Dynamics of Machines

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Module-4 Lecture-1

Balancing of Single Slider Machines

In the last class we have seen that even for simplest possible machine which uses a single slider mechanism, the exact representation of the unbalance force is a complicated task. However, the situation is saved by the fact that it is possible to make some approximation without introducing too much of error and the problem can be somewhat tractable. What we have done is that we have replaced the actual connecting rod by an equivalent connecting rod consisting of two particle masses at the two ends, at the slider pin and at the crank pin.

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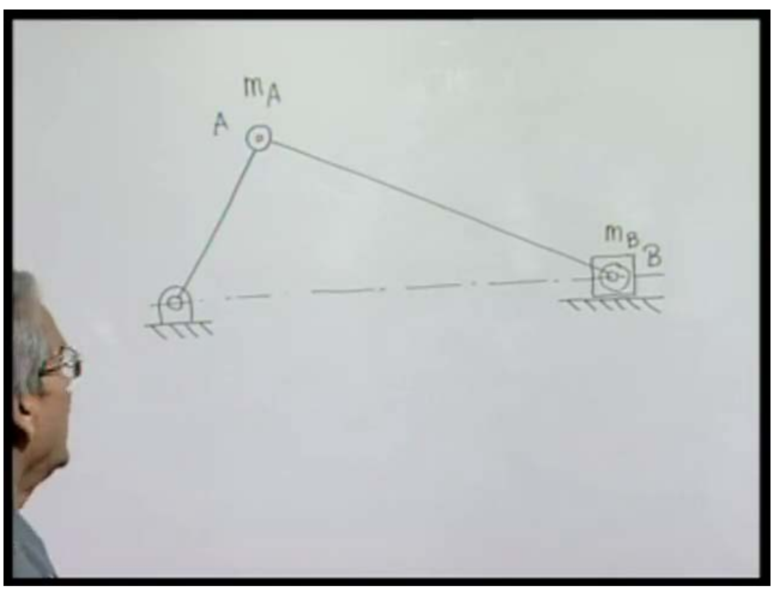

We have placed m_A at this point A and m_B at this point B, along with the slider and this becomes a massless rigid rod. In the process what has happened, all the matter contained in that machine is divided into two parts, one part, is purely rotating object, which can be easily balanced by providing a counter balance opposite to the crank. Once that is done, what remains is only this matter or this mass or slider which is executing this reciprocating motion. Then we have seen that if the acceleration of this the x 2 dots in this direction, then the inertia force, obviously will be acting in this direction. It is obviously simple to look at and much simpler to handle. But even then you should not forget that the expression for x 2 dot itself is again an infinite series. Even for a smooth operation, where the crank rotates at a constant angular velocity, the x 2 dot will be in infinite series containing all the higher harmonics.

We have seen that the inertia force or unbalance force is given by only its x component, so we call it Fx. Y direction unbalanced force because of rotating mass has been taken care of by this and this is equal to $m_{reciprocating}$ omega squared r into cosine theta plus x times cos 2 theta. It will have two parts. Or in a way we can say that force is acting in this direction, that means if Fx, that is the force acting on the whole system, as an inertia force in this direction. If we call that as Fx, then Fx is nothing but $m_{reciprocating}$ omega squared r cosine omega t plus lambda $m_{reciprocating}$ by four, two omega squared r cosine two omega. I think to indicate that it is not exact but it is approximate, as we have revoked or ignored the higher harmonies terms. This we have called the primary and this is called secondary. In most cases, as mentioned earlier, it will be enough for us to consider up to secondary continuous.

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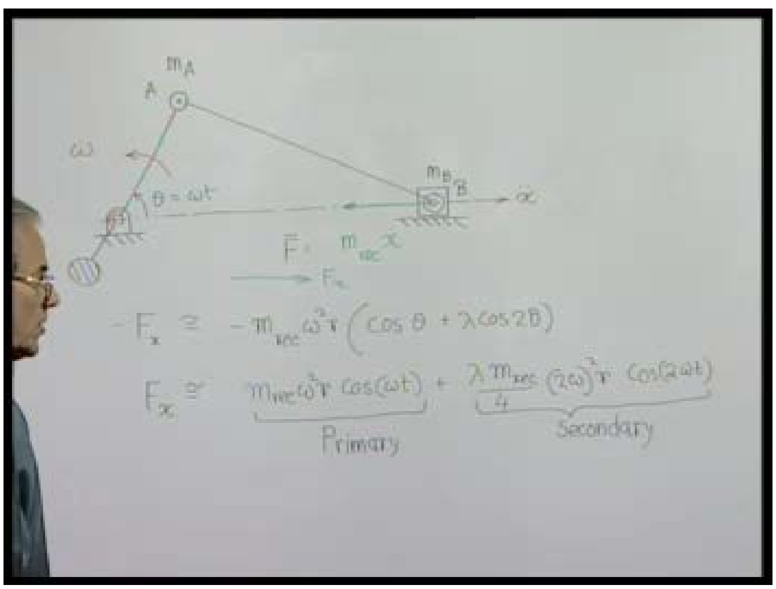

We go to such situation, where much higher accuracy is required, like say aero engine. Such cases, where we must have for more accuracy than normal, auto engine, are other machine like stamping machine, punching machine or compressor.

What we will do next, which will useful at a later time, try to represent the inertia force due to this reciprocating mass with the help of rotating counter, by the help of rotating vectors, as you have done for the harmonically carrying functions of time in many cases.

As you can see, this primary force if we consider that reciprocating mass has been kept at the crank pin and this is the crank, which is rotating with an angular velocity omega like the actual crank and therefore this angle will be omega t. If we consider initial condition, that means t is equal to zero, the crank was aligned with the center line of the engine or the system. Then what will happen, this rotating mass will produce an inertia force which is equal to and its component along the x direction. If you call it primary, this will be equal to $m_{reciprocating}$ omega squared r. Of course you have to reciprocating keep in mind that it is one this component along x direction that is what really exists. The cross component in the y direction is not existing.

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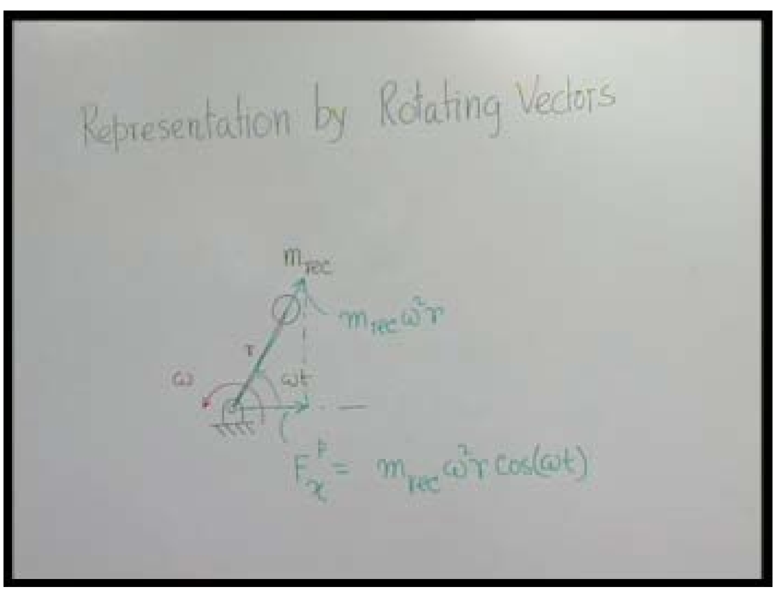

Now this will be useful only when we solve problem with multiple cylinder or engines and try to arrive at results employing the same kind of techniques what we have used in rotary balance. If this is the primary component of unbalance force produced in the system, what about the secondary? Now to present the secondary one, we use another crank and we keep the mass which is lambda $m_{reciprocating}$ m by four at distance r. So it $m_{reciprocating}$ by four into lambda at the distance of r, and we call this as secondary crank. This is primary crank. These terminologies we will useful at later time and let us allow this secondary crank to rotate at a speed double the speed of the primary crank. That means with a speed two omega. If you use the condition that a time t is equal to zero, secondary crank was also aligned with the x direction, then this angle will be simply two omega t.

What is the force here? This is two omega. The force which is produced by this rotating mass in this case is again a centrifugal force or unbalance inertia force. That magnitude is lambda $m_{reciprocating}$ two omega squared r by four. Its component, along the x direction, which we may call as secondary unbalanced force, which will be equal to lambda $m_{reciprocating}$ r into two omega squared by four into cosine two omega t. Thus we can see both the secondary and the primary forces can be imagined to be components along x direction of hypothetically two masses placed the tip of the primary crank, which is the actual crank and the secondary crank, which rotates at speed double of the primary crank, of an same radius. Then these components along the x direction will be what we get in the real system or actual single cylinder engine with connecting rod replace by hypothetically connecting rod of zero mass and particles at the two ends.

For balancing these forces, what we feel, is that this force is being produce by as a component of the force produce by these rotating mass, then what happens if you apply output if you a put counterweight to this. Now let us see, this is our single slider machine. The rotary part has been already balanced by a counterweight. I am now putting another counterweight, which $m_{reciprocating}$ r. That is, unbalancing kept here, the product will be equal to this. Then what happens, the unbalanced force produced by this is in this direction which is $m_{reciprocating}$ omega squared r cosine theta. This will produce a force $m_{reciprocating}$ omega squared r. This component will be $m_{reciprocating}$ omega squared r cosine theta. This angle is theta. Now what is happening is that the actual original unbalanced due to the reciprocating mass and the centrifugal unbalanced force produced by this balancing mass attached as a counter weight is going to in effect, the resultant system will now be subjected to an unbalanced force which is in the y direction.

This system now, the unbalanced force is in the y direction. The force in this x direction has been balanced but a force in the y direction has been generated. This is not an exactly satisfactory situation but sometimes this is useful where a force in this direction may less desirable than the force in this direction. For example, suppose the mounting of this machine or the engine is something like this, if we have the engine here, then a force in this direction may be less acceptable from the point of view of the vibration of the support structure. Whereas force in this the normal direction, in this direction, may be preferable because the support structure may have much higher rigidity in the direction normal to the ground. Therefore, if we can generate a force in this direction but completely remove the force in this direction, it is sometimes an acceptable solution. The other solution which of course can be considered is that you do not completely nullify the x component of the primary unbalance force but only a fraction of it. So in that case, this will be say some fraction let me say this kai $m_{reciprocating}$ r, where kai is less than one. So what will happen here, this force what will be generated will be kai $m_{reciprocating}$ omega squared r and unbalanced force in this direction is $m_{reciprocating}$ omega squared r cosine theta for $omega_t$ sine $omega_t$.

So then what happens finally, the system we will have a smaller force in this direction and in this direction. So the engine, represented by this outline, the unbalanced force in the engine in the x direction is this much, which is obviously less than this. But we have also produced another unbalanced force in the y direction, which is also less than this in magnitude. So sometimes this partial balancing is adopted to reduce the level or the amplitude of the accelerating unbalanced force. However, you can also stretch your imagination and think or consider a complete balancing of the unbalanced force.

If you have to do complete balancing of unbalanced force, then obviously two things you have to keep in mind. You have to neutralise the force without generating a force as you have done in this case, by adding a simple counter weight. Secondly you have to also keep in mind that in this way you have only neutralised the primary force. To neutralise the secondary force again, what we have to do is we have to create physically another crank, which will be the secondary crank and add another mass. But I think the system becomes very complicated.

However in the special situation where it is absolutely essential to neutralise as much force as possible we adopt such techniques. This technique is called often Lanchester balancing technique. This is our single slider system. What we did was we have attached a gear one of some radius. We have attached or connected an identical gear with this, like this, which is again a composite one, which is a smaller gear attached to it directly. So if it is gear one, this is gear two, this is gear three, this is gear four. So gear one and two are identical. Then, two and three are attached, three and four are identical. Now this angle is theta. What we do now is we attach two masses. These masses are $m_{reciprocating}$ r. These unbalanced quantities, which are given, r equal to $m_{reciprocating}$ r. That means the product of this mass and eccentricity they are equal to $m_{reciprocating}$.

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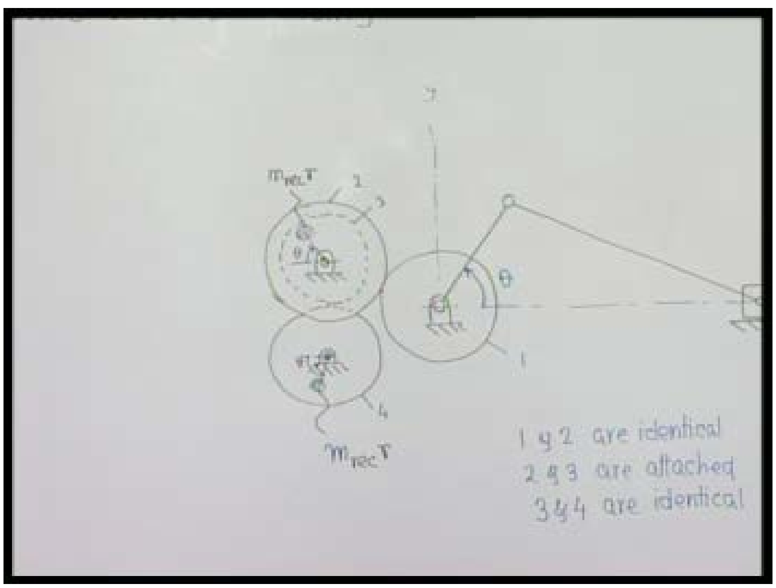

They are also placed in such a manner they make angle, that this line joining the respective centers with these masses, with horizontal line like this, they make theta. This is same as this. Now let us consider the motion. Since this is rotating in this direction with an angular of velocity omega, this gear two and along with that three, will rotate in this direction same speed omega. Since three and four are identical, this will also rotate with speed omega.

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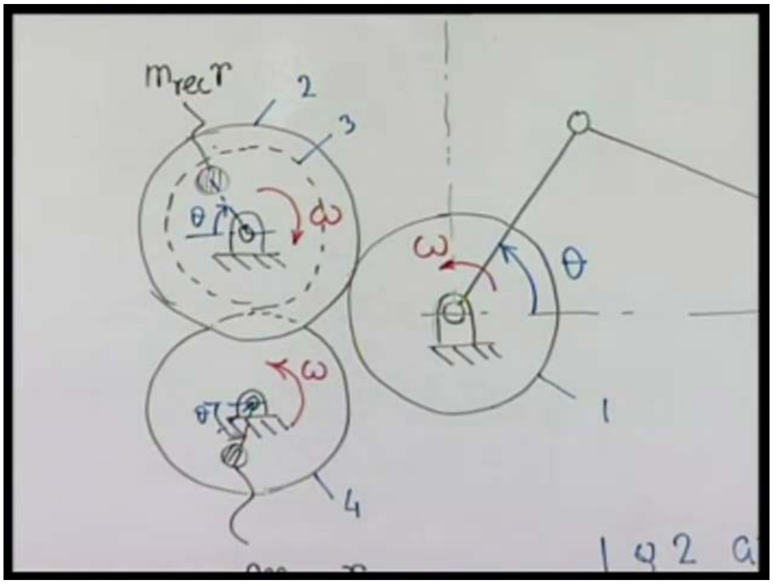

Now what happens, this will produce the centrifugal force $m_{reciprocating}$ omega squared r. This will also produce and the resultant of these two forces will be $m_{reciprocating}$ omega squared r cosine theta. The x component of this is in this direction cosine theta half $m_{reciprocating}$ omega squared r component of this along x direction is again half $m_{reciprocating}$ omega squared r cosine theta.

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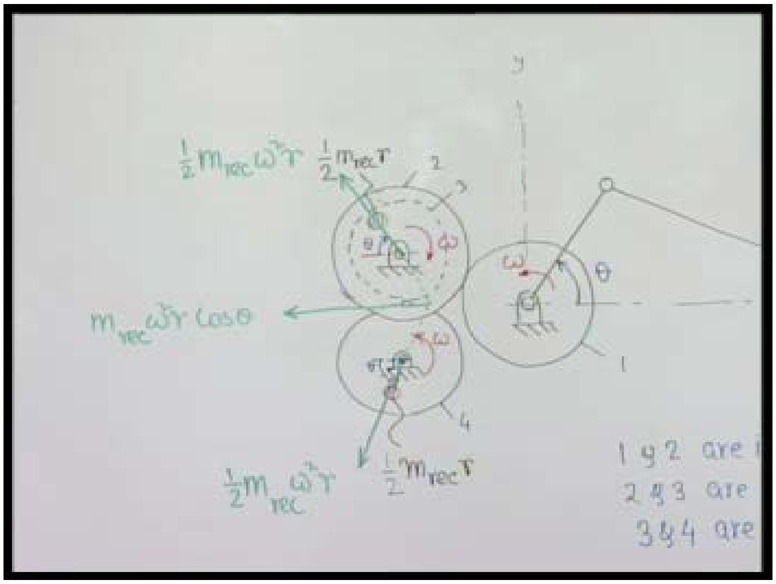

Combined that is the 4. This has produced this instant force which is. So this force unbalanced inertia force generated by this slider and the resultant of the unbalanced centrifugal force produced by this Lanchester Balance mass is placed here. We will neutralise the effects of each other. As you can see resultant of this and this will be in this directions and the y component of this and y component of this will be neutralised among them. You also have to keep in mind that they must be aligned so that they do not produce a couple. In this gear systems and this unbalanced mass system can completely neutralise the primary. If you are taking that much of an initiative to neutralise the primary force, it will not be appropriate to leave this secondary unbalanced force without any compensation.

In such cases, what we do is we put another pair of gears whose radius is half the radius of gear one and two. So this is five and this is six. Five and six have radii half of that of one. What will happen if this is rotating with an angular velocity omega in this direction? This is an idler here. This will rotate in this direction and this will again rotate in this direction and this will rotate this direction, but this speed will be now be two omega. These two gears will rotate at speeds double the speed of the crank. What we can do is we can attach some masses here, which is lambda by eight $m_{reciprocating}$ into r, same thing here, lambda by eight $m_{reciprocating}$ r. The angle is such, this is two omega t, similarly this will be two omega t. Time t is equal to zero there are all aligned here. This was here this was also here.

Since it is rotating at two omega speed, this will rotate to an angle two omega t, this will rotate to an angle two omega t in the opposite direction. What will be the force produced?

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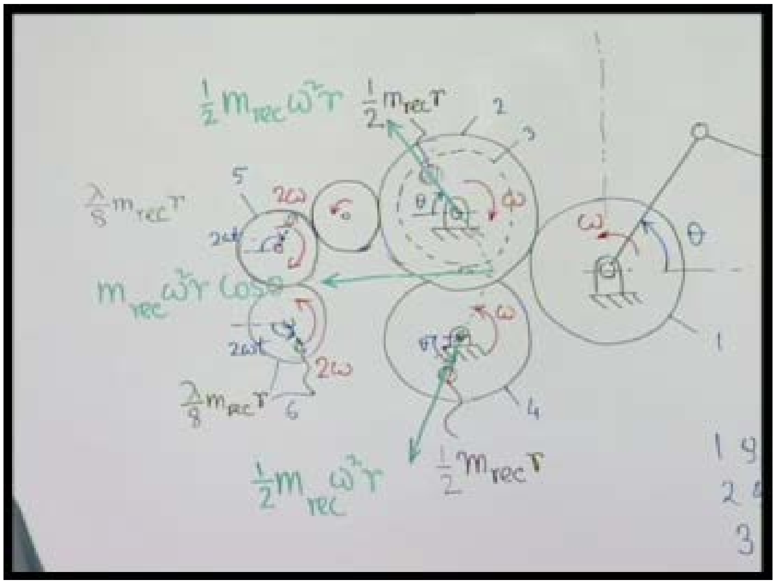

Here, of course, this force is lambda by eight $m_{reciprocating}$ r two omega squared. Same is this and the resultant in this direction will be cosine component of this, cosine component of this, y components r neutralised among them. It will be lambda by four $m_{reciprocating}$ r two omega squared cosine two omega t. The secondary force in this direction was lambda $m_{reciprocating}$ omega squared r cosine two theta. You can find that omega t equal to theta. So this will again completely, at every instant, neutralise the secondary.

The primary is being neutralised by this set of balanced masses and the secondary is being balanced at every instant by this set of balanced masses. This way, you can find that a complete balancing of the reciprocating unbalanced forces of a single cylinder machine can be achieved up to the secondary. However, as I mentioned, this active balancing technique developed by Lanchester is a very elaborate arrangement and it is done only in very special cases. In normal situation, we always try go for simplest possible mechanical construction in the form of counter weight, where you can either convert the horizontal or x component of unbalanced force into a y component or you partially balance it. So that little bit of x component all balance remains and little bit of y component is generated.

Now situation is much better, as you know when a machine is designed keeping its smooth functioning in mind, in such a way that we do not use a single slider system. If you use multiple slider systems then you can really appreciate the fact, that it will be possible for the unbalanced forces generated by the reciprocating masses in the various component software machines. That means each one composing one single slider system. They can neutralise among themselves. The whole system will be self-balanced. That is why we will find that in engines where you have more than one engine or compressors, where you have more than one cylinder. This technique is employed where the unbalanced forces of various cylinders or engine parts will be balancing among themselves. This we will take up in our subsequent discussions.