Dynamics of Machines

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Module - 11 Lecture - 2

During the last lecture, we solved the problem of forced vibration of a simple viscously damped system. We continue with the discussion on the topic which involves vibration of viscously damped system when subjected to harmonic excitation.

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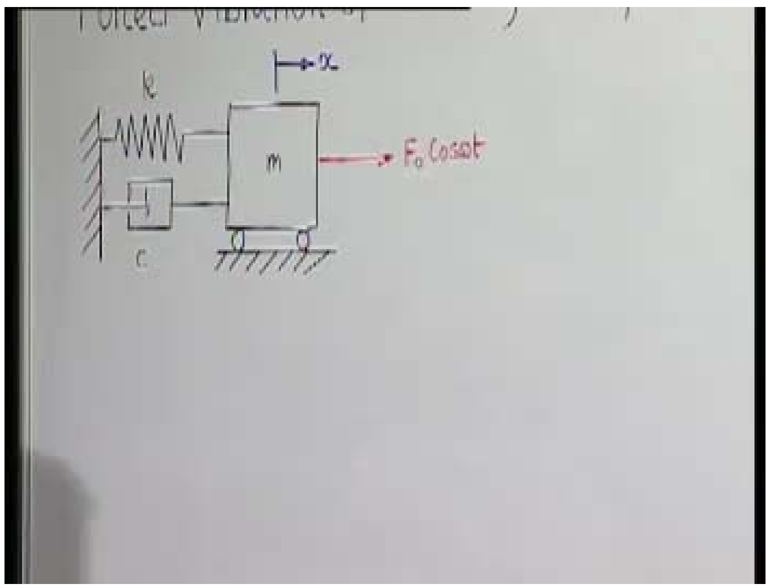

What we did? We wrote the equation of motion. We also noticed the point that, once this steady state starts; once the transient is over and damped out. The oscillation of this mass will be also a simple harmonic function of time with the same frequency but not necessarily at the same phase. So, we note that, the solution in the steady state (Refer Slide Time: 02:40) will be of this form. Next, what we did? We substituted this in this equation of motion and found out the values of x and phi that means the amplitude of the resulting vibration x will be F_0 (Refer Slide Time: 2:58) and the phase difference, tan of that (Refer Slide Time: 03:22).

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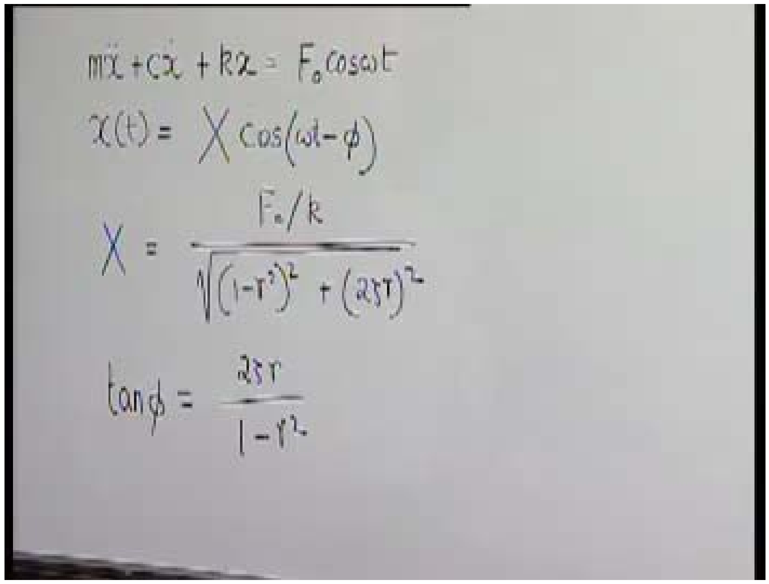

Then we noticed that the static deflection when the same force is applied statically, that is, F_0 has a constant force. Then the deflection of the system will be also a static deflection whose amount will be F_0 by k, but when the same force is applied dynamically, the maximum displacement or the formation of the system will be x and (Refer Slide Time: 04:00) this quantity we have already defined in the undamped situation as the magnification factor, which will be (Refer Slide Time: 04:16). Then we plot it. The response of the system, either x or the magnification factor, so, we start at r that is frequency r is equal to nothing but the frequency ratio when omega starts at 0 and gradually increase. Here, we have seen that, (Refer Slide Time: 05:08) this line represents magnification factor 1 and this line represents frequency ratio 1 and for various values of damping coefficient as damping increases and this particular line is for 0 damping.

When we plot the phase difference phi (Refer Slide Time: 06:01) as you can see the phase difference also starts at r is equal to 0, which means phi is 0. So, irrespective of the value of zeta, phi will increase, become equal to 90 degree at r is equal to 1 and asymptotically approach the value phi, because when r is equal to 0, this is equal to 0, phi is 0. As r gradually increases then it also increases. When r becomes 1 it becomes infinity. That means phi becomes phi by 2 and after that this becomes a negative quantity and obviously we will go to the second quadrant. As r tends to infinity, this tends to minus infinity and obviously this will tend to phi.

This was the kind of result we received. We have noticed one thing here that the peak value of the response of the magnification factor is not at r is equal to 1 as it was in the case of undamped situation. Let us find out where this peak will take place, that means, if we define this as the (Refer Slide Time: 08:04) r_p. That means the frequency response where the magnification factor reaches maximum value.

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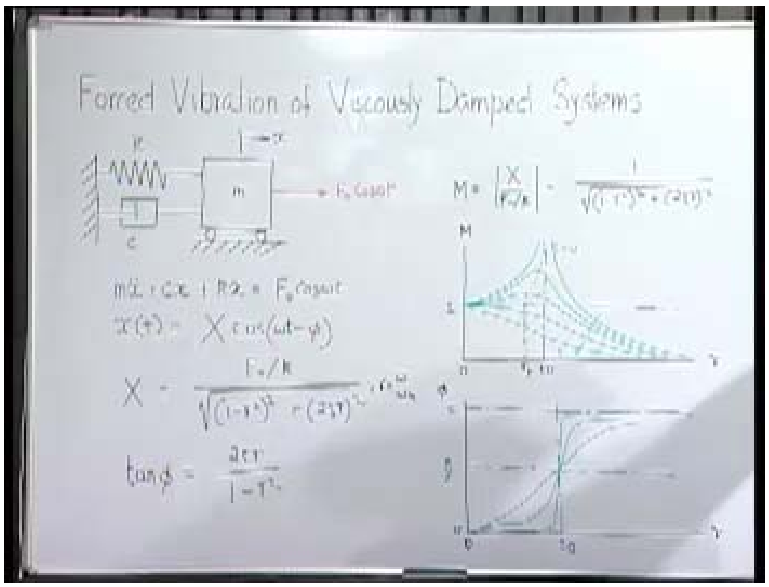

We can easily find out by trying to find out the maximum value of (Refer Slide Time: 08:20). So, del M del r will be equal to 0 at r is equal to r_p.

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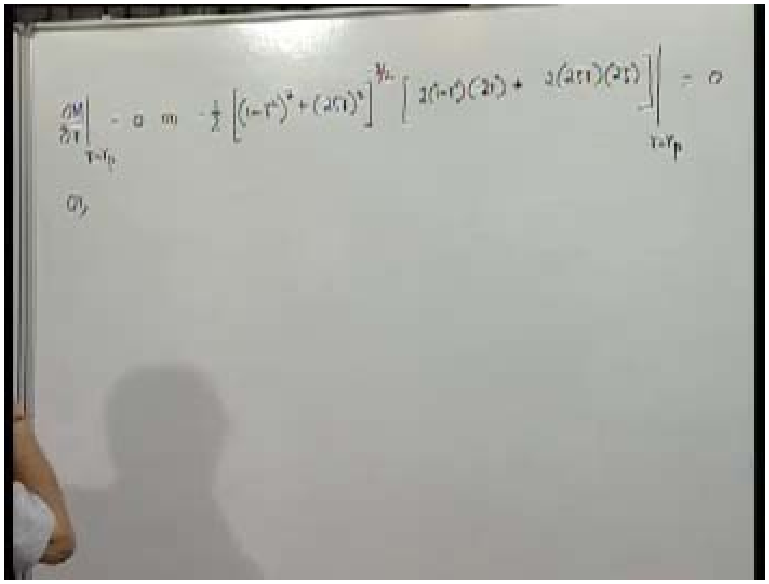

We know very simple when M becomes maximum, how much is this? It will be (Refer Slide Time: 08:40) this is del M del r at r is equal to r_p or we can say the numerator must be equal to 0, means 1 minus r square is equal to zeta square. I must here put r is equal to r_p, so the peak value of the frequency response, where the magnification factor is at maximum is given by this. When zeta is equal to 0, that is the undamped case, obviously r_p is equal to 1. There is one particular interesting situation which we will be utilizing at later time and therefore which is not out of place to discuss it here. When r_p is equal to 0 that means the peak itself is at the beginning then only it reduces, that means it is tangential here. What is the corresponding value of zeta? This will lead to 2zeta squared is equal to 1 or zeta is equal to 1 by square root 2, so with this damping factor the peak is here itself. Significance of use of these we will find at later time. Here also you will find that for zeta is equal to 0.707 we will find that it will be almost a straight line for a large part of its range then we find another very interesting thing, that when the frequency ratio is pretty high, then the deformation of the system in the dynamic situation is far less compared to what the deformation would have been had the same load been applied in a static manner that means if I apply some load here statically whatever deformation in the spring will take place, if I apply the same load with a frequency which is much higher than the natural frequency of the system the deformation of the system will be much less.

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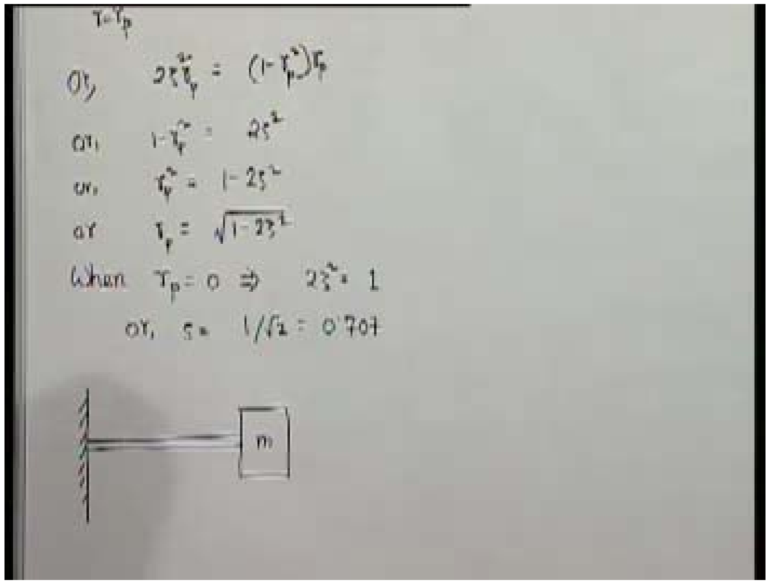

That is suppose if there is a cantilever beam subjected to carrying some kind of a mass here then if a load is applied here statically this phase which will be generated or strain which will be developed will be much less compared to the static situation if the same load is applied must have natural frequency. So, this is a very important observation and we will see that this can be used in design of system very effectively, where the system can be made much lighter compared to what it would have been if the load is static but, nevertheless we should keep in mind that the natural frequency of the system has to be designed in such a manner that r becomes much higher than 1. There is another way of solving this problem, which I will take up. The reason being that, it can be not only the remembered much easily but can be effectively utilized in a more complex situation which I will demonstrate, a vector quantity, harmonic quantity. Perhaps we have discussed this quickly in the past but we will remind our self about it

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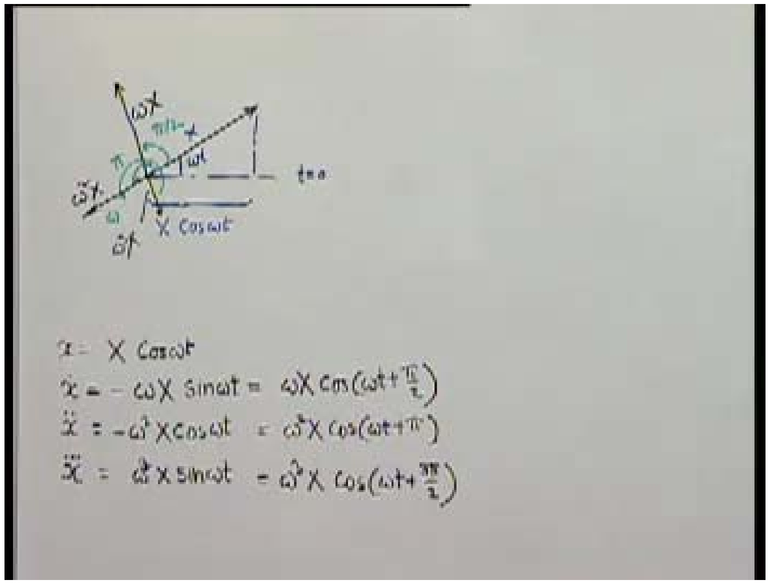

If a vector is called x, which is say cosine omega t, we can represent it by a rotating vector like this. It is rotating at a speed omega and so therefore its projection as we have mentioned before, where t is equal to 0. This will be omega t and this will be x cosine omega t if this is x. Now, what will be x dot or instantaneous velocity if we differentiate this with respect to time we will get minus omega X, which I can write again as omega X cosine omega t plus phi by 2 is nothing but minus sine omega t. Now, this is again the harmonic function of time whose magnitude is omega X and it is leading the x vector or the vector representing the position by 90 degree so x dot will be represented by a vector whose length is omega X and that is leading the position vector or position…, whether the vector representing position by 90 degree.

If we want to now represent acceleration, then we differentiate it once more, minus cosine omega t we will write as cosine omega t plus phi. This is again a harmonic function of time and can be represented by a vector whose magnitude is omega square x and that is leading the position vector by 180 degree, as can be seen, this is cosine omega t; this is leading cosine omega t by 90 degree; this is leading cosine omega t by 180 degree, with this we can also keep on doing it, say the third derivative will be (Refer Slide Time: 19:00) r derivating will be represented by this rotating vector. When these four vectors rotate like a rigid body with an angular speed omega their projection on that line is equal to 0 will be harmonic functions of time. This will produce something which will represent the projection; then this will represent the velocity; this will represent the acceleration and this will represent the jerk (Refer Slide Time: 19:29). Therefore, these vectors do not have any physical existence. All the time we have to keep in mind that always only the projections along this line is what we have in reality.

If we consider the free body diagram (Refer Slide Time: 19:53) the block, what are the forces, at any instant of time when it is displaced from its equilibrium position by x moving in this direction at a speed of velocity x dot and with an acceleration x two dot the forces are the external force. Now, these forces are harmonic functions of time, so we can represent them by rotating vector. First is the spring force, spring force will be like if x is there in this direction, then spring force is acting in the other direction. What we can do, we can represent this spring force first and let the spring force rotating vectors length be k into x. Therefore, its component along the line along which we will take the projection, it needs not be always this one, it can be something else. But whatever it is, it will be kx into cosine omega t, which is nothing but this (Refer Slide Time: 21:30).

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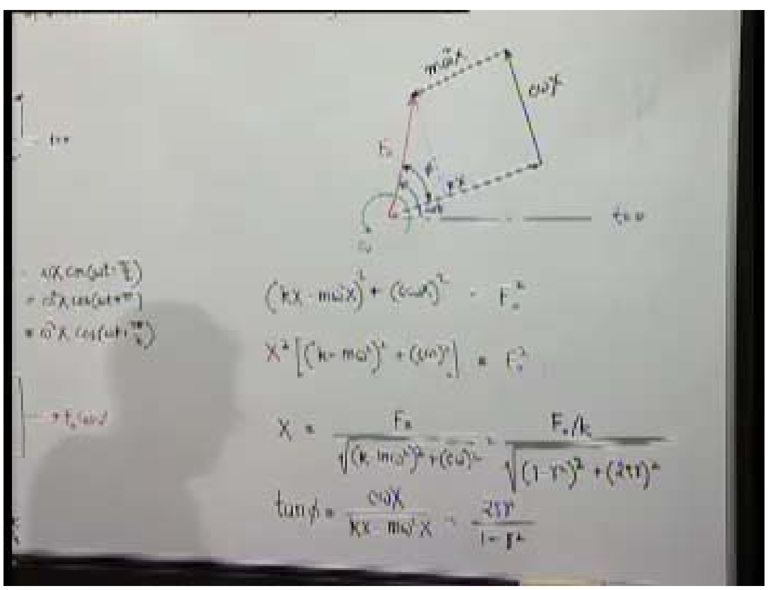

We have to now add cx dot. Now, x dot is represented by (Refer Slide Time: 21:39) this, so cx dot will be represented by a vector which is leading this (Refer Slide Time: 21:49).

So these two x vector c omega X vector together when you join, they represent the total force acting on this side. Actually, if we look into this equation, what we are doing, we are representing this equation (Refer Slide Time: 22:22) by a rotating vector diagram. So, kx with a varied cx dot, now, we have to add mx two dot. x two dot is represented by a vector (Refer Slide Time: 22:32) like this, so mx two dot will be represented by a vector, which is now leading the x vector by 180 degree and these three together will be equal to F_0 cosine omega t.

(Refer Slide Time: 23:01) These three must be equal to x_0 and the whole thing as it like a rigid body is rotating with a speed omega. Its projection, if t is equal to 0 is here, then this is equal to omega t, obviously this must be equal to phi, because x vector is represented by x cosine omega t minus phi, that means position vector is lagging behind the force vector by an angle phi, so this must phi. Once this diagram is ready, we can derive the results directly from this. Say for example, we can consider (Refer Slide Time: 24:06) this right angle triangle, where this side is how much, kx minus m omega square x. This is this side (Refer Slide Time: 24:21), if we square it and (Refer Slide Time: 24:27) this is equal to this itself, we square it and add, what we will get is nothing but F_0 square. Now, taking x common, taking it out (Refer Slide Time: 24:53) directly now we find x equal to (Refer Slide Time: 25:14). If we divide both the numerator and the denominator by k we get (Refer Slide Time: 25:34).

Now, here if we divide this by k, when it goes inside this, it becomes k square, again when it goes inside this it becomes k squared divide this by k means 1 minus m omega squared by k, which is nothing but omega squared by omega_n square, that is r. Now c. omega by k we have shown number of times is equal to 2zeta r. So we get the magnitude of response as we solve the equation of motion and here we got it directly from the diagram. The phase difference also directly comes from this diagram, which is obviously tan phi is nothing but c omega X by kx minus (Refer Slide Time: 26:48), this is nothing, if you cancel x from both the numerator and denominator and divide both numerator and denominator by k. c omega by k is nothing but 2 zeta r and k by k is 1 and m omega squared by k is nothing but r square.

Both the results, which we have to struggle a little bit and solve the equation we directly get from this diagram. This approach is very convenient; it also gives a physical insight all the time. For example, here we find that at r is equal to 1, the phase difference is always phi by 2 that means irrespective of the damping the phase difference between the excitation and the response is always 90 degree. Why it is so?

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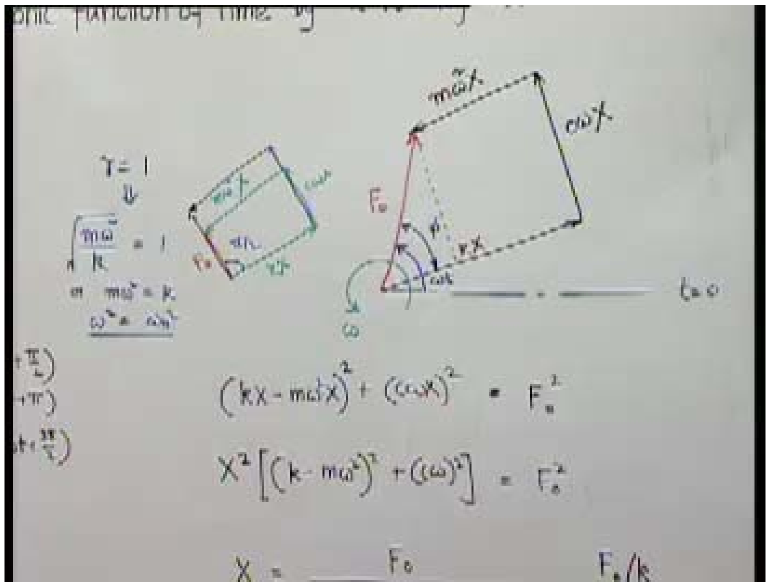

It can be very easily explained with the help of this diagram. Say when r is equal to 1, r is equal to 1 it means (Refer Slide Time: 28:07), m omega squared by k square root is 1 or m omega square is equal to k or obviously omega squared is equal to omega_n squared. But anyhow, that is not important, there is no resonance but k is equal to m omega square. If k is equal to m omega square, then how much will be this length be? This will be 0. That means m omega squared will be this that means this will come here and situation will be something like this (Refer Slide Time: 28:55). When kx is equal to m omega squared x because k is equal to m omega squared, which is very obvious that this will be a rectangle and this will be 90 degree.

It is irrespective of this damping c, it could be something like this. Suppose damping, we make a different damping, so it will go here (Refer Slide Time: 29:42). But again the diagram is going to be again a rectangle and this is 90 degree always. So, we find that at k is equal to m omega square, this spring force is exactly cancelling the inertia force or spring force takes care of the inertia of force. So, the externally applied force always takes care of the damping force and therefore the phase difference between the response and the excitation is always 90 degree irrespective of the amount of damping. This gets very clearly explained with the help of this diagram, which gives much better physical insight.

Another advantage of using this diagram is that more complex situations can be solved, which is without going into a very solution of very complicated differential equation. We will take up one example for our own practice and how to demonstrate that statement which I have just made. Sometimes what may happen, the dashpot or the damper may not be connected to the body and the fixed foundation on its two sides. May be one side is mounted or one side is connected to a point which has an elastic connection with the foundation. (Refer Slide Time: 31:34) This class of problem is interesting, so problem is this, that we have a system where now we find one end of this dashpot is connected to the body but, the other end is connected to a spring whose other end is connected to this foundation.

There are two springs, because the dashpot is mounted elastically and obviously therefore two string constants so we call this coefficient for this one as k_1 and the coefficient of stiffness for the other spring as k_2, and c and m are there. Now, here we will find that, first, let us try to see what will be the equation of motion

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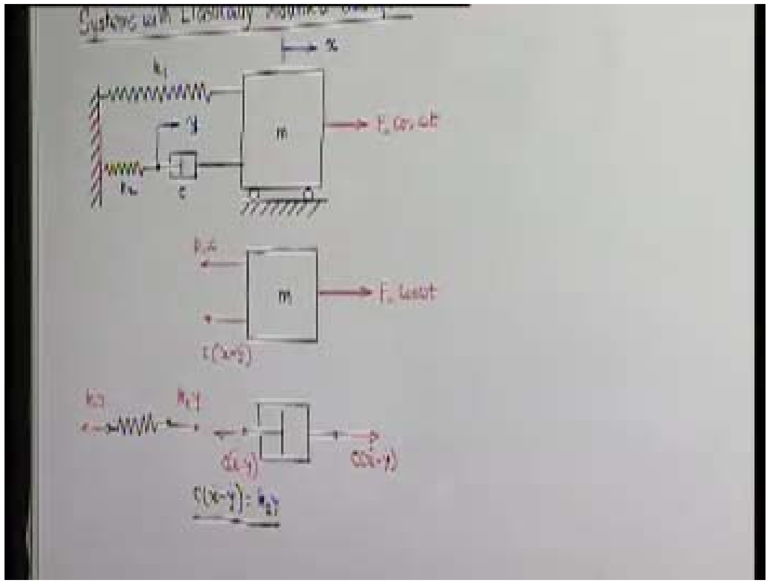

To write down the equation of motion when it has got gone to a displaced position x, what will be the damping force? The damping force is or the force excited by this dashpot will be c into the relative velocity between the piston and the cylinder. Velocity of the cylinder is same as x dot, but the velocity of piston is not 0. In the previous cases, the velocity of piston was 0 and the relative velocity between the piston and this cylinder body was same as x dot and so it was a cx dot. Now (Refer Slide Time: 34:36) this end of the system is also deforming, let us now call this as y, so this will be c into x dot minus y dot.

Of course the free body diagram of this dashpot (Refer Slide Time: 35:07), it is applying c into x dot minus y dot and the other end has to be the same. If we consider the spring k_2, this is obviously stretched, it must have stretched if it is deformed by y here. The force here will be nothing but… Now this equal to this is very clear and these are all mass less we should keep in mind. One thing we find from this is that cx dot minus y dot, which is the force acting here and k_2y is the force here, they must be same. So, cx dot minus y dot must be equal to k_2 into y. Now, the equation of motion for this we know is mx two dot plus cx dot minus y dot plus k_1x equal to (Refer Slide Time: 36:53) and the other equation we have got (Refer Slide Time: 37:00). Now, we get two variables and two equations as a typical procedure for solving this equation, is that we have to eliminate one.

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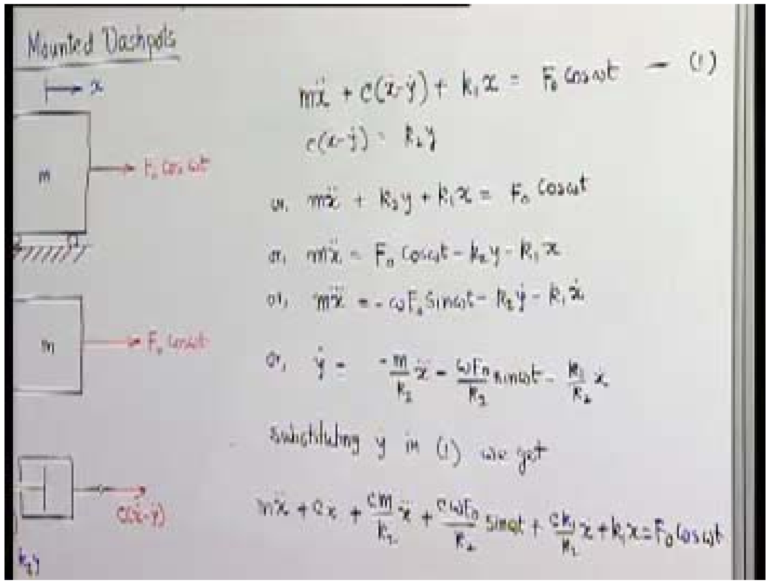

As you can see, eliminating y will be here (Refer Slide Time: 37:21), what we can do that, first let us substitute this is equal to this here or mx two dot plus k_2y plus k_1x equal to F_0 cosine omega t. We are solving it in great detail because this type of applied equation, which is a technique which may be useful in product equation or (Refer Slide Time: 38:00) Now we differentiate both sides of the equation with time once more.

So it becomes mx three dot (Refer Slide Time: 38:20), here we see now we can express y dot as a function of x and its derivative. If we now substitute this here, we will get this (Refer Slide Time: 39:17). It will be mx two dot plus cx dot minus cy dot, so we have to multiply by minus 1 and c it becomes (Refer Slide Time: 39:54) plus, this also be plus, this will be also plus. Now, we get an equation completely in terms of x and its derivatives. But we can see it is a higher degree equation, which involves x three dot, x two dot, x dot, x so on. So we can write like this.

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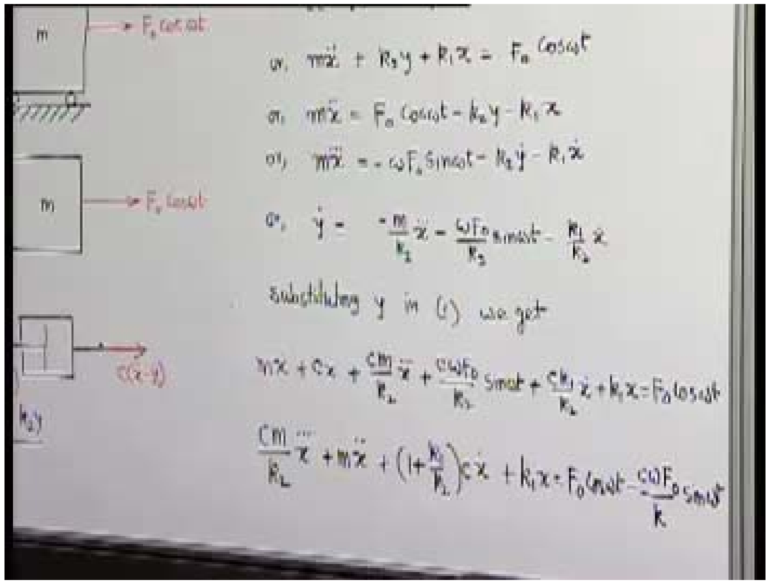

Solving this equation is going to be again problematic in the sense we have to assume x is equal to again some x into cosine omega t minus phi and substitute it here, and then solves the equations. Much simpler way could be perhaps, if we again try to use the rotating vector approach. We know that x (Refer Slide Time: 41:59) can be represented by a vector like this and we multiply it by k_1. So k_1x is this term, x dot will be leading it by 90 degree and we have to multiply that by into this (Refer Slide Time: 42:25).

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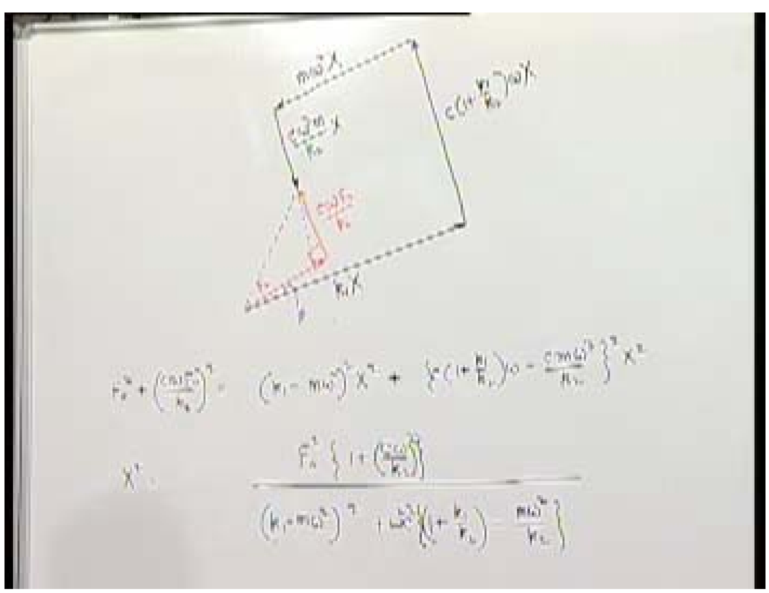

Now, k_1 by k_2 we can represent it by a quantity, may be ratio lambda, but at related time (Refer Slide Time: 42:34). Next, there is another term which will be now still leading the whole thing by (Refer Slide Time: 43:12) and this is the left hand side so this is the right hand side. Right hand side has now two terms, one we find is F_0 cosine omega t, which is F_0 (Refer Slide Time: 43:55). Another one is again minus sine omega t, minus sine omega t can be written as plus c omega F_0 by k_2 and minus sine omega t is nothing but cosine omega t plus pi by 2. Therefore, it will be again leading this vector cosine omega t vector. This will be (Refer Slide Time: 44:44) c omega F_0 by k_2, this is 90 degree. Obviously, the angle by which the displacement lags or response lags the force is phi. But our first interest will be to find out x, so we now use geometry and trigonometry. This one we get F_0 square plus c omega F_0 by k_2 square, must be equal to (Refer Slide Time: 45:34) this square plus this square. How much is this square? How much is this? (Refer Slide Time: 45:50) This much is this square, when you square it we get this.

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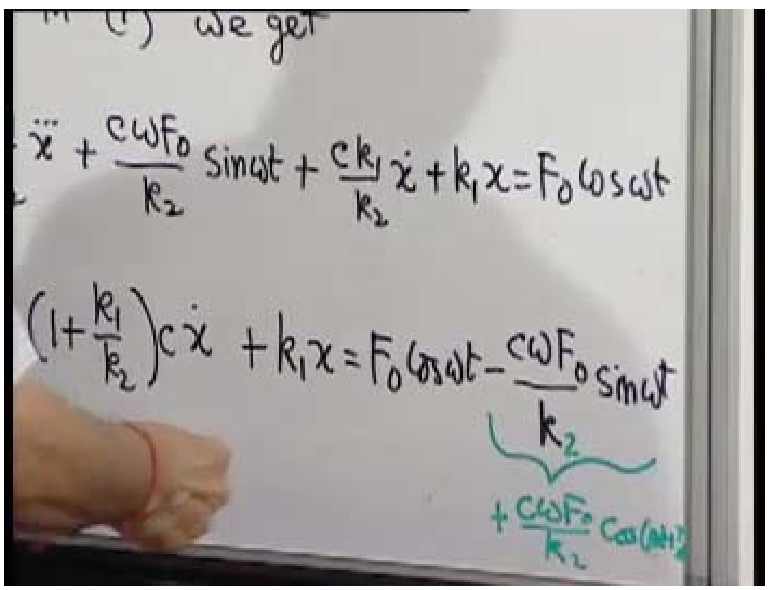

Therefore, now x squared is equal to (Refer Slide Time: 46:39), if we take F_0 square. It is a reasonably complicated term but still nevertheless we have found out x without following any equations from this diagram. We can also tell what to be our tan phi; tan phi is going to be (Refer Slide Time: 45:53) this divided by this. That also can be found out, so without going into the details I will give you the answer.

[48:08]….. **Video problem**… [58:00]

[48:08] …. **Audio problem** ... [48:45]

Magnification factor is x by x_0 by k_1 in this case if we apply some load statically, its deflection will be guided only by K_1 spring. We can easily see that because what is this in steady state or 0 velocity situation this will not transmit any force. The only resistance to deformation will be k_1F_0 by k_1 is the static deflection and that is given by (Refer Slide Time: 49:30) magnification factor is somewhat complicated. Now, there are two springs and their stiffness issues are represented by lambda. The natural frequency alone considered being undamped case square root of k_1 by m and static equation also obviously considered

Now, here this one we plot (Refer Slide Time: 51:15). Let us consider zeta is equal to 0, means, c omega by C_1. When zeta is equal to 0 then obviously there is no connection it becomes simple inverse system and its natural frequency is obviously square root of rho_1 by m and the response will be as we have seen before it will start from 1 and draw to 0 axis (Refer Slide Time: 52:07). Now, in extreme case, this is zeta is equal to 0. Another extreme case will be when this is in finite, means, it is rigidly connected. So, this becomes simply an equivalent to a system, zeta is equal to 0 means (Refer Slide Time: 52:34). Zeta is equal to infinity means (Refer Slide Time: 52:45) this is the equation. Obviously, natural frequency (53:00) for this it will again behave like a simple undamped case and the frequency response of the magnification factor and then (53:24) now one thing is very clear, at this particular frequency ratio the magnificent factor is not depending on the damping factor then easily we found that what will be the value of this. How do you find it out? That the magnificent factor when zeta is 0 is given by (54:08) when zeta is 0 magnificent factor is this but since it is beyond these we have to take the minus. And this must be equal to the magnificent factor is same or the other case zeta is infinite. When zeta is infinite then we can ignore this it becomes 2 lambda zeta r. Similarly, you can ignore this and it becomes 2 zeta below this and so finally what we get is (Refer Slide Time: 55:22) If you call this as r star we get lambda by This is for zeta is equal to infinity, zeta equal to so best value of that will be where this is the maximum value will be kept minimum. So, this is another strategy which is good example of design. If you want to design something for which the frequency of excitation is continuously varying over a wide range, then we can always design in a manner so that its maximum deflection will be minimum in the whole range if we design it in this manner. I think this was a good demonstration to indicate the advantage of using the rotating vector diagram. I have not noticed mainly this, this should be always indicated. So, we will proceed with our discussion for this is where the damping is not exactly of discussed nature what can be done in such situation simple analysis of (57:28).