Dynamics of Machines

Prof. Amitabha Ghosh

Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Module No. # 06

Lecture No. #02

Flywheel Analysis

In our previous lecture, we have seen that for a single cylinder engine the turning moment on the crankshaft where it is quite substantial. That is, if we plot turning moment for a whole cycle taking a 4 stroke cycle engine, we have seen that it will be something like this (Refer Slide Time: 01:23). Its average value will be as shown in slide. So, this is turning moment M and M_{av} is the average value of this. How to find out? First, let us find out this is one cycle and in this particular case, the crank rotation complete one cycle will be two rotations, it is two; that is angle theta will go to an angle 4pi.

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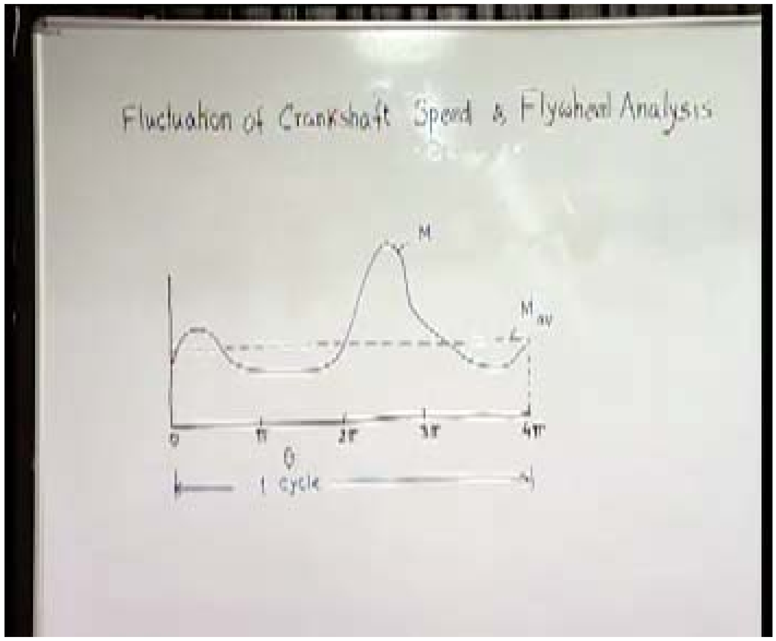

Now, what is total work done by the engine in one cycle? Because if it goes, crankshaft is rotated by an infinitesimal angle d theta; when turning moment on it is M, then the work done by the engine is M d theta. Therefore, in whole cycle the total amount of energy has been generated can be written as shown (Refer Slide Time: 02:55). In this particular case, that means for 2 stroke cycles E will be equal to 0 to 2pi M d theta and for 4 stroke cycles engine energy will be as shown (Refer Slide Time: 03:35). So, this is the amount of energy generated in every cycle.

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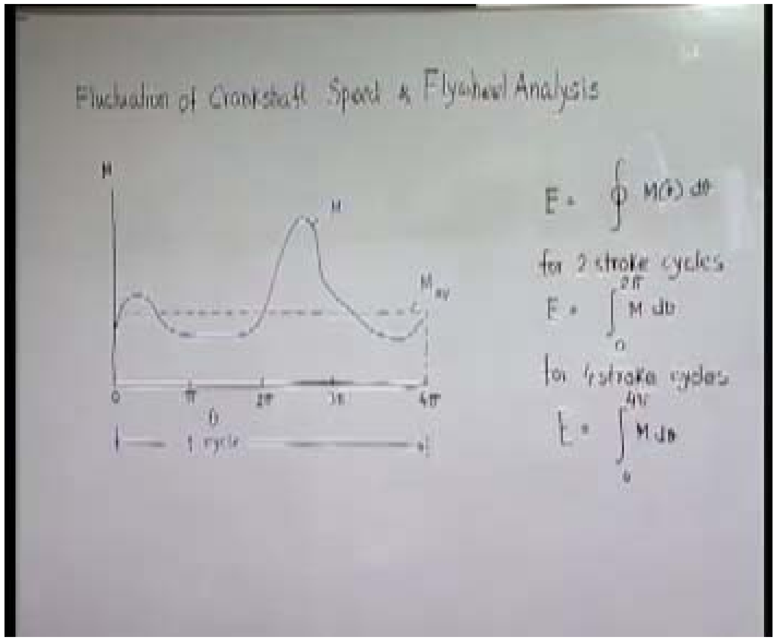

Obviously in steady state, we assume the same amount of energy is utilized. So, there is no accumulation of energy in the engine, continuously changing its speed; in case of starting, either it increases or when it is less than the power utilized at output end, then it will gradually go down; so in steady state operation what we have? The resisting torque which is constant you have assumed in this particular situation, it must be equal to the value in such a way that total watt consumed by the load is same as the total energy produced in every cycle.

So, the load M is same as M_{av}, which is nothing but energy produced per cycle divided by 4pi in case of 4 stroke cycle as shown here. That means, we can write it like this (Refer Slide Time: 04:44). Once, we know the turning moment diagram, it is possible for us to find out the average moment of which happens to be load of resisting moment. Now, we find that if we draw the resistance or resisting moment, then I think we find that at some places, this is above the resistance and in some cases driving torque is less then resistance.

Here what will happen? Energy will be added to the system so, the kinetic energy will go up till point B, after which driving moment becomes less than the resisting moment. So, energy is depleted and kinetic energy goes down up to a point C, where again your driving moment becomes more; that means, it starts accelerating system by adding energy till point d, where again the driving moment becomes less than the resistance and so energy is depleted reducing this speed again it goes back to point A.

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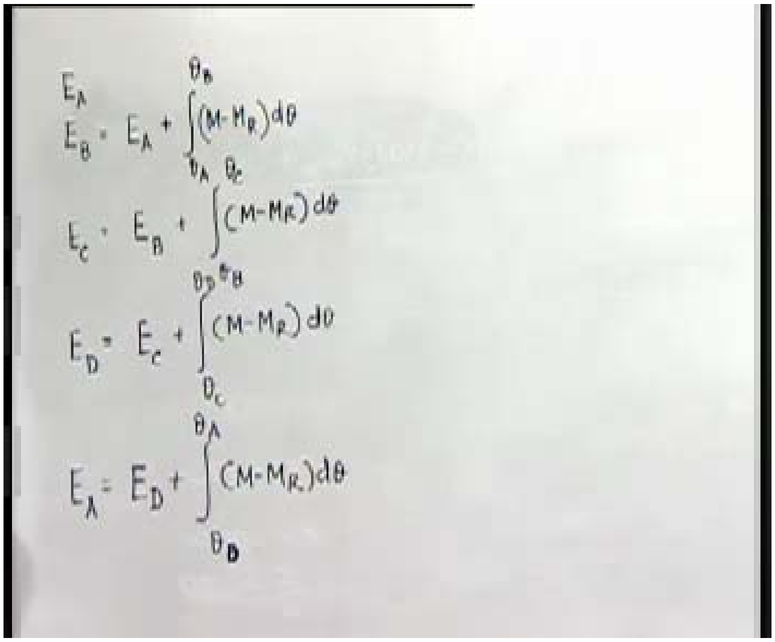

There we find that energy at A say E_A; energy means obviously mechanical energy of system that is the kinetic energy. So, at point B the energy will be whatever energy we have at point A plus the energy added. This is the amount of energy which is added in that first load which is indicated as plus. Similarly, energy at point C will be E of B plus theta B to theta C, M minus M_R d theta, but remember M minus M_R is the negative quantity and so actually it is negative. Similarly, at point E_D is the energy at point C plus M minus M_R d theta and then we finally come back to E_A which is again as shown in slide.

So, this is just one example; it can have many more slope and different, because remember, this is the case of testing the cylinder engine. If you have more than one cylinder then what will happen? The turning moment diagram of one cylinder and turning moment diagram of the other cylinder, they will superimpose with suitable axis. In case of 2 stroke 2 engines, obviously, they will be separated by 2pi angle and then the final turning moment will be a super imposition of that. If there is more number of cylinders then the turning moment of each and every cylinder will have to be added by suitably spacing them and the final turning moment diagram having many more loops than what we have here it is only one cylinder.

Now, let us say E_1 be the maximum energy among all this. Of course, you have to see that maximum energy can be at any of this point here or here and so on (Refer Slide Time: 09:36). Similarly, energy can be minimum also either here or here like that. Therefore, one of this must be highest and one of this will be the lowest, it cannot be anywhere in-between. Let E_2 be the minimum energy, let the corresponding angles be $theta_1$ and $theta_2$; so, what will be the difference between the maximum kinetic energy of the system and the minimum kinetic energy? What we call maximum kinetic energy fluctuation? That is the difference between the maximum and minimum value of the kinetic energy during the whole cycle will be nothing but as shown in slide (Refer Slide Time: 11:06).

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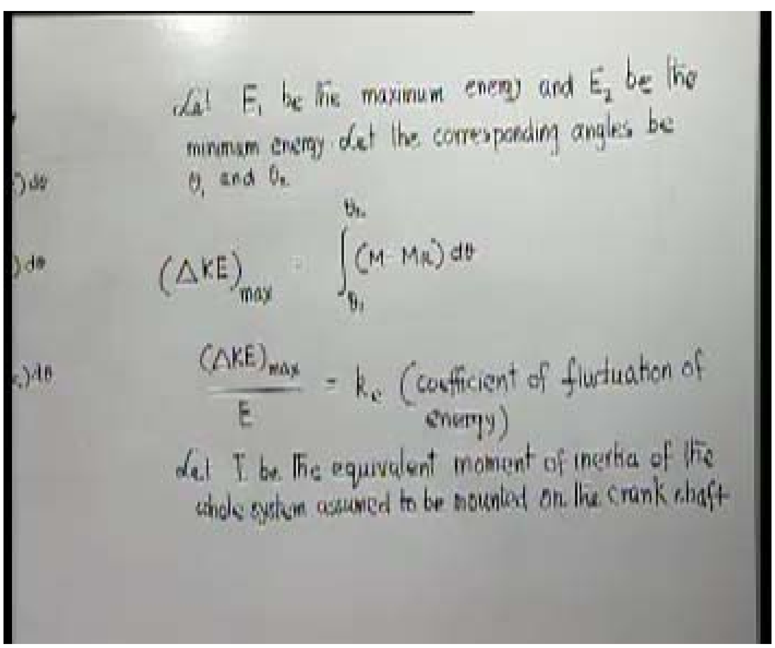

Now, the quantity with this maximum fluctuation; that means the difference between the maximum and minimum value of the kinetic energy what we call delta KE_{max}, if we divide this by the energy input R cycle as defined by this, it is called coefficient of energy fluctuation. What is the fluctuation of this maximum and minimum kinetic energy? Obviously, we can always write let I be the equivalent moment of inertia of the whole system assumed to be mounted on the crankshaft.

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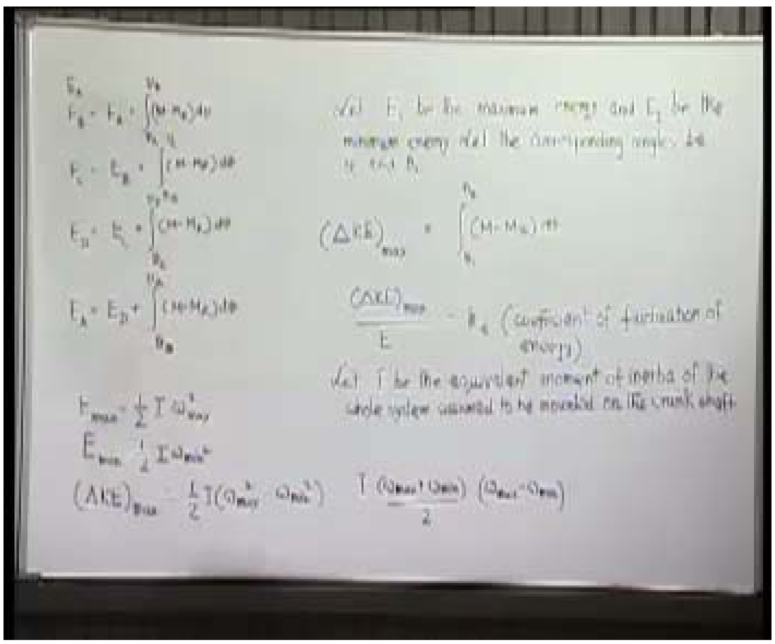

So, at any instant the kinetic energy of whole system can be now represented by this one; E_{max} is nothing but the maximum kinetic energy of whole system that is half I omega$_{max}$ square, omega$_{max}$ is the maximum speed of the crankshaft. Similarly, E_{min} will be half I omega$_{min}$ square. Therefore delta KE_{max} is nothing but half I omega$_{max}$ where it is nothing but the difference between the maximum and minimum kinetic energy. This can be written in this form; I into omega$_{max}$ plus omega$_{min}$ by 2 into omega$_{max}$ minus omega$_{min}$.

There is a fluctuation of speed obviously, what we can do this omega$_{max}$ plus omega$_{min}$, their average in most cases? Since, the difference is very small can be consider to be omega average. This can be approximately written as omega average or the average running speed of the engine. So, you can write this as I omega$_{av}$ square in to omega$_{max}$

minus $omega_{min}$ by $omega_{av}$ and this is as shown in slide (Refer Slide Time: 15:47), where the maximum variation in the speed divided by average speed is called coefficient of fluctuation of speed. This is an important parameter, which we prescribe, depends on the situation of kind on machine we driving with the engine.

Therefore, we find that if k_s prescribed, running speed is known which is the design speed; turning moment diagram is known, then in the steady state we can find out what must be the equivalent moment of inertia of the whole system. Generally, what is done? The system inertia is not enough and to provide or to make the system running smoothly, we increase the value of I substantially. How can it be done? The easiest way of doing it is that we attach a heavy wheel to be connected to the crankshaft. Normally that is called flywheel, because its function apparently is not seen very clearly, but what it does? It provides substantial amount of magnitude to the moment of inertia, attach to the crankshaft and for given fluctuation of energy, it produces much less fluctuation in the speed or in other words the engine run smoothly.

So most cases, you can say it is approximately equal to the flywheel moment of inertia because the engines connecting rod etcetera and the balancing masses they had, they have inertia, but the real contribution to I comes from the flywheel which is attached to the connecting rod. Therefore, you can now find out that, approximately we need a flywheel with whatever magnitude is found out from maximum fluctuation of energy divided by $omega_{av}$, where it is coefficient of k_s.

This is the way at E by which it can have very small fluctuation of speed by having heavy flywheel and therefore, in most engines we attached heavy flywheel to the connecting rod. As I mention just now, that the value of k_s depends on the kind of situation or the kind of function we are taking, I will just give you some idea about the magnitude of A in various situation.

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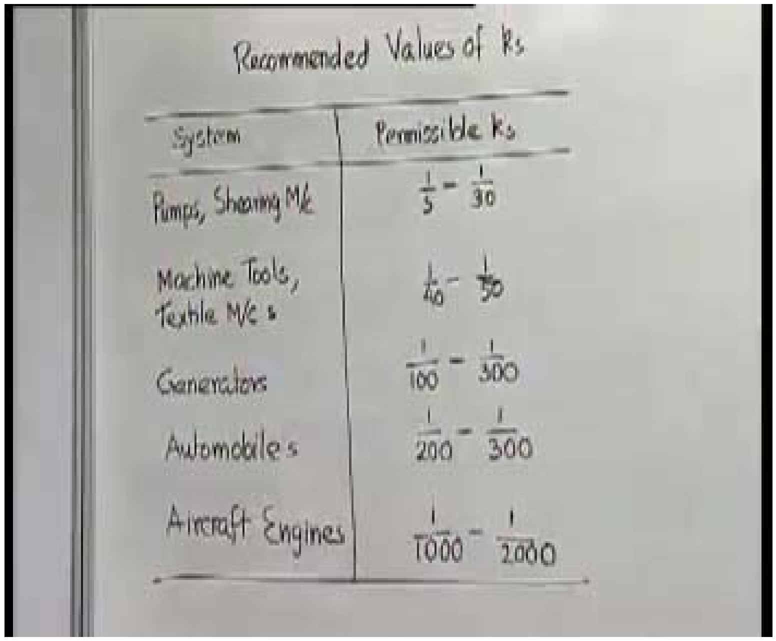

This table shows the permissible recommended values of coefficient of fluctuation of speed. In case of shearing machine, pumps you know the accuracy required much less and values of k_s in that range 1 by 5 to 1 by 30. Machine tools of different types, textile machine and such type of the situation, k_s can be permitted to be 1 by 40 to 1 by 50. In case of electric generator, generating set recommended variation of is speed as to be 0.01 to one third of that, an automobile and auto engine recommended value of k_s is this. Aircraft engine support will be far more accurate and there the speed fluctuation which is permitted is the minimum.

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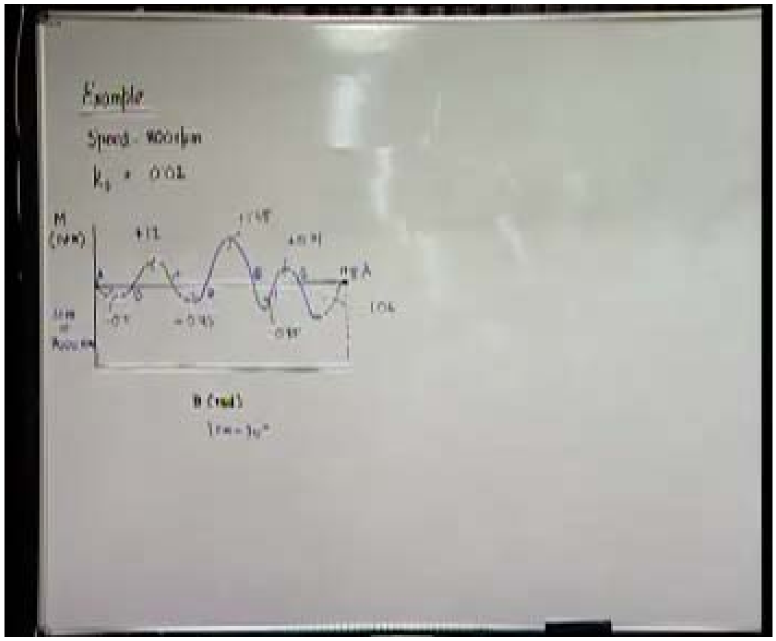

Let us consider how to use this by one example. Let us take example of an engine. So, the engine long set rpm is 800 rpm, permissible value of the fluctuation of speed is 2 percent and the turning moment diagram has been given. The turning moment in newton meters and theta in radians; this is one cycle (Refer Slide Time: 24:23). Here, what has been done? This point A, this point B, this point C, this point D, this point E, F, G, H it is same as A. Here of course, that it is again become negative that doesn’t matter.

The area when you plot it both in centimeter and centimeter in this direction this area found to be minus 0.5 centimeter squared, where this is plus 1.2 centimeter squared, this is minus 0.59 centimeter squared, this is plus 1.45 centimeter squared, this area is minus 0.85 centimeter squared, this is area is plus 0.71 centimeter squared and this is the minus 1.06 centimeter squared. So, if you plot this, that means here one, here actually scale will be 1 centimeter equal to 30 degrees and here 1 centimeter will be equivalent to 7000 newton meter; that is the scale we have used and using this scale, these are the areas measure by the planning.

Next, what we do? Let us find out what is the maximum kinetic energy and minimum kinetic energy has you have been doing. How do we do? Start with A - say energy at A point is E_A, energy at point B is E_A minus 0.5; we need not convert this centimeter square into energy now, we can do it at final because every where if we just keep it in terms of the area, we will able to identify the maximum and minimum and then we will convert to real energy unit.

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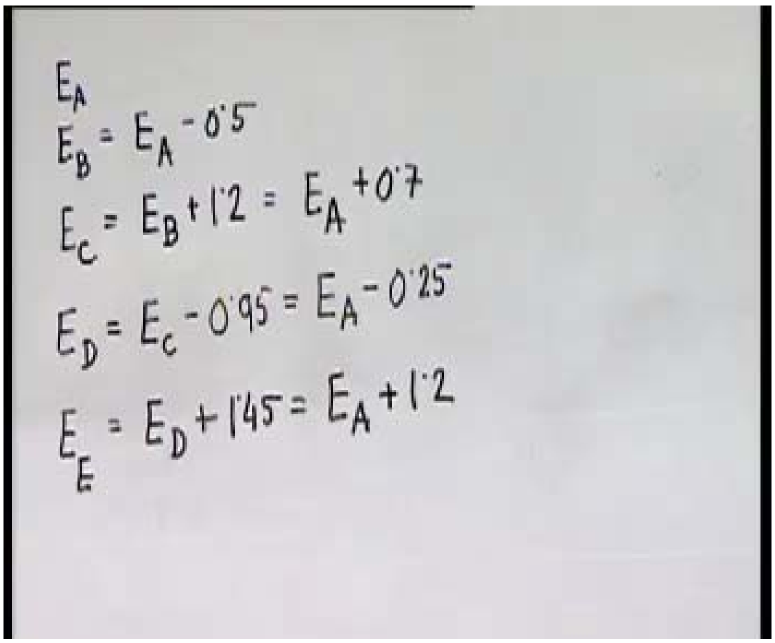

So, energy at this location B is E_A minus equivalent of energy which is this, E_C will be E_B plus 1.2 which is effectively E_A plus 0.7, E at D will be E at C minus 0.95 which means it will be E at A minus 0.25, E at E is E at D plus 1.45 which is same as E_A plus 1.2, E at point F equal to E at point E minus 0.85 which is same as E_A plus 0.35, E at G equal to E at F plus 0.71 which is equal to E at A plus 1.06 and E at H equal to E at G minus 1.06. You come back three point at 0.8 energy should be equal to A, let us see out of this which is the maximum and which is the minimum. So, it is not difficult to identify the maximum and minimum energy location; say you can compare all the value E_A with this and this; you find this as maximum and this as the minimum.

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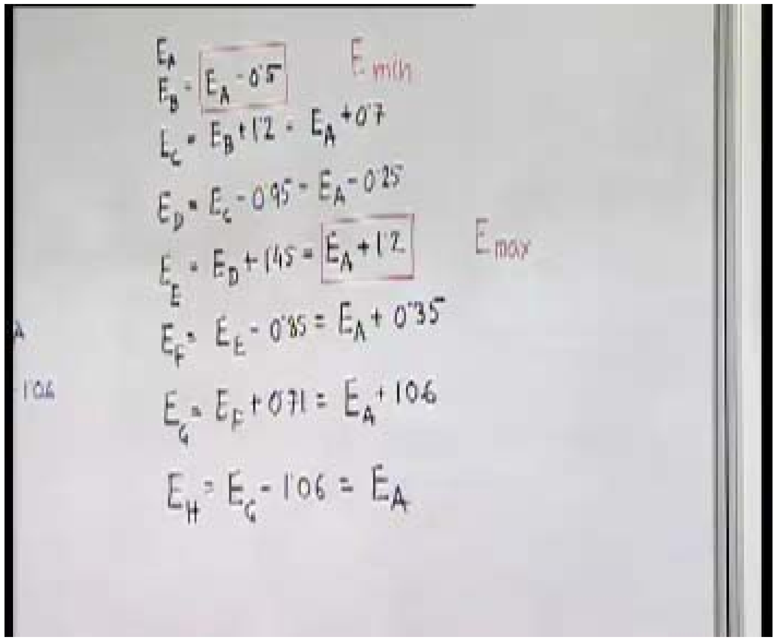

So, the maximum fluctuation of the energy which is nothing but E_{max} minus E_{min} which is equal to or equivalent to rather we can say 1.7 centimeter. How much one centimeter square represent? Now torque into theta, torque in newton meter into theta in radian is joule. So, one centimeter square is equivalent to 7000 and 1 centimeter is 30degrees in radians, it is pi by 6 joule, which is 3670 J in this scale. Here if you multiply by this, this is in joule which is equal to 6240 J.

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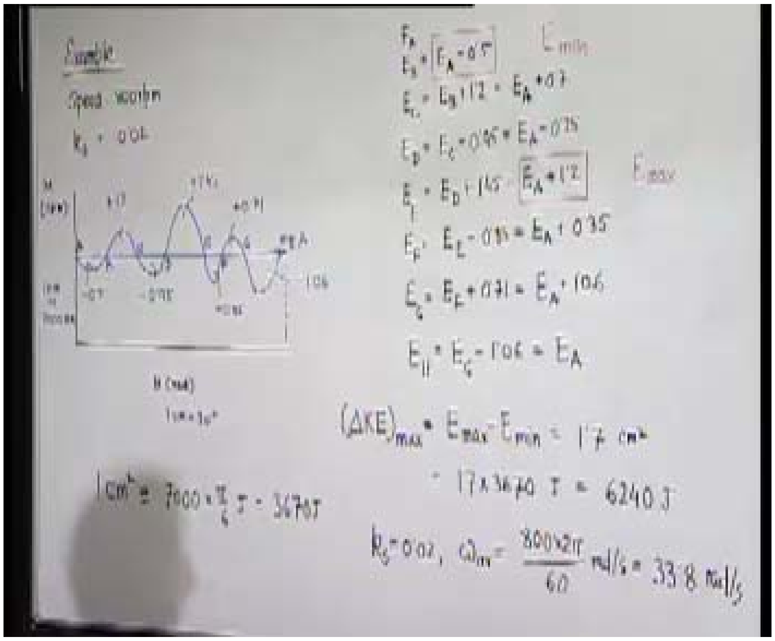

Now, our k_s have been specified as 0.02 and omega_{av} which should able to find out from this 800rpm; it is 800 into 2pi by 60radian per second which is 33.8radian. Therefore, utilize the formula right; that moment of inertia of flywheel will be approximately equal to as shown in slide (Refer Slide Time: 31:40). Neglecting the inertia of the other part of the machine and this is equal to 6240 by 33.8 squared into 0.02. What will be unit for this kg meter square? If you follow SI unit everywhere will get balanced and finally is equal to 44.3 kg meter square.

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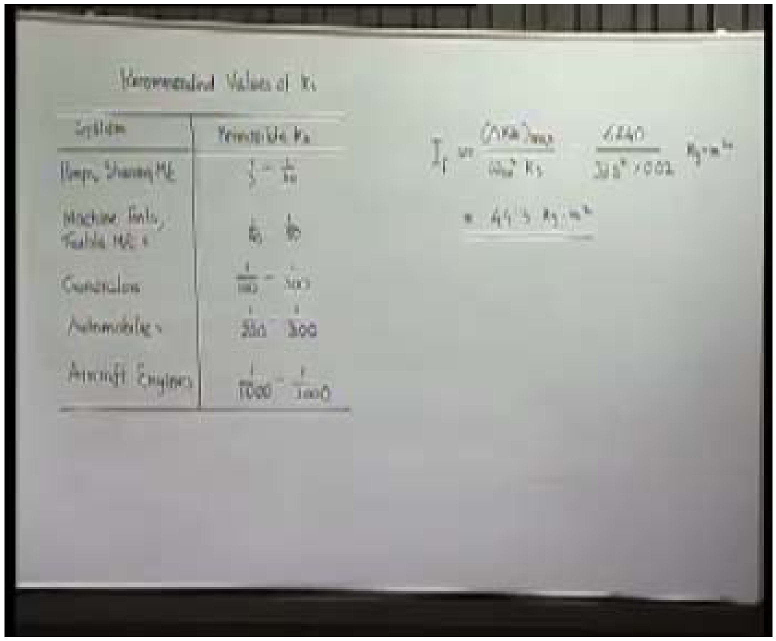

So, if we put or if you attach a flywheel with this much of moment of inertia, then the fluctuation of speed will definitely contain within 2 percent when the engine runs at 800 rpm and the fluctuation of energy or the turning moment diagram of the multi cylinder engine is as specified.

Let us now take up another example, which is of other type. Remember, that we mention that there can be two types of the situations. In one case, driving torque fluctuates and the load torque is uniform, just now as you have done. Another common example of fluctuation, we find when you use machines like punching or shearing machine, where the load fluctuates substantially and the driving which is done normally with help of motor is uniform.

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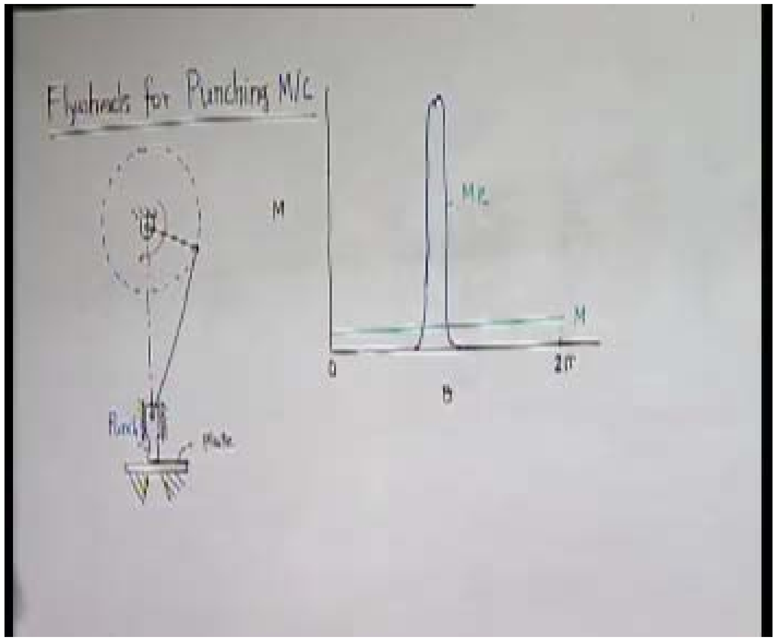

So, the basic mechanism involving in a punching machine is also a slide crank mechanism. This is the crank which is rotating and during this rotation you find that at a particular location, which is indicated like this (Refer Slide Time: 34:46); if punch just touches the plate it has to pierce and with little bit more rotation of the crank piercing is completed. Therefore, only during this period there is load or work done, rest of the portion of the operation in one cycle is absolutely free, there is hardly any torque.

That means if you plot, here one cycle is obviously one rotation (Refer Slide Time: 35:30); it will be 2pi and moment M. The moment except the point when it touches this, the resistance will be something like this and the amount of energy which is consumed here which is given by this area; that same amount of energy must be supplied by the motor during one rotation. So, the applied torque which is constant it will be this and this is M_R. Here, what we find that during this period if $theta_1$ and $theta_2$; the energy at here, this area must be equal to this area. What we can do here is that maximum fluctuation of energy can be written as this (Refer Slide Time: 37:35). If we call this total amount of energy as E minus the amount of energy, here you can call as E prime that is this; so, this is the fluctuation.

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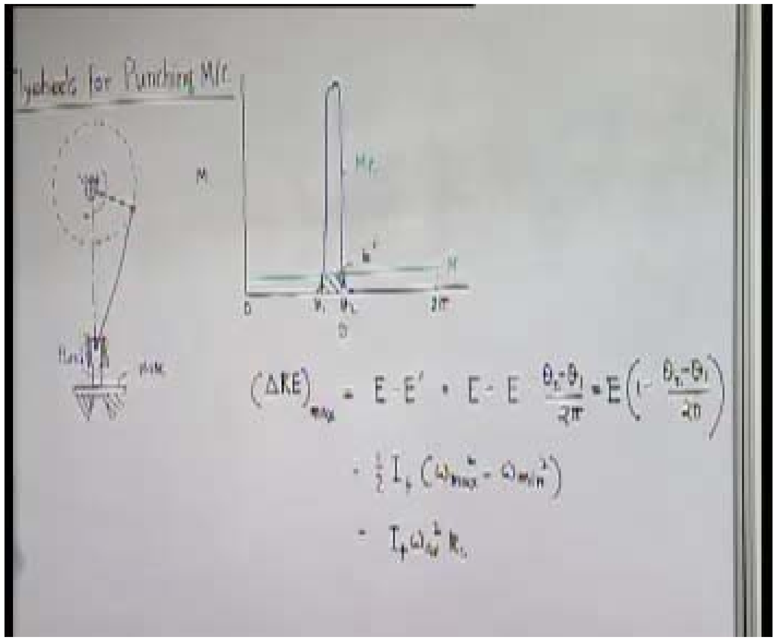

As you can see very easily, I think without going in to the detail operation of this will make some approximate approach. Approximate approach will be that how much is E prime; E prime we can easily find out from here; that the amount of energy here will be just into portion to the amount of energy in the whole cycle and this particular angle or the rotation. It can be written as total energy of whole cycle is E and what fraction it will be theta$_2$ minus theta$_1$ divided by approximately 2pi.

We can consider this or this is nothing but E 1 minus the rotation of the crank during which punching operation is complete approximately. This will be obviously, again equal to half moment of inertia; if we consider that is the flywheel which is the phenomena moment of the inertia I_f and you have seen that this can be written as I_f omega$_{av}$ square. So in this case, rather than going in to the detail of where the intersection point or finding in a complicated way; very approximate method to find out the fluctuation of energy in case of punching machine can be taken.

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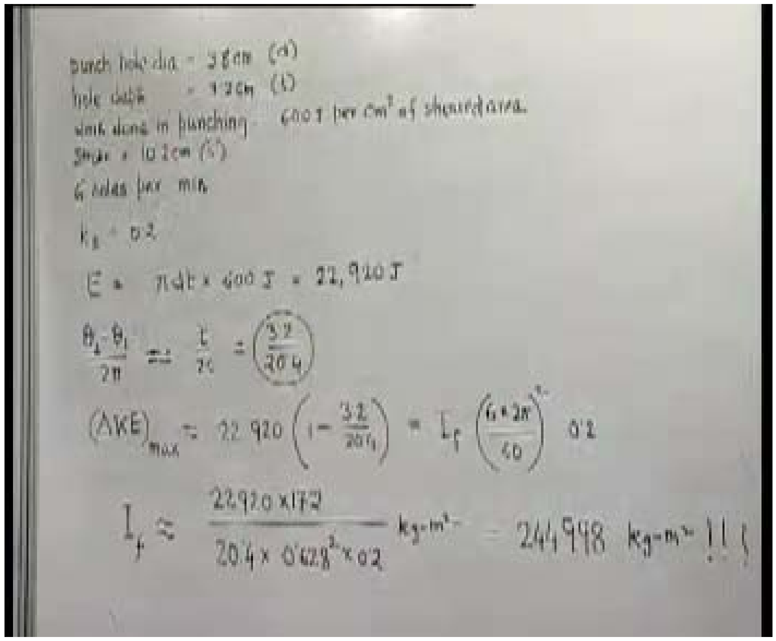

Now, let us take up a particular problem. The problem I am taking up is not only just solving numerical problem also demonstrate something which will be also very important as you will realize. Let us consider a case, where punch hole diameter equal to 3.8 centimeter, hole depth is 3.2 centimeter, work done in punching is equal to 600 joule per centimeter square of sheared area, stroke of the punch is equal to 10.2 centimeter, thickness sorry, thickness is not needed as it is same as the length of the hole or depth of the hole, 6 holes per min and k_s 20 percent because it is a punching machine, as you seen it can be if low as 1 by 5 in previous case.

Let us find out required flywheel to be put on the crankshaft directly, so that the position of fluctuation of it can be kept within 20 percent if this is situation. Now, energy required per punch per cycle will be how much? It will be sheared area which is nothing but pi d into t into 600 joules. This if you calculate, you will get 22,920 joules to punch each hole you will require so much of energy.

Now, $theta_2$ minus $theta_1$ by 2pi how much is this? These again we can do more accurate analysis by actually drawing the whole things. Otherwise, you can consider the movement of the whole things to be linear; that means the speed and displacement is having linear relationship. If we do that, then the amount of displacement which is required for punch during punching; that is the thickness. The total length of moment of this, which is twice this stroke 2s for that movement, is 2pi. Therefore, this we can write as t by 2s and that is in our case 3.2 and total length is 10.2 into 2 is 20.4.

So, maximum fluctuation of energy will be approximately equal to energy E, which is 22920 into 1 minus 3.2 by 20.4 and this must be equal to the moment of inertia of flywheel average omega squared; average omega we can find out because, it is hole per minute that means 6 rotations of the crankshaft per minute; that is 6 into 2pi by 6, so many radians per second, squared into k_s which is 0.2. From this, if you calculate the required flywheel moment of inertia it will be as shown (Refer Slide Time: 45:25). This by using SI unit we will get directly in kg meter squared. I think I have calculated; it comes something like 244998 kg meter squared; you see that we require a huge flywheel to maintain the speed even if we are allowing it to vary by 20 percent which is obviously not a physical solution.

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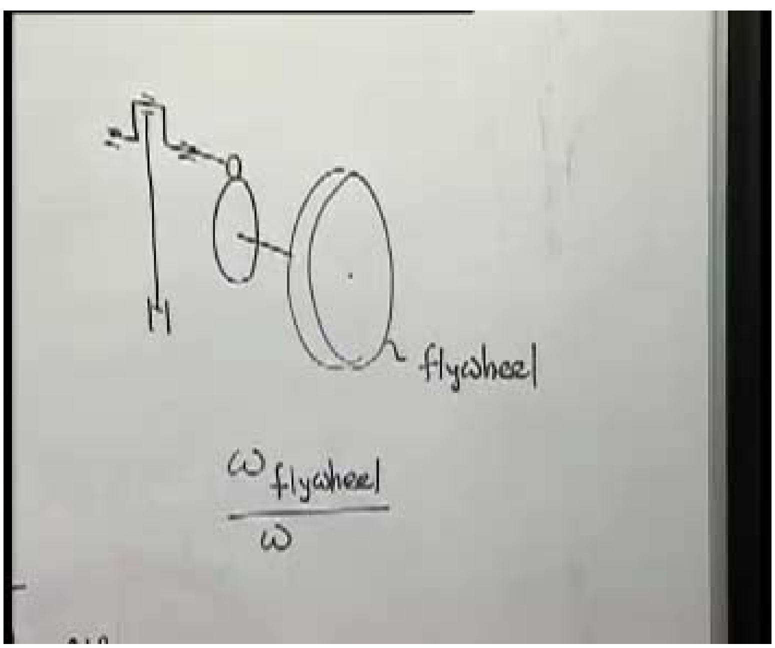

Now, this is solved in this manner. If you have this crankshaft and this is a punch rather than directly attaching the flywheel with this, we use gearing system and we attached the flywheel on idling shaft and omega$_{flywheel}$ by omega$_{crank}$ is equal to n, depending on the gear ratio. Now what happened? If I_f be the moment of inertia of flywheel, the system kinetic energy then become half I_f into n into omega squared.

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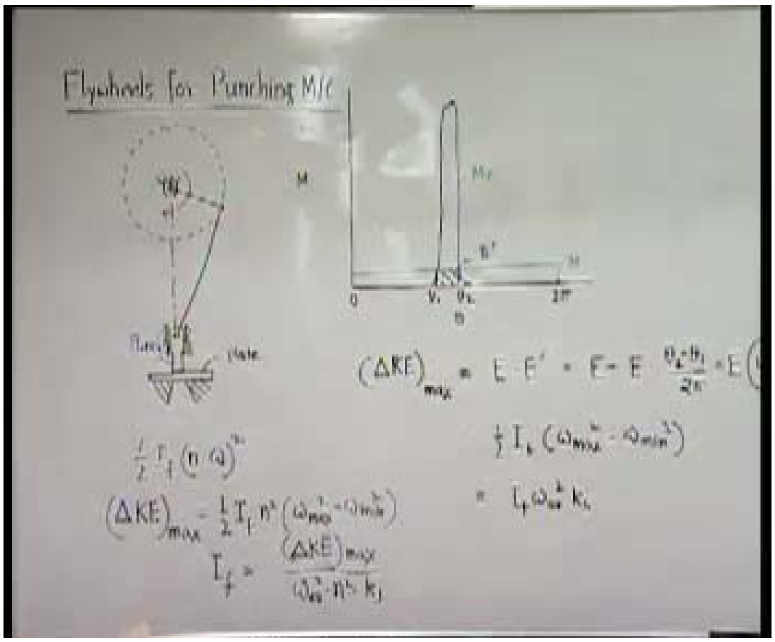

Therefore, maximum fluctuation of kinetic energy can be written as half I_f n square into omega$_{max}$ square minus omega$_{min}$ square. So, effectively what we get, I_f can be written as maximum fluctuation of energy by omega$_{av}$ square into n square into k_s. For example, in this particular case, if you put the flywheel on idling shaft which is connected to the crankshaft by gear front this gives a ratio of ten. Moment it happens, our moment of inertia of flywheel required becomes 2350 kg-m; that means, if you put rim weighing two and half tons at a radius of m which is quite possible flywheel flywheels are big then, I think your permitted fluctuation of speed will be maintained.

If you make this speed little bit higher, say if you want to make it 20 ratios, then the whole thing will be still half, it will be only 1.2ton of rim which is quite possible flywheel and at radius of m. This is what I want to demonstrate, that where you find connecting the flywheel directly on the operational shaft required or what you can do to make the requirement of the moment of the inertia of flywheel to be very heavy? You can use an idling shaft on which the flywheel is mounted and which is connected to the main shaft or crankshaft, with the gearing system or transmission system. That means what is effectively we are doing? We are making this real source of kinetic energy to rotate at higher level; that will make requirement of inertia to be lower and the machine will be multiply.

I think with this, there are more complicated cases and problems which we will not go into it. Only these things will be enough for you to remember; that the whole system or whole analysis is very approximate because a higher level of accuracy or better or more accurate analysis of this problem really does not mean much because our choice of k_s is a very ad hoc thing. Okay, let me allow only two percent variation, etcetera.

So, that of course is very ad hoc therefore, making a very accurate analysis of the whole matter is really not required. Only thing you have to keep in mind is that the fluctuation of speed in its operation is not allowed to go beyond that limit and this as a result, what is it doing? The whole system is running with a much uniform speed; that means again, nothing but smoothening of operation. As you have seen that removing the unbalance dynamic force will make the vibration of the system less and the system run smoothly; this is another way to do it, that where the actual operational speed also is allowed to maintain the uniform value as much as possible and this is the smoothening of the operation.