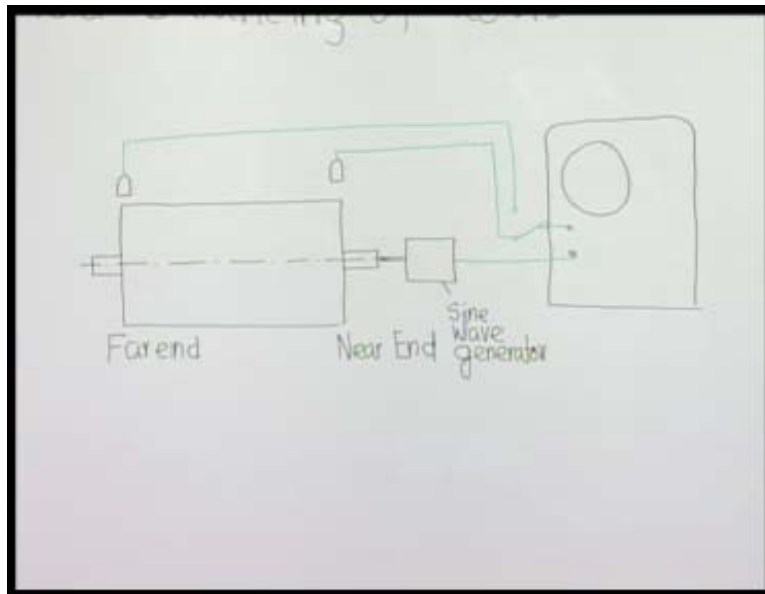


Dynamics of Machines
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Module - 3 Lecture - 4
Field Balancing of Rotors

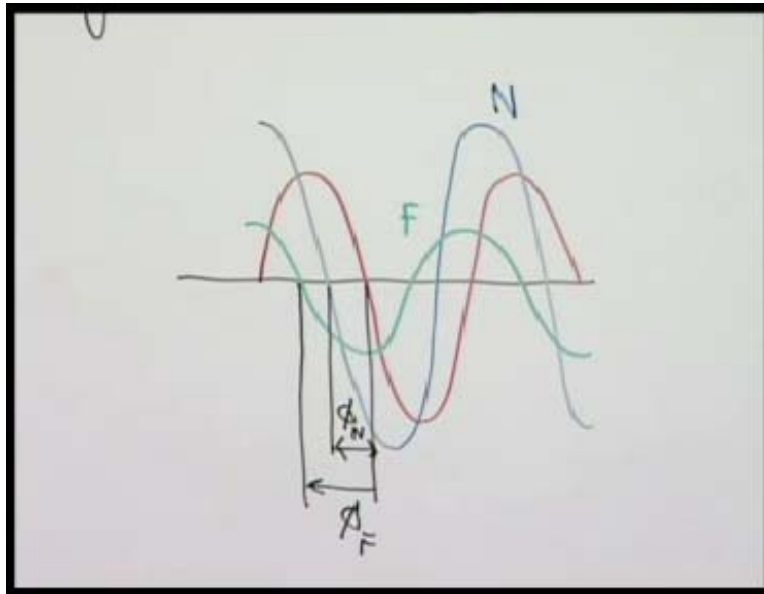
In the last lecture, we have seen how it is possible to dynamically balance rotating objects, which are of large size and it is not possible to place them on balancing machines. Now we have seen how disc-like objects of large size can be balanced in its own position that is what we call as field balancing. In today's lecture is that the same technique can be extended to rotor-like objects for complete dynamic balancing, keeping the object in its own position. Here also we keep the rotor in its own position and the instrumentation required will not be very elaborate, but just like in the case of discs, you can connect it to a small device, which we call sine-wave generator. We have pickups at the two ends and we have dual beam oscilloscope. So the sine wave generator is connected to one and the vibration pickups either connected to this pickup which we call the near end pickup, near-end means it is nearer to the sine wave generator and this is the far-end and this is the pickup.

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We can rotate the rotor at sufficient speed so that we get measurable vibrations through the pickups at the near-end and far-end and at the same time, the sine-wave generator produces a reference sine wave. The first test is first round. The first round is run around the rotor at a suitable speed ω to the near-end, so that we get measurable vibration. Then the near-end, as you know already that oscilloscope output will be giving a reference sine wave and the other one say near-end will be something like this and this for the far-end

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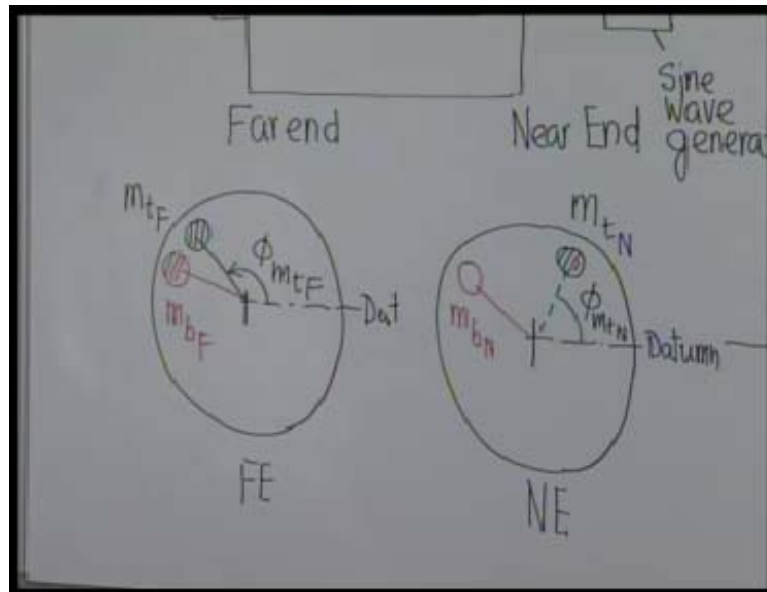
This oscilloscope signals tells that this is the phase lead of the near-end response or near-end sine wave, which you get and this is the far-end. The near-end response we get as N_1 vector and obviously, the reference is here. Reference vector is nothing but the sine-wave generator, so this will be ϕ_N . Similarly, at the far-end again the reference will be same, this will be ϕ_F and the vector representing the vibration of the far-end is F_1 .

So let us see again, what we have done, it is being rotated at a suitable speed so that the near-end pickup and far-end pickup produces reasonably measurable signal. So, the sine-wave generator is producing a signal as a reference. Then the vibration of the near-end is being measured like this and the vibration of the far-end is given by this. All of these are harmonic functions of time because it is a case of force vibration and in all cases the frequency of vibration is matching with the speed of rotation, so all these signals are of same frequency.

Next, we start the machine and let us see the near-end and far-end faces. This is the near-end face, this is the far-end face. We have a data or reference line marked and attach a trial mass m_t at the near-end at a known radius, at a known position. This is a data. After attaching this trial mass at the near-end face. Of course you have to keep in mind that

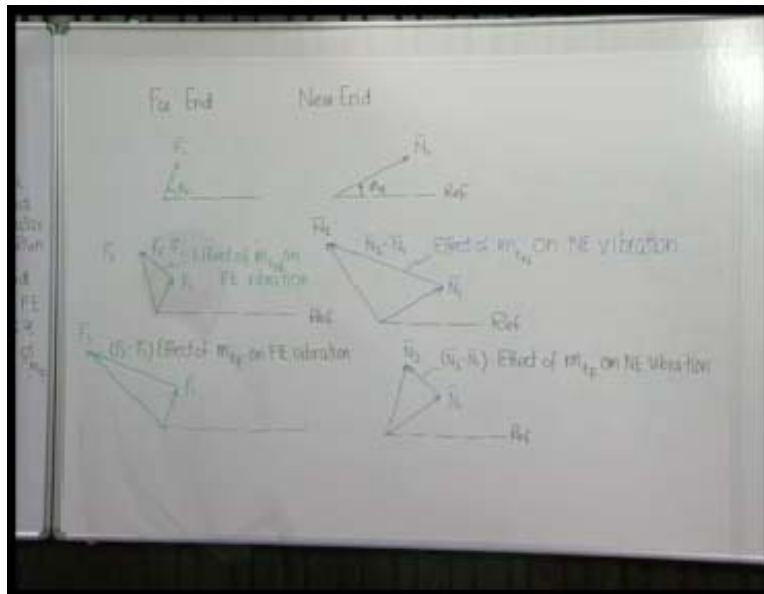
these are also the chosen planes for putting our balancing masses or taking out material just diametrically opposite location of the balancing mass as we have been saying in the previous lecture.

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Now, if we run the rotor at the same speed and again we measure the vibration of the near-end and far-end. What we will find is that the effect of this trial mass, which you place at the near-end, will be obviously more at the near-end of a vibration. The vibration of the near-end is now N_2 . This is the reference, this angle is the lead angle of the wave, which is found in the oscilloscope with respect to the reference sine wave signal. Now, when we did not attach the trial mass, the vibration of the near-end was this N_1 vector. So this vector: N_2 vector minus N_1 vector, represents the effect of on near-end vibration. This is the original vibration without any trial mass m_{tN} , this is the vibration of the near-end with the trial mass, so obviously this difference is the effect of the trial mass.

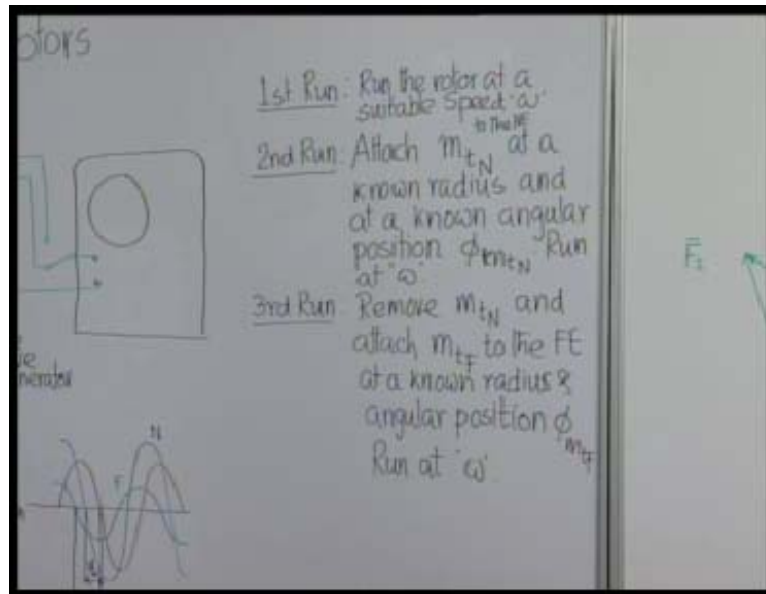
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Similarly, the vibration measured at the other end that is the far-end will be something like this. The original vibration without m_{iN} was F_1 , so therefore this extra, the increase that is F_2 minus F_1 that represents the effect of m_{iN} on far-end vibration. Remember these vectors have all rotating like rigid body in the same speed as this and real quantity is the component along this, this is the way we represent a **harmonically** quantity.

Next, we take out this m_{iN} . Remove it and put another mass and we call that mass as motion m_{iF} , at another known radius distance at a known position from a datum drawn on this. Then again we run at the same speed and the vibration at the near-end and far-ends are again measured both in amplitude and in phase.

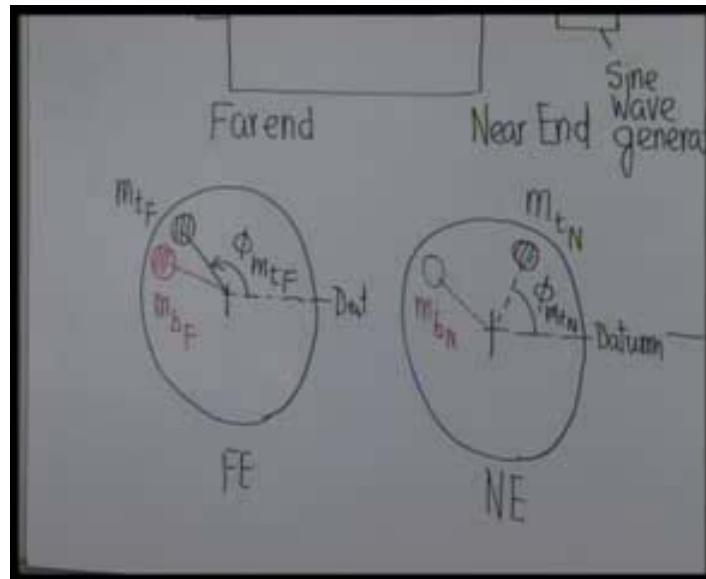
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Second run: attach a trial mass at the near-end at a known radius and at a known angular position given by $\phi_{m_{tN}}$ run at ω . Then the third run is remove m_{tN} and attach m_{tF} to the far-end at a known radius and angular position $\phi_{m_{tF}}$ run at ω . Now we again measure and we get N_3 vector. If this is the vibration at the near-end without any attached mass, N_1 vector, then this N_3 vector minus N_1 vector is the effect of m_{tF} on the near-end vibration.

Similarly, at the far-end the vibration is measured and we get the far-end vibration F_3 vector, the original one was F_1 vector without anything. This F_3 vector minus F_1 vector is the effect of m_{tF} on far-end vibration. Now let us see what we can do after this. What we have to do ultimately, we have to place balancing masses, like say m_{bN} and m_{bF} .

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How do we do it? We have to remember that once this m_{tN} is changed to this m_{bN} and m_{tF} is changed to this m_{bF} , then when both have placed there, the total vibration at the near-end and far-end should be 0. That is the effect of m_{bN} on near-end vibration and far-end vibration, plus effect of m_{bF} on near-end vibration and far-end vibration. When you sum total, the near-end vibration should be just minus N_1 to cancel the original effect when no mass was attached. The total effect here on the far-end should be minus F_1 to cancel the effect of the original unbalance present in the rotor.

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Field Balancing of Rotors

$$\vec{\alpha} = \alpha e^{i\phi_\alpha}, \quad \vec{\beta} = \beta e^{i\phi_\beta}$$

$$\vec{A} = A e^{i\phi_A}, \quad \vec{B} = B e^{i\phi_B}$$

$$\vec{A} \cdot \vec{B} = AB e^{i(\phi_A + \phi_B)}$$

$$\vec{m}_{bn} = \vec{\alpha} \cdot \vec{m}_{tn} = (\alpha m_{tn}) e^{i(\phi_{m_{tn}} + \phi_\alpha)}$$

$$\vec{m}_{bf} = \vec{\beta} \cdot \vec{m}_{tf} = (\beta m_{tf}) e^{i(\phi_{m_{tf}} + \phi_\beta)}$$

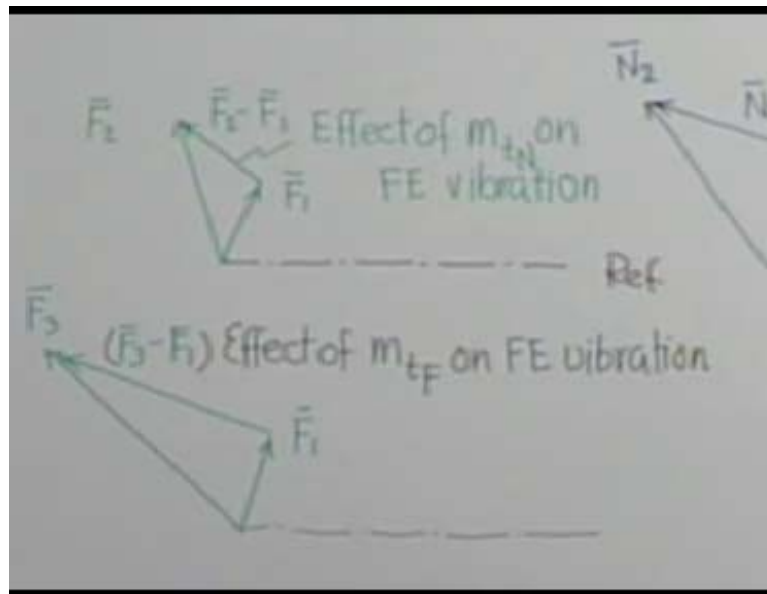
Now we have seen in the previous class that how we represent the vector quantities. We will have two vector operators: alpha vector, which is equal to alpha e to the power i phi subscript alpha and beta vector, which is equal to beta e to the power i phi subscript beta. When two vectors like: A vector is represented by A e to the power i phi subscript A and B vector is equal to B e to the power i phi subscript B, when you multiply these two vectors then what we get? We get a vector whose magnitude is the product of the two magnitudes and their phase is given by the sum.

If you remember this, then let us see that what we have to find out. m_{bn} , if you represent by a vector because it is a magnitude and it has an angular position that is direction, which is equal to alpha vector operated upon m_{tn} vector, trial mass position is known. So if we know alpha then we multiply m_{tn} by alpha to get m_{bn} and we will get alpha m_{tn} , magnitude will be the product and face-angular location will be shifted by e to the power of i phi subscript m_{tn} plus phi subscript alpha.

Similarly, the balancing mass at the far-end, vector m_{bf} can be written as product of vector beta with m_{tf} , that also can be represented by a complex quantity or a vector because that also as a magnitude and direction. This will be then beta m_{tf} e to the power of i phi subscript m_{tf} plus phi subscript beta. If you can find out alpha and beta, that

means you find out the quantity alpha and phi alpha and quantity beta and phi beta, then we can find out the magnitude and position of the balancing masses. But, what is the condition? The condition is that, when this is placed simultaneously, total effect at the near-end should be minus N_1 and total effect on the far-end should be minus F_1 . So let us see how we represent it mathematically. What is the effect of m_{tN} at the near-end? This one. What will be the effect of alpha into m_{tN} at the near-end? It will be alpha vector into N_2 vector minus N_1 vector. This is the effect of m_{tN} at the near-end plus what will be the effect of m_{tF} at the near-end? It will be beta vector into, what is the effect of far-end at the near-end? It is N_3 vector minus N_1 vector.

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This will be effect of m_{bF} at the near-end. This total should be minus N_1 vector, which was originally present. When everything is together, it is 0. Similarly, what will be the effect of m_{tN} at the far-end? It will be alpha vector into F_2 vector minus F_1 vector plus the effect of m_{tF} at the far-end, that is beta vector into F_3 vector minus F_1 vector. This total should be minus F_1 vector.

We now get, N_2 vector minus N_1 vector, N_3 vector minus N_1 vector. All these can be graphically found out. Then we have to solve this simultaneous equation. What do you do? You multiply this by F_3 vector F_1 vector. We multiply the second equation by N_3

vector minus N_1 vector. The standard thing is you subtract this or change the sign and add whatever you may say. What we get is alpha vector into N_2 vector minus N_1 vector into F_3 vector minus F_1 vector minus N_3 vector minus N_1 vector into F_2 vector minus F_1 vector. This cancels, is equal to F_1 vector into N_3 vector minus N_1 vector minus N_1 vector into F_3 vector minus F_1 vector.

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Rotors

$$\bar{\alpha}(\bar{N}_2 - \bar{N}_1) + \bar{\beta}(\bar{N}_3 - \bar{N}_1) = -\bar{N}_1$$

$$\bar{\alpha}(\bar{F}_2 - \bar{F}_1) + \bar{\beta}(\bar{F}_3 - \bar{F}_1) = -\bar{F}_1$$

$$\bar{\alpha}(\bar{N}_2 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1) + \bar{\beta}(\bar{N}_3 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1) = \bar{N}_1(\bar{F}_3 - \bar{F}_1)$$

$$\bar{\alpha}(\bar{F}_2 - \bar{F}_1)(\bar{N}_3 - \bar{N}_1) + \bar{\beta}(\bar{F}_3 - \bar{F}_1)(\bar{N}_3 - \bar{N}_1) = -\bar{F}_1(\bar{N}_3 - \bar{N}_1)$$

$$e^{i(\phi_{m_{tN}} + \phi_{\alpha})} = \frac{\bar{\alpha}[(\bar{N}_2 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1) - (\bar{N}_3 - \bar{N}_1)(\bar{F}_2 - \bar{F}_1)]}{\bar{F}_1(\bar{N}_3 - \bar{N}_1) - \bar{N}_1(\bar{F}_3 - \bar{F}_1)}$$

$$e^{i(\phi_{m_{tF}} + \phi_{\beta})} = \frac{\bar{\beta}[(\bar{N}_2 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1) - (\bar{N}_3 - \bar{N}_1)(\bar{F}_2 - \bar{F}_1)]}{\bar{F}_1(\bar{N}_3 - \bar{N}_1) - \bar{N}_1(\bar{F}_3 - \bar{F}_1)}$$

Finally, we get alpha vector. Similarly we will find out beta also which will be in this case of quotes you have to multiply this by N_2 minus N_1 and this by F_2 minus F_1 and then subtracting one from another we will get: (Refer Slide Time: 29:53).

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$$\bar{\alpha} = \frac{\bar{F}_2(\bar{N}_3 - \bar{N}_1) - \bar{N}_1(\bar{F}_3 - \bar{F}_1)}{(\bar{N}_2 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1) - (\bar{N}_3 - \bar{N}_1)(\bar{F}_2 - \bar{F}_1)} = \alpha e^{i\phi_\alpha}$$

$$\bar{\beta} = \frac{\bar{F}_1(\bar{N}_2 - \bar{N}_1) - \bar{N}_1(\bar{F}_2 - \bar{F}_1)}{(\bar{N}_3 - \bar{N}_1)(\bar{F}_2 - \bar{F}_1) - (\bar{N}_2 - \bar{N}_1)(\bar{F}_3 - \bar{F}_1)} = \beta e^{i\phi_\beta}$$

The diagrams below the equations show two circles representing rotors. The left circle has a reference point 'Ref' and two masses 'm_{IF}' and 'm_{IF}' at angles 'φ_α' and 'φ_β'. The right circle has a reference point 'Ref' and two masses 'm_{IN}' and 'm_{IN}' at angles 'φ_α' and 'φ_β'.

This is just solving simultaneous equation. What will it give you by solving the quantities by graphical construction ultimately is alpha e to the power i phi subscript alpha, this will be beta e to the power phi subscript beta. At the near-end, this is the reference or datum. Our near-end trial mass was here at known angle. We have to change this product as m_{IN} and you have to shift it by phi subscript alpha. This m_{IN} magnitude is equal to nothing but alpha m_{IN}, if you keep the radius same of course, because it is actually the product of eccentricity and the mass which is important but just to make the writing less complicated we are assuming that the placed at the same radial distance. So that is missing here, but otherwise you can all the time keep the product of the two as radial quantity.

At the far-end, if this is the original trial mass you have kept, then we have to shift it to this position m_{IF}, which will be same as beta into m_{IF}. Its distance angular position will be phi subscript beta. Therefore, you can see that even a rotor if it is too big, then it is not necessary to bring it to a machine. It can be balanced in its own location. This particular method which we have shown, where we have used a sine-wave generator and vibration pickup and a double beam oscilloscope. As you have mentioned in the case of disc like objects, here also you can solve the problem of field balancing, using only a vibration meter which will give you the amplitude of vibration. In that case, we have again put each trial mass diametrically opposite position and also at right angles, as you have done

in case of single plane balancing that is a disc balancing. Then we will have many more readings and the computation will be far more complicated, but you yourself can figure out the procedure how we can have or determine the position of the m_{bN} and m_{bF} for complete balancing using only a vibration meter.

The other last thing which we would like to mention briefly is that, so far we have considered these balancing problems where all these objects or the bodies were perfectly rigid, but in actual practical situation it may happen that a rotor may not be perfectly rigid and it may have some significant amount of deflection or elastic deformation. In such cases this method will not work. For that, we have to follow a methodology, which you call as the noodle balancing.

That method is more complicated, more involved and we will not discuss it here, but we just wanted to mention it, so that you are aware of the problem. The other kind of rotor-balancing problem is, where the mass and geometry of the rotor or rotating object is not a very definite one, as you find in case of washing machines, where we put all the wet cloth whose geometry or shape or mass are all uncertain. In such cases, a suitable technique is used for self-balancing of the rotating objects. That is also another very interesting technique of balancing rotating objects. We will stop discussion on rotor balancing here and from the next lecture we will take up the problems of balancing machines where the systems involve reciprocating objects.