Dynamics of Machines

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Module-8 Lecture-1

In the last few lectures we have seen, how we can make the running of engines, machines, mechanical systems in general, smooth by taking care of various fluctuating effects. Nevertheless, I think, we should keep in mind that in practice it is never possible to make the system to be completely free from any kind of fluctuating or dynamic effects. By dynamic effects, I mean to say, the existence of time varying forces in the system and if there be so, that means we have been able to reduce fluctuations, we have been able to make the smoothness quite close to desirable level but never perfectly smooth. Whatever, residual, fluctuating or dynamic forces in moment, etc, will be present will be result in oscillatory moments of the various components and members of system and the engine. These oscillatory moments, that means the fluctuating motion of a component about its neutral equilibrium position is commonly termed as vibration.

So, I think our last chapter or last topic of the discussion on perfecting the design of machines or mechanical systems will be vibration. Now you may ask the question that why is it necessary for us to study fluctuating motion or vibratory motion, because you have more or less taken care of various dynamic effects and only some residual amount is left? Therefore, let us first establish why it is so important to study the dynamic characteristics of mechanical systems or vibrations in mechanical system.

Let us consider a particular case, say a simple cantilever beam. It has a length l and cross sectional area b by 8, where 8 is the thickness and b is the width. This is subjected to a static force P_0. What will be the effect of this static force applied at the end of the cantilever? We all know that it is going to deflect and it will be something like this. Now, in this deflected position, what happens? We all know this outer or upper layer will be stressed, subjected to tensile stress? And the lower one will be subjected to compressive stress and we also know from our study of solid mechanics, this is the location where the stress magnitude will be maximum, say it is $sigma_0$.

Next, we take up the case where the same beam is subjected to a dynamic stress, but before we take up this comparison, let us also find out a situation where you want to design a cantilever beam, now what happens? Normally from our design procedure, if you know that the maximum stress developed here is $sigma_0$, and let it be so in this particular case, it is more than the maximum allowable stress for this material. From our experience in simple machine design courses, we know that the simplest possible way to take care of this will be that, rather than having a beam of height 8, we may slightly increase the thickness.

With this increase in thickness, this new stress will be developed here, something like sigma prime. We know that it will be less than $sigma_0$. If we add enough thickness to this beam, we can reduce this stress here, bring it below the maximum allowable level and that will be an acceptable design that is the static case. Now, in the dynamic case, as we have started here, it is same beam. It will have the same length, any length of course, thickness 8. Now the load is fluctuating, say simplest possible case like this (Refer Slide Time: 07:35), it could not even be harmonic but it is fluctuating.

If we try to take or experience from here and say that the response of the cantilever beam to this fluctuating force will be like this: that means, it will have lower most and upper most position because we are subjecting this to accelerating. Therefore, what will happen in this case, this point will be subjected to alternatively a tensile stress, when it bends like this and compressive stress, when it bends like this. We can now plot with time, the stress here is something like (Refer Slide Time: 09:00). So highest level is say $sigma_0$, alternately compressive stress. Now this also will become dynamic.

Now suppose we find that $sigma_0$, that is, maximum stress developed is more than allowable sigma of that material. So obviously, this design will not do and we may try to increase the thickness a little bit, as you did in this case, by an amount delta h and we expect that the new stress, that means sigma prime if this increased thickness will be fluctuating but in lower value, but it may happen, you may find that $sigma_0$ prime is still greater than $sigma_0$, which is definitely not according to our intuitive feeling that increasing the thickness, making the whole thing stronger so The same load will produce lesser stress, but in this case, when the load in dynamic, we will solve some problems in future, during this course of this program. That you can show that just in such case adding thickness and trying to make it stronger, may have reverse effect. That means, instead of reducing the maximum stress, when you increase the thickness and making it stronger in the static sense, dynamically it increases stress, increases the deflection implying that it is stiffness or strain has gone down. So the behavior or the pattern of characteristics of a system when subjected to dynamic loading will be very different from this case or the situations what we will find with static load. That is why, it is very important to study dynamics of the systems or mechanical vibration in a special way. We cannot extend or extrapolate the results obtained by static analysis into the dynamic.

Apart from this, we will find that there are places and situations where in a machine, vibration is deliberately introduced. We have seen that mixing of concrete or some such mineral processing industries, that vibration is generated to achieve the desired result and so you should also know that how to produce vibration? How to control it? There are some useful applications of vibration and thus the comprehensive study of vibration in mechanical system also becomes necessary to design some systems. Before we take up this study of vibration in a very methodical and systematic manner, it is desirable that will have bird's eye view of the whole situation. How many types of vibrations are there? What are the types of the systems which vibrate from all classification of systems from point of view of vibration, we will all look in to these things. So therefore, let us consider the types of vibrations.

Classification of vibration can be done from various points of view. The most common and important classification is like this, based on how the vibration is originated or the nature of vibration. So, this is a very common classification, but it does not show all the possible variations in the characteristics. We will see that now.

The first type of vibration is called free vibration. Free vibration means that the system has been disturbed from its equilibrium stable position and then it continues to oscillate for some time. You take a pendulum, we displace it from the equilibrium position, leave it and it oscillates. But that oscillation is without any disturbing agent acting on it and that is why we call it free vibration.

Next type of vibration is forced vibration. In forced vibration, the body or the system is subjected to dynamic loading, a force or movement which is fluctuated with time and as a result the system also vibrates. So, cause of vibration is present that we say as dynamic force or dynamic moment. They are called Forced vibrations.

There is another type of vibration, which is neither free nor forced, what I mean to say that here, this is definitely the vibration needs an external energy but it is not required to make the system vibrate by a dynamic force. Even as I am talking, the vibration of vocal cord there is no dynamic or fluctuating force which is acting on the vocal cord. This type of vibration is present in many cases. That means, when you by scratch something, it gives you a screeching sound, it means that it is vibrating, but it is vibrating not because of vibrating forces acting on it but due to the system characteristics.

I cannot show it here, not possible to show it here. So that kind of vibration, like say you pull the violin on one side, still vibrates. Sometimes, you write on the board with piece of chalk there is a situation where it will give you a screeching sound. When you are talking or when you are pressing the reed of the harmonium and pressing its bellow, it vibrates; there is a reed which vibrates at its natural frequency.

All these are examples of vibrations due to an external source of energy but without the need of dynamic vibrating force. There is another special case of vibration, it is somewhat similar to self excited vibration. Here, what happens is one of the parameter, we will discuss the parameters of vibrations very soon, one of them is dynamic. What is the parameter? There is no vibration force acting but say stiffness of the system is fluctuating with time. That is the situation, where again the system is vibrated and these are called parametrically excited vibration. I think there are some other types of process, but they are very special, we will not discuss those things here. These are the types of vibrations based on the source of energy. So, no source of energy, it is free. There is a dynamic force. There is no dynamic force but there is source of energy acting on it. Here, there is dynamic variation or time variation of one of the parameters of the systems. These are the commonly found vibrations.

Next type of classification will be based on nature of dissipation that means, whatever the energy whether it is energy conservative system or not, so such cases one can be undumped vibration, which means the total mechanical energy of the system remains conserved. There is no method by which it is being either reduced or increased, whatever the case may be, but the total energy remains conserved and there is no dissipation.

There is another case, which is of course in nature which is a more visible damped vibration. Damped vibration means energy reducing so in this case vibration continues. In this case, vibration goes dis…, which is what we normally find.

Damped Vibration can be again classified into two: one is discussed damping, this is the nature of dissipation, which is generally based on fluid fiction. We will discuss it in details or it can be coulumb damping. Damping where, there is solid frictions that means friction between two solid surfaces rubbing against each other. There also energy is dissipated and that type of energy dissipation is called coulumb damping. And other is material damping. It means, or you can see that if you take a piece of beam in space where there is no free, there is no air and you just vibrate it and leave it in space. We will still see that slowly the vibration will reduce. There, it is the internal mechanism in the material, just like histories, it is called also as history damping and the rubbing of grains all other crystal boundaries, they involve certain amount of energy dissipation.

Energy Dissipation means that the mechanical energy is converted into the worst form of energy that is heat, in most spaces times little bit that is converted into sound also. That means the mechanical energy is being converted into all other forms. Therefore, vibration problem can be handled according to the nature of energy dissipation.

Next classification we can have based on nature of the vibrating system. Nature means complexity. First, we have single degree of freedom. I will explain it later. Then you can have multi-degree freedom system and finally infinite degree of freedom, or these are also called elastic bodies. So the system complexity can also vary that it can also have a single degree of freedom system. Now, single degree of freedom means that, as you know from kinematics that only one independent quantity is needed to describe the configuration of the system. Multi-degree Freedom system means more than one such independent quantities are required to be specified by the configuration at any instant, and here we require infinite number of independent quantities or it is a continuous infinitely variable body.

Then, you can have the classification based on vibration type. We now go directly into the nature of the function, time varying function. So it can be simply harmonic vibration. It means a trigonometry function, sine functional, harmonically varying quantity that means any parameter what is representing in vibration, which is harmonical.

Next is anharmonic or we can say periodic, but harmonic. It is not a harmonic function of time. Say in this case, it is a simply sine or cosine function. Here it could be something which is periodic but not harmonic. Next is Random here of course, we are not going to discuss it, but periodic vibration is also very important. We will definitely not discuss it here. And finally, we will divide or classify vibration in another way, based on the system parameter characteristics. If the parameters are linear, we call it linear vibration. Here it is called non-linear vibration that means again it is based on the system. These are various ways the vibration phenomena can be classified into, because this overall view can help us when you take up study. You should start from simplest and gradually go to more complicated.

Let us look into the basic element and the fundamental properties that are essential for system to be in vibrating nature. Everything is not capable of vibration, for example, I take this and this is not vibrating. I take a football, I just kick it, it just goes, it does not vibrate. But I take a pendulum, displace it, it oscillates. Therefore I think there are some essential qualities required for the system to have the capability to oscillate. So, the basic element of the vibrating system, what are the basic elements? Why does a system vibrate? Most important thing, what is required for a system to have capability to vibrate is that it must have, generally a still equilibrium position. That means a position where it stays is not disturbed and it comes back to it, if disturbed. That means, if I have body and this is its stable equilibrium position. Now, what happens if I disturb it, say I push it one side. I push it to this side. Since it is in stable equilibrium position, it will try to come back. Therefore, it is essential for system to have capability to come back to the equilibrium position. That means its original position will be restored. This property is called restoration that is essential.

Suppose, if you take a ball and keep it here, I displace the ball, it will not be there. So that will not do. But if it is in an inclined or in a curved plane like, say a bowl or cylinder, then if I displace it, it tries to come back, that is restoration that can oscillate. But on a flat plane, if I take a cylinder, it will not oscillate. So this restoration is a property which always brings back the system to its original stability. This is what happens, when it comes here, it fixes a velocity and it comes here. But, since it has a non-zero velocity, it has inertia and so it shoots beyond this original equilibrium position and overshoots it and goes to this. This over shooting is possible, why? It is because of the inertia of the system. That means it must possess some definite capabilities to continue in this state of motion, which is the velocity when it comes to this, this continues in that, which means inertia and without inertia it will not go. So, obviously restoration starts acting on it in opposite direction. It will gradually come to a stop and come back to the position. But when it comes here again, it has a velocity and it will overshoot and go to other side. This keeps on happening, just like the pendulum.

Here pendulum comes back to its original position because there is gravity. This force is acting. If you go to space, I make a pendulum, displace it in one position, it will remain there. It will not oscillate. So, the presence of this gravitational pull towards the surface of the center of gravitation, that provides a restoration, for which it tries to come back to its original equilibrium position. But when it comes here because of finite inertia, it possesses some momentum, it overshoots to the other side. But then obviously, it comes to, where it is being acted by restoration and then comes back and this whole process continues.

If you consider, the situation where there is no inertia, mass is 0. Then restoration is bringing back here. When it comes here, there is no restoration, no force acting and since there is no mass, no inertia, it will remain there. These are the two essential parameters or properties of the system, which are required to make the system capable of oscillation and vibration.

There is another parameter which is not necessary for a system to have vibrating capability but nevertheless, all practical system will have that property that is Dissipation. By Dissipation I mean, in mechanical energy which we put into the system we need dissipated from an equilibrium position. Now that energy gets gradually dissipated to the surroundings by various methods, like there may be air here and air friction will dissipate energy. There were solid friction in the hinge here, that may dissipate energy and so on.

So, therefore though this energy dissipation is not essential for system to be vibrating in nature but it is present nevertheless. Then, there are three parameters which will play the predominant role in any oscillating system that is, the parameters are inertia, restoration and dissipation. In common language, this inertia is the mass of moment of inertia, if it is angular motion. Restoration, we call the spring stiffness in case of elastics or gravitational force in case of gravity base system. Dissipation, this is a damping which is the common language for describing dissipation. These are the three fundamental parameters of any vibrating system.

Now how do you handle vibrating systems mathematically? Now as you know that, whenever you solve a physical system, mathematically, the first thing what we do is, we develop a mathematical model. One must be keep it in mind, that whatever may be the subject, whatever may be the branch of engineering or science, whenever we are writing an equation to describe the behaviour or characteristics of a physical system, we are not really doing it for the actual system, which is in general, infinitely complex if we go to the molecular level. What we do, we make on approximate mathematical model which more or less behaves in a similar way as the actual system and you only solve the mathematical model.

Here also exactly that is going to happen when you try to solve the vibration of mechanism system. We do not solve the actual system, we actually write down the equation of motion for the mathematical model that we will develop.

Next thing, what we have to understand is modeling of a system. The most important or the basic idea behind mathematical modeling in case of vibrating system is lumping of parameters. The models we develop, or in this principle lumping of parameters are called lump parameter models.

What is lumping of parameter? As the language indicates that, these are the important parameters. We lump them. In a system say for example, take a cantilever spring with heavy weight at the end, this is a vibrating system. If we disturb here, it can oscillate like this. Where are these parameters located? See the system has obviously inertia, now where is that inertia present? In reality you can say, that the heavy particle of this, which is moving has inertia. Therefore inertia is distributed over whole but I think from our engineering common sense, we know that this particular thing, which was there is a mass which is of much less compared to mass of this, and the predominant job of this cantilever spring is to provide restoration to the system. That means if we displace it on its equilibrium position, it comes back to the original equilibrium position. That is the job which is being done by this.

What this block is doing, of course a block is also not a hundred percent perfectly rigid body. So it also has certain degree of velocity but so rigid that its velocity is so negligible. That its predominant role which is played which is just to provide inertia. That means, here we find, that we can say this is providing inertia that is predominately the mass. This is providing the restoration or acting as the spring and dissipation which is going because of the material damping in the beam which is undergoing distortion and friction with the surrounding air of this and this. All these things are causing dissipation.

Now what lumping means, this system is rather more or less equivalent to (Refer Slide Time: 14:20). Here we have lumped the parameters, lumped the mass as M, lumped the stiffness of spring k and the dissipation c. We will come to these things very soon.

Therefore, as you see in general what we will do in a mechanical system, we will extract the basic function or basic property of that. Who is providing or which part is providing the predominant inertia. Like that, this is providing the predominance to the restoration but it has nevertheless inertia also. Nevertheless, it also has some amount of dissipating capability because of it is dumping. But its main job is to provide restoration, so that is why we call this is the spring. This also has internal dumping. It has certain amount of stiffness and it has predominant inertia. Its primary job is to provide inertia to the system, so this is the precise principle of lumping of parameter and therefore this is the actual system and this is lump parameter system.

Before we take up more such cases, we should just have some little idea about these parameters, inertia. Inertia means, it will be a mass when the movement is linear or it can be moment of inertia I, when the motion is oscillating. Restoration similarly, can be simply, a spring or a torsional spring, this is a linear spring. Restoration can be gravity based, as we have already seen, not necessarily velocity based. We can have a basin like this, in which this is oscillating like this or this also oscillating into circle. Similarly dissipation again can be of three types, as you have seen, one is simply heat based what we discussed, there is a piston, some narrow hole and the piston moves in a cylinder. It is completely filled up with viscous fluid. So when we pull it, what happens? This side the fluid goes to this hole on the other side. And during that process it provides adequate amount of friction that is, reed friction which is proportional to the velocity. The other kind of dissipation of energy could be simply and this is represented symbolically by this symbol. This is called viscous damping.

There can be solid friction, as you know. Here there will be coefficient of friction mu. The solid friction is represented by this symbol and here the force is f. Here of course, the force will be c into velocity if v is the velocity. Here it does not depend on velocity. It is constant because it is solid friction. So these are the kind of parameters we have. Dissipation primarily is of two types. Material damping you cannot show, as it is hidden inside the material.

Restoration can be primarily elasticity based or gravity based in most cases. Inertia can be a linear inertia represented by mass and rotor inertia represented by fly wheel or a rotary cylinder. Here the stiffness is told k, here torsional stiffness K, stiffness sometimes can be recoil spring also. There is also angular motion which is resisted by coil spring. Here it is the gravity and these are the dissipative. So lumping a parameter, what we are discussing is important because only once you have lumped parameter, we can talk about forming an equation of motion, whatever you may say, that is the mathematics of the equitation relating to this property k, m and c and the displacement coordinate. We can derive some relationship between these or among these, which we solve and tell this is the systems response. Now, of course one thing is correct, that some times, suppose if you have thin still being like this and the mass at the end, we can tell that this is the mathematical model. We can have an example from the angular case. For suppose, if you have to consider vibration of cantilever beam itself like a reed. It is displaced, it vibrates, the lumping of the parameter becomes very difficult as you can see, because all the properties have been smeared into one body. Inertia is there all over distributed, restoration is provided by whole body elastic properties and dissipations also is dominant distribution with in the form of discrete damping, though there may be some damping when it moves relative to this surrounding to air. In such cases it is of course not possible to do lumping like this and get result though some times some very good approximate models have developed. Therefore, we have to identify which part of the body predominant property is what like say for example. There may be such cases like if you have this and may be another one and then obviously this part will be modeled as without damping. We neglect it and this part will be described by another. Modeling will be like this.

Now another important question we have to keep in mind while developing the model is how accurate it should be? Some people can model it like that you can divide it into four, and each one we try to represent by a spring mass system. It will give some result, which will be near to this. But obviously not exactly like the actual beam, but if you divide into more number of sections, obviously result is going to be more accurate. Therefore if you want to make the model very accurate, that representing the original physical system a very faithful manner, then it is going to be obviously, more complex. Solution will be difficult but the accuracy of the result also will be definite. It is otherwise a kind of give and take, a balance which one has to strike. That is how accurately the result is desired and based on that, you should develop the model. If the result required is not necessarily very accurate, it is pointless to go to the horrendous task of very accurate, model its last of complexities and spending a lot of computational time in solving the system. Therefore you should always find out how complex, how accurate the model should be.

Another important thing what we should discuss at this stage is degrees of freedom, which I mention is coming again and again. I think it is better to discuss here. The idea behind degree of freedom is known to all, I am quite sure this is related to our modeling. If you do not require very accurate result, in most cases when you do the parametric modeling, you can try to get a simplest possible single freedom system, a mathematical model they are simplest. So, therefore, we will see what degrees of freedom are.

The degrees of freedom are number of independent quantity required to describe the system's configuration. So simple thing, which is equivalent to (Refer Slide Time: 52:45) to be only one quantity is required. The displacement of mass from position, if so the degree of freedom is single degree. The system is something like this obviously we have to specify displacement of the two masses and it will have two degrees of freedom in this case. So degree of freedom in this case is 1 and in this case is 2. Similarly gravity base system we also can consider pendulum. We have to only give theta, only one quantity specified configuration single freedom system but if there are two pendulum joined like this, then obviously we have to specify, goes θ_1 and θ_2 independently, then the number of degrees of freedom are two, in case of elastic bodies for the beam.

When you deform in formal manner and to represent deform we have to express the displacement of infinite number of points describing the beam and therefore in such cases n is going to be infinity. All these are continuous points. Sometime it is not necessary that, one has to have more than one lump body to have more than one lump body, you should have more than one degree freedom. If you take object moving in two dimension space, say this plane of the black board and it is constraint to move with the help of three springs, then if it is confined to this plane, even a single particle like this will require two quantities to be specified, to give the exact description of the configuration. So it will also have 2 degrees of freedom, though there is only one particle.

Similarly, if this particle is resolutely supported so that, it can move in three dimensional space, even a single particle will have 3 degrees of freedom. Sometimes, if rotational motion of it is also there, that means this is supported by springs, here you know if you select any point, center of mass of the body, then it will require 2 degrees of freedom to describe the location of the center of mass and one more angular position, angular coordinate position to describe the orientation of this body. Though this is a single rigid body in case, it will have n motion. The number of degrees of freedom and necessarily not associated with the number of part, but in the most general case, one rigid body in three dimensional motion can have fixed degrees of freedom rigid body, the plane motion can have 3 degrees of freedom and the particle plane in plane motion will have 2 degrees of freedom if it is confined and played. However in such cases, we confine their motion is one direction, moving along with circular curve. Here it is moving along straight line, so in such cases they are not moving actually in a plane, that is it has a single degree of freedom.

Why we do modeling of system, another very important point, a margin and we should at least discuss the details, but briefly at this stage. That is, this parameter, what should be the nature of the parameter? Here again we continue discussion on the lumping of parameter and mathematical modeling. The important things in this modeling is that, if we take the case of simple spring mass system, its equilibrium position is this and its displacement coordinate, velocity and acceleration, they are x, x dot and epsilon. There are three forces involved as you can see, one is the force of inertia. If you clock with minus x2 dot, which is minus mx 2 dot and obviously this is a straight line. We can consider this straight line for almost all engineering problems, mainly because m remaining constant, unless the take the relative effect which is absolutely unnecessary for most of the problem solving in mechanical engineering, m is constant and so inertia force is a linear function of the motion parameter x 2 dot.

If we take the dumping force which is minus c x 2 dot, here again which I see that in reality, it will be a straight line up to some time. But then after some time, it may vary from a linear property when x velocity become very high, it may not remain laminar and it may not maintain that linear defended. Here we will find that we have a range in which it can be considered as linear but if we go beyond that, then the parameter which represents energy dissipations by discussions dumping cannot be treated as linear.

Finally, the restoration is again appears to be linear but you all know that if we split the spring in two parts, then it tends to become harder and harder. Therefore, the linear relationship between the force and dissipation stress is only up to certain point. Therefore, in reality now these are linear function. But when you develop the mathematical model we treat the whole system as linear. That means a is constant, c is constant, or this is proportional to x dot like that. They all are linear function of respective motion parameter. But in reality none of them are exactly the straight line relationship, but if we do not want to make a problem so difficult, I think you can skip all of this. We can consider that we are operating only in the linear range.

And that is the case, normally what will happen if we restrict ourselves to the solution of small oscillation. In general, when you solve a problem, we will have to keep in mind that the small oscillation which you are analysing, that is as small magnitude and that in most case we will rather justify assuming the system parameter you have linear relationship with the respective parameters.

We will now have to go to various other kinds of general discussion. I think we take up later, exactly one problem one by one, which is the type of excitation, the type of example which we have already mentioned, which can be harmonic but periodic, can be self excited. It can be basic excitation and force excitation and of course random. Next session we will start taking up the problem and pick up the problems one by one starting from the simplest and going to more complex.