Dynamics of Machines
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Module-1 Lecture-2
Rigid body motion: Dynamic Force Analysis of Mechanisms
(Graphical Method)

In the last lecture I mentioned that it is possible to solve a problem of dynamics by converting it into a problem of static equilibrium. Today we will first explain the basic principle behind this technique and later I will illustrate the technique with the help of an example. Here you have stated at the beginning that all the external forces and movements are keeping the system in static equilibrium. But when you consider the motion of a mechanism it is not in static equilibrium. How can we use this wonderful principle that each and every link is in static equilibrium and you can analyze each and every link separately for equilibrium, when the system is in equilibrium? But if the system is in motion how to equalize this concept of this principle, let us consider that next.

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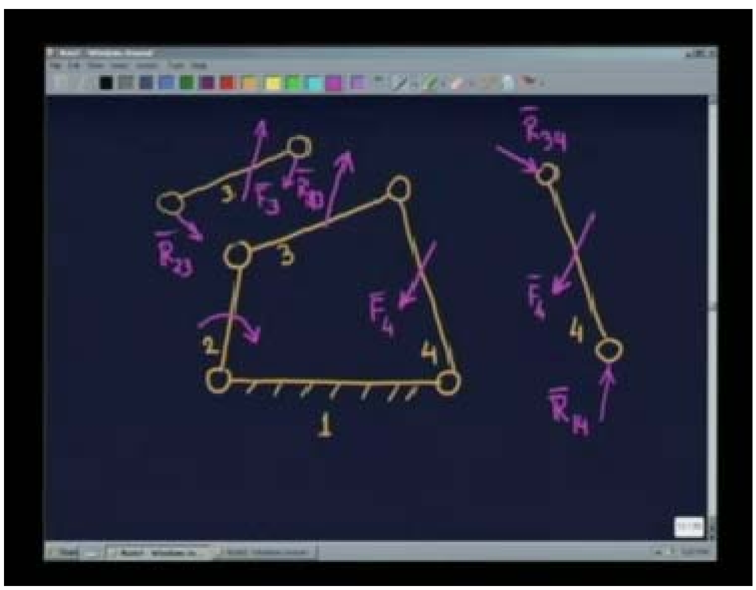

Let us come back to our problem that how we make use of this principle that a system in static equilibrium means that each and every member is in static equilibrium. How you solve a dynamics problem using that? We will go back to our basic dynamics course. I will remind you of the very widely used principle first developed by D Alembert and which is famous as D Alembert's Principle.

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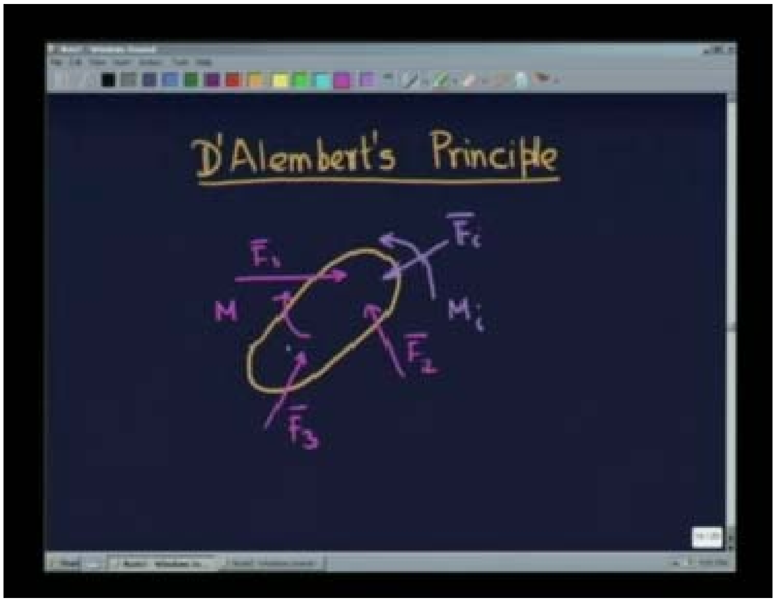

What is this principle? Just a small or brief recapitulation of that. If we have a rigid body or a body, D' Alembert's principle says that under the action of externally applied forces and movements, whatever may be there and also if you subject this to some hypothetical forces which I will explain very soon, which you call as inertia force and some inertia movement, then under the combined action of externally applied forces and externally applied couple, if we also add certain hypothetical forces which you call inertia force F_i inertia moment M_i then this body will remain in static equilibrium. Now to understand what these hypothetical forces, inertia force and inertia movement let us find out that.

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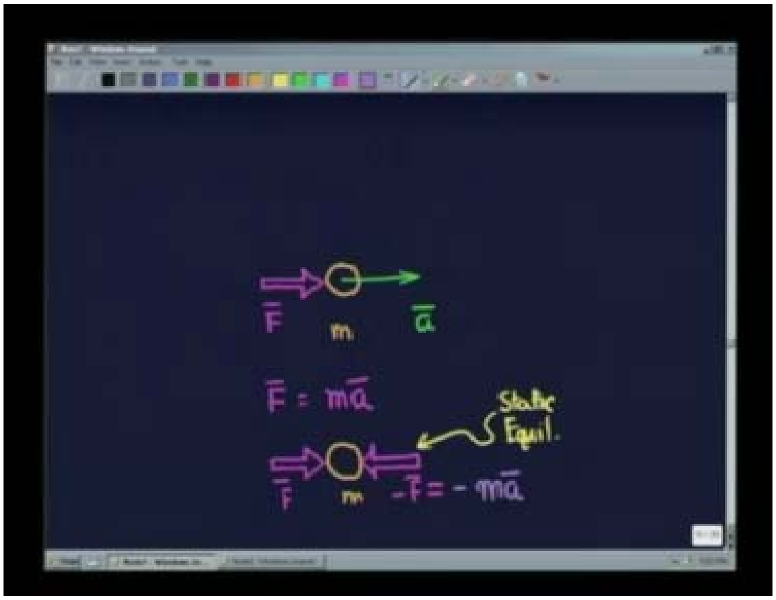

See if you take simple case of a particle. Now if this particle has an acceleration a in this direction so it implies that it must be acted upon by a force F which is given by mass into acceleration. Now it is almost trivial to you if I say that if this particle of mass M is acted upon by this force F, another force F – F, this system is in equilibrium. It is almost trivial statement. Therefore you can see that this minus F5 is nothing but what I was telling minus ma. If I tell that the particle is acted upon by a force F and another force minus ma, the system will be in static equilibrium or particle will be in static equilibrium.

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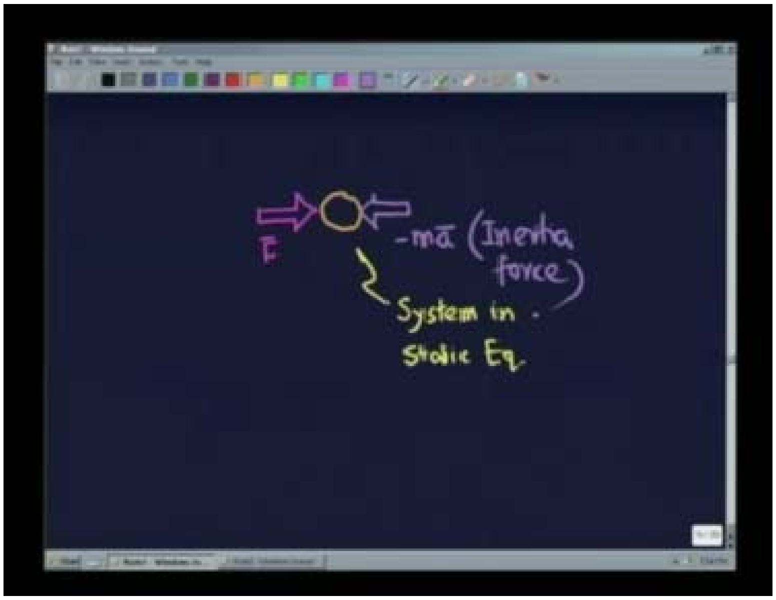

This is what is D Alembert's Principle. Though it is not just as trivial as its sounds but it says that to a body, if we subject the body to the actual external force along with a hypothetical force which is nothing but minus M into A which you call inertia force. The body will be in static equilibrium. Now this is for a particle. Let us just extend this to a rigid body or extend it to rigid body. So we draw our old rigid body who's center of mass is c, its total mass is M and its moment of inertia about an axis passing through the center of mass is I_c. Now if the resultant effect of all the externally applied forces or moments is represented by a force F. I have already shown that this will produce an acceleration of the center of mass which is A given by F by m and it will also produce an angular acceleration alpha which is given by F into E by I_c, where E is the perpendicular distance of the force or eccentricity of the force.

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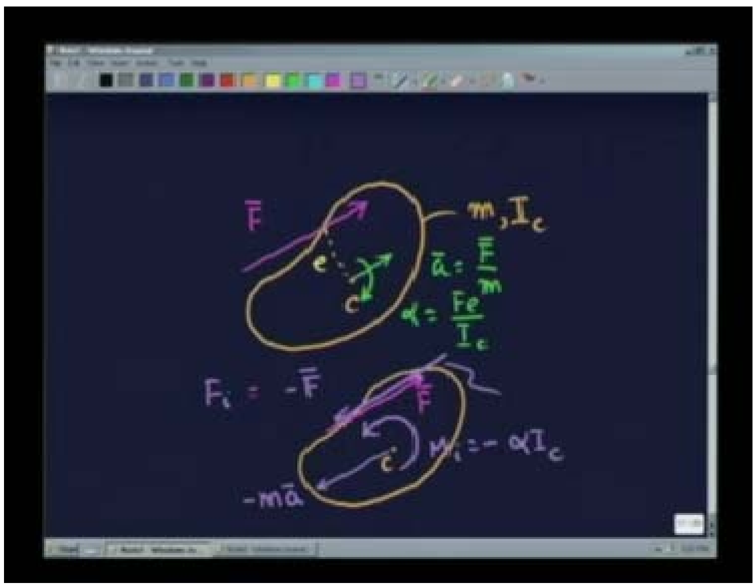

Now let us imagine what is that force which is equal to minus ma acting here and a moment minus F_E by I_c acting about an axis passing through the center mass. We know it is nothing but therefore the negative of the applied force which is nothing but the resultant of an inertia force minus ma and an inertia moment which is minus alpha I_c. Now when this hypothetical force is applied along with the externally applied forces, what is the resultant effect? They cancel each other or this F or which you can also call as F_I inertia force cancels the externally applied resultant force bringing the body static equilibrium.

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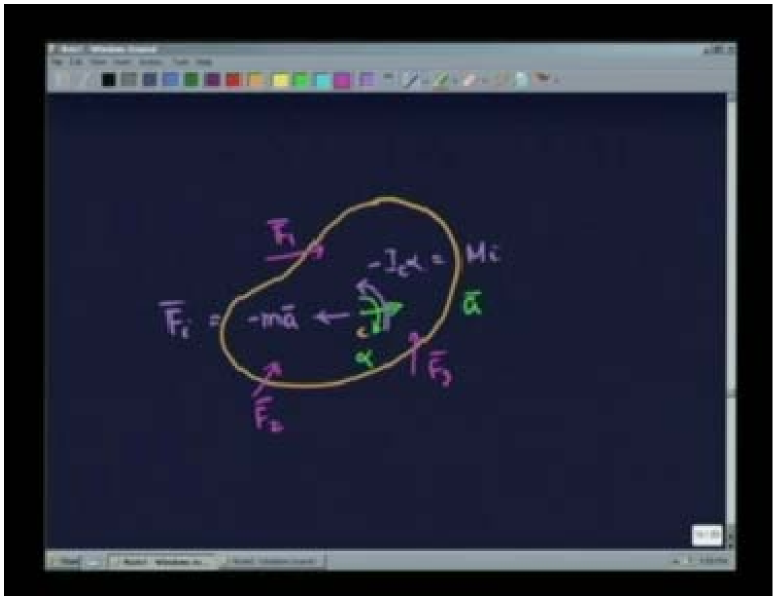

We can now say that if a body is under the action of a number of forces and the acceleration of the center of mass is A and angular acceleration is alpha. Then if I apply a force which is minus ma and a moment which is minus I_c alpha, then under the combined action of this inertia forces and inertia moment and the externally applied forces F_1, F_2 and F_3 which produce this motion, this system will be in static equilibrium. Because the resultant effect of these force and this moment is nothing but producing a force which is equal and opposite to the resultant force produced by these. That means they will nullify each other and the system will be in static equilibrium. So this is the technique which we will use this D Alembert's principle and we will add the inertia force of each and every member to it and then treat the whole system as a static equilibrium. Just like that suppose we go back to our original example which I am referring back again and again.

Let us consider a more realistic and more complicated problem or finding the loading on the connecting rod. How do you find out? Mass of this is given as M_2, mass of this is given as M_3, center of mass this is c_3, moment of inertia about an axis passing through the center of mass for the third body or coupler is I_c and of course I will not need that now. This can be some mass or some moment of inertia. The motion is given that means I know the velocity of this so this is having an acceleration A, now if this and this point is having an acceleration in this direction centrifugal acceleration, this has an angular velocity maybe in this direction and it may also have some angular acceleration in this direction alpha$_3$. And this center of mass has some acceleration like this so we know now the complete motion of the connecting rod. If you separate it out and now what will be the inertia force and moment on this we know that inertia force will act here which is nothing but mass of this link 3 into acceleration of the center of mass$_3$, this will be minus, moment will be I c_3 alpha $_3$ minus.

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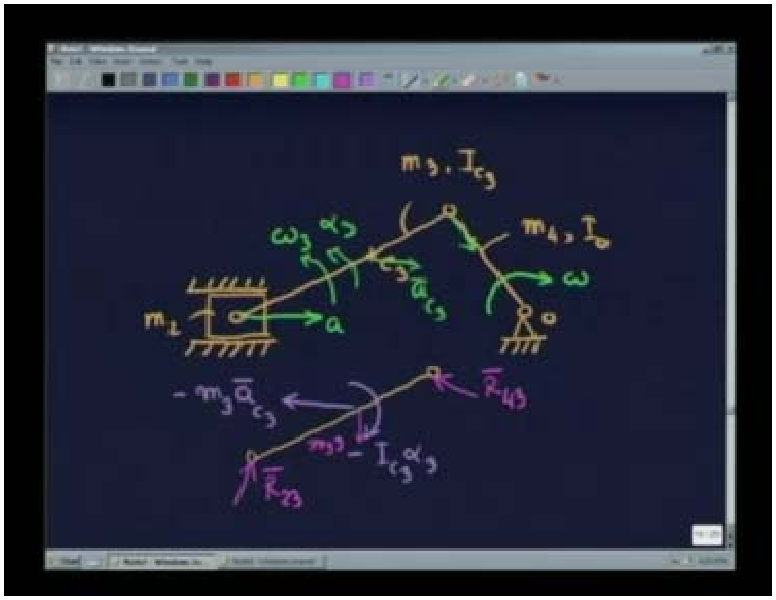

Now if we add these forces hypothetically, then under the action of these inertia forces and moment and the externally applied forces and may be weight, gravity. Under the action of these three externally applied forces to this member and applied inertia force and moment, the system will be in static equilibrium. That means the total force will be 0, total moment about any point would be also 0. So once we can come to this situation the whole problem gets converted into a problem of static equilibrium and there are various ways of solving this kind of a problem. We will illustrate that by solving one particular problem say maybe this kind of problem or a folding mechanism and so on.

We explain the whole procedure of finding the forces on various members and joints and contacts by definite example using the method explained so far. We will take the simple case of this slider time mechanism but the method is very general, you can apply it to any kind of mechanism with any number of links. First I will state the problem and completeness of stating the problem is very important in solving this problem at this analysis. The data which you have to give is all the dimensions that means the radius of this, length of this just say point is A, this point is B, this point is the fixed inch or the cranks of axis O, this angle at this instant we may consider to be theta and we mentioned that this is rotating with a constant angular velocity and a torque is acting on this say movement on link 3. Say we call this link as link 2, this as link 3 and this as link 4.

We should correct these two link. Our objective is to find out the total force on this connecting rod, why I am taking up this particular member, because you will notice that this member link 2 or the slider 2 executes a simple rectilinear translation system motion. This crank it executes a pure rotation with a center also a simple motion but this has both rotation and translation combined that is why I am explaining the procedure for finding out the loading on this connecting rod. The first step what you will have to do in solving this problem is determine the acceleration I mean to say the linear acceleration of the center of mass which you call a_{c3} and the angular acceleration of this link which you call $alpha_3$. Now that is a problem of kinematics which you have solved in number of problems and you can solve these quantities like this once the velocity of this is given. Next what we do? We consider only the link which we have to analyze and we calculate the inertia force which is F_i which is nothing but minus m_3 into ac_3. We also subject this to inertia which is nothing but minus Ic_3 alpha $_3$ which is the inertia for link 3.

Now we know that this and this ends will be subjected to link forces. You can also give some idea that, there is some external force acting on this maybe the weight of this. But I think to keep the problem less complicated we do not add any external force which could be easily done. But we know that there is some force acting here, which is the reaction force here from link 4 to link 3 and a reaction force from here which is from link 2 to link 3. We do not know these forces, but this reaction force, this inertia force moment that will keep the system in static equilibrium that means for this system total force will be 0 and total moment will be 0 that is the condition. Now since the magnitude and direction of this force both are unknown, the problem cannot be solved directly like this. We have to say for example if we take moment about this point A, then moment of this is known moment of this which has to exactly cancel the total effect is known but is direction and magnitude two quantities being unknown, it cannot be solved.

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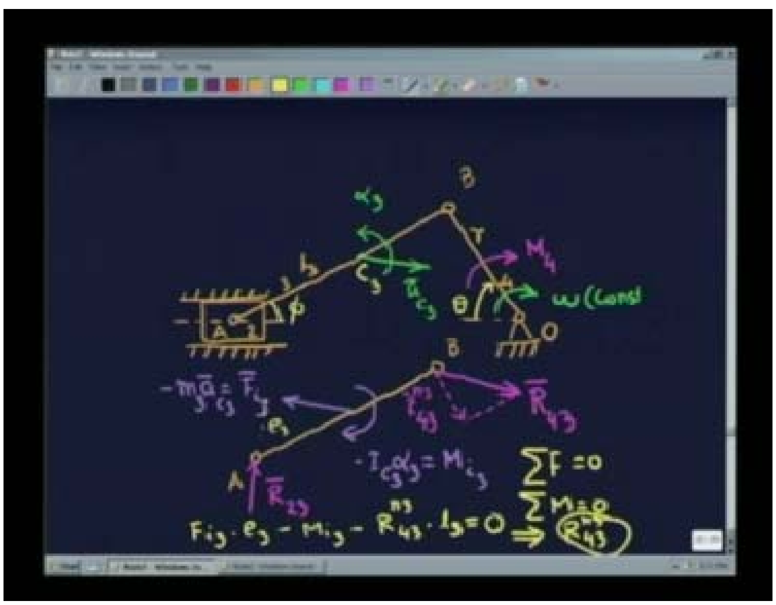

How you solve is this. Let us find out the component of R_{43} which is normal to 3 that we can find out. How? Taking moment about point A we can write that if this is the perpendicular distance of the inertia force that F_{i3} into E_3 minus M_{i3} minus R_{43} normal $_{23}$ into length of this member L. We could consider this as L_3 since we are giving, so we can consider this to be $length_3$.

The total moment about point A must be 0 because moment of R_{23} about this point is 0, so the moment of all other forces and moment ultimately must be 0. From this we can find out how much is R_{43} n3. Next let us consider link 4. Link 4 is also in static equilibrium under the influence of the external forces, torque and inertia forces. Now this is rotating at a constant angular velocity and if we do not put the gravitational forces then we know that the moment which was acting on this from outside was being acted upon this M_4 and some force here R_{14} and the force from this will be now exactly equal and opposite to R_{43} whose one component we have found out. Since there is no angular acceleration and the center of mass will have some inertia force which will be in this direction because it is pure centripetal acceleration, so inertia force on link 4 will be purely radial force. If you take the moment about point O the total moment must be 0. So we can say moment of this is 0. This produces a moment M_4 in the clockwise direction, this produces 0 moment and this produces how much moment? That we can find out by taking the normal component. This is R_{34} normal to 4 and this length dimension is L_4 which is nothing but r. So we get an equation which tells that m_4 minus R_{34} normal to 4 into L_4 equal to 0 or we find out R_{34} normal to 4 is known.

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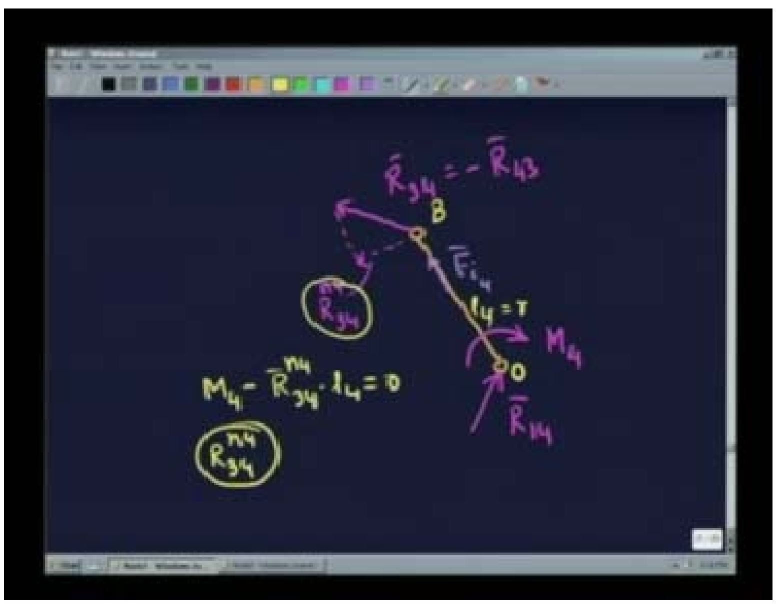

We have found out the normal to fourth link a component of this is being forced. Now let us consider this point B. We have seen that this is R_{34} and this is obviously equal and opposite which is R_{43}. Now we found out a component of this which you call R_{43} normal to 3. If you remember the link 3 direction was like this. We have also found out another component of this R_{34} which is R_{34} normal to 4 because link 4 of the crank was in this direction, so this is 90 degrees. Now you all know that R_{34} normal to 4 is nothing but minus R_{43} normal to 4 because R_{34} is just equal and opposite to R_{43} according to Newton's third law. We can say that R_{43} normal to 4 will be just equal and opposite to this. Now we find the situation that this is the component of 34 or 43 in a direction which is perpendicular to link 3.

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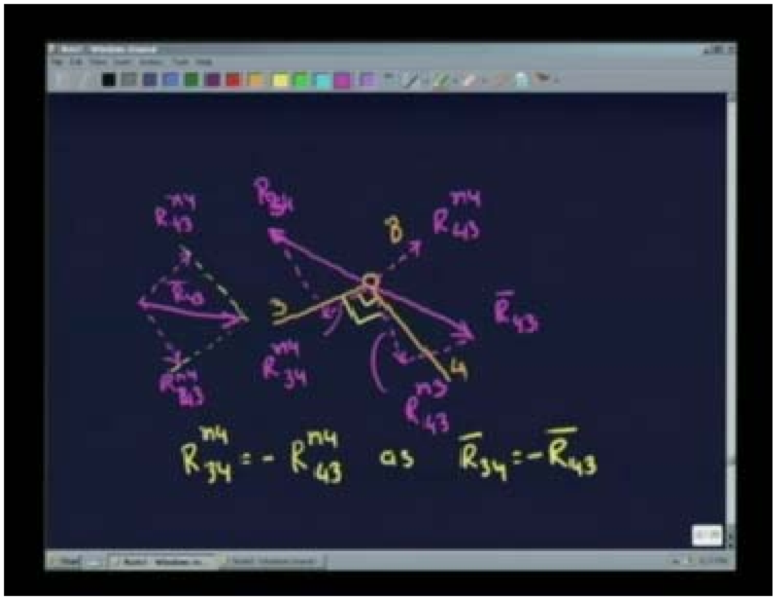

This is the component of the same force R_{43} in another direction which is perpendicular to link 4. If we know the components of a vector in two different direction this is R_{43} normal to 4 and this is R_{43} normal to 4. The vector will be nothing but, if you draw perpendicular from the tips where they intersect this must be the vector R_{43}. By considering the equilibrium of link 3 and equilibrium of link 4 and since they are interconnected at this location B we have been able to find out the magnitude and direction of the pin force or the joint force at B. Once that is done then when you go back to our original problem of finding out the resultant force on this, we have been now able to find out the resultant link force. We also know there is no other external force or anything but it is under the action of the inertia force F_{i3} and inertia moment M_{i3} and there is a force here we have to find out this point was B, this point is A. Now our job is to find out the last force which is still unknown on this. Now this can be easily found out because you know that the system must be completely force free. Therefore under the action of this force its total force should be 0 or we can say that if we add the three forces they will form a closed triangle this you know for vector mechanics for your benefit I slightly change the direction not to make them aligned.

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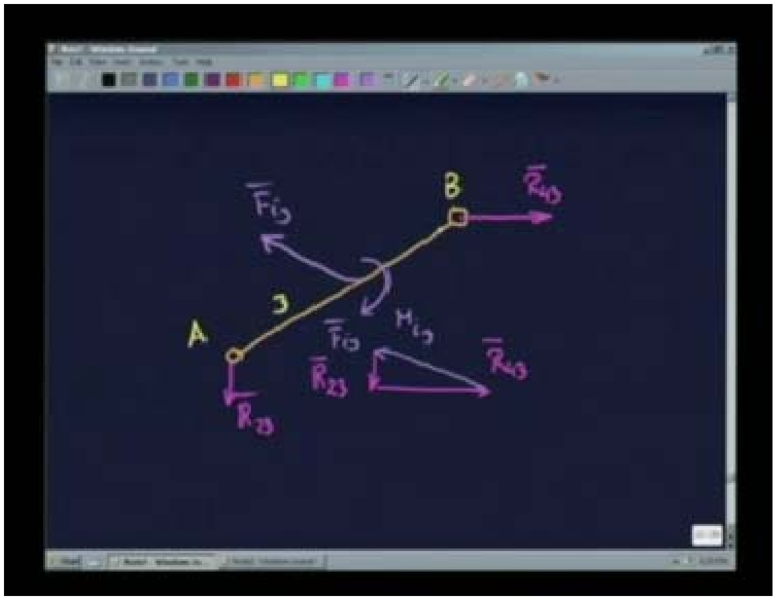

You will find this is the inertia force and then the force here has to be saw that it closes the triangle that will be R_{23} the force acting on this link from link 2. Drawing the force diagram which has three forces here the triangle can be completed and the last unknown force on link 3 at the joint A can be found out by this. Therefore now the whole link is under static equilibrium due to the influence of the externally applied forces at the two joints and the inertia loading. Of course the problem is very general, if there had been external forces and moments acting on the link they could be also taken care without any difficulty only if you had a slight more complexity of the problem.

This is the procedure that how graphically we can solve the dynamic force analysis of the force mechanism. Once that is known we can go back to link 4 we can go to link 2 and find out the total force on those 2 links. It is quite possible and once these force or loading is known, it will be possible for us to find out the bending moment etcetera force or force on any section of any member and if the material is known, the allowed stresses are given it will be possible for us to determine the cross sections not to allow because of this loading. This is just the beginning of the force analysis when a system is under motion due to some external forces and moment. In the next lecture we will start from this point with some more discussion on this and how to proceed further in the direction of design and maybe one or two more examples to illustrate the procedure much better.