Dynamics of Machines

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Module No. # 06

Lecture No. # 01

Turning Moment Diagram

In the previous module, we have seen how to balance the inertia forces in a mechanical system with an objective to make the operation of the system smooth, without any vibrating tendency; this not only improves the functioning of the system, but it also improves the life. However, the job is not complete; there are other sources or there are other reasons for a non-smooth operation of a mechanical system. In this module, we will take up that particular problem and also try to see, how the problem can be either minimized or removed.

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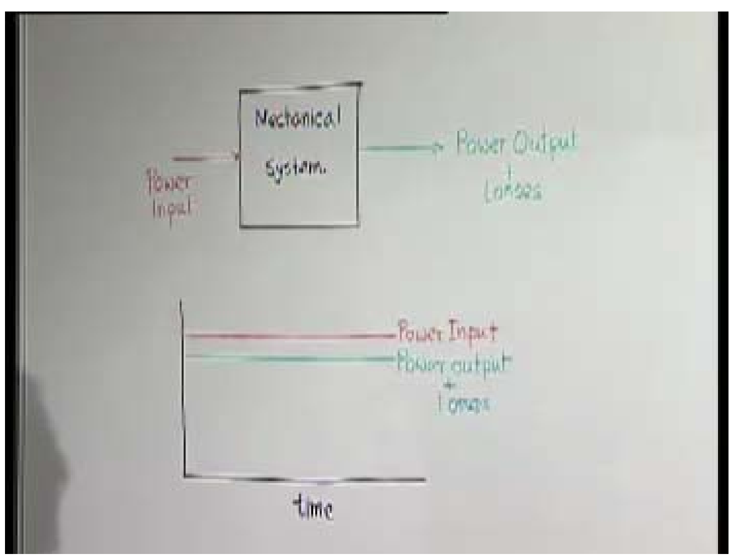

If we take a mechanical system, then its condition can be generally specified by two things. How much power input is there and how much power output plus losses are there. So, what is happening in a mechanical system? Irrespective of the nature of the system, there is power input from one side; that means, which is putting energy into system and the mechanical system is doing some work and spending energy. In between there will be losses because of friction etc., within the system.

Now, what are the different situations possible? If we consider that with time, the power input is something like this and the requirement of the energy for doing the task and the losses are something like this (Refer Slide Time: 03:22). We know that if we are putting energy at a higher rate then, the rate at which the energy being consumed, what is going to happen? We all know that the energy can never be destroyed; it is going to remain in the system, it will get accumulated in the system and obviously, a mechanical system can accumulate energy in the form of kinetic energy.

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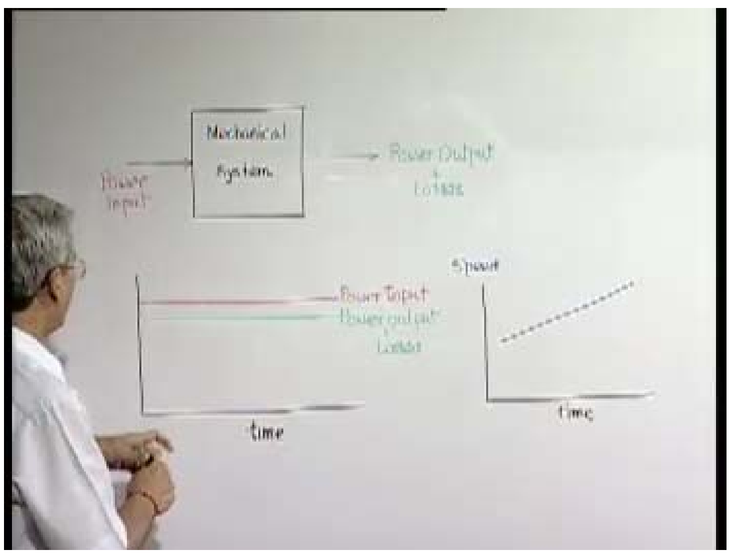

What is going to happen? The speed at which this mechanical system is operating will gradually change and in this case, we will find that with time the speed of the machine will gradually increase, because with time more and more energy is going to remain into the system and resulting in gradual increase in the kinetic energy; that is the speed of this. The reverse situation is there, where it can be less.

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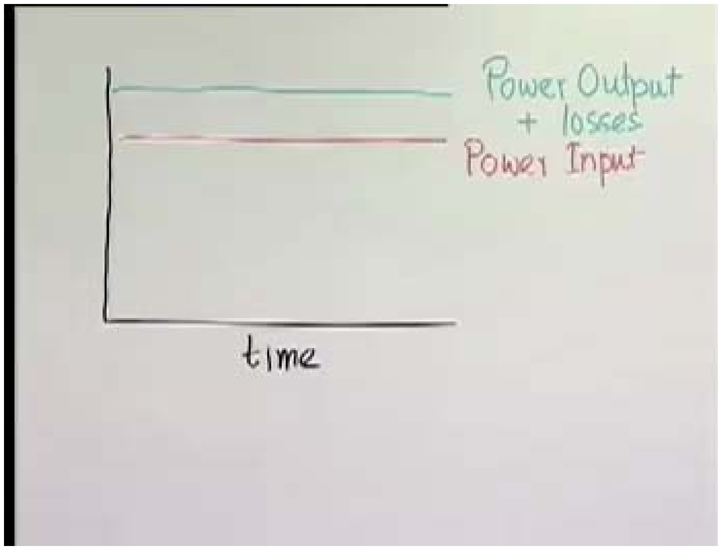

We just repeat the same diagram here. In this case, our input is less and power requirement or the expenditure of the energy is at a higher rate; this is the power input. The reverse thing will happen here; that means, whatever energy or the rate at which we are putting energy into the system, we are consuming energy at a higher rate and where from that extra energy is going to come? Obviously whatever kinetic energy is there in the mechanical system part of that will supply the energy. As a result, it can be easily understood that in this case, with time the speed of operation of the machine will slowly decrease - pretty obvious.

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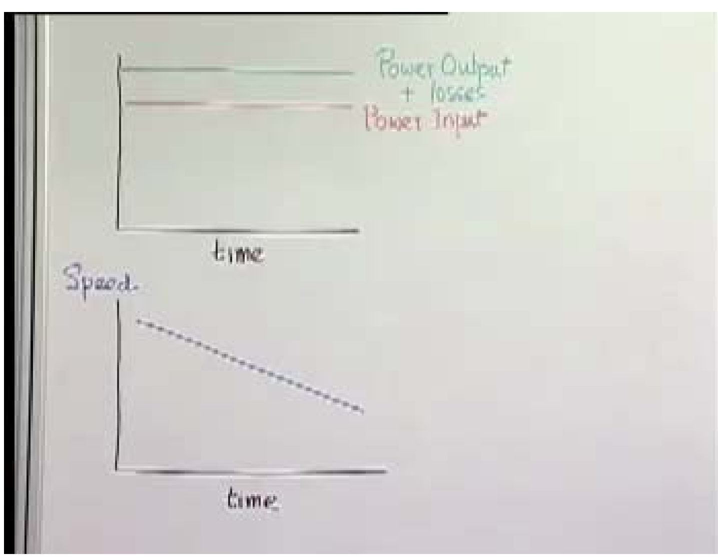

Both these types of operations are called unsteady operation. You can see that when we are starting an engine; initially the speed of the engine was zero then gradually it increases. During that period, we have to put more energy into the system to bring the level of operating speed of the system to the final steady level. Similarly, when we stop some mechanical system, what will happen? Your power output will be there because, work will be still done, but our input has been switched off and therefore, slowly this will come down. These are the transient periods at the beginning and at the end of the operation; but, during the actual operation of the machine, what happens? We expect the operating speed of the machine, overall must remain same.

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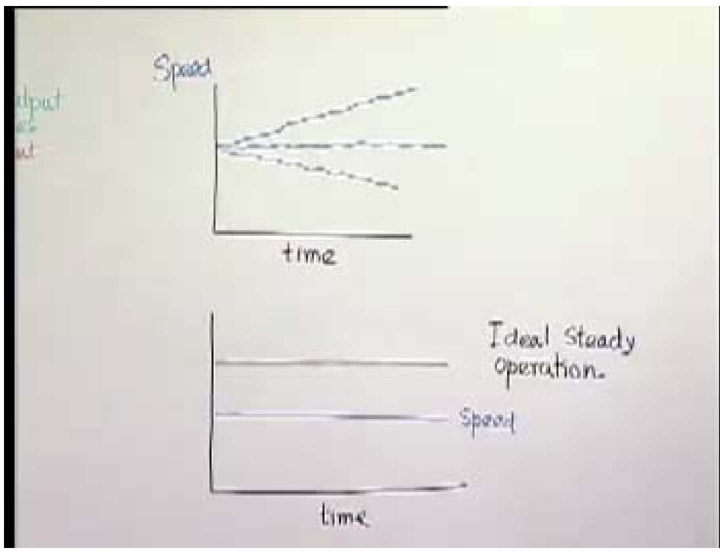

Though we have shown the diagrams like this, in real situation the speed is never that steady; this is the rising curve and this is falling curve, something like this. We are primarily concerned when the speed of operation of the machine remains constant; that is the steady operation. The ideal steady operation will be like this, with time the energy input is like this and energy output and losses as shown (Refer Slide Time: 08:18). This is the idealized situation for a steady operation and speed of this case will be just again constant. What you all can understand is, in reality such a perfect situation is never realized. What happens is the average of the power input and average of the power output plus losses will be same and speed will be also (()). Therefore, average will remain same though there will be fluctuation; what is that situation? Let us examine slightly in more detail.

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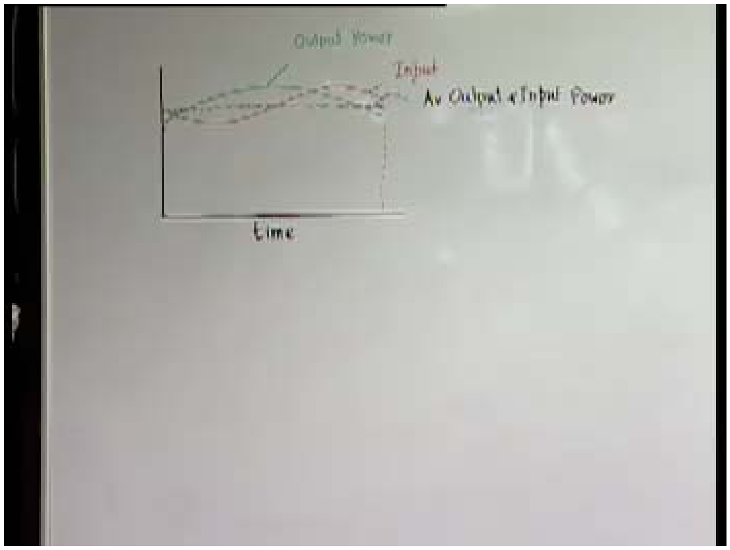

Let us take up a case, where the situation is like this. Time, input and output power variations are something like this. This is one complete cycle and this is the output power and this is the input power. What we can realize is that the average will be same for both the input and output power. Now, what is going to happen to the operation of the system that means the speed of operation?

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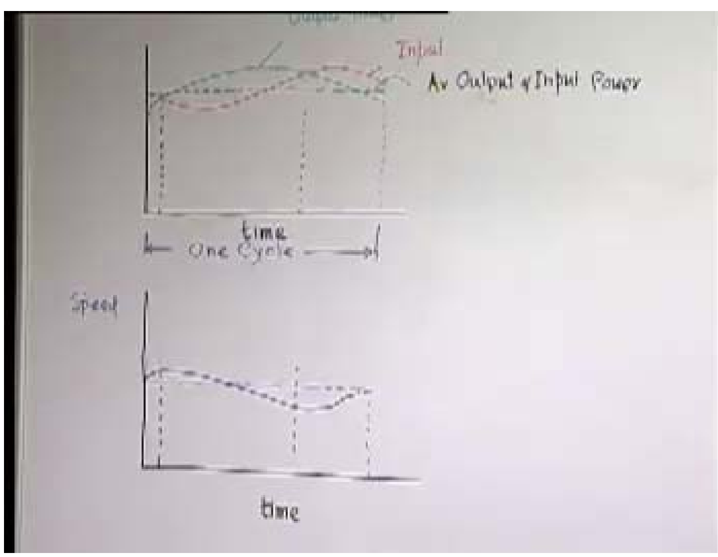

Now, we will see that though the average of both the input and the output powers are same; there are locations where, that means, at every instant - except for this instant and this instant - everywhere else the instantaneous input power and instantaneous value of the output power are different (Refer Slide Time: 11:32). So, what will happen is that depending on the situation, the system will either accelerate or decelerate except at two instants, where the input and output power are exactly the same.

What is going to happen during the period where the output power is more than the input power? Obviously, it will come down or slow down. Similarly, in this period that means from here to here, this is one cycle, because in steady state everything will be in cyclic pattern; this is one cycle, after which the pattern repeats (Refer Slide Time: 12:20). So here what will happen is that the input power is more; obviously, the speed will increase or this system will accelerate. Therefore, what situation will happen will be like this (Refer Slide Time: 12:50). The speed is constantly decreasing from here to here, because output is more and speed is increasing here, because input is more.

Therefore, you can see that in general, where the instantaneous values of output power and input power do not match at every instant, but the averages are same. Though the average speed remains constant, but the instantaneous value of running speed of system will change or fluctuate and there is a cycle after which whole things will repeat. This is the general situation of all the cases. Therefore, we will find that we have a non-steady or rather non-smooth operation; the operating speed of the system is fluctuating. In previous case, where we discussed the balancing of unbalance masses, the nature of non-smoothness was due to the shaking forces produced by the unbalance; but here, the non-smoothness of the operation is because the operational speed is fluctuating and this is the reason for that.

Let us consider the cases, which we will generally encounter. In general, we will find two types of cases. Of course, this is the most general situation, where both the output power and input power, they will both vary like this (Refer Slide Time: 14:13).

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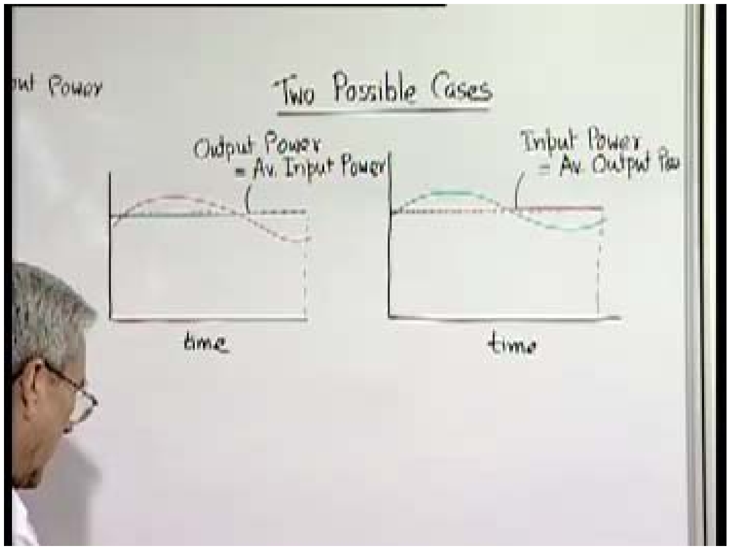

In general, we will take up the cases of an encounter. There are two situations; one in which the input is fluctuating and output power is uniform, one cycle and the other situation is idealistic situation of general nature is that the input is constant and the output is fluctuating. In both the cases, we have to keep in mind here because it is the steady operation; this is not only the input power, but this is all also equal to average output power. Similarly, here this is the not only the output power, but it is also equal to average of input power.

These are the two generally encountered situations in mechanical or machine dynamics, where in one case the input power is fluctuating; output power or the load is constant; whose average value happens to be equal to average of the input power, because otherwise the situation will not be steady. This speed slowly changes not only just fluctuate and the other possibility is that were the input power is more or less constant, but the output power or the requirement the load is fluctuating and again same condition for steady operation that the average of both are same.

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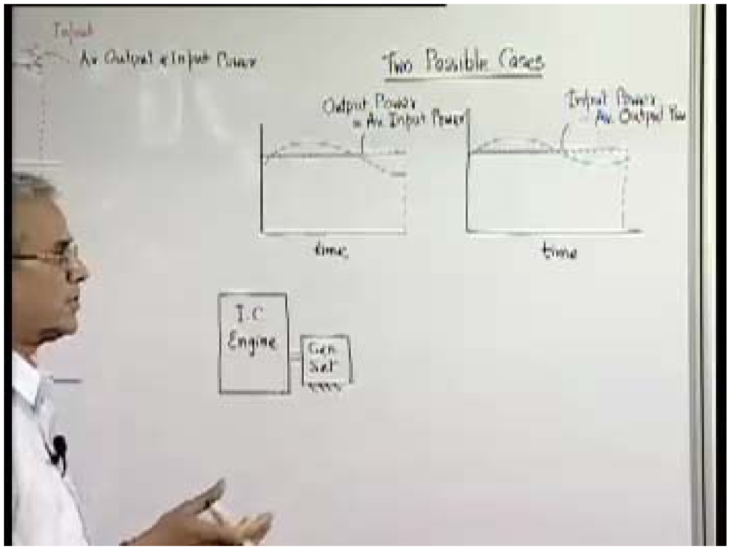

These two possible cases, we can think of examples. We get a very common examples, where the input power is fluctuating the output power is uniform or steady. A very common example of that will be an ic engine driving the generators producing the electric current and load is constant; this is the ic engine, say a diesel engine here you will find the mechanical torque produced on the crankshaft is fluctuating that is input power is fluctuating but the load or resisting load produced by generator is constant as our electric load is constant. Hence, this will represent this equation like this.

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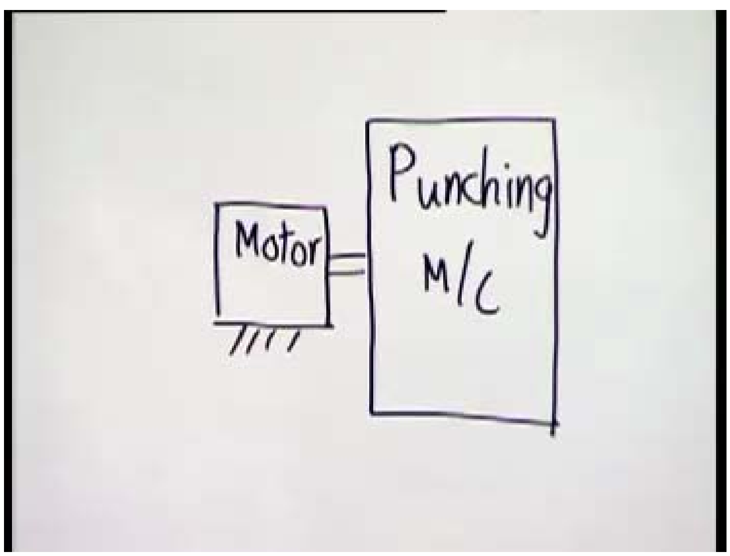

On the other hand, we take up a case where a motor is driving a punching machine; for example, a die punch machine, here what happens? The motor is producing a constant voltage and constant torque; so input is constant, but load requirement is fluctuating. Most of the time there is no load except friction then once the punch comes in contact with the plate or the sheet metal, immediately the load starts increasing. So, here you will find that the situation is some things like this; in such cases the situation is like this; in both the situation the speed will fluctuate.

Our objective of this discussion is to find out the ways and means to estimate the speed fluctuation and how we can keep it within tolerable limit? The point we have to keep in mind is that though the average speed remains constant the instantaneous speed varies and it fluctuates. Now depending on the operation, there are permissible limits to which we can allow this operating speed to fluctuate. If the operating speed fluctuates too much then the functioning or the performance may not be acceptable. Therefore, we will see, what can be done to keep that fluctuation of speed within permissible limit. We will first take up the most commonly encountered situation that is an engine driving a mechanical system either a pump or generator, say machine or some things.

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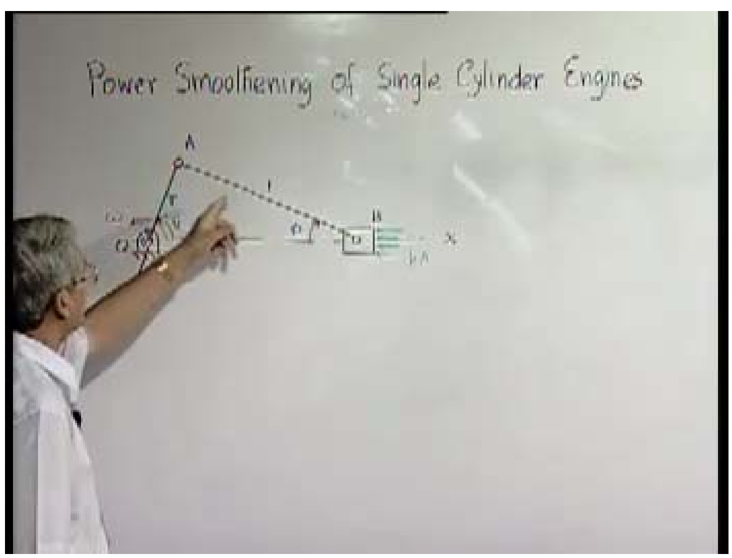

We take up a very commonly encountered problem of the nature; we are discussing is the power smoothing of single cylinder engine. What will do first is to find out the fluctuation of input power. In case of such a situation, we will again come back to our model; this represents a single cylinder engine. Now, what we have done here is, if you remember our previous lecture while discussing the unbalance of a single cylinder mechanical slider crank mechanism, we saw that it is possible to replace the mass of the connecting rod A B into two parts; one at crank pin here and other at the reciprocating mass here.

We have also seen that this mass and the cranks original mass are all actually purely rotating objects and therefore, to balance that we always generally put a balancing mass, but really it does not play any role in this analysis, which it will be doing. On other hand, the other opposition mass which goes here, that along with mass of the piston etcetera, in total becomes $m_{reciprocating}$ that you have seen. Now, what we will do here is to find out that a gas force, which has pressure at the instant of the gas within the cylinder multiplied by piston area A. This total gas force produces how much turning moment on this? The reason is that the turning moment M multiplied by the speed gives the power input.

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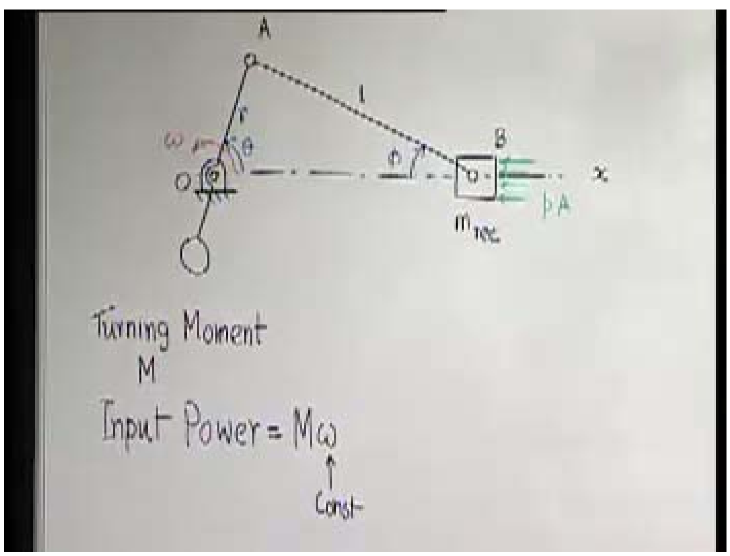

If we consider a case, where the input speed or the running speed omega to be approximately constant, then what really tells about the fluctuation of input power is also same as the fluctuation of turning moment which is M, because input power is nothing but M into omega. So, when we take this as constant then fluctuation of M or turning moment is the critical thing and that is what we have to find out. To do that we again take care of certain points like the lamda is r by l, which is much smaller than 1 and omega is approximately constant, it cannot be exactly constant.

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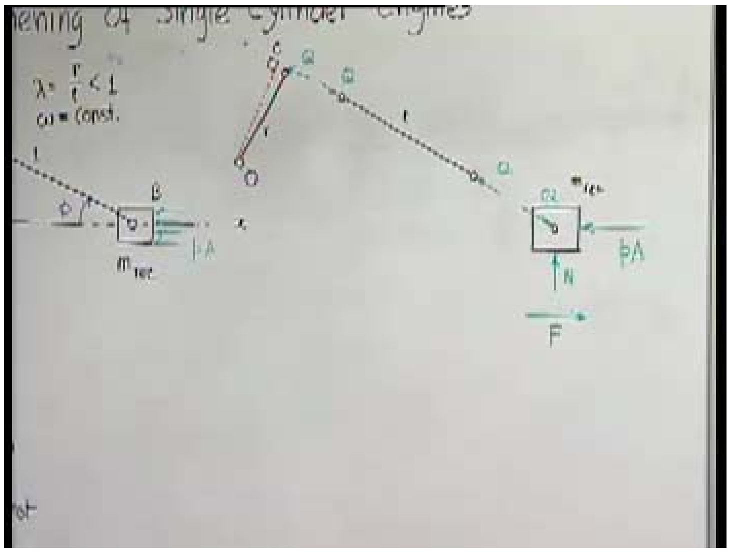

Let us draw the free body diagram of the various bodies; this is the piston, this is the connecting rod, this is the crank. Now, what the forces acting here is p into A. Since, you have replaced the connecting rod by a mass-less rigid rod and the mass of the connecting rod of this coupler A B being placed here and here; so effectively this is a mass-less rigid connecting rod. Therefore, any force on this has to be along this, we know that. If we call this force Q than equal and opposite forces will act at D, the force on the piston and the reciprocating mass will be in this direction and the normal reaction from the cylinder wall these three forces will be reproducing a resultant force in this direction because it can accelerate only along this direction which is x, so the total force along the y direction has to be zero according to Newton's law.

The Q which is aligned along the connecting rod has to be again balanced by equal and opposite force at the other end, because this body being mass-less it cannot accept any resultant force acting on them. According to Newton's second law or third law, a force at point A will act which is equal and opposite to this; we consider these as C. The turning moment is going to be this force Q into the perpendicular distance of this line of action of this force from the crank and that is the turning moment.

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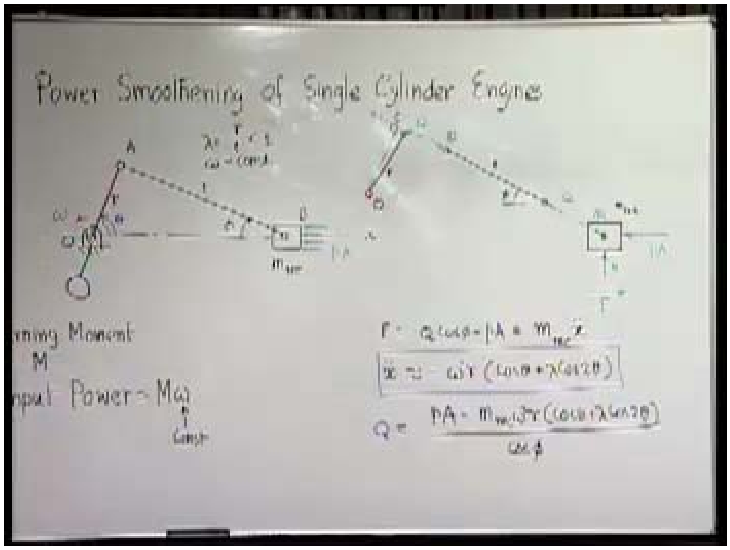

Our objective is to find out the turning moment M acting on the crank pin. Let us first find out Q. Now, the resultant force on this will be in this direction which is F; it has to be in this direction because the piston can accelerate only along x direction. We know F is equal to - now this angle is phi - Q cosine phi minus p A and that must be equal to the $m_{reciprocating}$ into the acceleration in that direction x two dots.

Now in our previous module, we had a detailed discussion on the acceleration of sliding body x two dot and we found out that x two dot will be this; this result I am not going to discuss again because we derived it in previous module (Refer Slide Time: 30:12), by using this result, we can write that Q will be approximately equal to p A minus $m_{reciprocating}$ omega square into cosine theta plus lamda into cosine 2 theta. We have found out the force Q. What will be moment produce by this force? That will be Q into the distance; how much is this distance?

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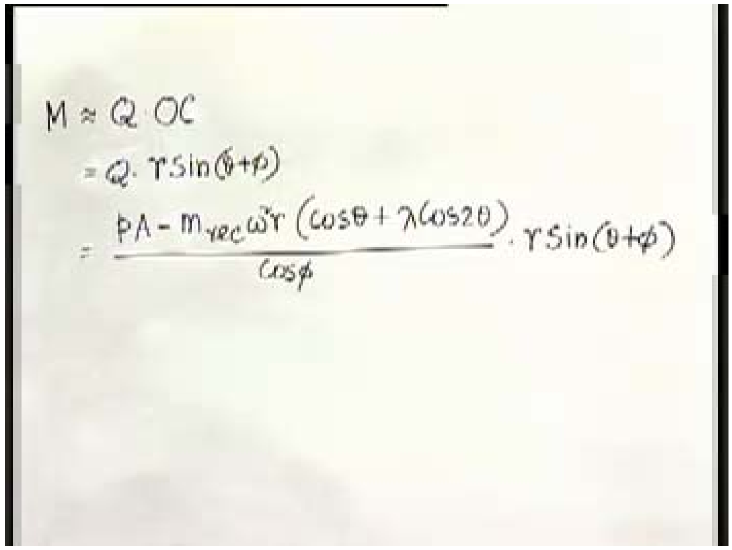

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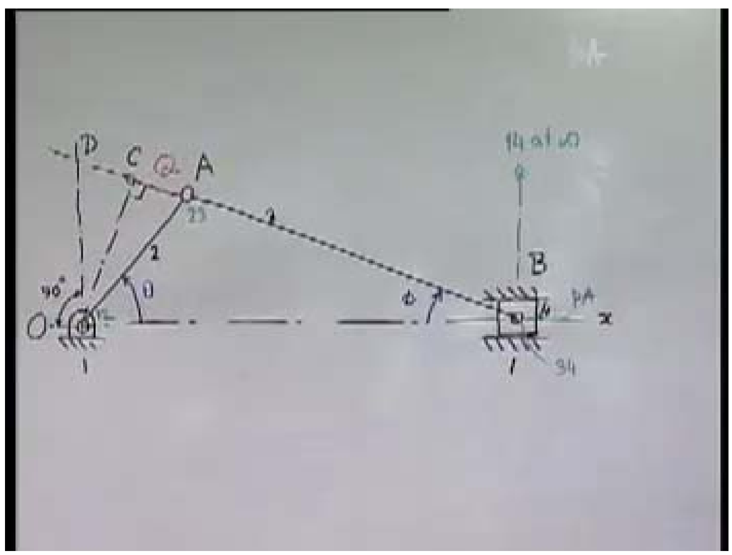

If you take up this right angled triangle; this angle is theta, how much is this angle? Its phi; this total and this angle is theta. So, this perpendicular distance OC is nothing but r sine theta plus phi from trigonometry; this angle is theta plus phi, hence it is nothing but r sine theta plus phi. Further, you can write this is equal to (Refer Slide Time: 32:36).

Therefore, this is the expression for a turning moment, which you can see is actually the function of theta - that means it varies theta - but, I things here we have other quantity which also varying with time is phi. Let us try to see, if it is possible to express turning moment purely in terms of theta. The turning moment diagram expression, we have got is seen as a function of theta and phi, because this gas pressure here acting on the cylinder is also a function of theta, as you will see. However, it is may be desirable to express turning moment completely in terms of one variable theta, because theta and phi are dependent.

We can do it in this particular way, say the moment of this force Q about this point O is obviously the magnitude of Q into the perpendicular distance from this crank pin point OC. Then, you will see that we can find out this point D, which is nothing but the intersection of the extension of the line D A and with the perpendicular drawn on the line of slope that is the x-axis, this is the point D. We can easily show that this point D is nothing but the instantaneous centre or relative instantaneous centre between link 2 and link 4 that means the slider or the piston.

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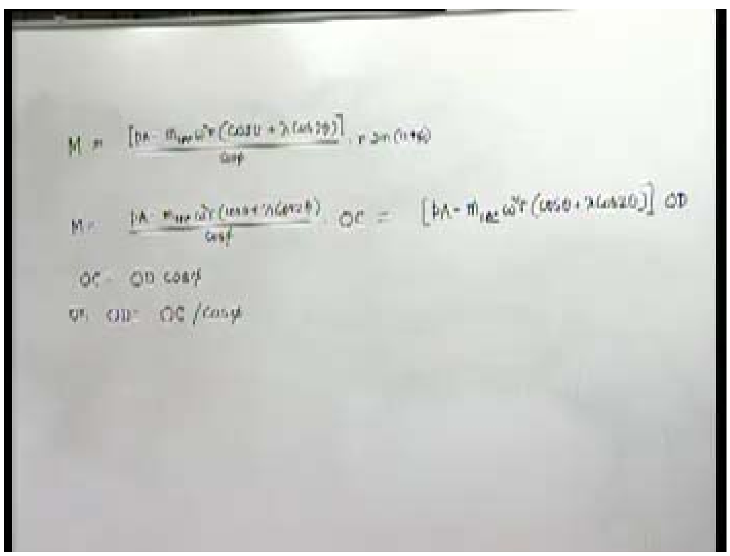

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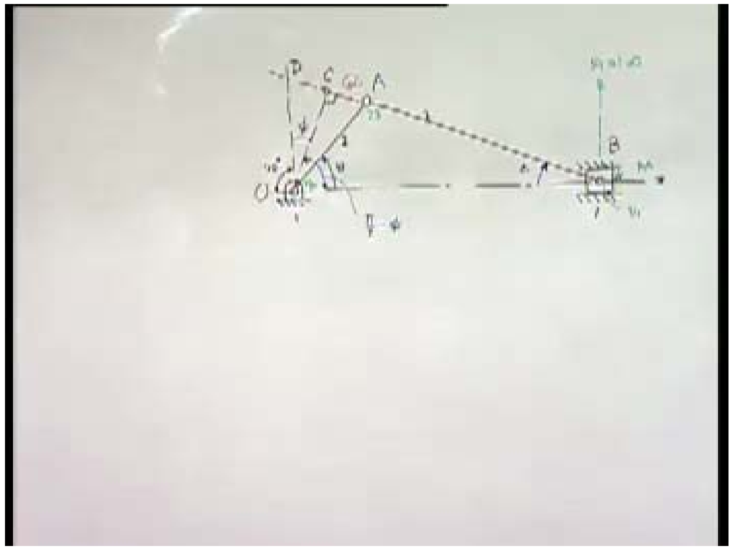

We know that turning moment is approximately the force Q which is p A minus $m_{reciprocating}$ omega squared r cosine theta plus lamda cosine 2 theta divided by cosine phi, this is the force Q, into the perpendicular distance OC. Actually, in this r sine theta plus phi is nothing but OC. Now from this right angled triangle we find that this angle right angle triangle O C B; this is the right angle; this is angle phi; this angle will have to be 90 degree minus phi and since this is 90 degrees, this angle is phi. (Refer Slide Time: 35:30).

You can write that OC is equal to OD cosine phi or OD is equal to OC by cosine phi. Therefore, using this we can now write this is equal to this (Refer Slide Time: 36:43). Now, OC by cosine phi is nothing but OD. You have to now find out OD as a function of theta; then our objective will be fulfilled.

Now, you are familiar with the technique of using Aronhold Kennedy's pre-instantaneous center theorem for finding out the relative instantaneous centre which is not known. Now in this slider crank mechanism; this is fix link 1; this is crank link 2; connecting rod or coupler is link 3 and the piston or the slider is link 4. So directly, we get the relative instantaneous center between link 1; that is the frame and link 2 as this hinge, between 2 and 3 is this hinge, between 3 and 4 this hinge and the relative instantaneous center between fixed frame 1 and 4 will be at infinite in a direction perpendicular to axis x, as indicated here.

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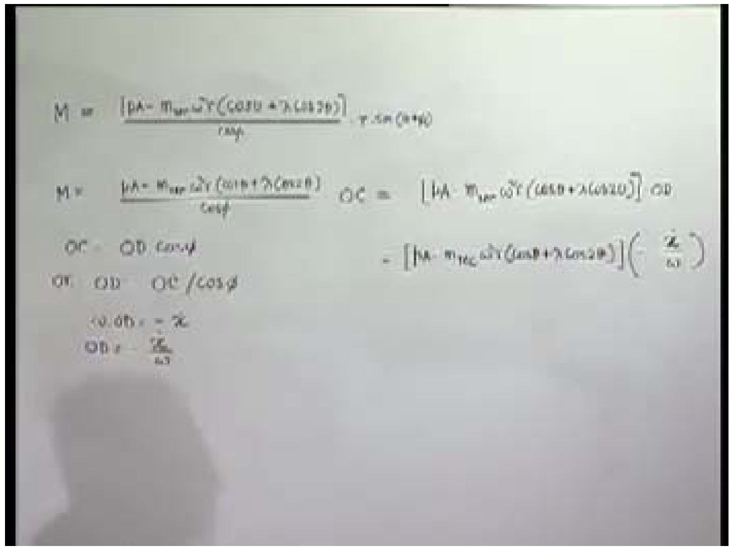

Now, we know from Kennedy’s theorem that to find out an instantaneous center between two objects can be done by using this theorem that the three relative instantaneous center between three bodies must lie in one straight line. If you want to find out the relative instantaneous center between link 2 and link 4; that means 2 4, which we have to find out, how can you find out? Now considering link 2, 3 and 4; we know that 2 3, 3 4 and 2 4 must lie on the line joining 2 3 and 3 4; this is line 2 4 on this. If you consider links 1, 2 and 4 then instantaneous center 1 2; relative instantaneous center 1 4; if you add the other relative instantaneous center between these bodies 2 4 multiply on this particular line.

If you join 1 2 with 1 4 which is at infinity in this direction, we will get nothing but this line 2 4 on this and therefore, the solution is 2 4. Now, what does it mean? It means that a point attached to link 2; but located here, has the same velocity as a point attached to link 4 located at the same point. That means, what is the velocity of point on link 2? Link 2 is having angle of velocity omega. So, the velocity of this point will be omega into OD and velocity of slider - because it is a translating body - is also same as this and we know that this is nothing but x dot. The velocity of this point has to be the velocity of the slider because, it is the translating body and velocity of all points of the translating body 4 is x dot, but unfortunately we have taken x in this direction; positive x dot will be in this direction whereas, with given omega value of omega into OD in this direction. So, we can write that omega into OD equal to minus x dot or OD is nothing but minus x dot by omega using this here again we can write is equal to pA minus $m_{reciprocating}$ omega square r cosine theta plus lamda cosine 2 theta and OD will be equal to minus x dot by omega.

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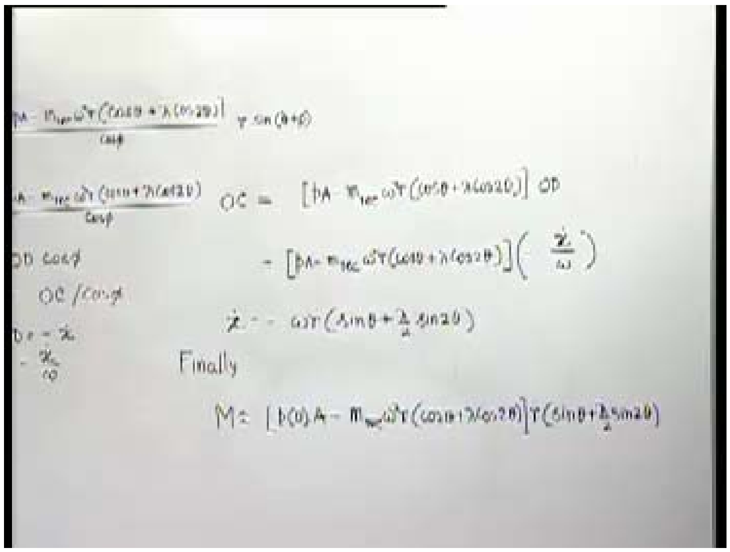

Now, let as recall our memory from previous module where we derive the expression for x dot in term of theta, omega and r. We know that x dot was equal to minus omega r sin theta plus lamda by 2 sine 2 theta. So finally, we get turning moment is approximately equal to pA; p is the function of theta that is the gas pressure, A is cross fictional area of the cylinder of the engine $m_{reciprocating}$ omega square r and minus x dot by omega is nothing but r. Thus, we find turning moment as the function of only one variable that is the crank rotation angle theta, how it look like let us see.

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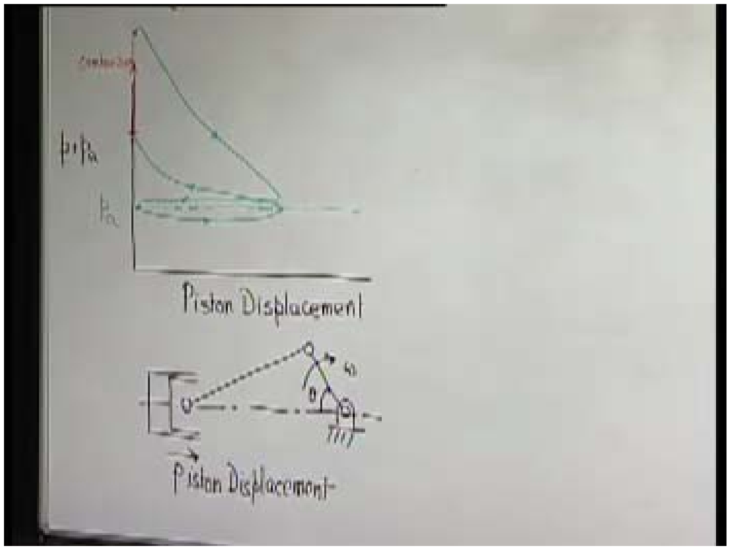

To do that first, we will consider most commonly used engine that is a single cylinder 4stroke cycle engine, which is operating on auto cycle; that is a petrol engine, let us see. As mentioned, let us take up the case for a 4stroke cycle petrol engine; why we need this is a here most of the things are just purely of geometric nature except this gas pressure; this is the gauge pressure that is the pressure above atmospheric in the cylinder which is actually also fluctuating thing which changes the theta.

How it changes? That we can see here, we look into the auto cycle. Now here, suppose we know that the piston displacement is in this direction and since the cross sectional area of the cylinder is constant, is in a way also represents the volume of the gas inside. Actually the pressure here and volume in the diagram of a petrol cycle or auto cycle, how it looks like?

One thing here on this side will give piston displacement and on this side, we will have p plus p_a. If this is the atmospheric pressure, then you know first stroke is the suction stroke. Where the pressure inside will be slightly lower than the atmospheric pressure, partial vacuum is created; then there is compression stroke; the pressure increases because the valves are being closed. Then there is firing and this is the combustion; this is the firing and next is the expansion, where piston the comes down and the last slope is exhaust (Refer Slide Time: 46:40), where again the piston pushes out the burnt out gas and again to push it out into the atmosphere, pressure has to be slightly however.

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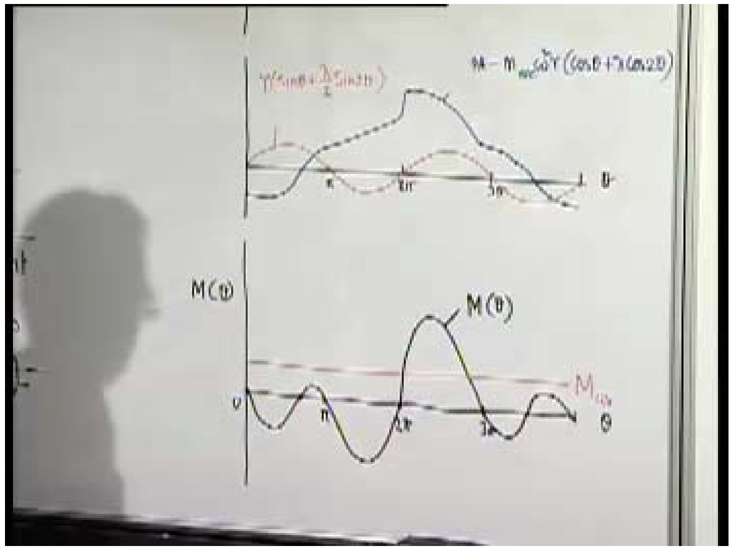

If these be typical auto cycle on which the petrol engine runs, what will be the nature of the turning moment? Since it is 4stroke cycle total range or value of theta for a complete cycle will be four points p or p A, whatever it is, may be let us see for p A. We can see, it will be nothing but these multiplied by areas; the nature will be more or less same. If we spread it out, it will look something like p into A. Now, what we have to do? We have to subtract this; that is inertia forces - this is nothing but, the inertia force - and that is going to be something depending on the speed. Now, we add these two; when we add these two, it should take a shape like this (Refer Slide Time: 50:00); this will be p A minus $m_{reciprocating}$ omega square r into cosine theta plus lamda into cosine 2 theta just the summation of the two. Now we have to multiply this by r into this quantity all this quantity changes in this form (Refer Slide Time: 50:45).

Finally the turning moment will be nothing but the product of this and this; just we can draw the final turning moment diagram, you can see this is the 0; so obviously, this is point, which we have again this is 0, obviously just to pass from this; then this is negative; this is positive; so it will be something like this; then this is again 0; it has to pass through this and here this is negative; this is positive; so whole things up to this will be negative. Now, both are positive; so it will be now positive but, it will again become 0. So therefore, it will be something like this and here again one is negative one is positive up to this; so it will be 0 and here both are negative; therefore, (()) (Refer Slide Time: 52:45). This is the final turning moment. Thus, we can see that the turning moment fluctuates and average value of the turning moment will be something like this; or if you think it is slightly on the higher side, perhaps it will be somewhere here (Refer Slide Time: 53:12).

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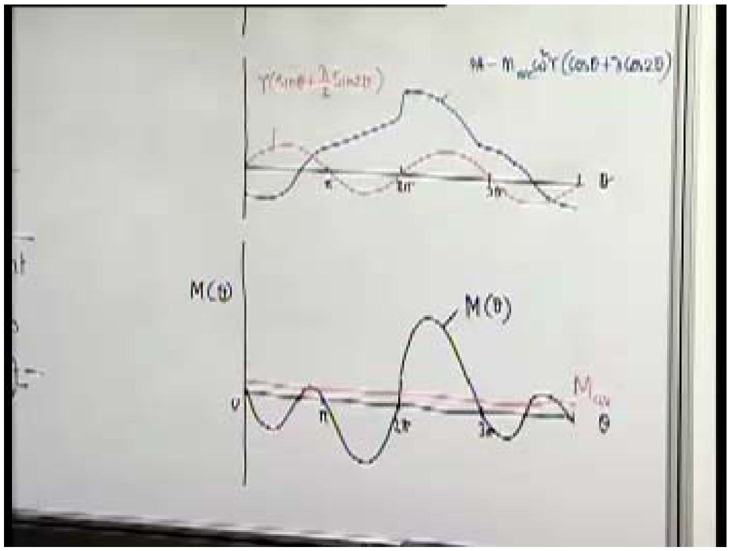

That means, total area bounded by the black curve and the red line above and below will be same. It is quite clear that in such a situation a substantial amount of fluctuation of energy, here the load is more than the input up to this; then slightly input is more up to this; again load is more than input; then here input is more than the load and so on (Refer Slide Time: 54:25). Therefore, there will be substantial amount of fluctuation of speed and such.

In the next lecture we will see, how we can find out the resulting fluctuation in the speed of operation of the engine and also we should be able to find out what is to be done to keep the speed fluctuation resulting from such a situation, within permissible level.