Dynamics of Machines

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Module No. # 05

Lecture No. # 01

V & Radial Engine Balancing

In the last session, you have seen that it is possible to neutralize the unbalanced force produced in one reciprocating mechanism system, may be an engine. By arranging a number of such identical systems in a suitable manner, the basic principle in this type of arrangement is that, the unbalanced force produced in one cylinder can be neutralized by that in the other one. Apart from inline engine, another popular arrangement with reciprocating or slider crank arrangements is the V Twin Engine.

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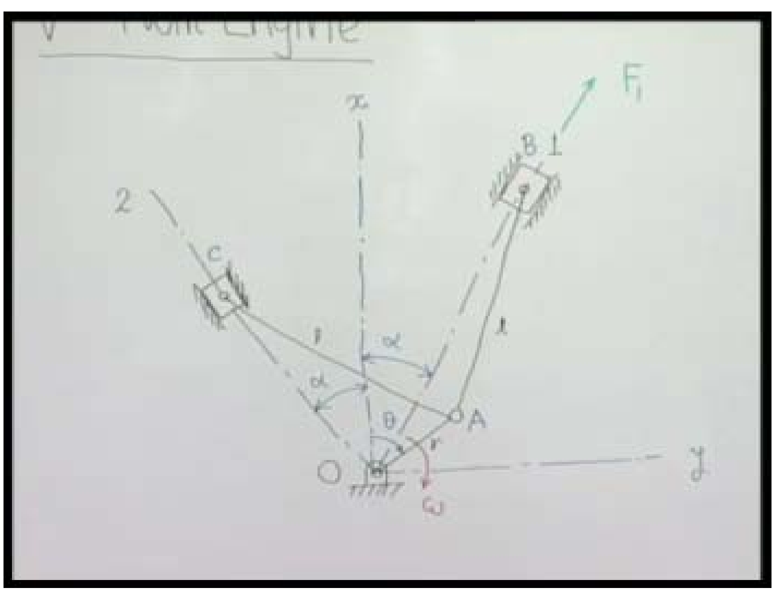

It should also be remembered, though we are referring to these systems of engines, there can also be compressors and other types of machines. In V Twin Engine, two identical cylinders rotates a common crank, if both of the connecting rod are identical with length l, then crank is called as common crank.

This is the common crankshaft; these are the common cylinder piston systems or sliders (Refer Slide Time: 02:25). Let us call this as cylinder 1 and this as cylinder 2 and this as the centerline of two engines. For whole engine this is the main centerline, which bisects two centerlines of two cylinders; let us call this as y-axis and this as x-axis, where this angle is alpha, since it is symmetrically placed this will be alpha.

We call the angle made by crank - common crank with this x-axis of this engine centerline, which is called as theta. As it happens, this is rotating at a speed omega (Refer Slide Time: 03:15). The common crank pin is A; this one is called as B; this point we can call as C. When this runs, we know that this particular cylinder or this particular engine will generate unbalanced force in this direction, which you should call F_1. As you know that for all slider crank arrangement, the unbalanced force, after the rotating unbalanced part of course, which is taken care by counter balance, the reciprocating mass produces an unbalanced force along the centerline of that element.

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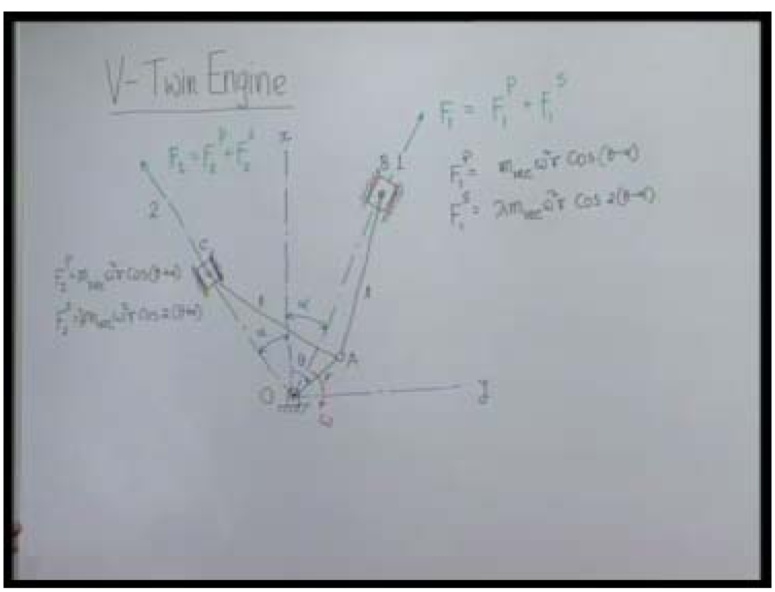

Similarly, for cylinder 1 - this is the direction, for cylinder 2 - this is the direction for unbalanced force to be produced. They will have both primary component and secondary component (Refer Slide Time: 04:44). We know that for primary component for this cylinder in this arrangement it will be, F_1 power p, equal to $m_{reciprocating}$ omega square r into cosine; angle made by the crank from the centerline will be this much and that is equal to theta minus alpha (Refer Slide Time: 05:30).

The secondary component will be lamda; twice of this (Refer Slide Time: 05:40). Similarly here, F_2 power primary will be along this; magnitude at this instant will be (Refer Slide Time: 05:54); an angel made by the crank from this centerline is theta plus alpha; secondary one is (Refer Slide Time: 06:12). Now, if you want to find out the resultant unbalance along x and y, then what you have to do? We have to take x component of this primary and add that with x component of this primary, this will give you the engine's total unbalanced force in x direction in this primary one.

Therefore, F_x power primary will be nothing but F_1 power primary. Component along this, any force along this, will be cosine alpha. We have to add x component of this force, which is again, secondary y component will be, y component of this force F_1, which is equal to F_1 power p sin alpha minus the y component produce by F_2 (Refer Slide Time: 07:46).

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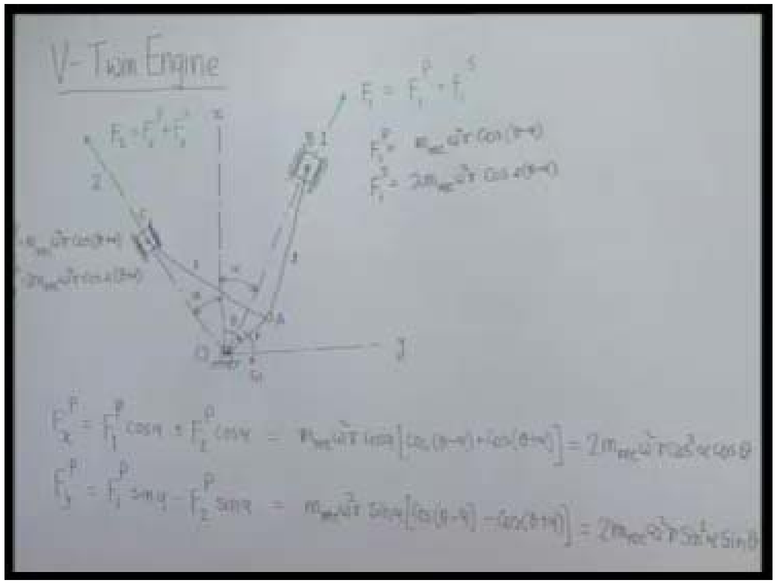

If you expand this we get; this will be $m_{reciprocating}$ omega square r cosine alpha and what will remain inside is cosine theta minus alpha plus cosine theta plus alpha (Refer Slide Time: 08:25). Further expansion of this leads to 2 $m_{reciprocating}$ omega square r into cosine square alpha into cosine theta, because sin theta and sin alpha term cancels. Similar way, if we take this as common; inside we have cosine; by expanding and simplifying we get (Refer Slide Time: 09:25).

The whole primary is unbalanced since theta is rotating with the constant angular velocity. If we consider t is equal to 0 and common crank was aligned with x-axis, then theta is nothing but omega t, which is harmonic function of time that means, a fluctuating unbalanced force is in x direction. In y direction, there is an unbalanced force, which is also fluctuating in center frequency.

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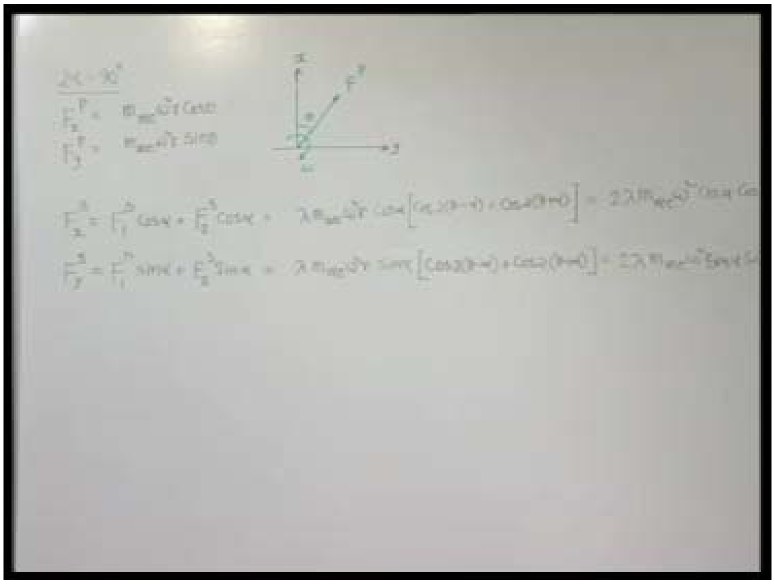

There can be a special case where 2 alpha is equal to 90 degree, this is called as 90 degree V Engine. In that case, what happen is F_x power p is equal to $m_{reciprocating}$ omega square r cos theta. If F_y power p is equal to $m_{reciprocating}$ omega square r cos theta (Refer Slide Time: 11:03). It is very interesting to note that the resultant unbalanced force for 90 degree V Twin Engine is nothing but a force, which is rotating along the crank. That means, for this primary force, this is the x-axis; this is the y-axis; we will see that the resultant unbalanced force at any instant is F power p; this angle is theta, it is rotating at the same speed along with real crank or actual crank.

Since the unbalanced force is represented by rotating force, it is very easy to balance the sheet by putting a counter weight in case of 90 degree V Twin Engine; so at least primary is completely taken care off. If you now consider the secondary, then secondary force in the x direction will be F_1 power s cosine alpha plus F_2 power s and this is equal to lamda $m_{reciprocating}$ (Refer Slide Time: 12:57). By expanding, finally we will get 2 lamda $m_{reciprocating}$ omega square cos alpha cos 2 alpha cos theta ; y component will be as shown in slide below; I can split this but for a sake of completeness.

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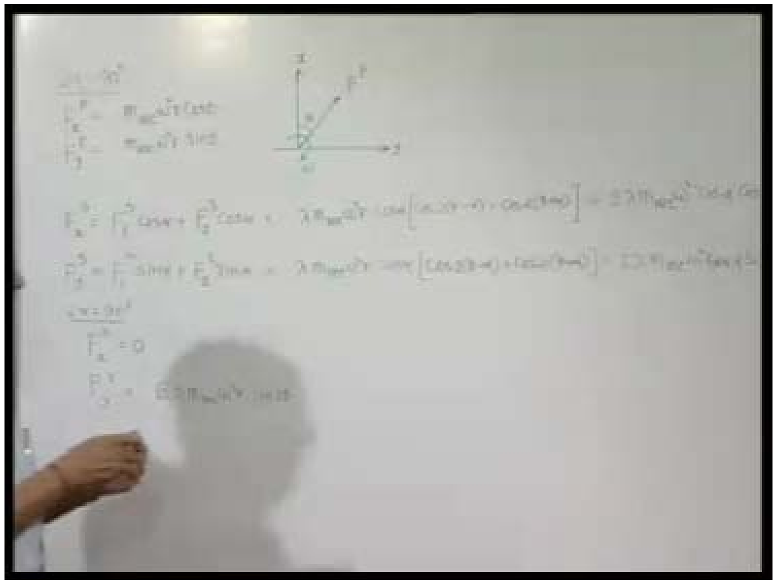

For 90 degree engine, if we find 2 alpha is equal to 90 degree, if F_x can become 0 then F_x power s is simply equal to 0 and F_y power s is equal to root 2 lamda (Refer Slide Time: 15:13). In case of this particular engine alpha is equal to 45 degree, x is equal to 0 because cosine 90 degree is 0.

The y component exists; it is a harmonic force which is fluctuating at a frequency twice that of the crank rotating so it is 2 omega. To balance the secondary force you have to take help of some active balancing system, may be you can indicate it like this; this is the plane containing two centerlines and obviously, this is the axis of the crank (Refer Slide Time: 16:47).

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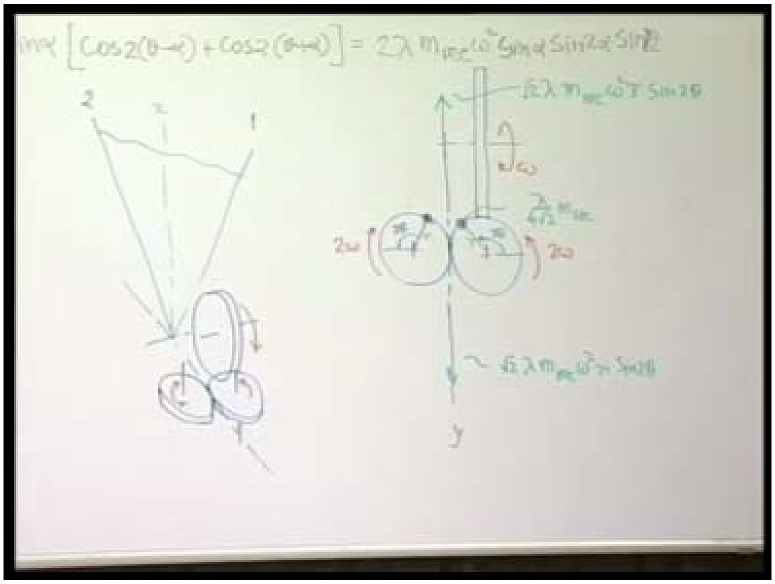

Here what we will do, we will attach a gear to the main crankshaft and then two gears they are all called helecoid gears; this rotates like this; this rotate like this and obviously this will rotate like this; this rotates with a speed of 2 omega. The diameters of the two gears are just exactly half; whereas this rotates as omega (Refer Slide Time: 18:24). We have placed two unbalanced masses in such a manner that when theta is equal to 0 that means, when common crank is along x-axis then the two masses will be here.

Since, it is rotating double the speed the angle made with this direction will be 2 theta in both the cases; the amount of unbalance force will be generated if the two masses are lamda by 4 root 2 $m_{reciprocating}$; all this is (Refer Slide Time: 19:13). So, both of them will generate this force; it will be lamda $m_{reciprocating}$ omega square r; this 4 get cancelled; this is 4 omega square and multiplied by 2; so here it is root 2. The original unbalanced produced by two engines are the same; here this component will be sin 2 theta, which we already know.

Thus, at every instant the force developed by the engines can be balanced by two centrifugal forces rotating in the opposite direction. Since, the objective is to pack as many cylinders as possible in a smaller volume that gives maximum power density to an engine; there are arrangements in which these can be placed online. We will not discuss that but I will just indicate how the arrangement is.

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There will be a common crank or crankshaft; this will be one v; this will be another v; another v, like that (Refer Slide Time: 20:42).Each v will have two engines or two cylinders, this v will be placed in inline conduction. In the last session I showed that for six cylinders, four stroke cycle engine, it is possible to arrange the firing order in such a manner that crankshaft will be in such a shape that the primary, the secondary force and moment are completely balanced among themselves. Here also, we will find by suitable arrangement such as engine, may be eight cylinder engines, each we will have two and there are four wheels. It will be possible to completely balance up to second order the unbalanced forces and moments.

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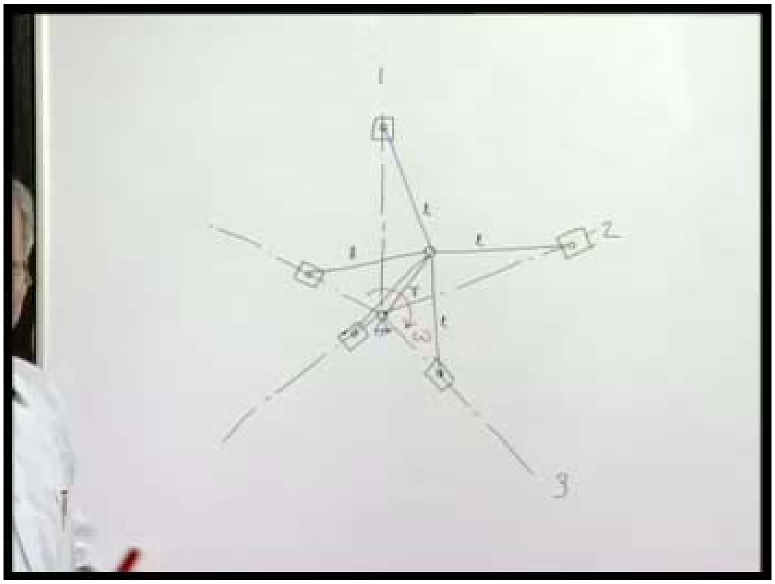

Next, we will take up one more type of arrangement which is fundamentally different from these two; which is at least having substantial importance in the aviation industry in the past. We will take up the cases where this arrangement is such that the cylinders or cylinder centerlines are placed readily in the form of star, it will look like this (Refer Slide Time: 22:50). If we want 1, 2, 3 and so on; all this we will drive with the common crank; these are the pistons of various engines; this is the common crankshaft; this is the common crank. Obviously, all the connecting rods will be of identical length and all the pistons will have identical masses and so on.

Here, this crank is rotating in this direction with an angular velocity omega (Refer Slide Time: 23:53). Now, such engines are generally used for aviation industries that means, for driving the propellers of planes. Being in that situation, two things happened; one is the rpm at which these engines rotate or run are much higher than many of the stationary IC engines. Secondly, since such engines are used for aeroplane, one has to be more careful in design, more perfect. Therefore, we will try to balance even higher order unbalanced forces and higher order harmonics. In IC engine, we are satisfied only up to second harmonic that is, up to secondary unbalance forces utilized. In this case, we will try to see if it is possible to utilize even the higher harmonics.

Before solving the problem, we will solve it both analytically and graphically, we will be satisfied if we can take up any one of these methods. I will use the graphical method to analyze the situation of unbalance or balance in a star engine. These are called as star engine or also called as radial engine. Before we do that we have to develop another techniques which is quite useful, this technique is called as the method of direct and reverse crank.

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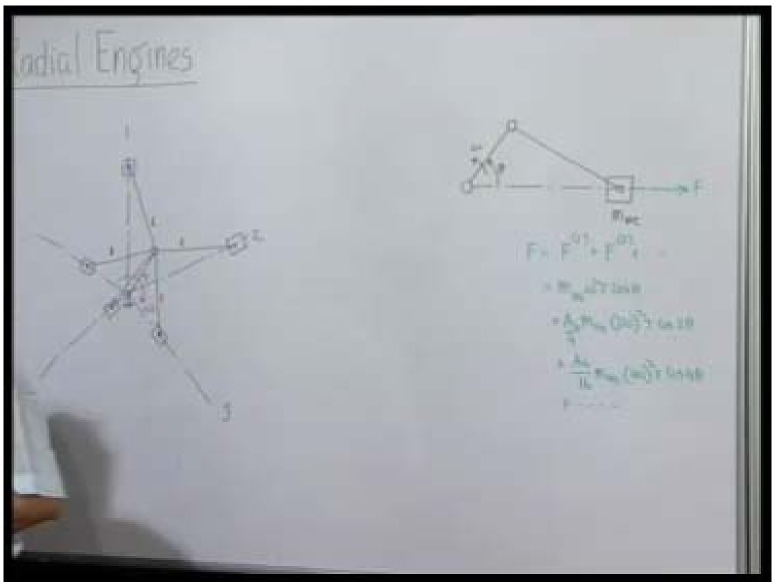

We have seen that the unbalance force produce by this reciprocating mass, we have to keep in mind that rotary part has been balanced; the unbalance force which it generates will have the entire higher harmonic. We know that the unbalance force in this direction is F, F can be written as first harmonic - the primary, second harmonic - the secondary and so on, which means $m_{reciprocating}$ omega square r cosine theta plus A_2 by 4 into $m_{reciprocating}$ cosine 2 theta plus A_4 like this, all are the higher harmonies which are present (Refer Slide Time: 26:53).

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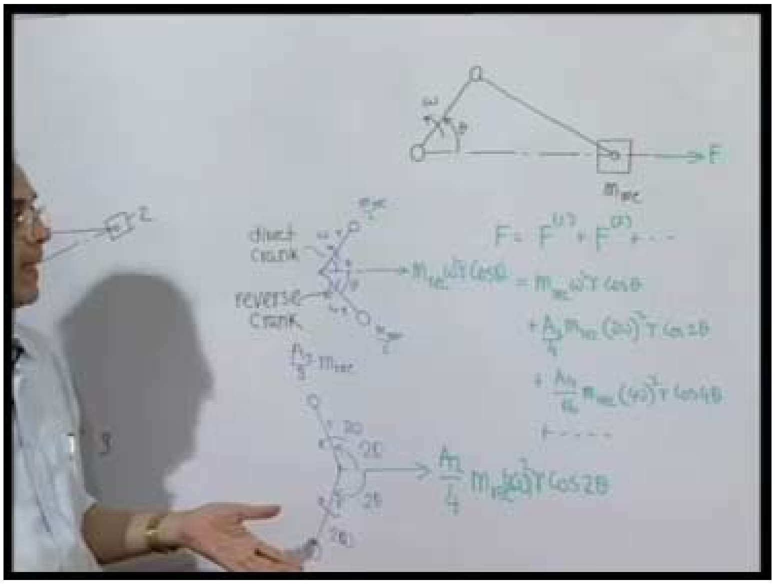

This can be represented in another way, what you can say, this primary is produced by the centrifugal force produced by two unbalance masses; both are equal; $m_{reciprocating}$, $m_{reciprocating}$..They will produce how much? I am sorry; they produce $m_{reciprocating}$ by 2 omega square r; this component will be cosine theta of that. Similarly, this centrifugal force will be $m_{reciprocating}$ by 2 omega square r into cosine theta; the resultant forces will be this one. Similarly, for higher harmonies we will do the same things, if you consider secondary and put here as A_2 by 4 or 8 $m_{reciprocating}$ at this r and their rotating force is double the speed, this angle is 2 theta; this angel is 2 theta (Refer Slide Time: 28:46).

The force in this direction will be how much? It will be A_2 by 8 into $m_{reciprocating}$ 2 omega whole square into r into cosine 2 theta and same things here. Together they will produce A_2 by 4 $m_{reciprocating}$ 2 omega whole square into r cos 2 theta. By this way, all the harmonics of this simple harmonics forces of different frequency can be produced by two unbalanced rotating masses, which is rotating in opposite direction and they are placed in such a manner that their inclination is either of this or accordingly the twice or four times of the crank angle (Refer Slide Time: 29:44).

Therefore, this is completely hypothetical situation. Here, this crank is called as the direct crank; this is rotating in opposite direction, so this is called reverse crank. As you can see the direct crank and reverse crank should be always a mirror image about this center line of a particular engine of a particular cylinder. You can imagine that the whole actual machine has been replaced by only corresponding direct crank and reverse crank. All the direct cranks are rotating in one direction with speed omega, not omega that means, primary cranks are rotating with speed omega, secondary are rotating with speed 2 omega, revere cranks are rotating with similar speed but in opposite direction.

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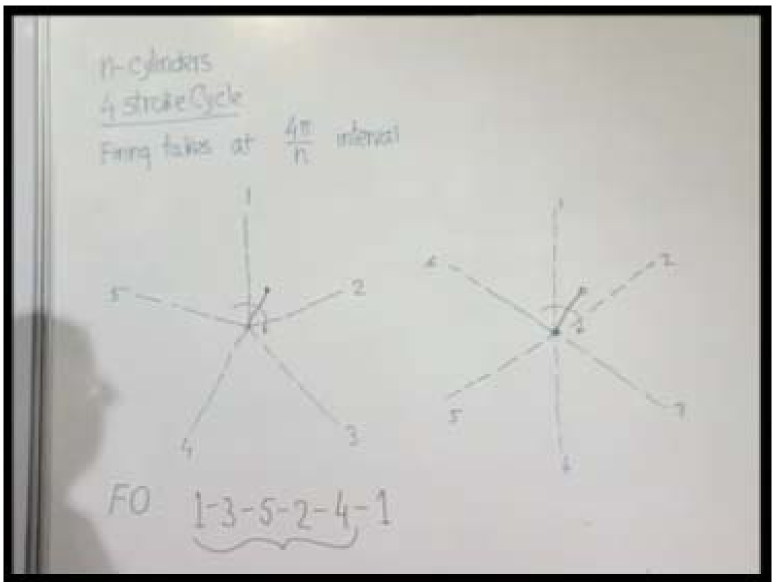

If you apply this technique to this machine then what will happen? I will draw it. Another thing we have to keep in mind that here is the firing order, if you have n cylinder, if we consider four stroke cycle then firing take place at what angle? 4 pi by n interval (Refer Slide Time: 31:36). If this is the first; this is the second; this is the third; this is the fourth; and say if five cylinder; say let us also have six cylinder; there are two cases, let us study the situation, this is 1, 2, 3, 4, 5, 6; here it is 1, 2, 3, 4, 5; here 5 cylinders and there you have 6 cylinder.

Let us see a very interesting thing, here we have common crank, here also we have common crank, this is the direct crank of primary of all the cylinder, because that always matches with the actual crank and actual crank is same for all the cylinders (Refer Slide

Time: 32:56). Now, which one is first? If it rotates in this direction say clockwise, let us see the firing order of first one.

What should be angle between two consecutive firing? 4 pi by 5, 4 pi by 5 means this one. Next one, when this comes here the piston will be at top most position; there can be firing so 3 will get fire (Refer Slide Time: 33:40). Next you need to go to this; another 4 pi by 5, then again this cylinder will be at top most position with maximum composition situation and there can be firing. Now 5, then again 4 pi by 5; then it will be next; 2, it will come here, 2 will get fire and then again 4 pi by 5, it will come to 4 and this will get fire, after that again this is over and again it will go to 1; say a complete cycle of firing is there.

If you have two stroke cycles, what would happen, after 1, 2 would have fire, 3 would have fire, 4 would have fire, 5 would have fire and then it will complete one cycle, because the consecutive firing angle could have been 2 pi by 5. So here, there is no problem, but here if you see it is two stroke cycle engine, but no problem 1 will fire then 2 will fire, 3 will fire, 4 will fire, 5 will fire, 6 will fire; when ever crank arranges itself along the particular centerline then that engine will fire.

I will complete one cycle in 2 pi rotation; but if it is four stroke cycle then what will happen? The angle between consecutive firing will be 4 pi by 6 that is, 2 pi by 3 that is, this one (Refer Slide Time: 35:00). First this will fire; then this will fire; then this will fire; then again this will fire; this will fire; you can see that 2, 4 and 6 will never get a chance to fire if we maintain planning at equal interface of time.

Therefore, one thing is very clear, if I use four stroke cycle engine then number of cylinder should be odd, if it is odd then there will be no problem in getting all the cylinder fired. In even number of cylinder, alternative cylinders do not get a chance to fire if we want to maintain equal angle between firing. To maintain the uniformity, in general we will find in radial engines we have odd number of cylinder that is something which you have to keep in mind.

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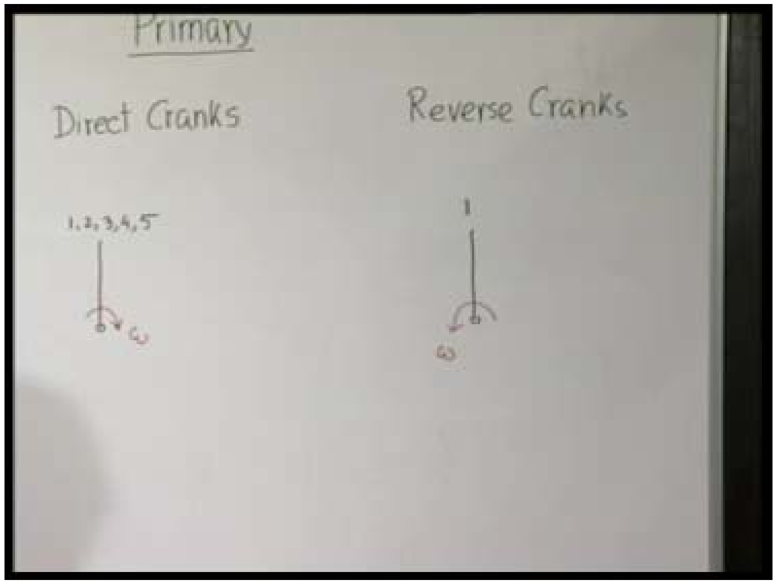

Let us see with the help of an example, let us say five cylinder engine, we will not take two stroke cycle, because they are not generally used. Let us see the direct crank of primary; primary unbalance; so direct cranks. You know the direct cranks are nothing but the actual crank for cylinder 1, cylinder 2, cylinder 3; all direct cranks are the same as 1, 2, 3, 4 and 5. Reverse crank: reverse crank are nothing but just the mirror image of the direct crank about that particular cylinder centerline.

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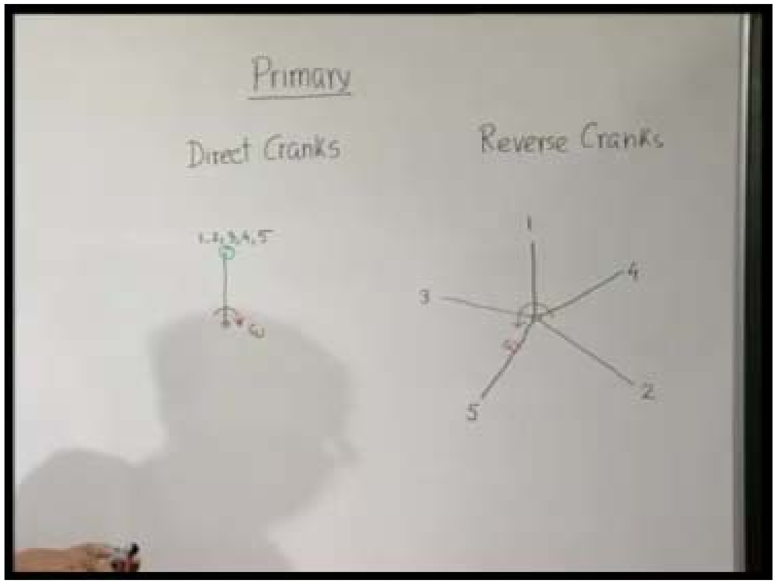

For cylinder 1, since it is along the centerline this will be the position for crank. You have to keep in mind that in the direct crank all are rotating at same speed, because they all are primary in this direction with speed omega. Here, they all are rotating in the opposite direction with speed omega (Refer Slide Time: 37:54).

What will be the reverse crank of the cylinder 2? Direct crank being here, so reverse crank will be definitely its mirror image that means, along 3. This will be the reverse crank of cylinder 2; because here it is the direct crank, so reverse crank will be just here. For 3, the mirror image will be along 5, for 4 it will be along this 2 and for 5 this is the direct crank, reverse crank will be along (Refer Slide Time: 38:46).

With the reverse crank of these 5 engines or 5 cylinders, what you can do? In the direct crank, this is an actual crank at particular instant, say at 1, then all the direct crank are here and the reverse crank will be the mirror image about the particular cylinder centerline. Since the direct crank is like this for all the cylinders, for cylinder 2 the reverse crank will be mirror image of this, it will be along this, so it will be like this (Refer Slide Time: 39:20).

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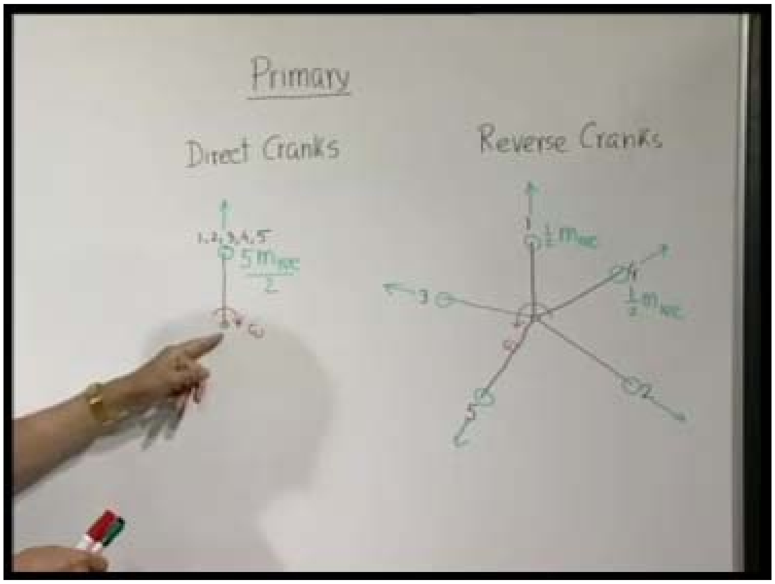

For 3 if we draw, we will find that centerline is bisecting this angle, so this will be reflected here and 3 will get here; similarly for 4 and 5, these are the reverse crank they are all rotating in the opposite direction at the same speed omega. Now if we place, how much mass you have to keep? It will be $m_{reciprocating}$ by 2, so five of them; it will be 5 $m_{reciprocating}$ by 2. The centrifugal force produce in this direction will be 5 $m_{reciprocating}$ omega square r by 2. On the other hand, replace half m $_{reciprocating}$ here, half m $_{reciprocating}$ here and so on. Then this centrifugal forces produce by the imaginary masses, which is utilized among themselves.

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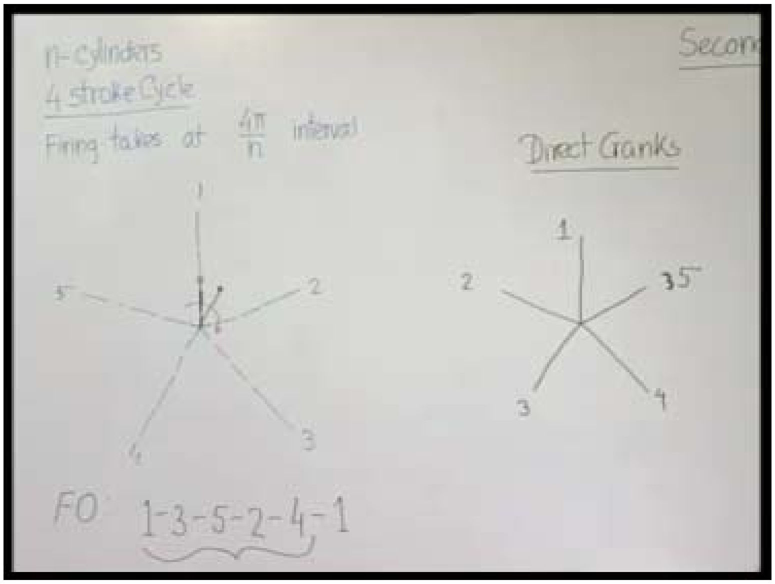

Thus the only remaining primary unbalance force is 5 by 2 $m_{reciprocating}$ omega square by omega square r and that can be balanced by placing counter weight just opposite to the actual crank; this is okay.

If we consider this second harmonic, then we will have situation like this (Refer Slide Time: 40:45). Let us consider the second harmonic secondary, the direct cranks and how they will look like. For 1, the angle made by this when it is here, so it is 0; so 2 into 0 will be 0; so the direct crank of cylinder 1 will be the same as that of 2. What is the angle made by this? It will be 1, if we call this as 1 unit then it will be 1, 2, 3, 4 unit; twice of that, which means 8 units; so it will be 1, 2, 3, 4, 5, 6, 7 and 8. The direct crank of cylinder 2 will be here, for 3 it will be 1, 2, 3 unit as the theta; so 2 theta will be 1, 2, 3, 4, 5, 6, it will come here; for 4 it is only two units, so it will become 4 unit, so it will go here; for 5 it is 1 unit, it will be twice, it was secondary.

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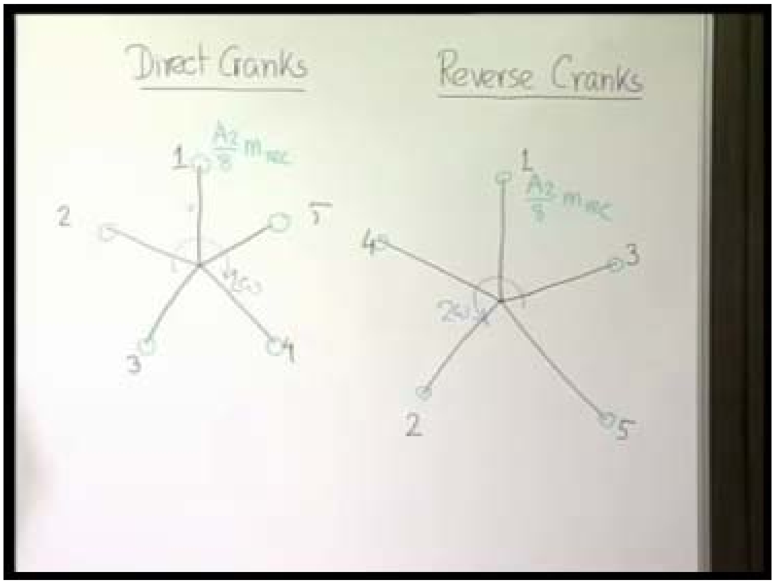

If we place then how much mass we have to place? A_2 by 8; this is r; same here; same here; same here; same here (Refer Slide Time: 42:45). This is direct crank; they all are rotating in this direction at speed omega. The centrifugal force produced by the eccentric masses placed at the crankpin of the direct crank of the direct secondary crank; you can see they are balancing among themselves.

If you now see the reverse cranks for first cylinder, it is the same reflection, because, the crank is along the; it is here (Refer Slide Time: 43:44). For 2, direct crank is here, so reverse crank will be here; for 3, direct crank is here, so reverse crank will be here; for 4, direct crank is here, so reverse crank will be here; for 5, this is centerline, direct crank is here, so reverse crank will be here. These entire reverse cranks are rotating at the same speed; sorry, this is 2 omega, 2 omega. If you now place again 2 by 8 $m_{reciprocating}$ at each crank pin, the centrifugal force produced by the masses attached to the reverse crank will also balance among themselves. Effectively, this means that secondary is balanced.

Following the same procedure, if you now take the fourth order and find out the direct and reverse cranks, then you can find out situation that whether both the direct and reverse crank indicate a completely balanced cycle or not. By this way, you can find out the balancing situation of an engine with multiple cylinders placed or arranged in the star formation, this is very convenient way of finding the balanced situation. Theoretical method is also there, where you can get all the results together, but I think this will be good enough (Refer Slide Time: 45:39).

Here, you also get some physical insight of whole things; it will be nice exercise for one to really see that in which level we first get the unbalanced force. We will find that for n cylinder engine up to n minus 1 th harmonic will be balanced except the primary, because we remember that the primary direct cranks are all aligned and that centrifugal force can be easily balanced by a counter weight. After the primary is taken care of up to n minus 1th order and all unbalanced forces will be neutralized among themselves.

I think this is the situation which we really wanted, where even hard harmonic are balanced, because the aeroplane engine need to be far safer and they should be subjected to lesser vibration. This kind of more rigorous condition is essential, because speed of operation of the radial engine in generally much higher.

We will stop here, we will not go to more complicated situations; cases where rotors are flexible or similar things. In radial engine, IC engines, we have seen what is the situation. We have also seen how we can actively attach some mechanism to an engine to produce unbalanced forces, which can balance or neutralize unbalanced forces of the original engine. We have also seen the importance of the firing order. Lastly, we have seen that in case of radial engine with four stroke cycle operation, we must always have odd number of cylinders, if we want the firing to be uniformly placed and all the cylinders are to be fired.