Dynamics of Machines

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Module -7 Lecture – 2

We have already seen that, it is essential to have some control over input to an engine, to ensure that, it is run at the desirer operation speed when the maximum efficiency is achieved. We have also gone through quick classification of the different types of governor mechanism which used to be in practice which are also used some times. Today, what we pick up is the commonly used governor mechanism based on speed, centrifugal force and controlled by gravity. So, I have also mentioned that the first ever governor mechanism was designed by James Watt for steam engine in the 17th century. It was also one of governor that belongs in the class, centrifugal governor but controlled by gravity.

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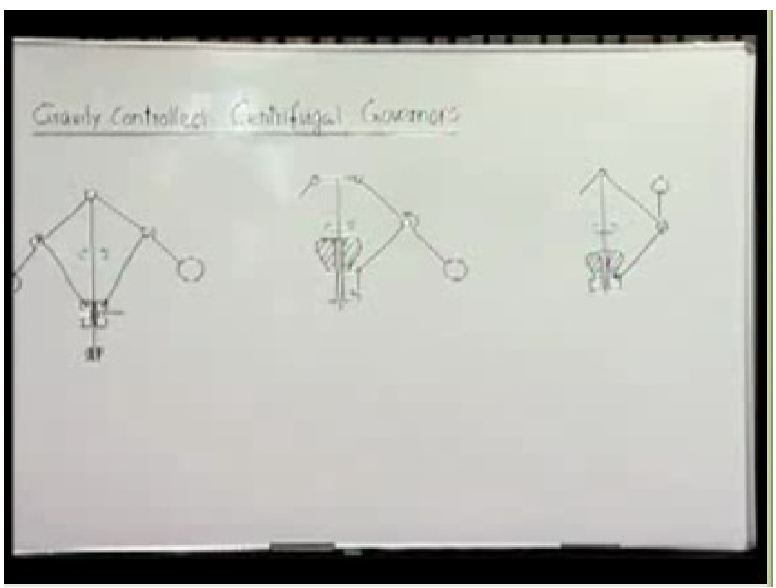

If you commonly use gravity control centrifugal governors, I will just indicate here. These are the basic scheme of three different types of gravity control and centrifugal governor. This is family of watt governor (Refer Slide Time: 05:09) where these are the spherical balls which try to fly off when it rotates and that tends to move slide up and down causing the lever which operates all mechanism to control the input energy or input power for engine. A little variation of this it has sudden difficult is particularly at higher speed level what governor tends to become insensitive.

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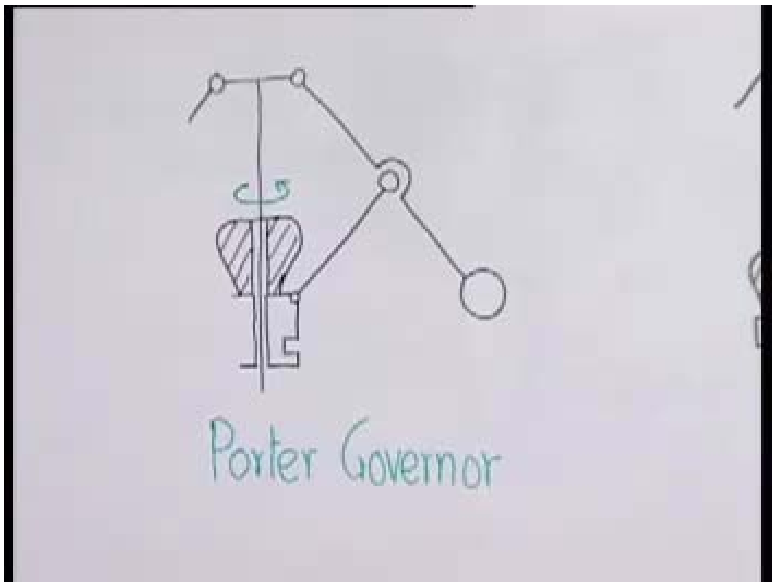

So, to improve its sensitivity and performance the next improvement was by Porter and we call it Porter governor. Here, the only difference we find, the real main difference is that there is a heavy weight not just sleeve and this weight can act as the controlling agent and you can see later that, it can improve efficiency and improve sensitivity at a speed level. Basically, it is therefore a Watt governor with a very slight difference. One thing we should also mention whether it is on the axis or away from the axis, all these things are matters of details vary from one another. To prove the sensitivity, other change was made to have Proell governor. Here, the spherical balls are not placed on this pendulum like member but, instead it is on the lower link which is extended beyond the hinge point here and this pair is mounted on this part of this extension. This improves the efficiency further and of course you also have a central weight to control (Not Audible: 07:23). This provides much higher sensitivity. These are the developments through ages is the lower efficiency. So, what will we do is now we will take up an analysis of Porter governor and Proell governor. Watt governor we will not do because there is not much different with this only whenever we can make this weight of the central mass kept here as 0 it becomes same as a watt governor.

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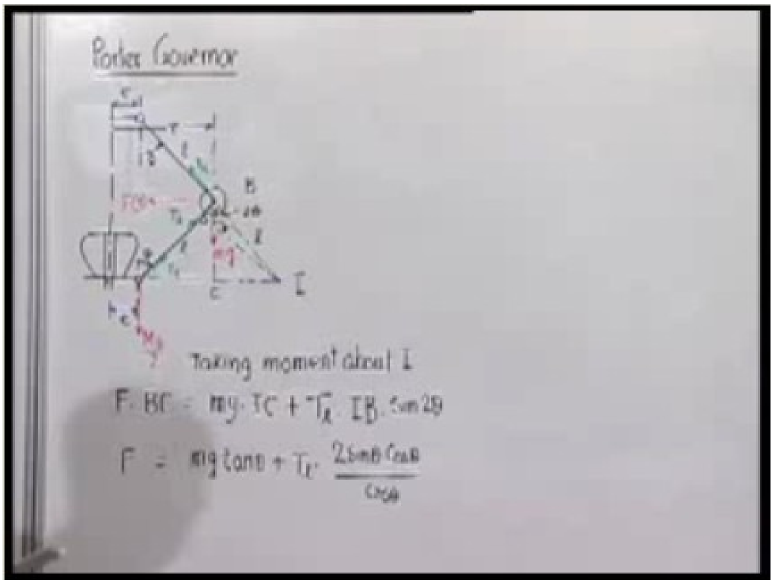

We will draw only one side because the whole thing is symmetric and will therefore draw also on one side. We will take up a particular case where this length is 0 clerical mass is kept here. They are both hinged here and sleeve is here and this space is symmetric in the sense that here this distance is e, here also e. The lengths of both the links are same, l. We call this end as B and at this particular situation radius at which the balls are revolving around the central axis is r, makes an angle theta obviously this angle will also be theta. We can also see kinematics we can find out at this point moves in a direction which is perpendicular to this (Refer Slide Time: 10:36) and this point of this link moves in this direction. So, velocity of this point has to along this velocity of this point will have to be along this. So, the instantaneous center at this point of this link is at I. The forces which are acting: weight of the ball and of course, here, if the mass of this is Mg but there are two sides, only half will come to this side. These are the external forces. Internal forces you can regenerate that means tension in this you can say T_u the upper link and tension here in this you call tension in the lower link T_l and obviously tension in the lower link here and here will be equal and opposite. Now, what we do is let us take the moment of the forces acting at B.

Now, what we can do is that, effectively control force you know always which we have assumed to be a force representing all the restoration mechanism like gravity here reacting like a force in the direction. So, if that, be the case then, then F, if taking moment about point I, F into BC perpendicular distance will be equal to mg which is the force here and moment about this will be IC plus T_l into IB this is IB (Refer Slide Time: 13:48) and this is theta this also is theta. That means, this angle is 2 theta. It will be… explaining it further, this point (Refer Slide Time: 14:06) moment of this force T_l will be T_l into the perpendicular distance from I which is this. If this angle is 2theta then, this is nothing but IB into sin 2theta. The control force that means, this force what we are doing is try to find out the expression of the control force. Control force means here, the effect of control will be representing the effect of other forces what other forces are acting on this: one is gravity, one is T_l and other is T_u. Therefore, if we take the moment about this (Refer Slide Time: 14:51) instantaneous center the moment of actual force acting on this here, must be equal to the force or moment what would have been produced if all this force is represented by horizontal force.

Therefore, this into BC will be same as the moment of other actually acting forces about this point. The moment of T_u about this point is 0 because it is passing through point. Moment of this (Refer Slide Time: 15:20) and moment of this weight is this. So, the control force F can be expressed as (Refer Slide Time: 15:28) IC by BC this is IC this is BC so IC by BC is nothing but tan theta this theta, because this length is l, this length is l, this length also becomes l because, this angle is symmetrical and this angle they are same. Therefore, IC by BC is nothing but tan theta this is this. BC by IB is nothing but cosine theta. We also know…Therefore the question is how much is T_l that is still not known, but, if we consider the equilibrium of this T_1 and another T_1 on this side, they are supporting this total weight mg we know $2T_1$ and what is the vertical component cosine theta is equal to mg. Using this, finally we get the control force which is the function of r equal to (Refer Slide Time: 17:15). Now, of course, you have to write in terms of r.

So you can write this as: g and tan theta you can write (Refer Slide Time: 17:43) this length how much it is r minus e, this much, how much is this l square minus this is r minus c square is this. This by this, nothing but tan theta that is what we have written and therefore now F expressed purely as a function of r, there is no other variable there. For any other speed the equilibrium is established by a condition which is this control force must be equal to the tendency to fly off, which you call centrifugal force we must match. Therefore, we get relationship between the speed and (Refer Slide Time: 18:57) this is just to equate this with this and we find out omega square so you will get it because here hardly a calculation is involved. Now we get that for a given speed omega when it runs, the equilibrium value of radius at which the square must be and correspondingly the location of the sleeve will be.

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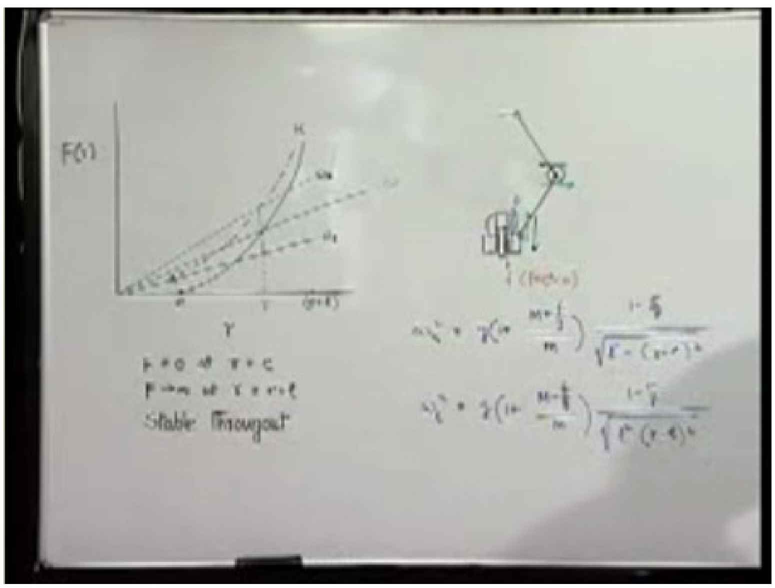

Now, let us plot to see how the control force looks like for this particular type of governor a Portal governor and with this kind of configuration. So, we have an expression for the control force as a function of r. One thing is clear that when r is equal to e, Fr is equal to 0. It starts at location which you called r is equal to e and then its goes. There it becomes infinite when the denominator is 0 that is, r minus e is equal to l or r is equal to e plus l. It tends to becomes infinite and this is the control force for Portal governor with this kind of configuration. Therefore, we find if we now plot the centrifugal force, which will be just a line for a particular omega and slope of this line will depend on omega because slope is nothing but m omega square. This will be equilibrium position. If the governor rotates at this speed, then its radius at which the ball will be this intersection. You also find the other two things: one is F is equal to 0 when r is equal to e and F tends to infinity when r is e plus l. This diagram makes (Refer Slide Time: 22:16) it very clear that whatever may be this speed of rotation of governor. The intersection point at the equilibrium position, the slope of the control force card always more than the slope of the centrifugal or the reflecting force and that is the condition for stability. It is stable throughout, the governor will be stable all along we do not have to worry. Only one thing, we can discuss here is this derivation is fine. Only one thing we have not been able to take up and let us take up that. You have not considered when sleeve moves we have assumed it perfect no friction perfectly smooth situation. But in reality it is not the case, there will always certain amount of sliding friction involved because it is a sliding motion of the sleeve on the central weight along the central rod which is the central axis of the governor.

So, when its speed is increasing, it will try to move up and so whatever friction there may be, so when it is speeding up that means, speed is increasing than is increasing tendency move up and so obviously sliding friction on the sleeve, will be acting in the direction so this is a friction. So, what is the effective of friction on the whole thing, we will find effect of friction is nothing but to add to the mass m here. So, therefore we will get that if we add, that means if we show that the total force in this direction is the mg plus F. Each one will be mg plus F by 2. As a result, the control force curve will appear to… I am exaggerating upwards (Not clearly audible: 48:50). Suppose at that instant it is speeding up so the control force curve the force will be here which is, total force F and you have to add certain amount with this here. What you will find there that, omega u square, that means when it is going up at the upper limit. At this particular rotation therefore, radius that means the governor will be at one location though this speed may vary because, friction will take care of that variation. So, at this radius (Refer Slide Time: 25:53) when the governor is rotating what is the maximum speed at which the governor can still be at this location is this point this will be upper limit of the speed at which governor can remain at this radius r. Similarly, what is the lowest possible speed when the governor… so therefore there is always certain amount of insensitivity, that means the speed of the governor can change from omega l to omega u but still governor will remain at the same location that means, the governor will become slightly insensitive.

We can find it out, this will be as I mentioned, g into 1 plus… effect of the friction force is nothing but to add the force in this direction. Total force is mg, which now is mg plus F. Therefore what we do we add m plus f by g because g has been taken out. So, the expression for the upper speed limit will be this (Refer Slide Time: 27:20). When we try to find out lower speed that means the speed is decreasing in this direction, tends to move in this direction and obviously then, this wants to go in this direction. Obviously, friction force acts in this direction that means, now it is opposes the weight it will be mg minus A. This upper limit can be found out from the same expression only when mg is replaced by mg plus F, then what it is going to be g into m plus f by g, m will replaced by m plus f by g.

Similarly, when mg is replaced by mg minus f when it is decreasing and sleeve is going in the downward direction then m will be replaced by m minus f by g. We get upper and lower speed limit at a particular operation radius and what we can do therefore, you can find out insensitivity omega u square minus omega l square by omega square is equal to…. subtract that and then divide by omega which is the mean speed where you will get this (Refer Slide Time: 29:20) will be simply 2f. Omega u minus omega l by omega, if you use this, will be approximately equal to f by… and this is called the coefficient of insensitivity or detention by friction or both. Effectively frictions are making system little bit sluggish in the sense the governor configuration does not change just by any amount of change in speed. It remains static in the particular range of speed omega u omega l.

Obviously, you can see that how much there is the insensitivity depends on particular ratio that means, difference between these speeds that is operating speed or average speed which is this. When f is 0 as you can see it is directly propositional to the friction force.

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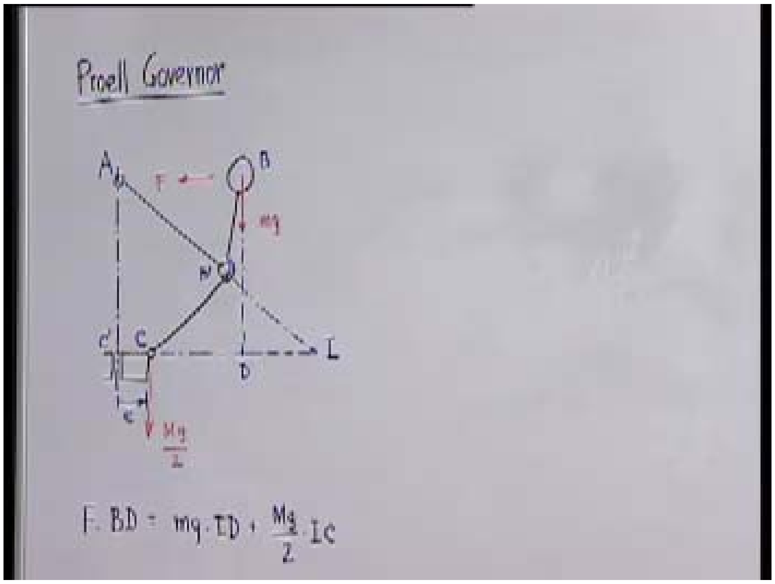

Let us take up the other governor that is the Proell governor that is animprovement on Porter governor. Let us take the particular situation and our representative control force which is hypothetical force as if acting in the radial direction like this. Let us give name to various points. Let this point be where is particular case kept is about and the axis particular situation it may be even IF from the axis that each and every cases has to be solved separately. This point, also c and this point, this is prime and this is e, this point is B prime and this point is B. So, the instantaneous center will find out as before I. This is a particular configuration at situation, at particular speed. Here of course, the force acting on this and that is half the total weight. The other half will be other side. Here, what we have in this particular case again is, If we take the moment of the representing force F about I, this is F into BD is the distance perpendicular distance form point I. It must be equal to…, but actual force acting is mg or if you consider the whole system here, any force acting here will pass through this its moment will be zero, for these two forces moment you can take, it will be mg into ID plus mg by 2 into IC (Refer Slide Time: 34:27) and therefore the control force which is a function of r will be actually mg ID by BD.

In general, we may not be able to write nice closed form expression for these ratios, when the symmetric etc., somewhat lost. You could do it in the case there because both the points were located there, the lengths were equal, and all kinds of symmetries conditions were satisfied. But in generally, it may not be… so best thing is to draw and then measure these dimensions and find out the value of F at that particular configuration when it is at a distance r. This is actually instantaneous radius of rotation. At equilibrium, we know the condition is if F equal to (Refer Slide Time: 35:57) if we at a particular r if we can find F you can find the speed at which the system will be stable, at that particular radius. Best will be to take up particular example.

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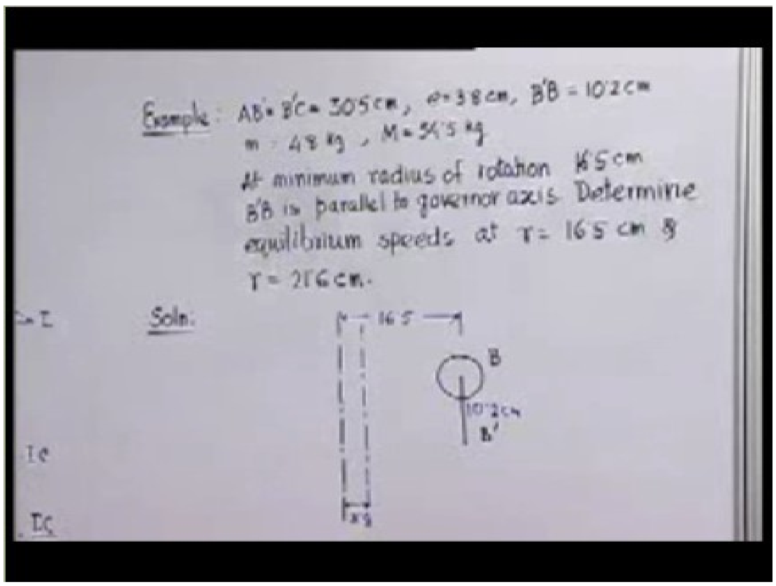

Let us take an example, we take a particular case where AB… two link lengths are same value of e is 3.8 centimeter. The extension B prime B on which the balls are clasped, mass of the rotating balls is equal to 4.8 kg, and this sleeve weight gives some idea about the order of magnitude of the values involved all how heavy. These are huge systems because very large engine has to control that is the, whole river mechanism has to be operated. So, we need very substantial magnitude. It is also mentioned that what we have do to is determine equilibrium speed at r is equal to 16.5 centimeter and r equal to 21.6 cm.

This example tells how to handle such problem. Let us first find out the equilibrium speed when the radius of rotation is equal to 16.5, which is the minimum position. Also, when this extension of this lower link the B rime B is parallel to AC prime that means the governor axis, how to draw it, it is best to solve graphically. To solve it first, we select any point at the center of the ball. Since we know that BB prime will be vertical that means parallel governor axis in this lower position. What we do is we draw a vertical line, whose length… point B is given the prime so point is 10.2 centimeter? You can find out this point. Now, we know the distance of this wall from the governor axis. This length is 10.2 and this line we draw which is at 16.5. That is the value of r. Now draw another line on which point c will be located and we know that it will be at a distance 3.8 centimeters from the governor axis. Next, you know that point c will be at a distance at 30.5 centimeter from B prime. You can easily cut this length, to make this point C and obviously, cut this point to get a, both cut the point at (Refer Slide Time: 41:53). Once it is done then, extend this over perpendicular I. You can measure, if you can do it, we will find that ID comes out as: 17.2 centimeter. IC is equal to 29.8 centimeter. BD is equal to 37.6 centimeter. With these values you can substitute here we know the m we know M and you know IC ID in BD. Therefore F can be found out easily and that obviously we can get this equation.

So what we will get, omega square is equal to 16.9 square radian per second, which will ultimately give us 161.5 rpm at r is equal to 16.5. If we go to the other value r is equal to 21.6. To solve it we have to draw this again and do that. What we do is (Refer Slide Time: 42:58)…. This is the governor axis and at distance of 21.6 somewhere it could be point B. We also know that point C will be located on a line which is 3.8 centimeter. Now one thing we should notice that, in Proell governor, this is one rigid link. Therefore, this distance BC also remains constant. This angle cannot change one rigid link. We have already found out distance BC on the first drawing, which is going to be same for each case. We can now cut this point c measuring this BC, as this one. Once this is known obviously, we find B prime easily by drawing the right triangle. This will be 30.5 centimeter and this will be 10.2. These two points I can easily find out equation.

Now, we can find out this location A and then the other dimension can be found out. This will be I point D, you will find that ID is equal to 20.2; IC equal to 38.8 and BD will be equal to 35.4 centimeter. While these things are substituted in this control force expression and equated to the centrifugal omega square r, we find omega come out as, 18 radian per second which is equivalent to N is equal to 171.5. Therefore, we can find out speed at various radii at which it can rotate in stable manner. I think this will be enough for our discussion of gravity control and centrifugal governor. I think next will go for further improvement which have come, see here you can see that we have certain disadvantage if we depend on the gravity for all our controlling operations. We have large force; we have to have a large mass which will make the system heavy and so on. On the other hand I already mentioned in the previous lecture that much better control system can be much more compact and lighter etc, when we use springs for controlling the governor mechanism. Next we will take up the spring control centrifugal governors.