Dynamics of Machines

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Module-10 Lecture-1

In the last class we have started our discussion on free vibration of damped single degree freedom system. We have seen that depending on the system damping, it can have either an aperiodic motion or a periodic motion. When the damping is about a critical value, then the system performs non oscillatory aperiodic motion, of which if the system is disturbed from its equilibrium position. On the other hand, when the damping is lower than the critical damping for that particular system, then the system executes oscillatory motion but not with constant amplitude, it decreases exponentially.

What we will do today is, we will continue our discussion on single degree freedom systems with damping, and investigate little bit more of the kind of or the nature of motions of such systems with damping, more than the critical undamping at the critical level. We would also like to find out what is that mean to finding out critical value of damping coefficient for a particular given system.

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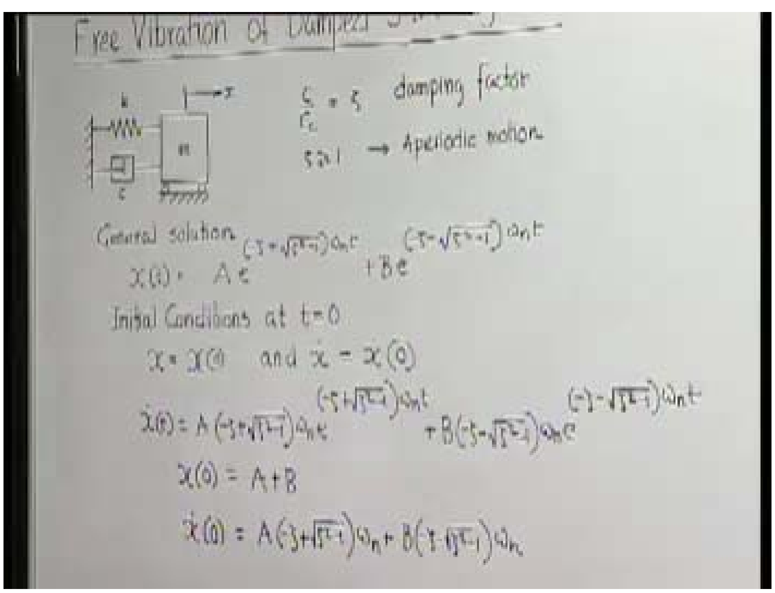

This is the system we have been studying, and we continue our discussion on that. When we also know that we have found out the ratio of the damping coefficient and the critical value of the damping coefficient can be written as in one-dimensional quantity, which is called the damping factor. When zeta is more than equal to 1, it leads to aperiodic motion. We will investigate little bit more on the motion of such systems, when zeta is equal to or more than 1.

The general solution for x can be written like this (Refer Slide Time: 03:45), where A and B are to be determined from the initial conditions. Let us give the initial conditions like this x is equal to x(0) and x dot equal to x dot(0). Once these two quantities are given, the subsequent motion will be completely known. This is x and x dot will be this this (Refer Slide Time: 04:54). Now, when we put this condition, let us say that x is equal to x(0) at t is equal to 0.

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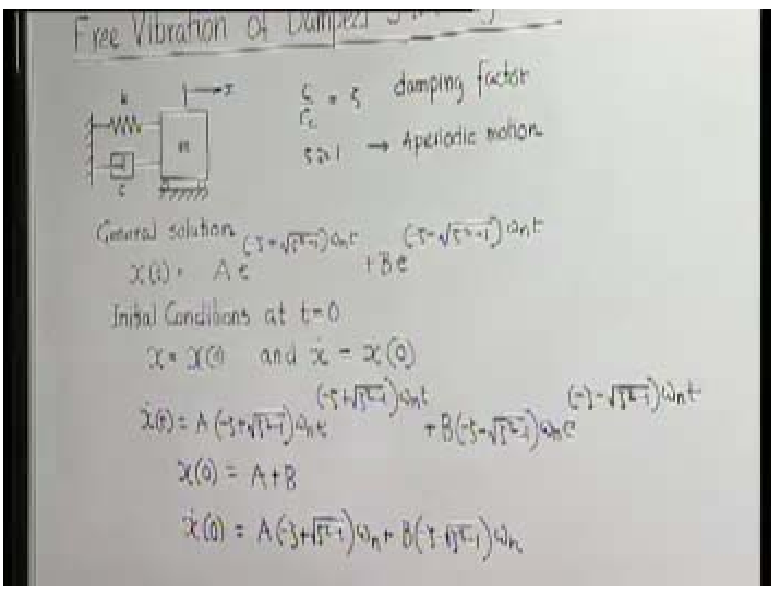

t is equal to 0 means this will be equal to 1, this will be equal to 1, or x(0) equal to A into 1 plus B. Similarly, when we put this condition, that is at t is equal to 0, again this is 1, this is 1 and what remains is equal to x dot 0. We have two equations with two unknowns A and B. A and B, this is a state of simultaneous equation, and we can solve, instead of wasting time in determining this in a long way, I will just give the results.

Obviously, the solution can be written by using A and B, substituting the values of A and B here. I am not writing it because it will take lot of space. Of course, we can see here, both these terms in x(t), this one is a quantity which is negative real number. Because this is less than zeta but positive but this is minus zeta. Therefore, this quantity will be negative and this term will be decay in time as I mentioned earlier same is the case here.

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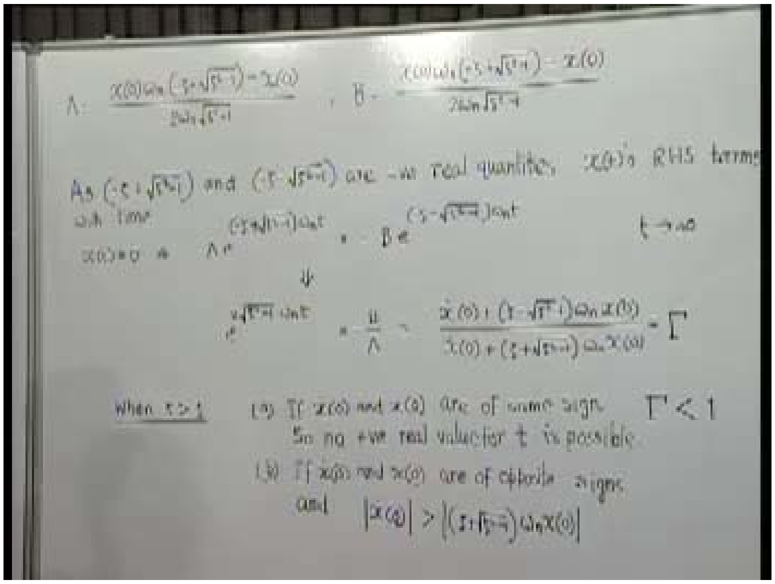

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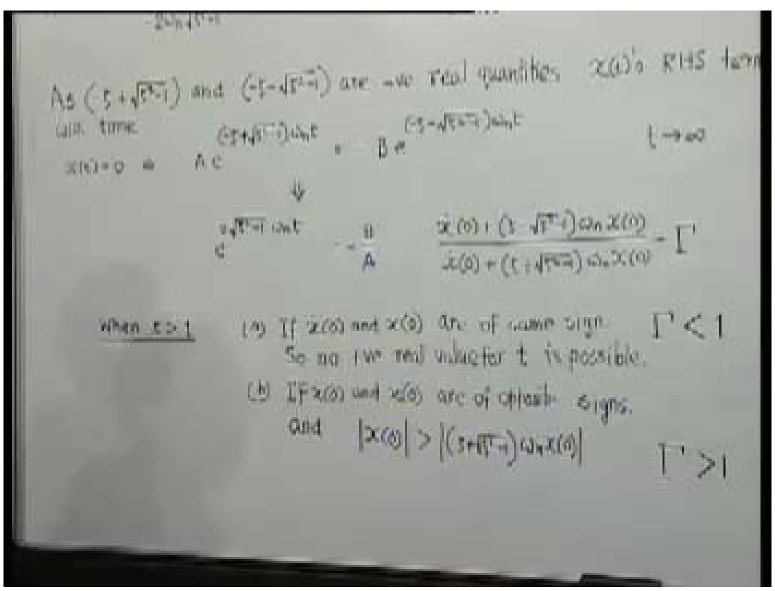

This is less than 1, but both being negative, which is obviously a larger negative value than this and this will also decay exponentially. Therefore, we can get that the system will asymptotically decaying. The question is, as minus z plus zeta squared minus 1 and minus zeta minus 1 or negative real quantity, this is fine. Now our question is what happens when it decays when t tends to infinity.

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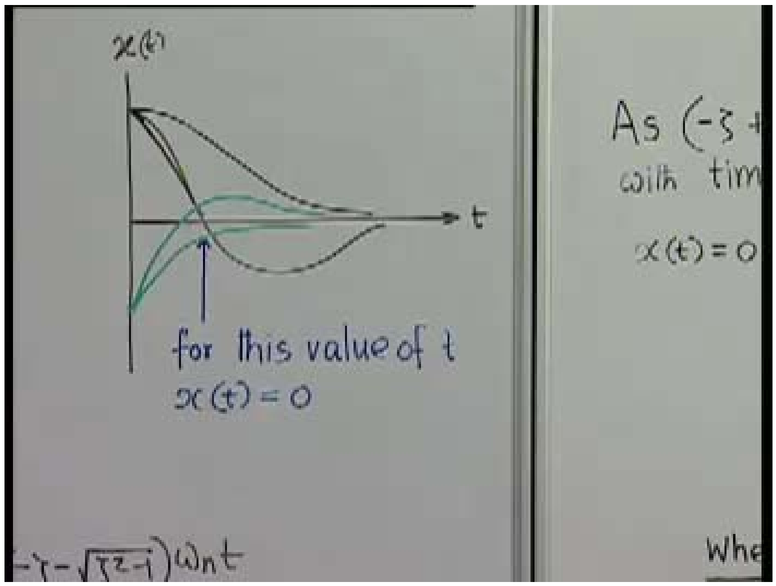

Now it will decay like this or it will decay like this that is something which is of incursive. That means whether there is any value of t, any real positive value of t, for which x(t) is 0, therefore the question is that for this value of t, x(t) is equal to 0. Let us find out if and when such a thing is possible. If x(t) is 0, that means what do we get if x(t) is 0, then obviously this mean that, A e to the power minus zeta plus zeta square minus 1 omega$_n$t is equal to minus this (Refer Slide Time: 11:45). If we can find out a positive real value of t from this condition, that is the value where x(t) can be 0, means that the mass will cross the equilibrium position.

If we now try to see that, first of all e to the power minus zeta omega$_n$t, e to the power minus zeta omega$_n$t will cancel from both sides. This is will give us e to the power 2 equal to minus B by A. Because I take this e to the power minus this, when you bring it to this side, it becomes e to the power plus. This becomes plus minus B by A, or which is nothing but, so one thing this condition is satisfied for two values of t, one is when t tends to infinity, then this is 0 and this is also 0, the condition is satisfied.

The other value of t for which it will be satisfied will have to be found out from this. Now rather than solving for t, let us find out the conditions under which if real positive value of t is possible. Since this quantity is positive, $omega_n$ is positive, so t is also positive. If t is positive real quantity, then e to the power this thing will have to be a quantity which will be more than 1. Therefore, these are what under what condition these will be greater than 1. That will give us a real positive value of t that means to crossing the equilibrium position will be possible. Let us find out that. First of all when zeta is greater than 1, obviously the whole thing we are doing. If x dot 0 and x(0) are of same sign.

What happens, if they are both either positive or both negative, then what happens if they are both positive? Then it is as such, and you can see that this quantity is definitely smaller than this quantity. As a whole, the numerator is smaller than the denominator and the whole quantity is less than 1. If we consider that this is equal to say gamma capital. Then capital gamma is less than 1, and no positive real value for t is possible or it will never cross the equilibrium line, basically it means that.

Now if x dot 0 and x(0) are of opposite signs then what happen. You will find that if suppose x dot 0 is positive and x(0) is negative, this is positive then this is negative, means which will be x dot 0 minus a negative, which is because the negative quantity means minus this quantity. This is positive and this is again negative.

You will find that, as such there cannot be any decision that whether gamma will be less than 1 or more than 1. But there is a further condition that if, or and magnitude of this then what happens. That means magnitude of this is more than magnitude of this. Then if this is positive and this is negative, if magnitude of this is anyway a smaller quantity than this. This will be a positive quantity minus some quantity which is less than this. What will happen, the upper part and this magnitude of x(0) is more than this. That means when this is negative sign, is put in the bottom, we will get a positive quantity.

Here of course, since x dot 0 is more than these in magnitude, x dot 0 will be more than this also in magnitude because this is smaller than this. What we are getting is that it will be x dot 0 minus the quantity, and below x dot 0 minus a quantity which is larger than this. What will happen, the numerator will be a positive quantity but and denominator also will be a positive quantity. But this will be less than in magnitude, so that means gamma will be more than 1.

If x dot 0 is positive x(0) is negative, if it is other way round x dot 0 is negative and x(0) is positive, then we can take minus common here, minus common here, this cancel to ultimately getting in the same thing. Therefore if x dot 0 and x(0) are of opposite signs, and x dot 0 is of sufficient magnitude to be larger than this quantity in magnitude. Only then there will be possibility of 0 crossing like this. This is for the phase when x dot 0 is that means like this.

Similarly here, if you have x dot 0 is negative, but we have to give a positive velocity, then again you will find. But if you give a lesser velocity, will go like this. We have to give sufficient high velocity, so that it rises and that condition is obtained from this. For over damped situation, the motion will be always aperiodic, and the possibility is that it may cross the equilibrium position only once. The condition is there that the initial velocity and initial position should be of opposite sign, and initial velocity should be of enough magnitude to be more than this quantity. That is, when it is possible for the mass block to cross the equilibrium position.

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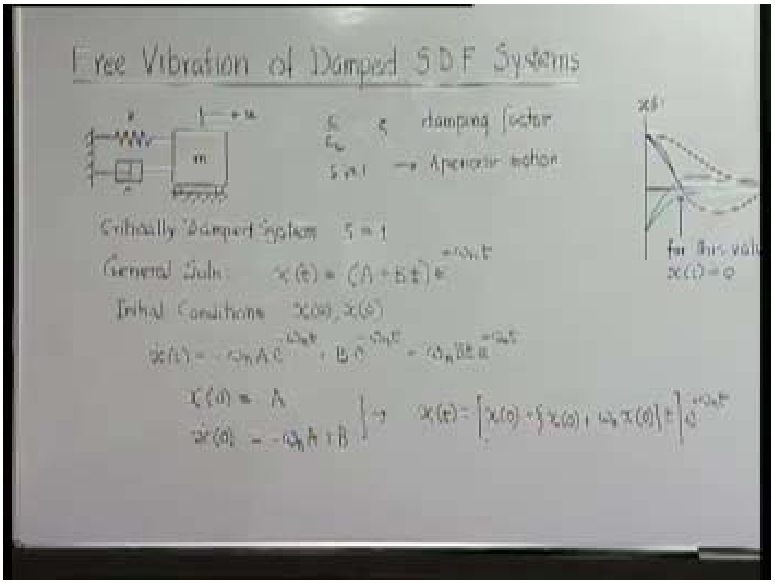

If the system is critically damped general solution in this case, we have already seen, will be e to the power. Now zeta is equal to 1, so initial condition will be simply x(0) and x dot 0 as t is equal to 0. We can write that x dot t is equal to minus omega$_n$Ae to the power minus omega$_n$t plus Be to the power omega$_n$t minus omega$_n$Bt, by differentiating this quantity with respect to time 1.

If you put the initial conditions that x (0) is equal to A, when you put t is equal to 0, this becomes 1 and this becomes 0. So, simply this and x dot 0, when you put t is equal to 0 here, this term goes, this becomes B. This will be minus, so these two equations can be solved very easily to give the final value. The general solution becomes of the form, A is this. You find that as t increases now, this is our exponentially decaying term. Finally, even this term is increasing with t, but this is increasing linearly and this is decreasing exponentially. Eventually it will also decay exponentially, just like the case where you have over damped situation. Only thing here, we know that it will return to the equilibrium position at the quickest possible time.

Here also, let us find out that whether it can cross the equilibrium position. That means whether x can be 0.

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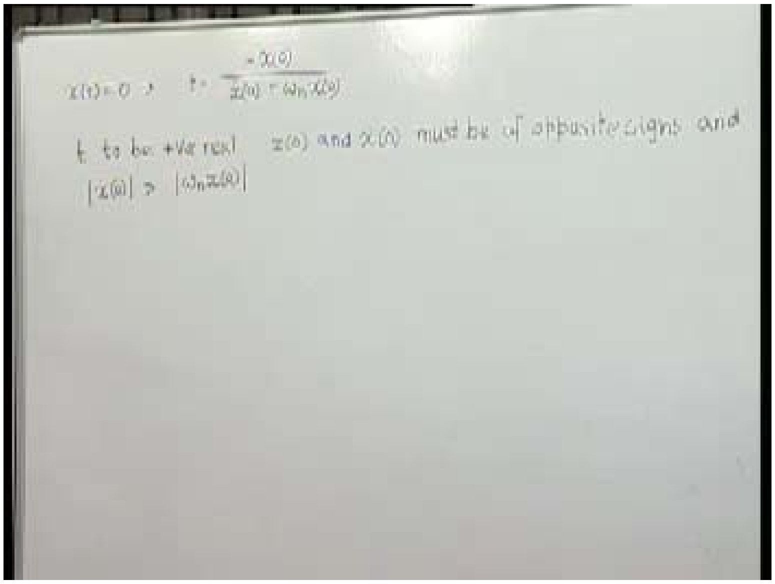

Here x(t) will be 0, means two conditions. When t tends to infinity or as before, there will be another situation when this is equal to 0. This is equal to 0, means t is equal to this. Again, t to be positive x(0) and x dot 0 must be of opposite sign. That is not enough as before that means the velocity has to be sufficiently large, and magnitude of the velocity has to be more than magnitude of this. Here the case is simplified little bit, but the basic logic is the same, that when they are of same sign, that can be both positive and both negative. We will find that it is never possible to make it a real positive number. If they are opposite, then it is possible that, and x dot 0 is more than this quantity, you can easily sure that t will be positive real.

In that case, that will cross the equilibrium position only once, because t can have only two values. Once t is at infinity, allow this particular value of continuity. The next question is to investigate the critical damping, and in this kind of a system the critical damping is given.

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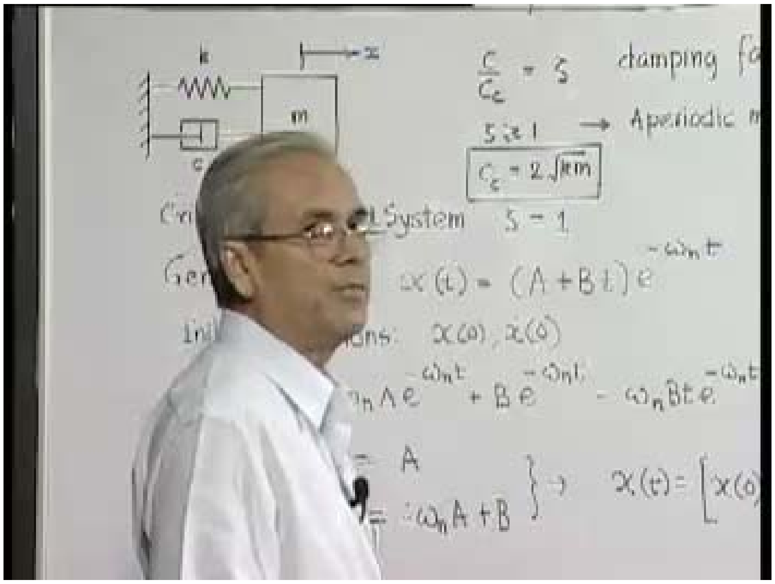

However, the mechanical system may not be always of this form, it can be of different configuration. How to find out the value of the coefficient, which corresponds to the critical condition? We should be able to find it out, let us now take up the task of finding the critical damping coefficient for systems. Here it is the system which we have already done, and we know the differential equation for this is, mx two dot plus cx dot plus kx equal to 0. We know that the characteristic equation will be mx square cs plus k, obviously the critical damping coefficient is obtained by this (Refer Slide Time: 32:20). In this case what is going to be the equation of motion?

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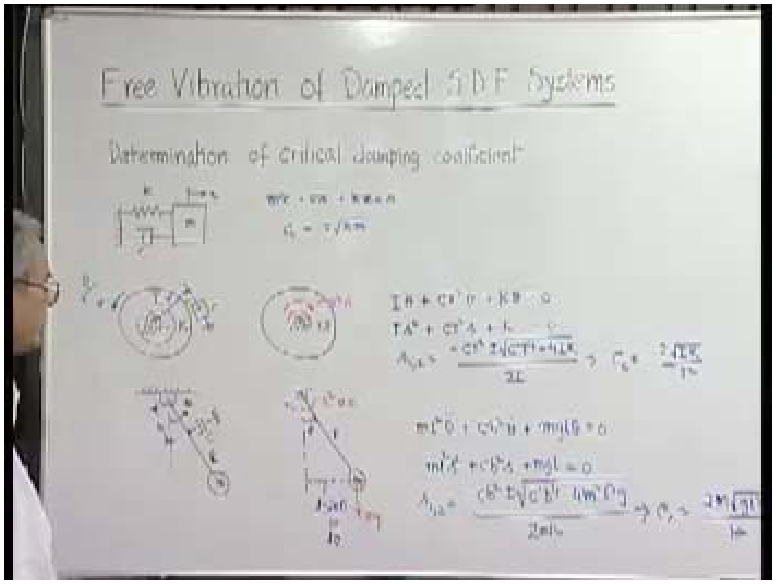

We will find that the system is a pear rotary motion. Therefore, let us consider only torque. What was the torques acting? The spring torque, suppose which is going in this direction, the spring moment will be capital K into theta. If it is rotating in this direction with a velocity theta dot, then what will be the velocity here? Radius at which this damper has been faced is r linear velocity of the dashpot system will be r into theta dot. The damping force will be c into r into theta dot and its torque here will be r square into theta dot into c.

Again there will be resisting torque, which will be c r squared theta .That means the force generated is velocity here, that is r into theta dot into c. The torque or moment of that force here, because you are writing only a moment equation. It will be multiplied by this arm and that is again r, which therefore makes this. The equation of motion we get is I theta two dot, theta two dot in this direction is equal to minus this plus or minus this, that means if you bring to one side.

Assuming the solution in the same usual form like e to the power st, the characteristic equation will be e, I s squared plus c r squared s plus k equal to 0, $s_{1,2}$ will be and this, gives us the critical value means, this has to be 0. c_c becomes equal to 2 square root of I k divided by I squared.

The last case, the gain is an angular motion. The forces are, one is mg and when it is moving theta dot, velocity of the piston is b into theta dot. Force will be c into b into theta dot, and its moment here will be c into b into theta dot into b. Therefore b squared theta dot into c. The moment of the damping force is given by this. What will be moment of the respiration force? It will be moment of this force, if the length here is l. This is l into sine theta, as the sine is this theta. But we always remember this thing, we always consider small oscillation. This is approximately equal to l into theta, sine theta is theta, otherwise you will have a non linear equation. Therefore, the moment of inertia of this is, ml square into theta two dot plus cb square theta dot plus mgl theta is equal to 0. Obviously, ml square s square plus cb square s plus mgl equal to 0, is the characteristic equation. These are the two roots and obviously the critical value of c will be that, for which this will be 0.

These are the simple cases where the critical damping coefficient expression will depend on the system. We will take up now a proper example to solve.

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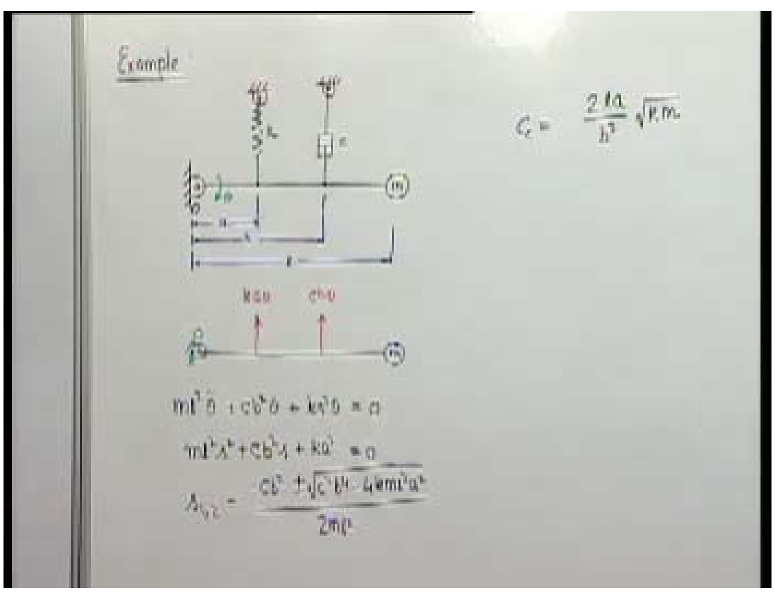

The system is like this, a mass less rigid rod is hinged at o is approximately horizontal position, and carries a mass concentrated mass m at the distance from the hinge. A spring is attached of thickness k, at a distance b from the hinge. A viscous dashpot of damping coefficient c is attached, and the system of course is in equilibrium under gravity. If we displace it from this equilibrium position, what will be the critical value of the damping coefficient c, so that it just stop oscillating it execute a periodic motion.

We should remember the cardinal principle. We have that, since the system is in equilibrium under gravity and we measure the displacement of the configuration from that position. We forget about gravity and we measure the displacements everything from the equilibrium position itself. The free body diagram, suppose you consider that, it undergoes a displacement theta from the equilibrium position.

How much is this space here? It will be, a into theta or small theta so the spring force will be ka theta. Similarly, what will be velocity of this point b into theta dot, the force acting here will be c into b theta. The equation of motion we can now write about point o m l square theta two dot, will be equal to total moment in that direction, which will be minus this into a minus this into b. Therefore, if you bring into the same side, we will get plus c b square theta dot and plus k a square theta equal to 0.

When you take moment about this point, we have to multiple by this by a, when you take the moment about this point you have to multiply with (43:09). Our characteristic equation will be the two values. Obviously, the critical value of damping coefficient will be such when this is 0, so c_c is equal to this. Therefore, we have to keep this in mind that for each and every individual configuration for a system there exist, a different expression for the damping coefficient, which will never consider damping coefficients. Critical value is always given by this, each time which has to be found out.

So far as the application of the critical damping coefficient, we have already mentioned, you understand that the practical applications in all such cases, where you do not want the system to cross the equilibrium position or rather approach the equilibrium position with negligible velocity. Now to sum up the matter concerning viscosity damped single degree freedom system free vibration problem, we have seen that cases can be underdamped. In underdamped situation the oscillation of the system in the case like this.

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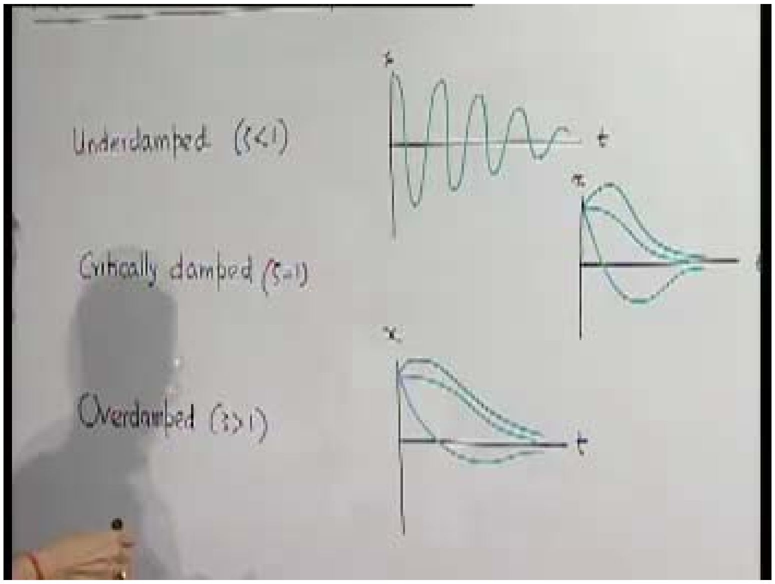

If the system is critically damped, then the system can behave like this, that if you push it like this, it will go like this. If you just leave it here, it will go like this and if you push it down hard enough, it will go like this. Finally in case of over damped phase shift cases, if you push it hard, it will go like this. If you leave it like this, it will go like this and if you push hard enough. The characteristics of the under damped, critically damped and over damped, they look similar. Only thing you find that, the decay is slower or the approach time for the equilibrium position is always longer in case of over damped system.

The characteristics is being same here the only difference that it approaches the equilibrium position in the quickest possible time. In case of under damped system, it decays exponentially and decay is measured by quantity called logarithmic decrement. That is the definition and value of delta is approximately equal to 2pi zeta, when zeta is very small. This is a major of the decay rate, and we have also seen that decay in viscously damped system can be also indicated by the amount of energy dissipated per cycle, divided by the maximum energy in the cycle. That is also related to delta and those derivation we have seen.

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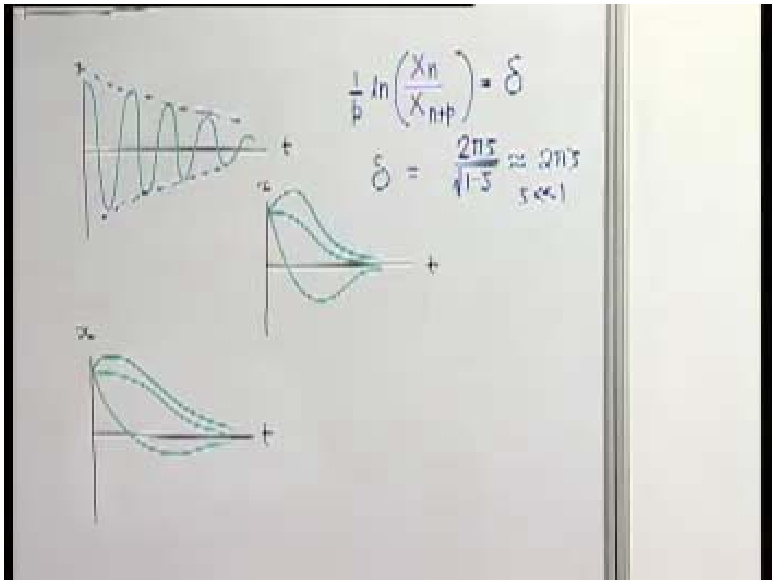

The viscously damped single degree freedom system, and their vibration characteristics, I think we have to take up the case of the system, where we have solid friction. That means coulomb damping, and the next lecture we will take up the single degree freedom systems with coulomb damping. It is the vibration.