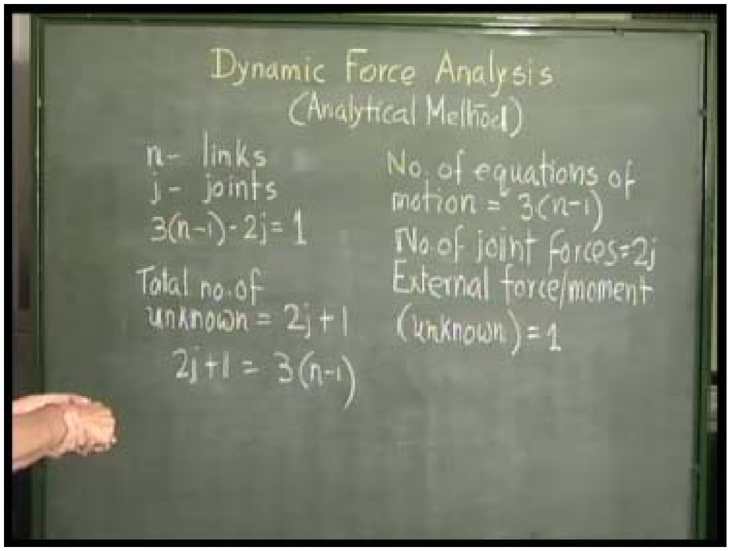

Today's lecture will be on dynamic force analysis by analytical approach. In principle it is the same, we write down the equations of motion for each and every moving link. Since in plane motion, every rigid body will have two linear components of accelerations of its center of mass. One component of angular acceleration; we will have three equations of motion for each such rigid body. If there are n numbers of links, j number of joints, then you know that for constraint mechanism the following relationship has to be satisfied.

We will take up only simple cases in this lecture, our objective is to find out the unknown joint forces and one unknown externally applied force or moment. If the system taken is a constraint that is, when there is only one degree of freedom then the number of moving links will be n minus 1, the number of equations you will get is equal to 3 into n minus 1. Since, each joint will have two components of forces we will get, the number of joint forces is equal to 2 into j, the externally applied, is unknown or to be found out will be 1.

To find out 2j number of unknown joint force components and 1 unknown externally applied forces moment, the total number of unknowns will be equal to 2j plus 1. From this relationship, which has to be satisfied for a system with single degree of freedom, we know that 2j plus 1 is equal to 3 into n minus 1, which is the number of equations of motion. It is very clear that you will have many equations when there are many unknown quantities to be found out, so the equations can be solved and results can be found out.

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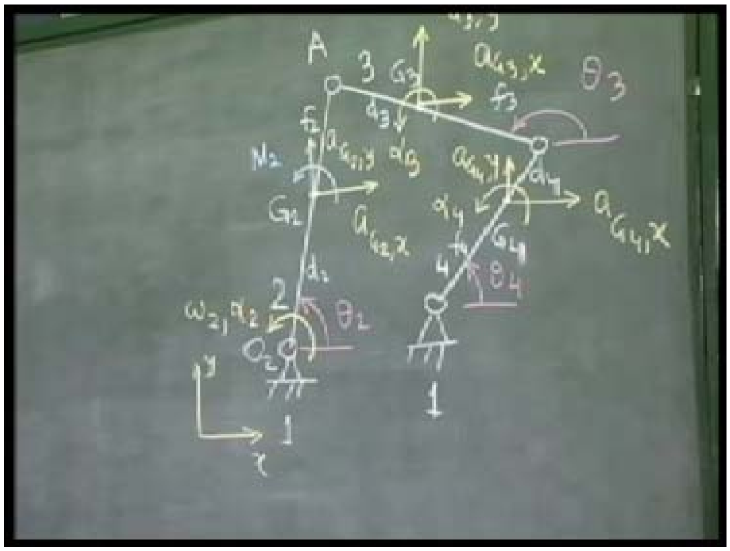

Lets us explain the procedure with the help of an example of the four link mechanism. Let our mechanism be this; this is the moving link 2, this is the moving link 3, this is the moving link 4 and the fixed link we always call as 1, as you know.

At this particular instant the angles made are theta$_2$, theta$_3$ and theta$_{4,}$ the lengths of the links are known, the location of the respective center of mass is G_2, G_3 and $G_{4,}$ we also give the dimensions for this part that is, from O_2 to G_2 as d2; G_2 to A as F_2; A to G_3 as d_3; and this part F_3; this is d_4; this is G_4. (Refer Slide Time: 09:19)

Let the externally applied forces, components are M_2, say let M_2 be the only externally applied force in this case, even if there are other forces acting then they have to be prescribed. So, to keep the problem simple for explaining the procedure, let us assume that only one external force is acting, in this case it happens to be a moment.

We also know how we have done in the previous course of kinematics, if the angular acceleration of one of the members in this case -let it be link 2- which is given as alpha$_2$, and its velocity is prescribed as omega$_2$ and then it is possible for us to find out the acceleration of various mass centers. (Refer Slide Time: 11:12) Let the acceleration components of the mass centers be $a_{G2,x}$ and $a_{G2,y}$. We have assumed, x in this direction, y in this direction; the acceleration of center of mass as $a_{G3,\ x}$ and $a_{G3,y}$; acceleration of this points components are $a_{G4,x}$, $a_{G4,y}$; angular acceleration of this is already prescribed, so we can find out the angular acceleration of other links. This figure now contains all the kinematic parameters like the acceleration components, the configuration and also the externally applied moment, we should draw the free body diagram of each and every moving link before we write down the equations of motion of each of these.

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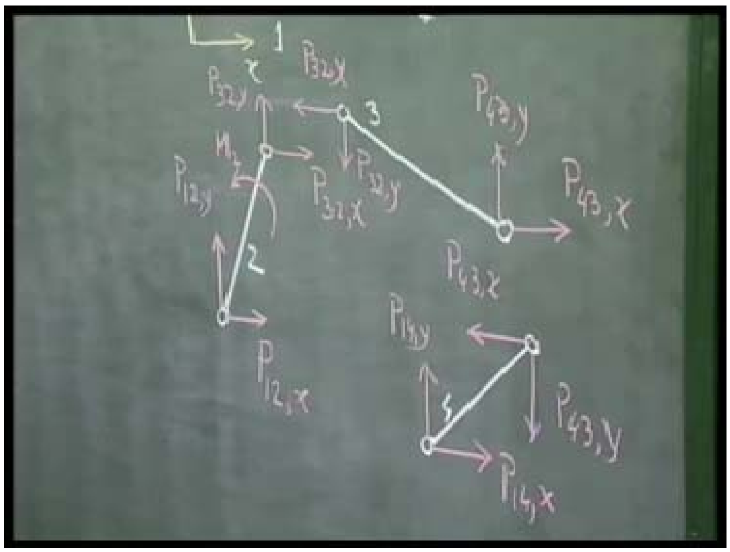

Next, let us draw an excluded view of link-mechanism. The joint forces, say this particular joint which connects link 2 with link 1, the components of forces be $P_{12,x}$ and $P_{12,y}$, it means it is the joint force from link 1 on link 2, in the x ((component)) direction and y direction.

Similarly, we write here $P_{32,x}$ that means force from link 3 on link 2 its component along x, and $P_{32,y}$. We know that the force according to Newton's third law between two bodies will be always equal and opposite .Whatever force which is coming on this link, from link 3 at this point exactly opposite and equal forces will act on the other link. (Refer Slide Time: 14:30) So the force in the x direction at this point will be $P_{32,x}$, but in this opposite direction this will be $P_{32,y}$. Similarly, the force on link 3 from link 4 will be

$P_{43,x}$, this is $P_{43,y}$. You can always see that the first subscript is which the body is applying the force and second subscript is the body on which the force is acting.

On this point again, it will be just equal and opposite forces is acting here. (Refer Slide Time: 14:35) Here again, it will be $P_{14,x}$ and $P_{14,y}$. Over and above this, there is one moment acting on link 2 which also we have to find out. So, these are the unknown force components, which we have to determine that means, how many are unknown; there are two here, two here; so four; two here, so six; two here, so eight; one and two.

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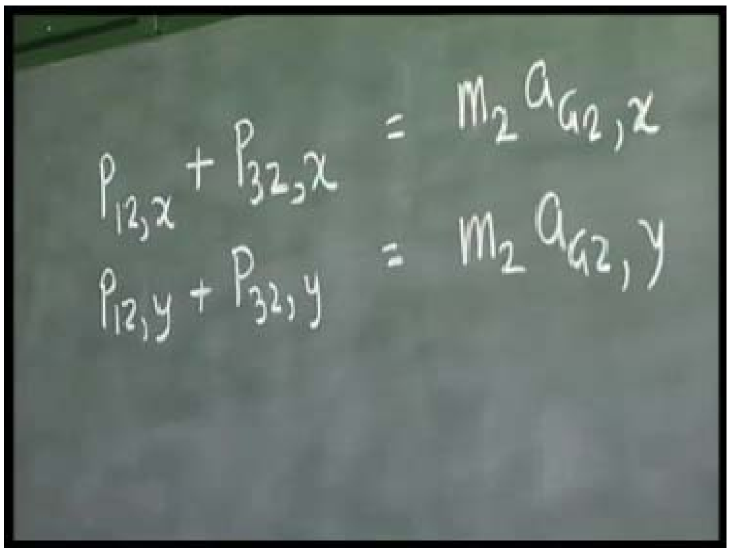

The equation of motion of each member can be written by Newton's second law. For the second link in the x direction the total force acting is equal to $P_{12,x}$ plus $P_{32,x}$, this must be equal to mass of the second link into $a_{G2,x}$. We should remember that there can be externally applied forces apart from m_2, but we have to prescribe them, we cannot find more than one external force components. Instead of m_2, we could apply a force in this direction or a force in this direction, when more than one force is acting then only one can be kept as an unknown quantity. (Refer Slide Time: 16:23) Here for the sake of simplicity, we have not applied any other force except the only one, which we have to find out.

In the y direction the equation of motion will be $P_{12,y}$ plus $P_{32,y}$ and that must be equal to the total force in the y direction, where $m_2a_{G2,y}$ is given by the mass of the body into the acceleration of the center of mass in the y direction. If you consider the angular motion about the center of mass which is here, then the total moment will be how much, $P_{12,x}$ multiplied by this (Refer Slide Time: 14:35), that is the arm momentum. Since, this is d_2 and this angle is $theta_2$; this momentum will be d_2 $theta_2$, the moment about the center of mass of this component of this force will be $P_{12,x}$ d_2 sine $theta_2$. This component will produce a moment in the opposite directions so, you have to put a minus sign $P_{12,y}$; so the moment arm will be d_2 cosine $theta_2$.

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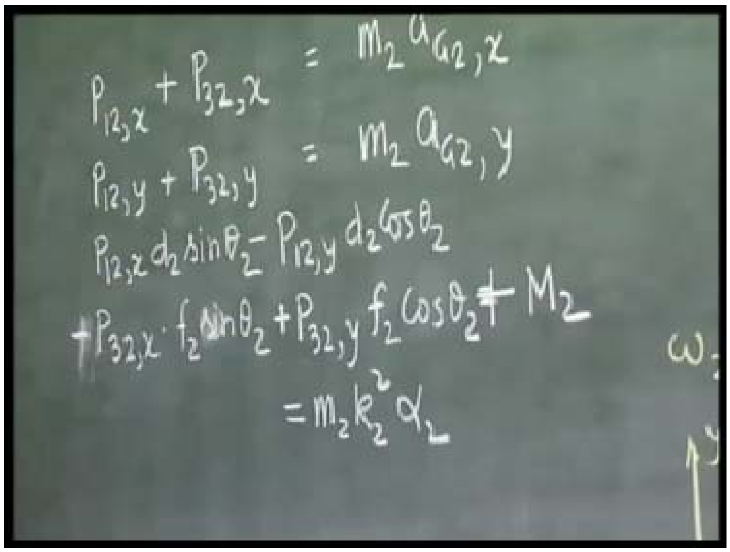

This force component will produce a clockwise moment so it will be $P_{32,\ x}$; this length is F_2; F_2 sine $theta_2$ is this; (Refer Slide Time: 18:46) this is also in the clockwise direction, so it should be minus; but this one is again in the anti-clockwise direction that is the direction in which it is accelerating with angular acceleration $alpha_2$; this is the total moment, when you add the externally applied moment $M_{2;}$ and this whole thing is nothing but the moment of inertia of the object about its centroidal axis, which is equal to M_2 into K_2 square, where K_2 is the radius (( )) of second link into alpha. Thus, you can see that we have three equations for the second link.

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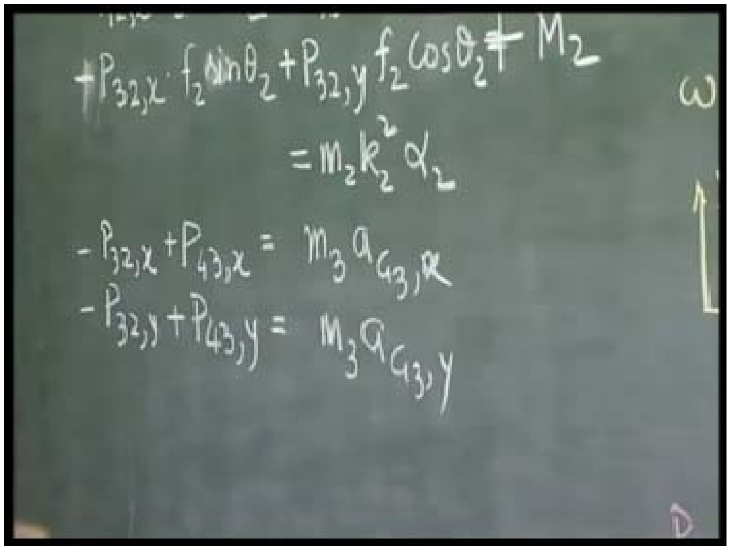

Exactly in the same way, when you go to other links, we can write down similar equations like minus $P_{32,x}$ plus $P_{43,x}$ which is equal to m_3 $a_{G3,x}$, minus $P_{32,y}$ plus $P_{43,y}$, which is equal to m_3 $a_{G3,y}$. Then by taking moment it will be, $P_{43,x}$; this is F_3; this length is d_3 and this angle is theta_3. (Refer Slide Time: 20:40)

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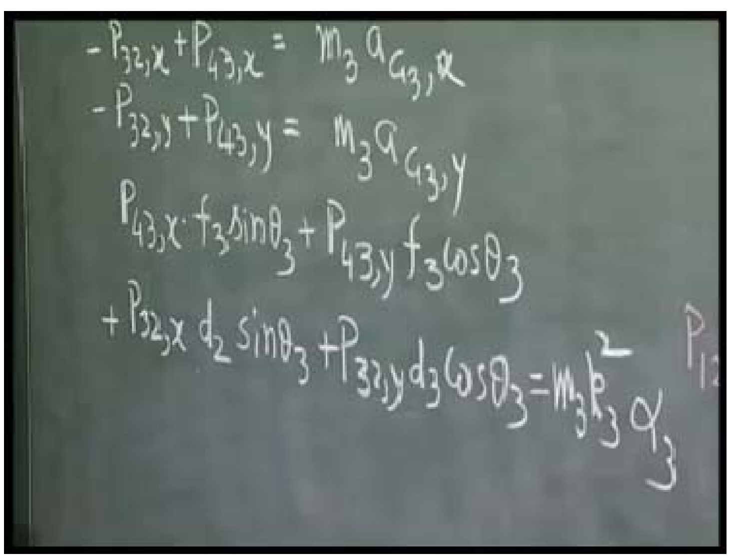

Therefore, the momentum is this; it will be equal to F_3 sine theta_3; this is the moment of this about this. $P_{43,y}$ will also apply a moment about this, with this as the momentum. (Refer Slide Time: 21:21) This also will produce clockwise moment $P_{32,x}$ d_2; momentum will be this; this is equal to d_2 sine $theta_3$; this will also produce a anti-clockwise moment that is, $P_{32,y}$ d_3 cosine $theta_3$. There is no other external moment acting here, so the total moment in the anti-clockwise direction is this; this must be equal to moment of inertia of this link about its centroidal axis that is m_3 k_3 square, where K_3 is the radius (( )) and $alpha_3$ is the angular acceleration.

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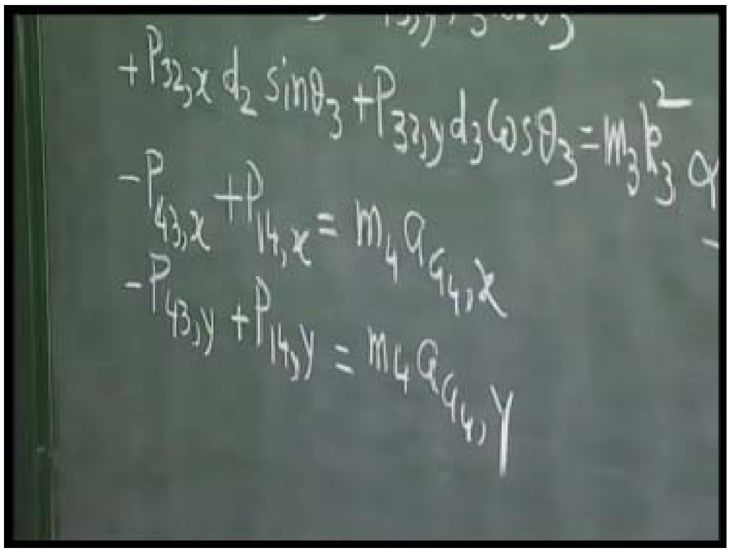

I have already mentioned that since the kinematics is completely solved we all know the acceleration components and the angular accelerations. For the last link, exactly in the same procedure, we will get total equation in the x direction as minus $P_{43,x}$ plus $P_{14,x}$ this must be equal to m_4 $a_{G4,x,}$; in the y direction, we will have minus $P_{43,y}$ plus $P_{14,y}$ and this is equal to m_4 $a_{G4,y.}$

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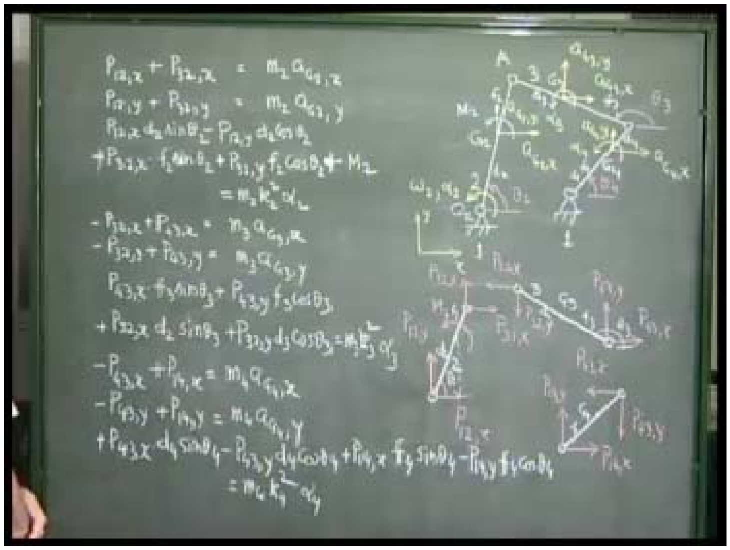

Similarly the moment equation, which means the total moment about its center of mass, this will be minus $P_{43,x}$, sorry this will be plus because this will be in anti-clockwise, so $P_{43,x}$ into d_4. Moment arm is this much, this is d_4 and this angle is theta$_4$, so d_4 sine theta$_4$. (Refer Slide Time: 23:42) This one will produce a clockwise moment; this will be minus $P_{43,y}$ d_4 cosine theta$_4$. This will again produce an anti-clockwise moment that means, a moment in the direction of alpha$_3$ or alpha$_4$ and momentum will be F_4 sine theta$_4$. The y component will produce a clockwise moment that means, opposite to the algebraic positive direction of alpha$_4$. This represents the total moment acting on link four, which as to be equal to moment of inertia which is m_4 K_4 square into alpha$_4$.

We have one, two, three, four, five, six, seven, eight, nine equations; nine linear equations, in nine unknown quantities such as $P_{12,x}$, $P_{32,x}$, $P_{12,y}$, $P_{32,y}$, then $P_{43,x}$, $P_{43,y}$, P_{32x}, P_{32y}, $P_{14,x}$ and $P_{14,y}$, We have already drawn all those forces which are unknown; M_2 is also the other unknown quantity, we have set of nine equations in nine unknown quantity they all can be solved, the unknown force M_2 and all other unknown joint forces can be determined.

The advantage of this methodology is that once these set of equations are written, which we have not shown it here because it is a part of kinematic analysis, you should get the expression for alpha$_3$, acceleration of the mass centers in terms of theta$_2$, theta$_3$, theta$_4$, theta$_2$ dot, theta$_3$ dot, theta$_4$ dot and the input acceleration as theta$_2$ dot which is equal to alpha$_2$.

Once that is done, computer can solve the equations and give you the values. Only thing what you have to do is, you have to keep on changing one the parameters like theta$_2$, say for example, you know that for every configuration the answer can be found out with the help of computer, this gives the whole system a complete cycle of motion, so you can find out the unknown forces, and this is the advantage of the analytical technique. The answers will be quite accurate depending upon the accuracy of the input values, depending upon the accuracy of the physical parameters, which we have given and there are no other limitations. You can also solve these problems sometimes by keeping in mind the optimization of machines and its optimal design, the minimization of forces, and the maximization of some other quantities, whatever it may be.

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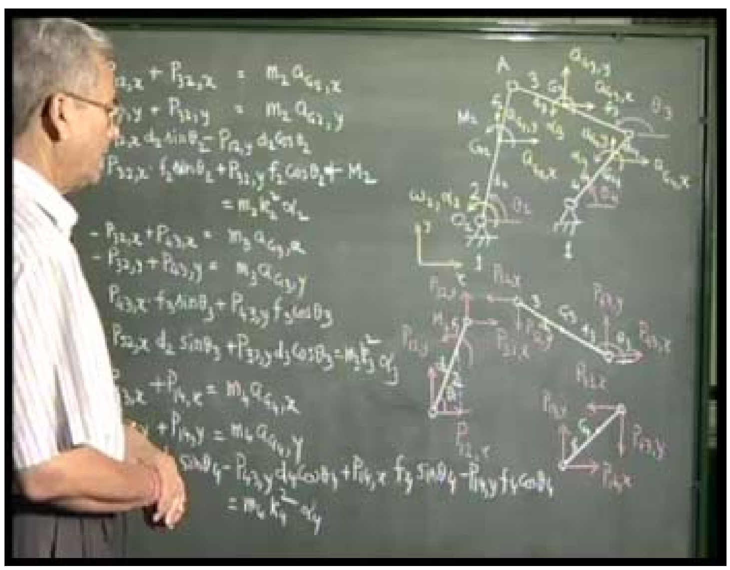

In the graphical procedure, it becomes extremely difficult to repeat it hundreds of times for getting analysis of one complete cycle. Once the forces are known, then of course a designer can design the bearings and other joints suitably so that such forces can be sustained. It is also possible, but we are not going to do here, once you have force on each link, it is also possible to find out these traces which they are subjected too and their design can be taken care off.

This brings us to the end of module one, in this our objective had been the dynamic force analysis, which means a system in which motion is given, system parameters are known, and we were able to find out the unknown forces in the joints, when one unknown force quantity which causes this motion is prescribed. As I mentioned that if the number of degrees of freedom is more then, we can have more number of externally applied forces which are unknown and that can be solved.

In the next lecture, we will start a new module where we will start studying the motion of rigid bodies in three dimensions, because what you have seen here is that the whole system of motion is containing one plane or parallel planes so the situation was very simple. But we will see rigid bodies which has many new characteristics and many new properties, when we consider the motion to be in three dimensions.