Dynamics of Machines

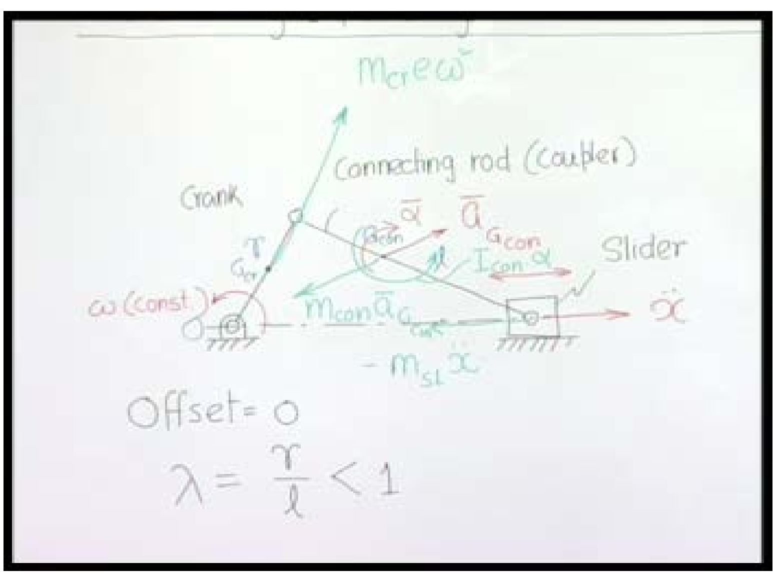
Prof. Amitabha Ghosh
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Module - 4 Lecture - 2

Single-Cylinder Engine Balancing

We have discussed so far systems where bodies rotate about an axis and you have seen the effects of unbalance present, how it can develop dynamic forces on the supports. We have also looked into the matter of balancing such systems. Next, you have to consider other types of mechanisms and machines, where the components do not involve only in rotation, but have other kinds of motion. Of course, the problem of balancing or even the problem of investigating the balancing of mechanisms is a very complex problem. However, in a large number of situations, mechanical machines, systems or engines, they employ the slider-crank mechanisms. So next, what we will do? We will take up those cases where the system constitutes of a single slider mechanisms, which you call slider-crank mechanism in kinematics.

(Refer Slide Time: 01:36)



This single slider machines are very common in large types of machines, then compressors, engines, they all employ this mechanism. So, I think it will be appropriate to take up this particular case and investigate it unbalance and also look into the ways and means of balancing the unbalance present.

A typical single slider machine will look like this. This is the crank, which rotates with a constant of angular velocity as a special case, omega, which is constant. This rod object, which connects the crank to the slider is called the connecting rod or coupler. I hope you are all familiar with this mechanism, where the crank continuously rotates and the motion of this crank is transferred to the slider with the help of the connecting rod and the slider executes a reciprocating motion. We find there is one body that is the crank, which is making a pure rotation, so it is a rotating body whereas, the slider is executing a pure translatory reciprocating motion, so it is a reciprocating body. The connecting rod, which causes the serious problem is a general floating bodies which execute rotation and also translation. Depending on the situation, sometimes this can be input like a compressor or a machine like, stamping machine or punching machine or sewing machine. This is the input member, where power is given and this is the functional part, which does the job.

In case of engine, for example on the other hand, this is the input body where gas pressure produces this kind of motion, which ultimately rotates the shaft connected to this crank, which is called the crankshaft and that drives the output system. Here, the dimension normally as such as we have shown. Here, the crank radius is r and Cg of the crank is say here. The connecting rod and its center of mass is somewhere here, the center of mass of the slider is here and it is really not important because for a translating body the location of center of mass has no consequence. The length of the connecting rod may be l and another important thing, which you should notice in this case, that you have treated a case where the slider is reciprocating along an axis, which passes to the origin O, which generally is not the case, it could be here or here. That means, with certain amount of offset, but since offset cases are not frequently used, so we are using the most frequently used situation, where the offset is 0.

Therefore, we have to keep these things in mind that offset is zero and also you should keep in mind that the connecting rod length and the crank ratio is r by 1 is less than 1, which is also generally the situation. We have seen that connecting crank is purely rotating body, slider is purely translating body and the connecting rod is executing both rotation and translation. What will be the nature of unbalance forces? Since, there are three bodies let us examine the problem for these three bodies. Here, the unbalance force is simply the inertia force of a rotating object, which is nothing but the centrifugal force. Therefore, the unbalance due to this crank will be nothing but the centrifugal force, which is m_{crank} into e omega square. We have seen this in the previous cases of the totally unbalance.

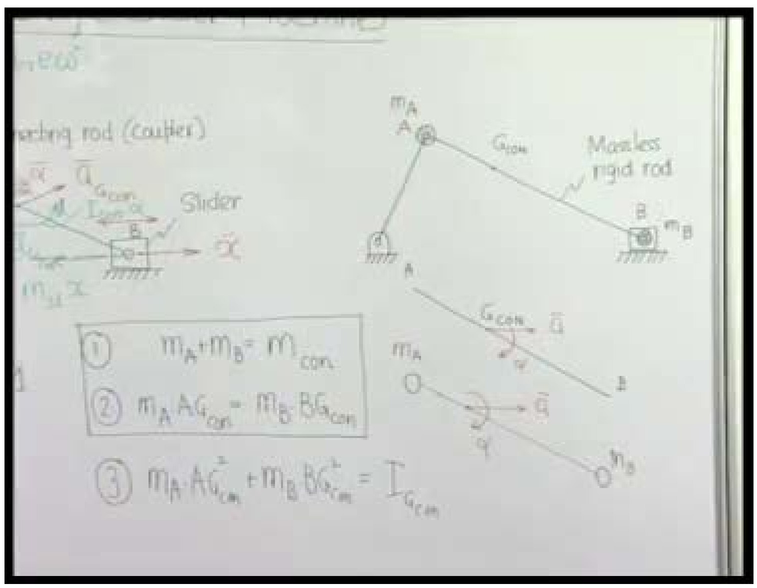
On the other hand, the unbalance force due to the slider, which at this particular instant may be having acceleration in the direction say x two dot. So, the inertia force of the slider will be minus m_{slider} into x two dot, but in the opposite direction, so it will be in this direction. To find out the inertia forces due to the connecting rod, we have to see, what is the instantaneous acceleration of the center of mass? We can call this as acceleration of the center of the mass of the connecting rod, also perhaps there will be some kind of an angular acceleration, alpha of the connecting rod. At this instant, therefore the unbalanced force produced by this will be felt by the support of the bearing structure.

It will be $m_{connecting\ rod}$ into $a_{G\ subscript\ connecting}$, that will be force and it also produce unbalance movement, which will be nothing but the moment of inertia of the connecting rod into alpha opposing the same support. Therefore, you see that even for the simplest mechanism like a single slider machine or single slider mechanism, the total unbalance effect, what the support of the whole machine is going to experience will be rotating unbalance force due to the connecting rod or due to the crank, a reciprocating or oscillating unbalance force, which is m_{slider} into x two dot at a particular instant and x two dot continuously change direction and magnitude. So, this force also will be a fluctuating force.

A very complex force, which is a force generated by the connecting rod, which is $m_{connecting}$ a_G at this instant, but this a_G of the connecting rod is also continuously changing and fluctuating, so this will be also continually changing in fluctuating force. Angular acceleration you can say the connecting rod is making an accelerator motion like this between its extreme positions up and down. So, this will produce a moment due to its angular acceleration at any instant. So, it is a very complex situation. What you can try to do is consider an equivalent system, which is kinematically equivalent but which is represented only by a purely rotating body and purely a reciprocating body.

In other words, what I mean to say, that we replace the connecting rod which is the problem maker, because it is a floating body, by two concentrated masses or particles placed at the two ends, that means here and here.

(Refer Slide Time: 11:52)



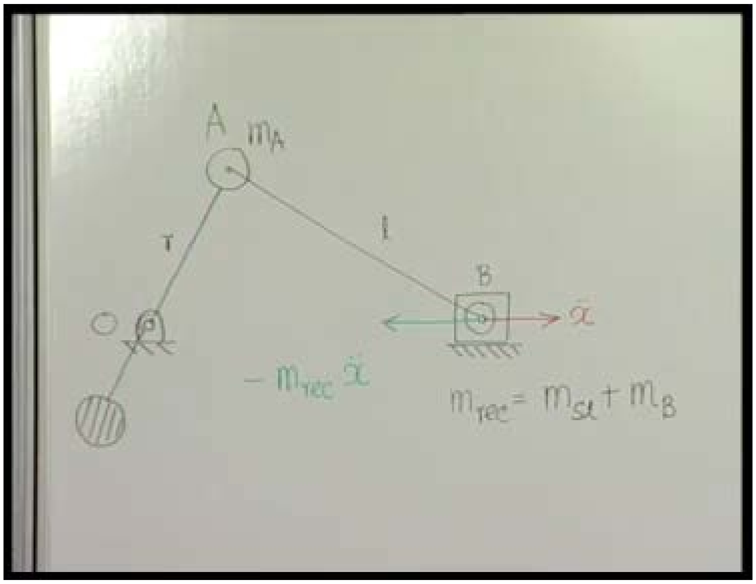
What we propose to do next is we have replaced the actual connecting rod by a hypothetical connecting rod and the hypothetical connecting rod consists of two concentrated masses at this end, which is m_A and this end, which is m_B. So, that this actual connecting rod becomes dynamically equivalent to this, so that the distributed mass here is represented by two concentrated masses. But in doing so, we keep the dynamic equivalency in time. What will make this dynamically equivalent to the actual connecting rod? We say a particular object is dynamically equivalent to other object, if same set of forces and moments produce the same set of motions. So, how do we achieve it? Suppose a force and moment acting on this produce acceleration of this center of mass and on angular acceleration of the object, so a set of forces and moments produce acceleration here and an angular acceleration. The same set of forces when applied here should produce the same acceleration of the center of mass at the same location and same angular acceleration, then we will not able to feel the difference between these and this in matter on dynamics. Of course there may be other changes, but so far as our discussion on unbalance source is concerned, it will not show its effect.

Now, how can we achieve that? This equivalence will be possible when the total mass of the replaced system m_A plus m_B must be equal to the total mass of the connecting rod. This is the first condition and the second condition is the location of the center of mass here and location of the center of mass here should be also be unchanged, so that the moments produced about center of the mass due to certain acting forces also remain same. To satisfy that, ensure the center of mass is at the same location as $G_{connecting}$. The same set of moment, which is produced about this should produce the same angular acceleration. That will be possible when the moment of inertia of this equivalent system about the center of mass will be same as the moment of inertia of original connecting rod about the center of mass. That is, if $I_{G\ subscript\ connecting}$ is the moment of inertia about the center of mass of the original connecting rod, it must be equal to this. So now, what we have done, we have placed two concentrated masses, not at any two locations, but at this which you call the crank and this slider. The locations of A and B that means, where we have placed concentrated masses are fixed.

Therefore the location of the G is also fixed. What are the only unknowns available to us, for our choice is m_A and m_B. AG, BG they are all decided. So, if there are only two unknowns which will have to satisfy three equations, it is not possible, we know that. Therefore, the only thing at our end which is possible is that, we satisfy only two conditions out of these three and hope that the error introduced will not be very serious.

It has been found that if we satisfy these two equations and this is not taken care of exactly, then the effect or the dynamical mismatch between this and the replaced system will be minimum. Therefore, what we have to do? Now onwards, we always treat the connecting rod has been replaced by two concentrated masses one at point A, other at point B. In such a way that if total mass is same and the location of centre of mass is also the same. The moment we do that our m_A and m_B can be calculated and from these two positions.

(Refer Slide Time: 19:00)



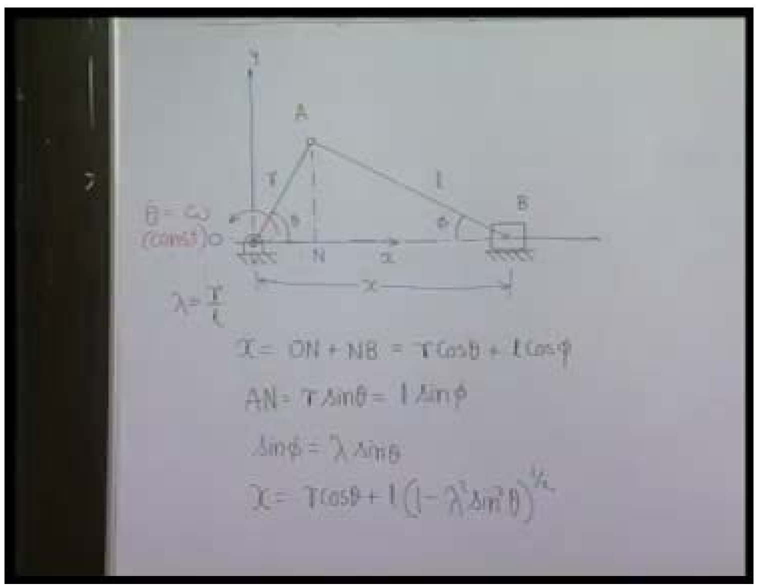
So now what we have? We have an object here, the original crank and concentrated mass at location A and this object is only rotating object. There is another matter, which is here which is the original slider and the portion mass of the connecting rod at location B. This becomes purely reciprocating mass. The original system is now replaced by a system which is composed of purely rotating object and purely reciprocating object which is definitely much simpler and we will now attempt to handle this problem.

From this point onwards, you should also keep in mind that if force produced by this, purely rotating force can be easily taken care of by placing counter weight. Therefore, this counter weight placed at opposite end of the crank will balance not only the original crank but also the opposite mass of the connecting rod m_A here. The only thing what remains that can produce unbalance effect are reciprocating mass, which is $m_{reciprocating}$ is the mass original slider plus opposition mass of the connecting rod at location B.

Therefore, the only unbalance force, which will be $m_{reciprocating}$ into minus x two dot where x two dot is the acceleration at this instant.

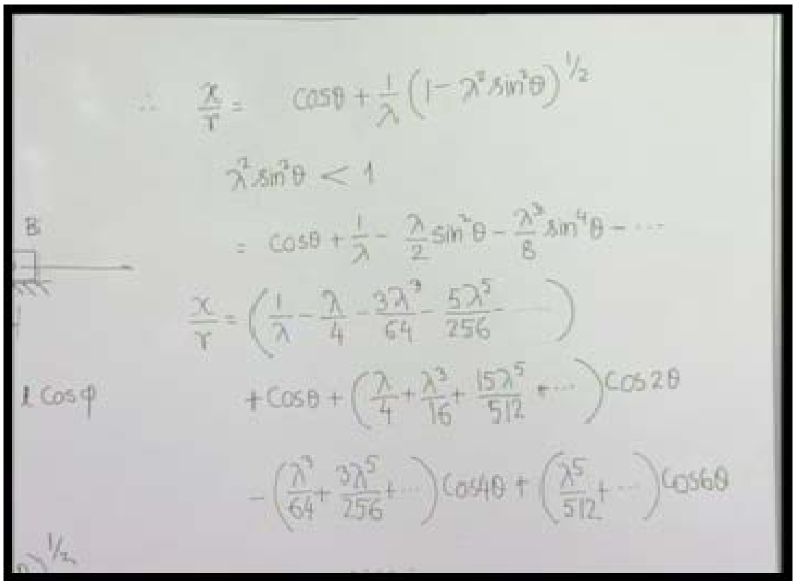
We should keep in mind that x two dot in a positive sense in the direction. The moment of inertia force will be in this direction that will be ambulance force. When x two dot is negative, then of course this force will be in other direction. Therefore, now we will have only unbalance force in this system when rotating part is taken care of by a counter balance is the only reciprocating inertia force. So next, we have to find out magnitude etc of this reciprocating unbalance inertia force and to do that we need to have the expression for the acceleration of the slider.

(Refer Slide Time: 22:20)



This is our problem, which is the slider-crank mechanism, now we are not bothered about mass etc. Our objective is only kinematic analysis. What we have to find out is the expression for the acceleration of the slider at any instant, which is given by the angle theta, when this crank is rotating at a constant angle of velocity theta dot which is equal to omega. Now, here we know that this distance from the origin slider, which is x. x can be written as obviously ON plus NB and that is equal to, how much is ON, it is r cosine theta plus, how much is this, it will l cosine psi. Theta and psi of course are related by the expression which we just know saw. If you drop a perpendicular from a and this line, so this length AN is equal to r sine theta and is also equal to l sine psi. Therefore, we find and we can express sine psi as lambda sin theta, of course lambda is equal to r by a. This cosine psi, we can then replace x will be, r cosine theta plus, l into lambda, therefore it will be 1 minus sine square psi, that is lambda square to the power half.

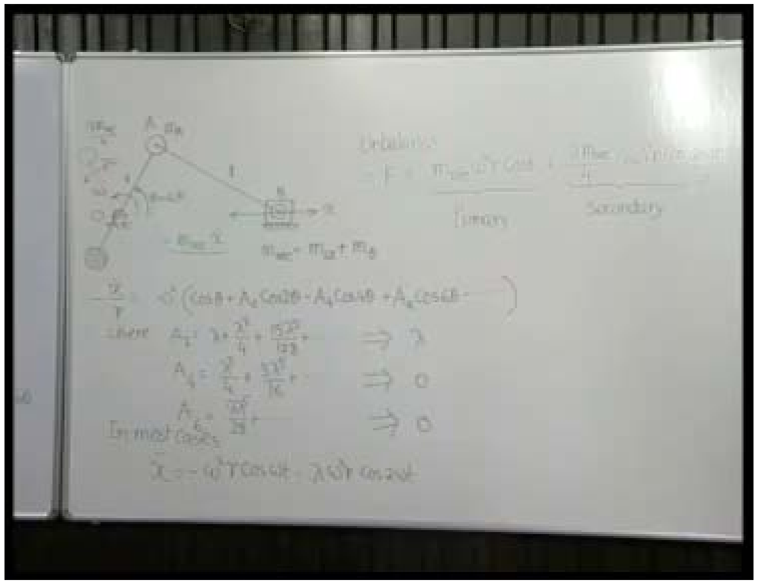
(Refer Slide Time: 26:22)



This we can write as, x by r will be equal to cosine theta plus 1 by lambda, 1 minus lambda square sine square theta to the power half. Now, lambda square sine square theta is definitely less than 1. Therefore, we can expand it in the form of a series. If you expand it we will get, equal to cosine theta plus 1 by lambda minus lambda by 2, sine square theta minus lambda cube by 8 sine to the power 4 theta and so on. There can be some trigonometry manipulation done, which I will not go into it, but you can show that x by r can be written in this form (Refer Slide Time: 27:46).

What we have done? Is that, in place of sine square theta sine four theta sine to the six theta and so on, you replaced the whole series in term of cosine theta, cosine two theta, cosine four theta, cosine six theta, which can be consider as the various harmonic components, it is nothing but a series. That means, where the x by r has been represented in the form of infinity series with harmonic terms. This is the constant term this is the first harmonic, this is the second harmonic, this is the fourth harmonic and this is the sixth harmonic and so on.

(Refer Slide Time: 30:07)



Our object is to get acceleration. So what we have to do? We have to differentiate this x twice with respect to time. Doing that what we will get? After we have differentiated this twice with respect to time, we can expresses the acceleration in terms of an infinite series consisting of only primary and then all the even harmonies. The coefficient of the various harmonies tells us about the strength of the harmonies, that means, the amount of continuation that harmonies makes to the acceleration term. So, you can see, even for a simplest possible case like a single slider mechanism, the acceleration of slider is actually complex quantity represented by an infinite series.

If you have to work with the whole series, which is going to be a serious problem. What therefore you should consider is the practical situation? In most practical situation, the speed with which the cranks rotate like normal engine, stamping machines or compressor. In such cases, what happens? The speed is such (Refer Slide Time: 32:53) and the lambda value is less than 1. We can ignore any term containing higher order of lambda. That means we can ignore from lambda cube onwards. If lambda is say 0.5 or 0.3, so lambda cube by 4 will be 0.3 to the power three, so it is approximately point 0, divided by 4, so it is 0.01, so 1 %, immediately you can ignore this terms. The moment it happens, we are considering only up to lambda, at the most lambda square, but lambda squared is not here. We ignore from lambda cube onwards and when A_2 is equal to lambda so this becomes simple lambda, A_4 obviously becomes 0 and A_6 which is the lowest, which is 0. Therefore we need not consider beyond the second harmonic. In such cases that is also a situation with large number of cases. So, in most cases, we can express x two dot as minus omega square r into cosine omega t.

You can see this angle is theta, which is nothing but omega t, if you assume t is equal to 0, crank was aligned with the cylinder center line, minus lambda omega square r cosine 2 omega t. So the unbalance force is given by a force, which is equal to m reciprocating x two dot, so x two dot is minus sine. Now, it will be $m_{reciprocating}$ omega square r cosine theta, plus lambda $m_{reciprocating}$ by four (Refer Slide Time: 36:19). This is nothing but harmonic force, whose magnitude is equivalent to a centrifugal force, where the reciprocating mass is placed at the cranking and the crank rotates at shown speed omega. Then its component along this cylinder center line x direction is the primary component, which is the primary. This force, which is the second harmonics, we call the secondary. Again we considered as the component along x of that centrifugal force when a mass of reciprocating into lambda by four is placed at the tip of the secondary crank. Secondary crank will be of something of the same radius, but which rotates at a speed which double the speed of the primary crank and which was of course aligned with this t is equal to 0, so this angle is automatically 2 omega.

So therefore, the unbalance of a single cylinder or single slider machine can be reasonably well approximated as a force along the center line that is x direction, which consists of two components, one is the primary, other is the secondary. We will need to consider the harmonics, when the speeds of operation are much higher, when the accuracy desired is much higher. For example, the aero engines and in such cases, they are of course will be essential for us to consider the higher harmonics part, otherwise in most of the situations of engines and machines, we do not have to go beyond this secondary component.