Dynamics of Machines

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Module No. #07

Lecture No. #03

From the examples we have discussed, it was very clear that if we provide enough capacity to the governor mechanism then it can manipulate valve mechanism or other control mechanism for a system. It is necessary to put a substantial amount of weight which makes the governor bulky, very heavy and so on.

On the other hand, I think the possibility exists that, if we take the help of spring to generate at least a part of control force, because the weight of governor mechanism which is added to sleeve and its primary function is to bring governor to lower most position or to effectively developed a control force, which takes the rotating sphere towards the axis of rotation. Another class of centrifugal governors has been developed, where the springs are the predominant factors or agents for creating control force.

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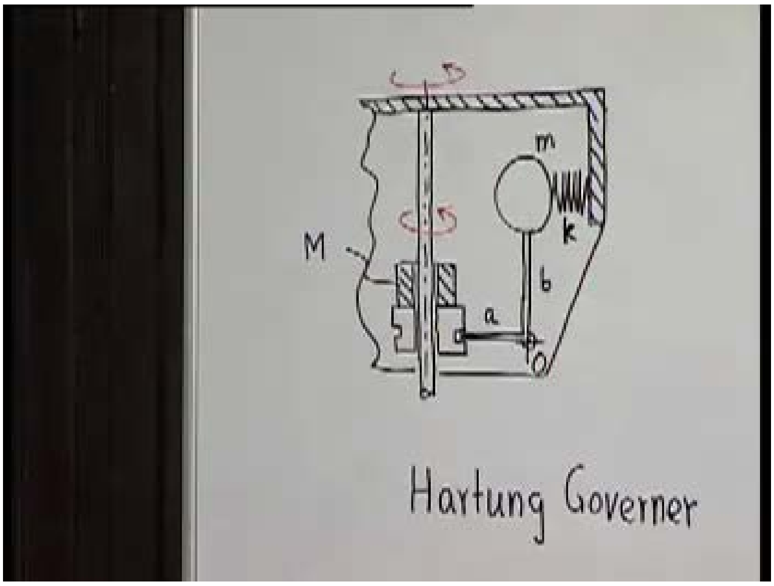

Today, we will discuss two important such governor mechanisms, which fall in the class Spring Controlled Centrifugal Governors. This class of governors also operates on the basis of the reflecting force, which is nothing but the centrifugal force, which are controlled predominantly by springs.

There are two types of governors which are commonly used. The basic stage representing this; I will draw only one half of this (Refer Slide Time: 02:38). In this type of governor we see that the rotating sphere is connected to a bell crank lever, which is hinged at this point that we call it as O, which is connected to cover that is rotating along the shaft. The whole body including this one is rotating just like this (Refer Slide Time: 04:25).

This ball is pressed against a spring, which rest one side in to the casing, other side to the ball so that it applies a pushing force in this direction. The sleeve is going to be operated by this lever that ultimately controls the ball mechanism. So, this is the type of spring that controls the governor. Of course, there is also some mass M, this is mass m; this is the spring of thickness k; length of this arm is b; length of this arm is a and this type of governor is call Hartung governor.

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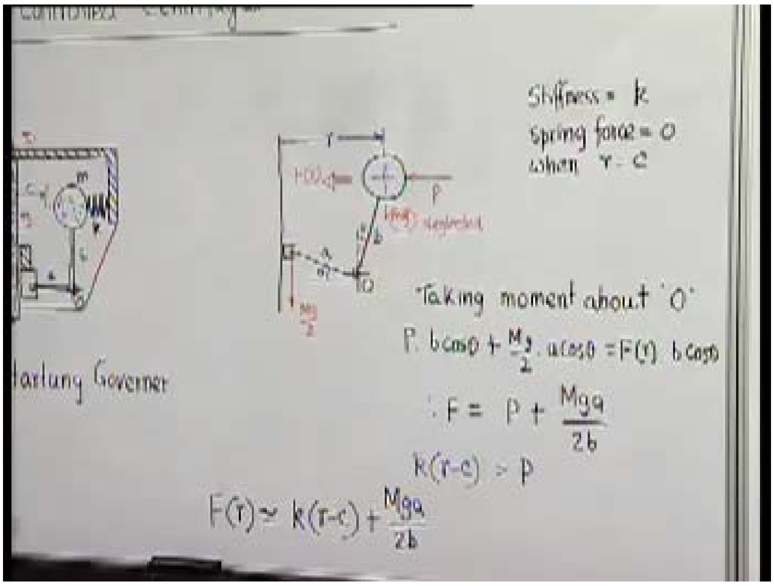

Schematically the deflected position is going to rotate at a particular speed; this length is b, this length is a; at any instant this angle is theta (Refer Slide Time: 06:11). Now, the force acting here from the sleeve will be say P depending on the initial condition and the amount of total compression. The half of the sleeve weight will be Mg by 2, other half will be taken care by the direction.

We also mention here that at any instant, of course you should also remember r is nothing but radius at which sphere is rotating. Now, we have to find out what is the effective control force expression that means effect of the sleeve weight, effect of the springs together or maybe gravity also, but we neglect gravity in this analysis, but it is also there.

So, what will be the equivalent control force as a function of r? Let us find out that, once if you find out the equilibrium position then it will be given by the condition that m omega square r is equal to F r. We have seen that stiffness of the spring is k and spring force is equal to 0, when r is equal to c; that means its natural length is such and the fitting of the location of the hinge etc is O, when r is equal to c then spring forces is 0.

Let us try to derive the expression for control force, as you have done before we will do the same thing that means, to find it out what we will do? The moment produce by equivalent force about this point O will be same as the moment produced here by the actual forces.

Therefore, by taking moment about O, what is the actual force that is produced? It will be P into; now this length is b; this height will be b cosine theta, plus this weight is Mg by 2 force; the arm length will be a cosine theta and that must be equal to equivalent control force (Refer Slide Time: 09:18). All the things are removed and equivalent control force is produced, its moment will be b cosine theta. From this, we can write F is nothing but P plus Mga by 2b.

Now, we also know that p is 0 when r is equal to c, when r is equal to r then what will be p? It is the compression that the spring has undergone. The original length of the spring, let this be represented by this one (Refer Slide Time: 10:18); that means this is p; so the compression, when it comes here to r, compression of the spring is r minus c and that is multiplied by the stiffness with this spring force R of r minus c is equal to p.

Finally, the control force F r is expressed as k into r minus c plus Mga by 2b (Refer Slide Time: 11:01). But, we have to keep in mind that this is approximate; why it is approximate because we have ignored the effect of this Mg assuming that this mass is not large.

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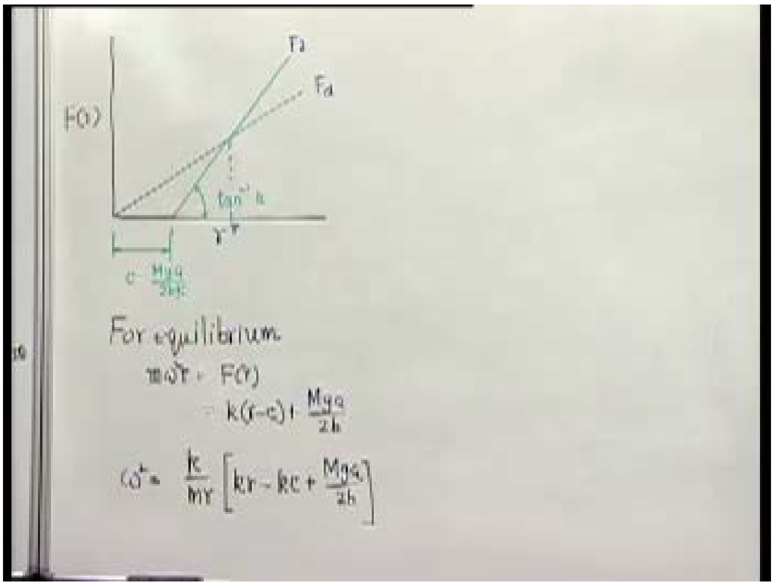

If this is the expression of the control force, how does it look like? If we plot it, then clearly you can see that it is a straight line. If we plot F versus r, then it is a straight line because this is a constant, this is a constant and simply k r is the term which is varying with r (Refer Slide Time: 11:50). Therefore, this angle is nothing but tan inverse n in suitable units. Intercept length is that when F is equal to 0, r is equal to c minus mga by 2bk; at this value of r, F r is equal to as shown in slide.

Now, we will find the equilibrium condition for equilibrium, we know that only thing which we have to do is m omega square r has to be equal to F r; or this is equal to k into r minus c plus Mga by 2b and this gives us the equilibrium speed at r relationship.

Therefore, we can find out the equilibrium speed very easily, because this will be m omega square r, obviously this is the equilibrium radius r for a particular value omega (Refer Slide Time: 13:55). As the speed increases r increases; it is quite obvious. We see there is a possibility because this quantity that is the original length of the spring, stiffness of the spring, the length of these bell crank levers a and b; these are all adjustable things, which we can control at our will.

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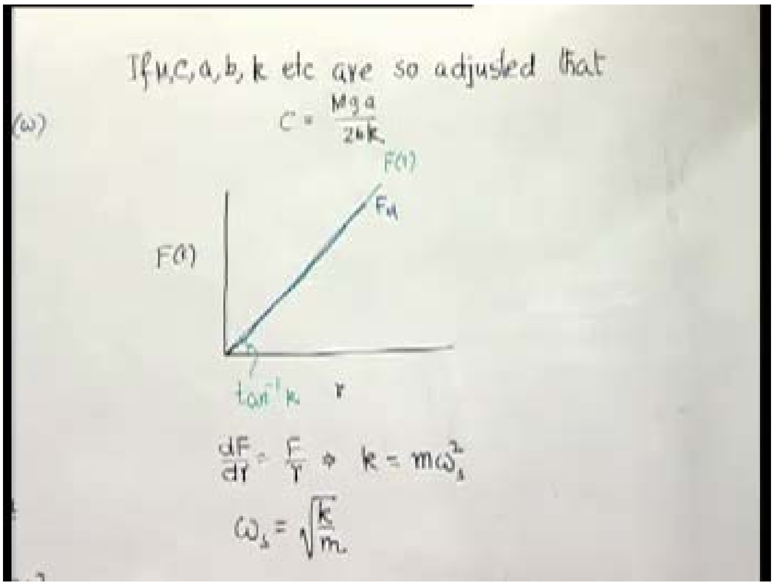

If that is done, if M, c, a, b, k etc are so adjusted that c is equal to Mga by 2bk; if we do that then what happens? then this becomes 0, the control force diagram will now look like this (Refer Slide Time: 15:32). What does it indicate? The system can have equilibrium condition only if the deflecting force also coincides with that. This indicates that we have already seen this kind of situation which is called isochronous situation or the governor is isochronous at the speed.

We find that dF by dr is equal to F by r, this is the condition for isochronous and dF dr is nothing but k. F by r is equal to; F is equal to Fd at equilibrium, which is m omega square r; m omega_s square by r is m omega square this is a isochronous operation, we call it omega_s. So, isochronous operation of this is achieved and isochronous speed is given by this quantity. Obviously isochronous means, it is very sensitive and so on. By suitably adjusting various parameters, we have lot of parameters at our disposal and therefore M need not be a very large value as you can see, so we can make the whole thing compact and sensitive.

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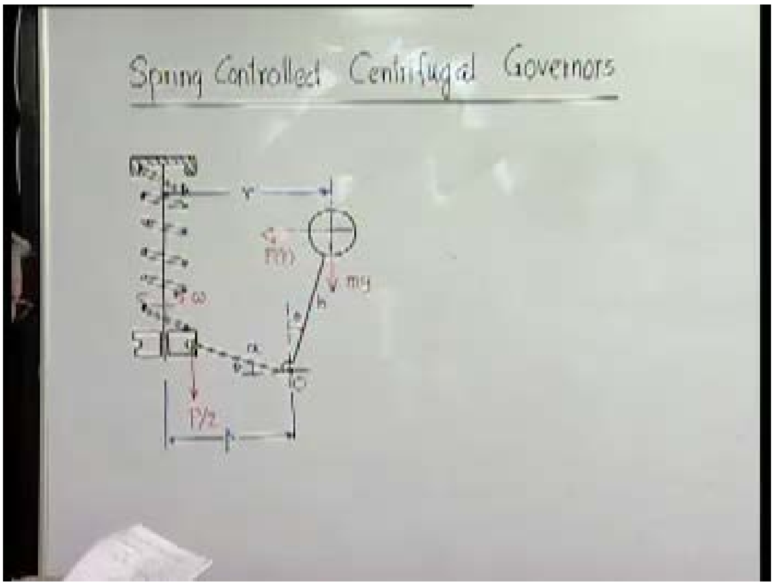

We will consider another type of spring controlled governor, which is also used in many cases. In configuration of that I will draw the schematic diagram; this is the spring. Here, the sleeve is actually pressed against the spring, which is being prevented going up by this cap, which is actually covering the central axis of the governor mechanism. The sleeve moved up and down, but the sleeve is subjected to the spring force in downward direction instead of gravitational pull of the large mass, which normally kept here. Rest is very similar to the other kinds of governor that is a bell crank lever.

At the free end there is one spherical mass m, at any instant the radius at which the sphere rotating is r, lengths are this, at any position at a particular rpm when it is rotating is inclined like this (Refer Slide Time: 21:25). Again, our usual procedure we will follow that means, we will now try to find out an expression for the control force. Therefore, control force if you consider ,whatever moments have been produced by the actual forces which will be same as the moment produced by the effective control force.

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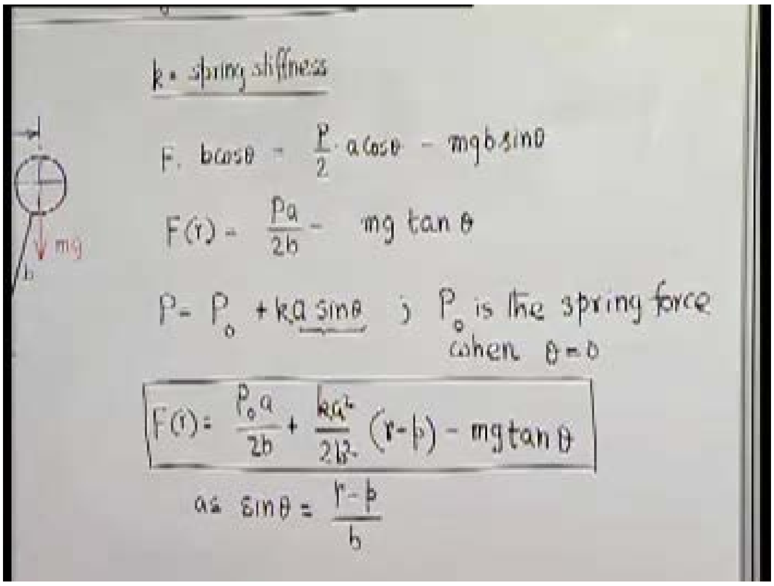

We will take k is equal to spring stiffness. Now, F into; if you take moment about this point O, which is nothing but b cosine theta that must be equal to the actual moment produced by the actual forces. So, this will be P by 2, why P by 2? Because half the force of spring is being taken by this bell crank, other half will be P by 2 into a cosine theta minus this moment, which is in the opposite direction and it is mgb. This distance will be sine theta (Refer Slide Time: 22:50).

So, we get the expression for the control force as F of r is equal to pa by 2b minus mg tan theta. We know that spring force at any particular position will be given by P is equal to P_0. Now, P_0 is the force when angle theta is 0, corresponding to that position at that time this spring force was P_0. When angle is theta, this is; the further compression of spring is a sin theta and that is multiplied by stiffness, which is the further spring force; the total spring force is this, where P_0 is the spring force (Refer Slide Time: 24:18).

Therefore, the expression of control force is P_0a by 2b plus ka square by 2b square into r minus p minus mg tan theta. Now, we have also used the condition a sin theta, this one, so how we have done it? Because we know as sin theta has been replaced; how much is sin theta? Sin theta is the same as sin theta, now if this length is p, this is r, this length is p minus r and this is b; p minus r by b is sin theta r minus p by b.

So, using this in this expression this and this, we finally get the expression for control force (Refer Slide Time: 25:48). If you want to plot, we will have certain problem. So for up to this is concern it is fine, but theta and r relationship is not that straight forward. So, what can we do? Let us do first approximation in which we ignore this effect of this mass, then how the control force looks like?

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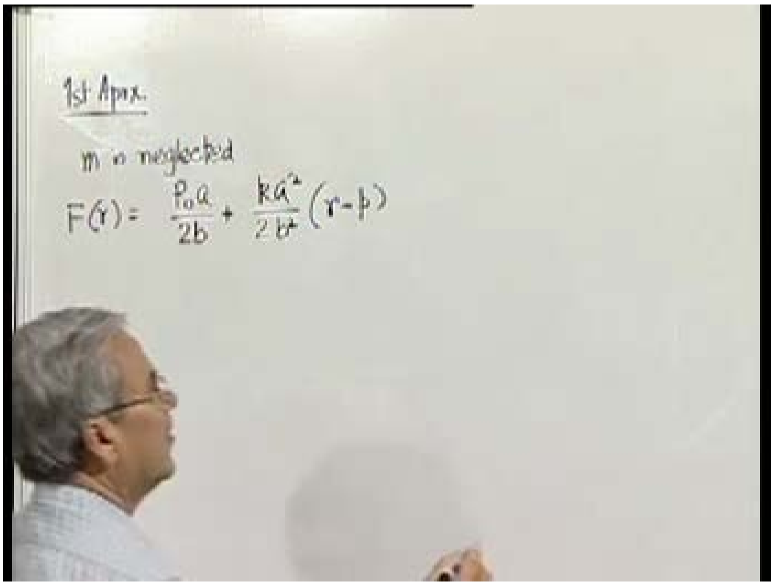

So, the first approximation we will do for mg is neglected, mass is not substantial and then obviously control force becomes simple linear function.

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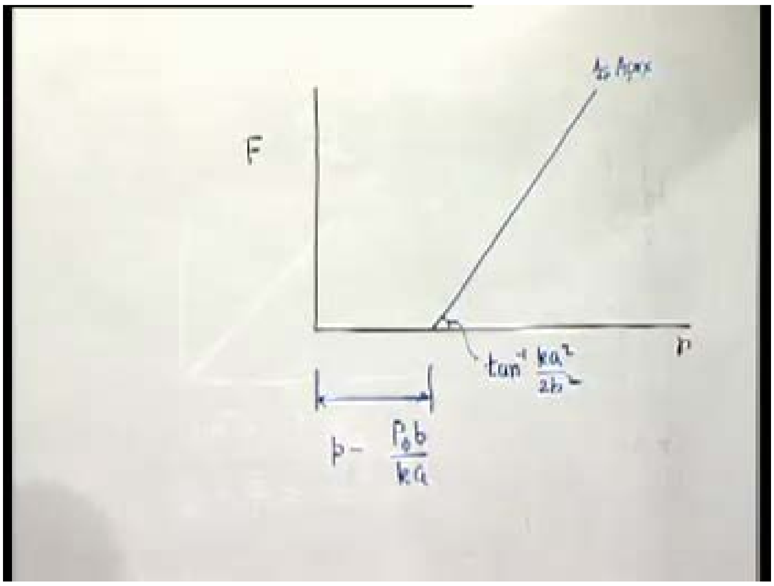

This slope is given by tan inverse ka square by 2b square. You can also find out this; this will be simply, when F is equal to 0, how much will be r? r will be P_0 a by 2b minus p sorry plus p, it will be p minus P_0b by ka, it will look like this. If you can consider this effect of the weight is negligible we will get this (Refer Slide Time: 29:25).

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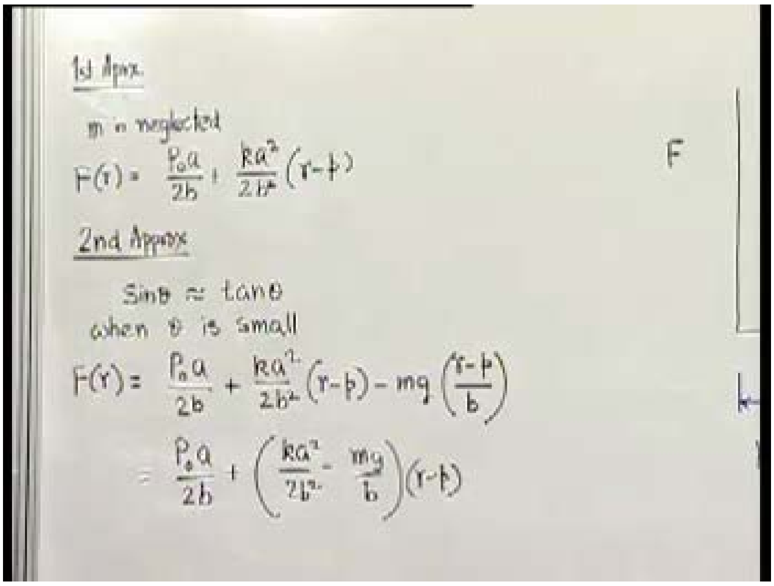

Next, if we want to improve our situation, suppose we want to make not that big assumption. In second approximation what we will do? We will get tan theta is approximate equal to sin theta, which is possible in case of thetas value being small. If we make this approximation then we will find expression of Fr will be (Refer Slide Time: 30:19). Now, this tan theta can be replaced by sin theta, which is again nothing but r minus p by b.

Now, we can take the terms containing r as common and finally expression for the control force should be as shown (Refer Slide Time: 31:00). If you plot the second approximation you will see that it is still another straight line, because it is linearly depending on r. Only thing now is the slope of that straight line, which is dF by dr and that is going to be a square by 2b square minus mg by b that means, the slope was a square by 2b square, so it will be somewhat less.

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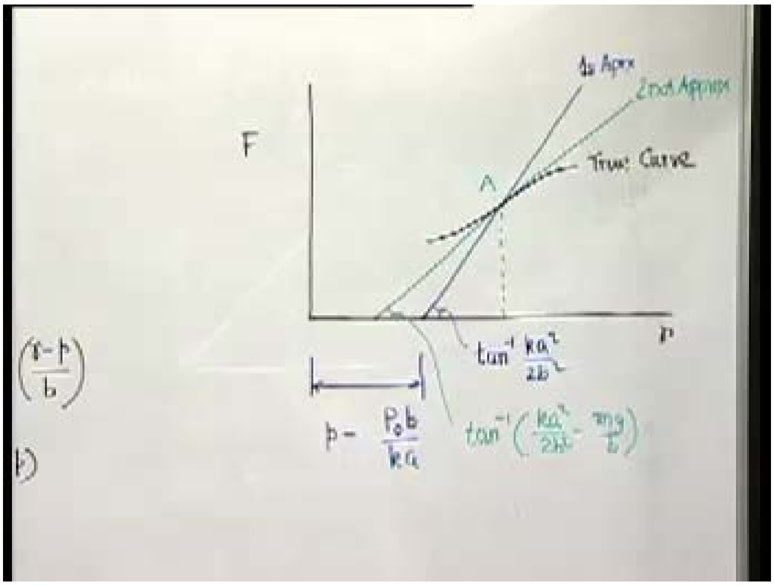

Therefore, you will get another straight line whose slope is slightly less than this. Therefore, at this point you may call it as A, the two curves; the result of values of these two curves. If we do the exact analysis, then you can show the two curves will be something like this (Refer Slide Time: 32:40). The result of an exact analysis is if we plot F we will find this will be curve, where it will pass through the same point and its slope will match with the slope of the second approximation, because when theta is very small then this approximation becomes exact and so this slope will match.

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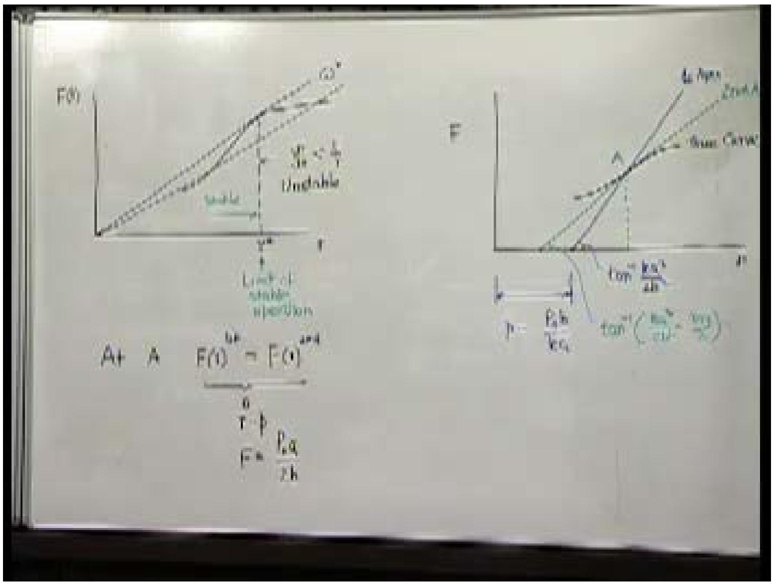

So, if we examine this more carefully, then the exact curve is something like this. If we draw a tangent to this F r curve, what does this indicate? It indicates a particular speed; that this is nothing but the deflection or deflecting force, which is the centrifugal force (Refer Slide Time: 34:17). It is obvious that - this is the speed - at which the operation has to be for any equilibrium operation; if this has to be below than this. Otherwise, F r can never be equal to be centrifugal force, as you can see that. It is also very clear from this, that this is the critical r or radius operation beyond which the operation is unstable.

I hope, you remember that if we take a speed something like this, then the equilibrium is given by this (Refer Slide Time: 35:08). Now, here it is obvious from this nature that dF dr is less than this region F by r; the dF dr is this slope and F by r is this. Therefore, it is unstable, whereas if you operate here, you can see dF dr is more than the F by r, which is nothing but the slope of this is F; this is r; this is stable and limit of stable operation is this (Refer Slide Time: 36:15).

Now, one more thing you should find out from this is that this location of this point A. The point A means what? That the two curves; first approximation and second approximation, they give the same value. Now, it is very clear that they will be same value of the force and that value will be equal; that means, at A, F r first approximation is the same as F r in the second approximation. This will be possible only when r is equal to p. We will see that only in such cases the two approximations will give same value of the force, which is nothing but values of this will be (Refer Slide Time: 37:30).

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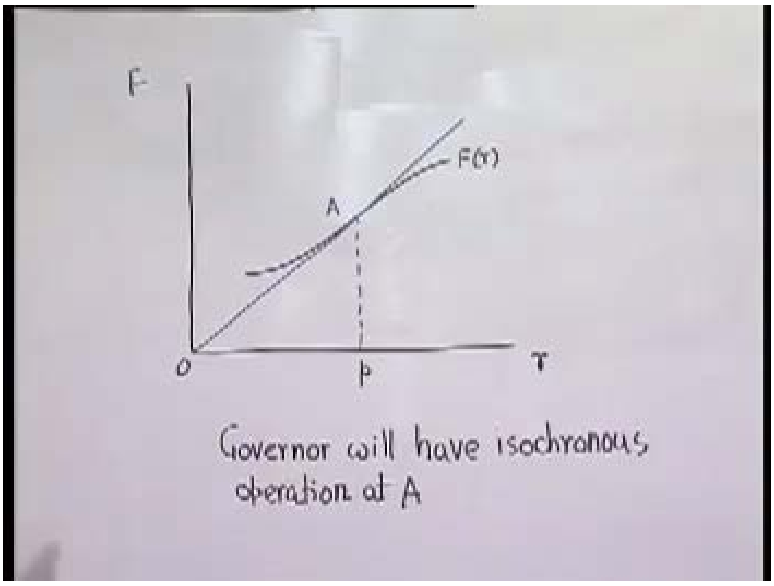

Therefore, location of A is indicated by the value of r which is p; p is given here; is distance of hinge form the axis of the rotation, small p. Now, if it happens so, that p is such; that this location of A is such, that if you join or pass a tangent to that, it will also pass through point O. This implies that at this location, it will have isochronous operation, because here dF dr is this and F by r is also same. Therefore, it satisfies the condition and the governor is very sensitive and obviously sensitive means that, it will operate better.

Now, another important point is that we are improving; we are trying to increase the sensitivity, but there is a limit to that and we have to be very careful about increasing the sensitivity as much as possible and that particular situation where sensitivity is high, we may or the governors are susceptible to a particular phenomenon, which we call hunting and let us see what is that.

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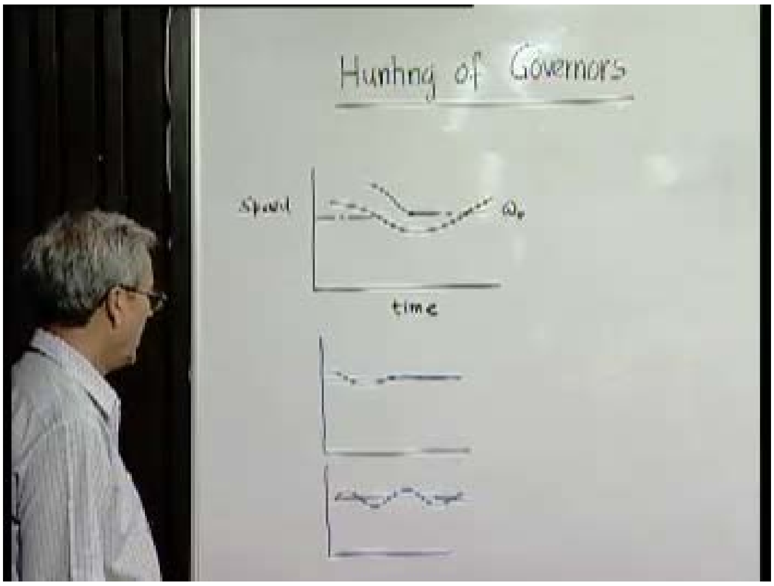

Now, we have seen how a governor functions. When system is running, suddenly due to some reason, may be a change in the load or change in input whatever it may be; suddenly may be the load is less, so governor will try to speed up and as you will try to speed up, we are controlling or cutting down its input. Therefore, then it will again start coming down to the required level of speed, but due to the finite inertia of the whole system or mechanism it does not stop when it reaches the required speed that means, graphically you can explain. If we plot its time and speed, so this is our required speed and we are somehow here (Refer Slide Time: 40:50).

So, what we will do? We are trying to cut down the input so that speed reduces. When it comes here to our desired operational speed, it does not stop here because, the whole system will have some inertia, it continues; it goes below the desired speed but again since it become less than this and speed input will increase and then again slowly its will change its range and it will try to come here. Again due to inertia, it will overshoot and this will cause the whole system to hunt, it will not asymptotically stop. The ideal situation would have been we start here and then we reach this, but that will not happen.

The phenomenon of hunting becomes more prominent in certain cases particularly, where the natural frequency of the system matches with the frequency with which the power or the requirement load etc are fluctuating that is a serious thing. In normal case, it will be like this it will gradually come down and get damped, as it happens with vibration (Refer Slide Time: 42:05). Say, it is suppose to form like this but, if the fluctuation in the load etc is also continuing then this will also continuing. So, this will continue following the load fluctuation. When the two frequencies that is the frequency which the load is fluctuating and the natural frequency of the system they match then the problem can be quite severe.

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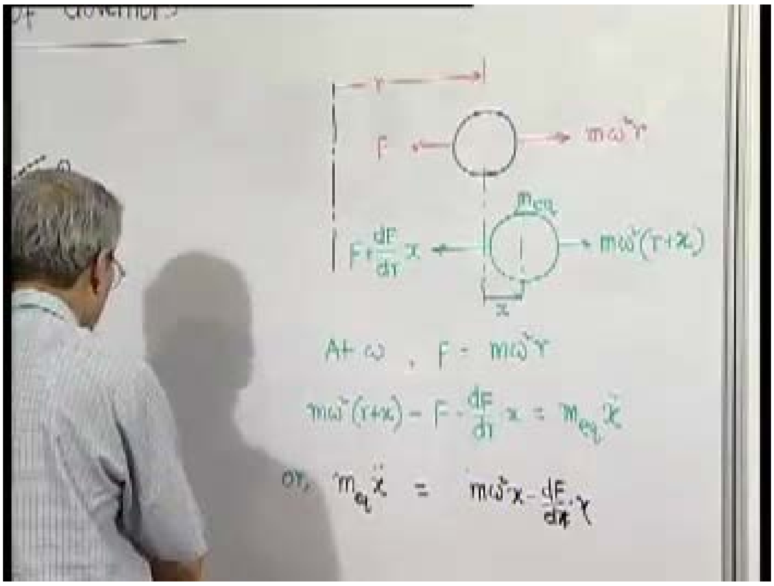

Now, how to find out the natural frequency of the governor, its operation? We can do an approximation analysis like this. This is the spherical ball and the forces are as we have seen that in the equilibrium position; this is F and this is m omega square r; this is r. Now, due to some disturbance it has shifted here (Refer Slide Time: 43:43).

So, this force will be now, what will change? It will be force at r plus the rate at which force changes with r and the amount of change in r is x. This is m omega square - which is same because speed has not changed only the displacement has been provided may be - r plus x. If, at operational speed omega by definition F is equal to m omega square r that was at equilibrium. What we have done? We given a displacement and you have to see how it changes; what frequency, that will be the natural frequency of the system; that means, if a governor operating at a speed omega at a particular r, if you displace the spherical mass and then we will find that this spherical mass will oscillate about the original equilibrium position that we will give us the natural frequency.

Now, one thing we have keep in mind, the mass of the whole system represented by this is not the mass of the sphere, because there is certain amount of matter present in the linkage, certain amount of matter present in the sleeve, all those things (Refer Slide Time: 45:11). All the equivalent amount of the inertia is accumulated at this point and therefore, we consider that its mass is equal to $m_{equivalent}$ or equivalent but, the centrifugal force which is produced, which is trying to displace the whole thing is actually the mass of the spherical ball on which we call m.

So, what is the total force in this direction? It is m omega square r plus x minus F minus dF by dr into x and this total force in this direction must be equal to the mass of the whole inertia of the system represented here, into its acceleration. The displacement from the equilibrium position is denoted by x. So, the acceleration will be denoted by x two dot. Now here, we get or $m_{equivalent}$ x two dot is equal to m omega square r minus F is 0. So, what remains is m omega square x minus dF by dr into x.

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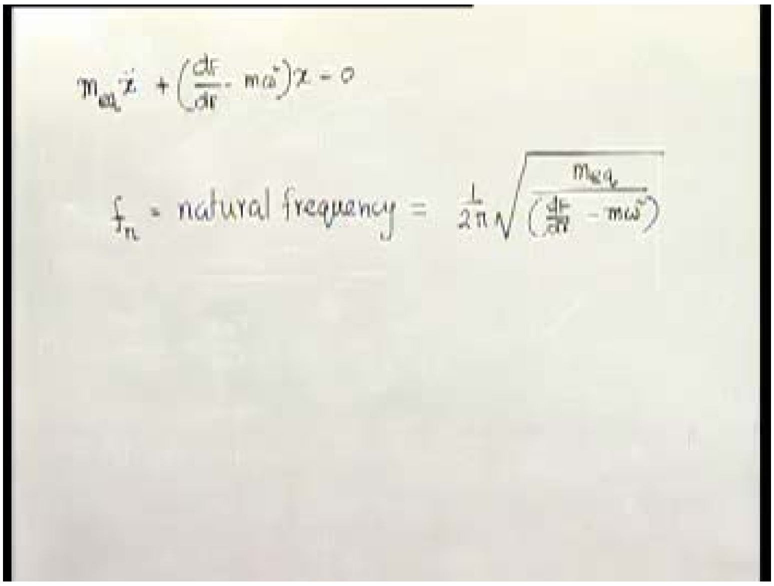

So, rearranging this we get (Refer Slide Time: 46:55). We know the solution of this equation is when of course, dF minus dr omega square is a positive quantity, then it is a harmonic function of time. We get with the natural frequency of oscillation is given by 1 by 2 pi as shown (Refer Slide Time: 47:58). This is a standard equation x two dot plus k x is equal to 0, so the natural frequency is square root of k by m which happens to this and natural frequency in half will be 1 by 2 pi multiply

Once, this frequency become very close to the frequency which the load fluctuates then the whole system will undergo in a rigid manner; that means, the whole governor and whole engine system will hunt or fluctuate in its speed and the operation.

I think, this brings us to the end of our centrifugal governors, but we remember that we mentioned, it is possible to achieve much higher sensitivity or response if we base our operation, not on the change in velocity but rate of change of velocity that is acceleration, rather than waiting that velocity will change from one value to other and taking that change and making our system to be based on that. Better situation you may get, if the rate of change of velocity which we can sense at any instant, if you can sense that and operate our governing mechanism on that.

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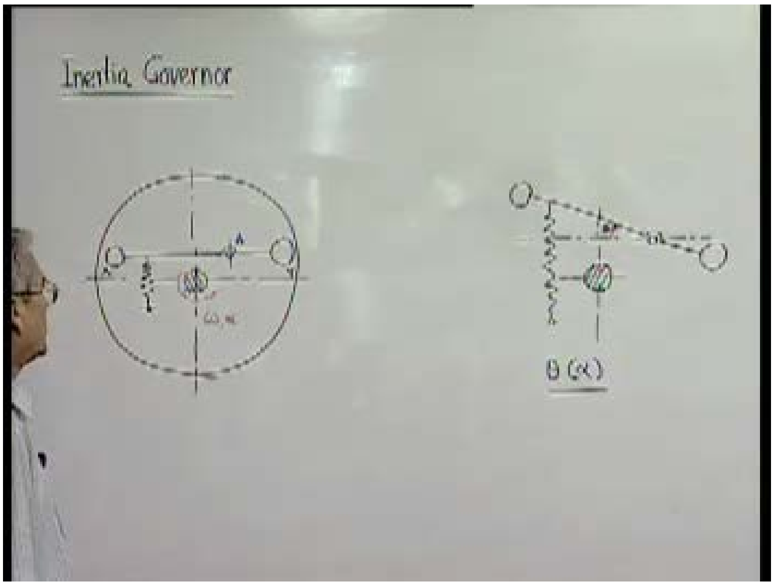

So, I just give one quick example of inertia governor (Refer Slide Time: 49:58). This type of governor is called Inertia governor or flywheel type governor. I am not going into the detailed analysis; I will only indicate the basic features of such system. This is like a disk mounted on the top and is rotating in this direction at speed omega. Now, there is a lever hinged at location say, let us call it A.

There are two masses mx and my at these two ends. This is again controlled by a spring which is attached to this location here. Now, what happens, if it tries to accelerate that means, suppose load has increase or decrease and the system is trying to accelerate; that means, there is an acceleration alpha. If it tries to accelerate in angular direction what you can do? You can again analyze the system by bringing it back to an inertial frame using D' Alembert's principle.

So, everything in this will be acted upon by inertia torque in the opposite direction. This is very standard technique of solving problems in an accelerated frame of reference. I will not draw the whole thing, but what it is going to be now, if the whole system here everything is subjected to an opposite inertia torque then this bell crank lever of this lever will obviously rotate about this hinge point. So, it has rotated by angle theta and this change in configuration can be sensed, can be picked up to operate the ((valve)) or input mechanism.

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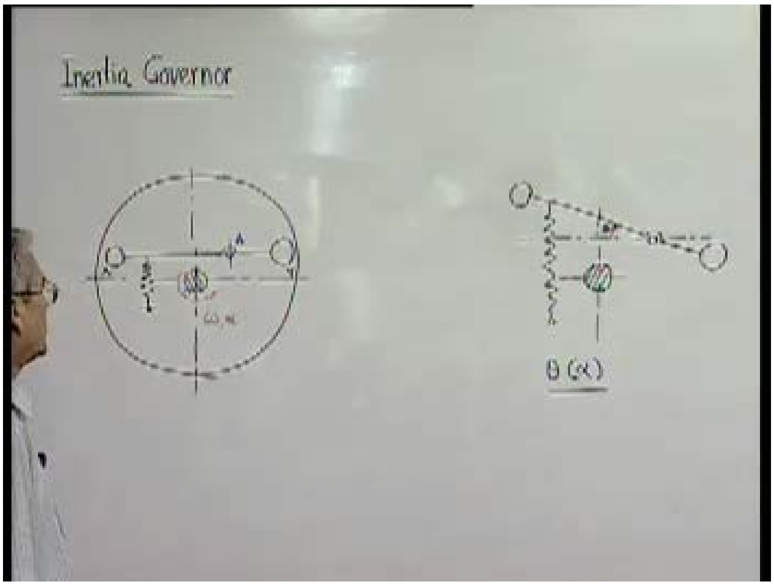

Here, we will find that this, we will not going into derivation. You can definitely try to find out that what will be the relationship between theta, which will be a function of alpha. So moment, it tries to accelerate as soon as the load changes, immediately it will come back to position, it does not have to wait till the speed changes to some suitable value and then sense, it is definitely much quicker in response. This type of governor called inertial governors or some time fly wheel governors.

So, I think this will be interesting exercises, we assign various dimension and then find out an expression of this angle rotation in terms of function of alpha or in terms of alpha. There are other mechanisms by which you can have speed control in small devices, as I mention in such cases, there is not much scope of controlling the input because the input may be a spring which we have wounded and that spring load is gradually reducing and you want to maintain the rpm same, so they are used as energy dissipative systems, we call. So, just a basic principle I will explain, there are various models using the same principle.

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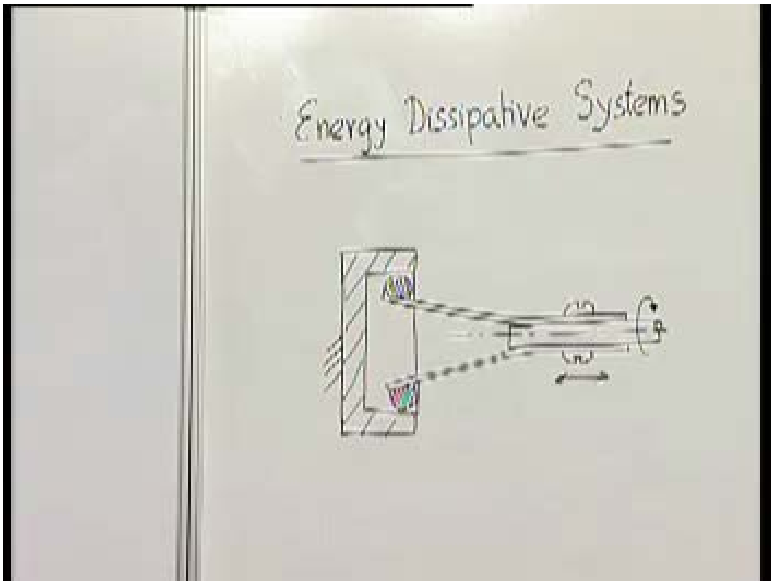

This is rotating the shaft coming from the system and there are two leaves spring very thin spring, at the end there are two brake shoes kind of thing. The length of this can be adjusted so that sometimes it go nearer to this up and sometimes it can be down.

Now, if the speed of this increases or decreases, whatever it may be, then what will happen? This centrifugal force acting on this will try to - if it increases – try to fly it off. So, the moment it happens, what will happen? It will press against this fixed cup and more energy will be dissipated. So, this larger torque coming here will again lower down the speed. Similarly, on the other hand, if the speed tries to decrease then obviously as it decreases then this will come nearer and the pressure here will be less and less torque will be developed here so, speed will increases.

In this way, what will happen? That system will not be allowed to go beyond some designed speed which is decided by adjusting the position of this (Refer Slide Time: 57:11). If you take it too much and obviously, we will find it will press harder, if you will take this side obviously, this will come below and it will press with lower pressure or with lower force. This kind of systems can be very small and miniaturized and in many small toys, the similar system or in a modified form also sometimes it is found; but, basic principle is same. The moment speed tries to increase, more energy is dissipated more fiction torque is dissipated.

This is another way of controlling speed, which is used in many devices. So that brings us to the end of our discussion on how to control speed of a system when there is major change in either the output load or there is the change in input condition. Therefore, if you provide a proper governor system or the engine will run at right kind of speed for which system was designed, implying that the efficiency with which it works will be higher.