Dynamics of Machines

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Module No. # 01

Lecture No. # 01

Rigid Body Motion

From the brief introduction to this subject, Dynamics of Machines, we have seen that machines are primarily interconnected rigid bodies. If you have to study the Dynamics of Machine, the first thing we have to understand that how rigid bodies behave under load or under forces. That is, studying the motion of rigid bodies forms the first taped words of the subject, Dynamics of Machines. We start today studying the motion of the rigid body.

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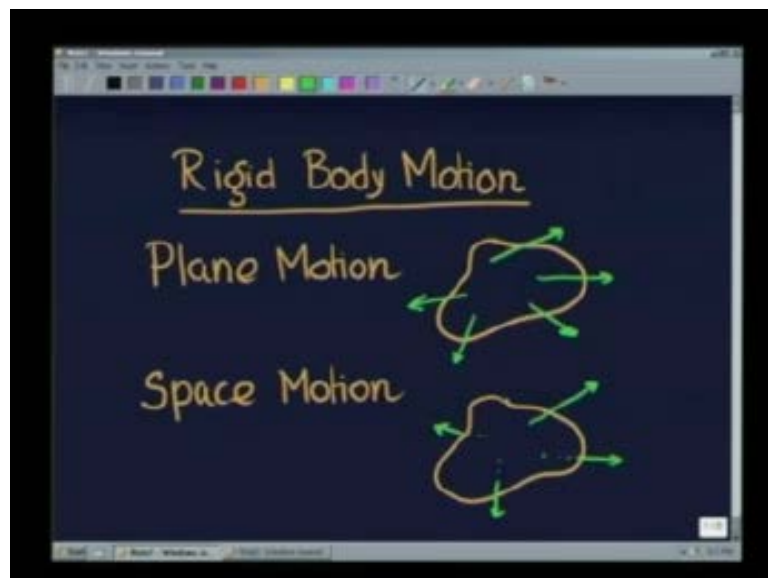
Rigid body motion can be studied under two headings: the first; the simpler case, which you call plane motion and the more complicated type of rigid body motion, you call

space motion. When do you call the motion of a rigid body to be a plane motion? It is the situation where all the particles of the rigid body move in different directions no doubt, but at any instant of time, all the velocity or displacement vectors lie in one plane or in parallel planes.

On the other hand, in case of a space motion of a rigid body the velocity vectors of different points will not lie in different directions or lie in the same plane or parallel planes. So, studying plane motion is much simpler because you can project the whole motion parameter or all the motion parameters like velocities, displacements and accelerations on a projecting plane or the plane of the paper.

Next, let us see again, what are the different possible types you can have in the plane motion. Let us first study plane motion.

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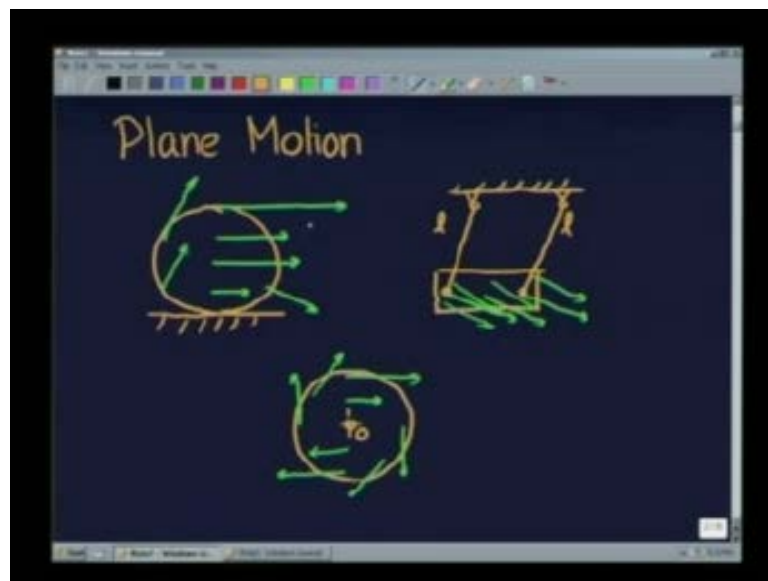
What are the typical examples, say a wheel rolling on a horizontal surface here you know that the velocities of different points will be different, but they will be all lying in parallel planes. Similarly, we can have another example. If you hinge an object and suspend it by two equal links, then again we know that this object will undergo a motion which you can call plane motion, but again there is a subtle difference between these.

I will take another simpler case, where a wheel is rotating about a fixed hinge. In that case, the velocities of various particles will be like this (Refer Slide Time: 03:59). There

are three types of plane motions as you can see here; one is where all the particles on the rigid body they have exactly same velocity, same motion, identical situation. In another case, it is a more general situation; it is neither a pure rotation like this nor a pure translation like this, but a general motion.

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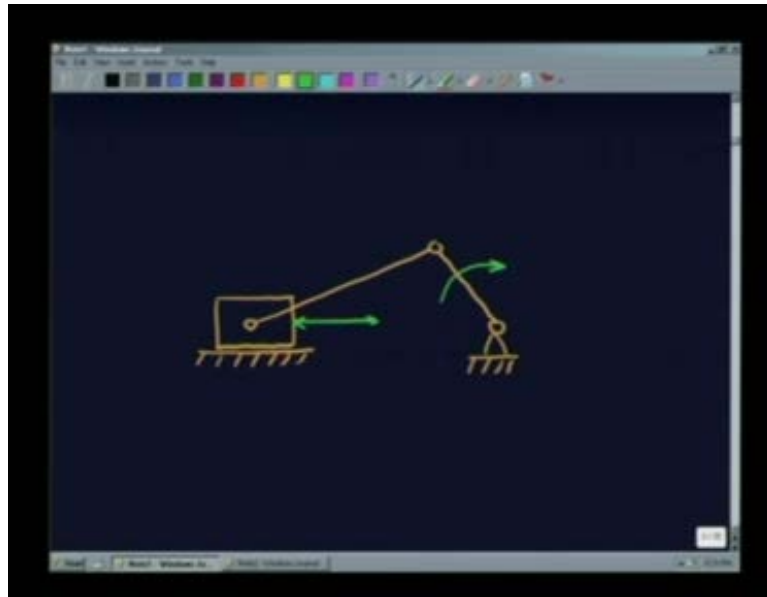
For example, we take our familiar example of a slider crank mechanism, which we have discussed in our first introductory lecture. Now, we study the three rigid bodies which are interconnected and they are moving. You can see that this particular rigid body which you call crank is undergoing a pure rotation about a fixed inch point O. This particular rigid body is undergoing a pure transitory motion like this (Refer Slide Time: 05:10), whereas, this one is undergoing both rotation and translation that is a general type of motion. A machine consists of number of rigid bodies interconnected and each one of them undergoing different types of motions. Of course, as I mentioned earlier in this particular example, all the particles of the system they have their displacements, velocities and accelerations lying in parallel planes, so that the whole motion picture can be projected on to the projecting plane.

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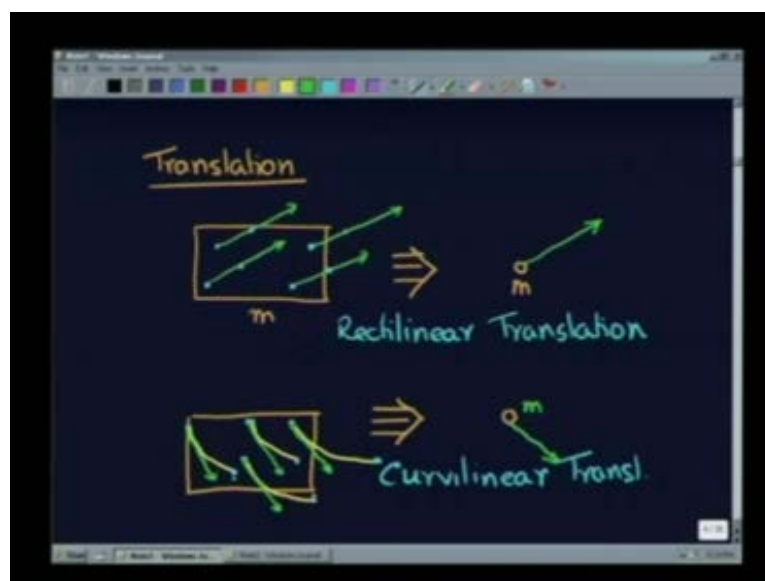
The simplest of all these will be the case of translation. A Rigid body where each point of this rigid body they have exactly same displacement, same velocity, same acceleration. So, effectively what we can do? Then this is more or less equivalent to a particle with the same velocity or displacement as the rigid body, but the total mass of this is same as the mass of this particle. Therefore, cases of pure translation a rigid body may be considered to be concentrated in a particle with the same displacement parameters, velocities and accelerations. This is pure translation.

Translation can be of two types: this is a translation which we call, rectilinear translation, why? Because we find the path of each particle is in the straight line. That means, it was here; it has gone there. It was here; it has gone there. It was here; it has gone there. It was here; it has gone there (Refer Slide Time: 05:47).

There is another kind of translational motion is possible, where each particle of a rigid body undergoes a different kind of trajectory which is not a straight line, but it is same for all. That means, a particle which was here; it as gone here; particle which was here; it has gone here; particle which was here; it has gone here and so on (Refer Slide Time: 07:27). Here also at any particular instant of time, the velocity of the particles will be all identical, but the trajectory of the particles will not be straight lines like this, so this is called curvilinear translation. The simplest type of motion that is translation can be either rectilinear or curvilinear, in both these cases, since at every instant all the particles

undergo identical motion, we can treat the whole body as one single particle like this of the same total mass as this original one.
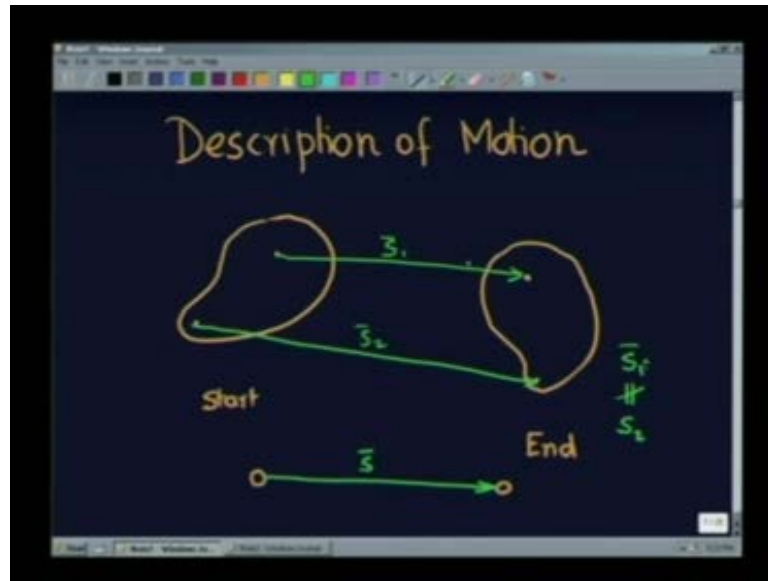
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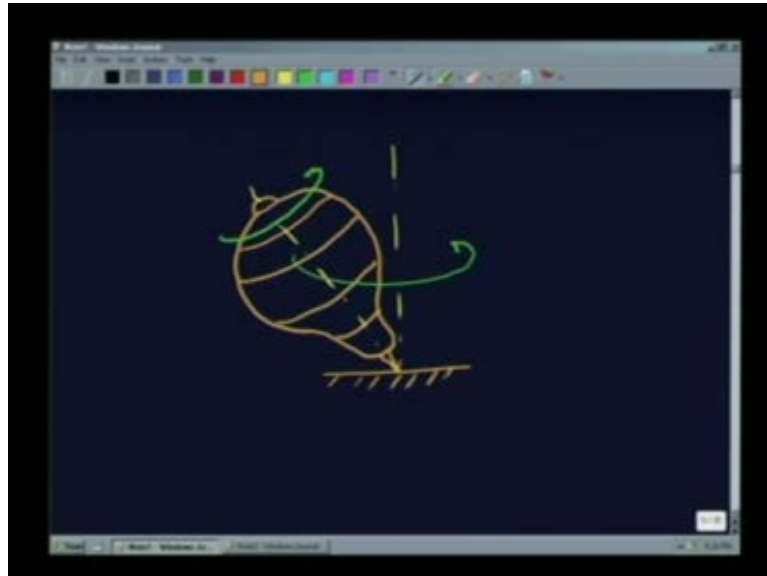
Let us consider an example of space motion. Before, I proceed further with the case of plane motion, it is also appropriate for us to understand what we call by a space motion? The simplest example of that will be a spinning top with which we are familiar. We know that this top undergoes a spinning motion at a very fast rate and the axis of this undergoes a precessional motion about a vertical axis like this (Refer Slide Time: 09:05). In this case, we will find that no point on this body or all other points on this rigid body their velocity vectors, acceleration vectors, etcetera at any particular instant of time do not lie in single plane or in parallel planes. This is a very common example of space motion, but in engineering we have cases where a particular system will undergo a motion where the velocities, displacements and acceleration of various points do not lie in the same or parallel planes.
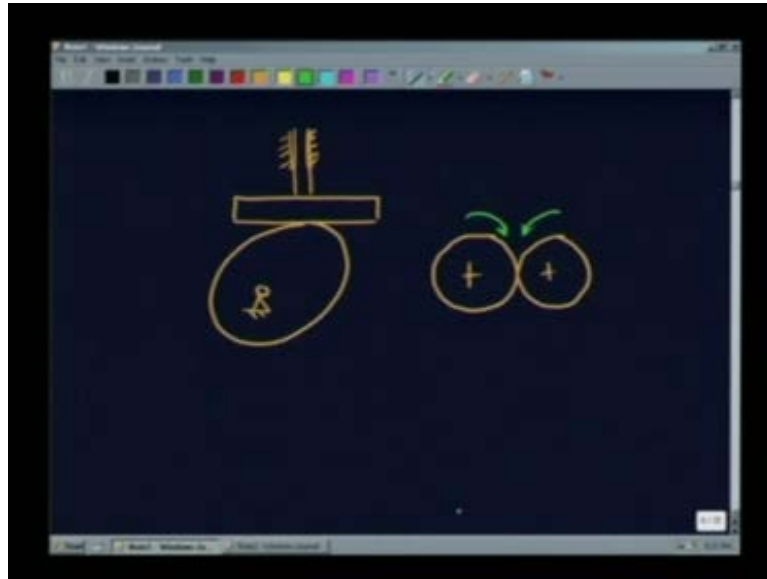
Let us come back to our study of plane motion; <mark>we study plane motion</mark> primarily because a very large number of engineering systems undergo parallel motion problems. As for example, we have shown our slider crank mechanism, we take a simple crank follower mechanism, here again we will see that all particles which are describing this system undergo motion where the velocities, displacements and acceleration lie in parallel planes. We can have motion of a gear or a wheel <mark>or a</mark> transmitting power between two shafts, again you will find that all the points undergo motion where the motion parameters lie in parallel planes. There are very large numbers of engineering problems or systems which involve plane motion and so, let us first start studying the problem of systems involving plane motions. Before we start studying the plane motion problems of a rigid body, let us see that how the description of the rigid body has to be done or rather how we describe the motion of a rigid body in plane motion?

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The next thing we will study in little depth is ==Description of Motion==. What I mean to say that one object is here and after sometime I find it has gone here, this is the start and this is the end or a finish (Refer Slide Time:11:29). How to describe the motion of this rigid body? In case of particle there is no problem, ((in case)) motion of a particle it was start here and end here. I can simply say that the displacement of this motion is this much and denoted by vector, but in this case, it constitutes a large number of particles and each particle may have different displacement vector. For example, suppose if you take this point, its displacement is this much (Refer Slide Time: 12:01). If you take this point, its displacement is this much, so they are different, this is called s_1 and this is s_2, then we know s_1 is not equal to s_2. So, how to describe the motion of such rigid bodies that is what is our problem? To do it, let us see that how we describe the motion of a rigid body.
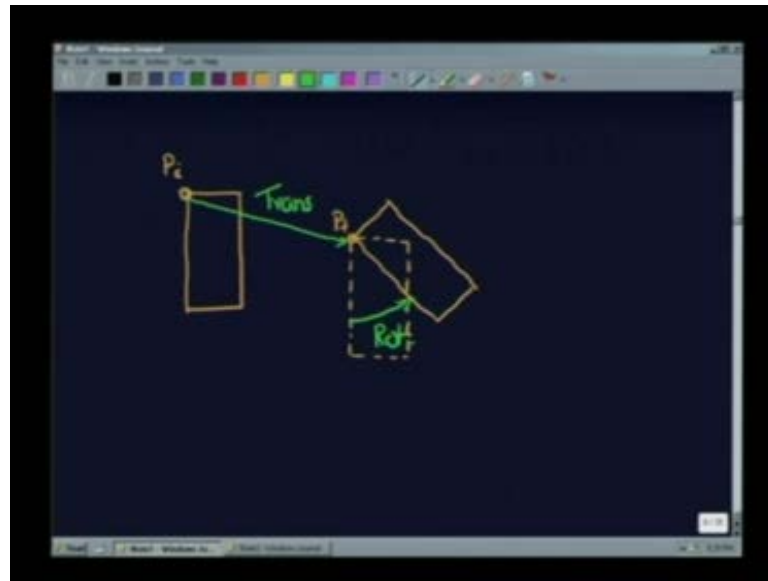
First let us take this rigid body, I am drawing a simpler body for better understanding and the same body goes there. We can generally show that we split this motion into two parts, how? First we take any point here, let us say for example, consider these as the reference points; it can be any point on this, this point P_i initially has gone to this point as P_f, so this point displacement is this and if we just subject this body to a pure rectilinear translation then the position of this body would have been like this (Refer Slide Time: 13:37). So, this is a pure rectilinear translation of the body from the initial and intermediate position and then keeping this reference point whatever I have selected, we give a rotation to the body from this position to this position, so this is the translational part and this is the rotational part. So, any motion of a rigid body can be split into these two pure motions; pure translation and pure rotation with one point fixed and that will provide us this general motion of a rigid body.
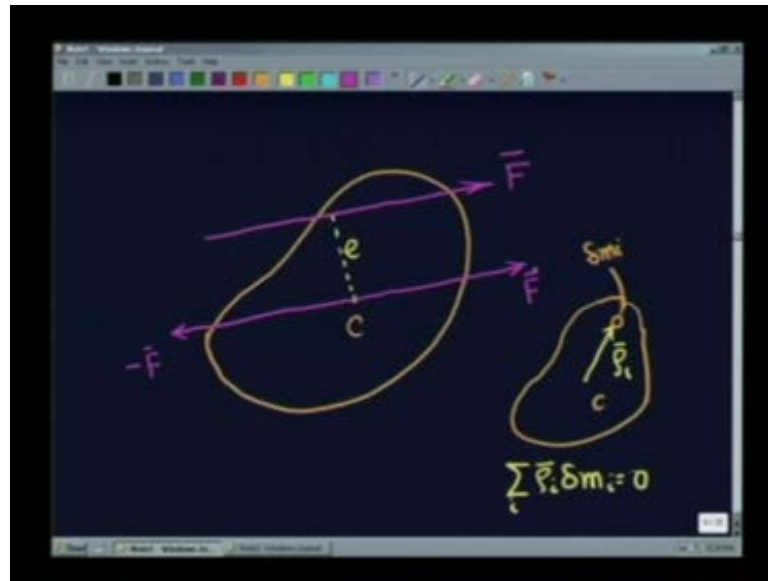
Now, we will find out the effect of a force on a rigid body, because ultimately, as you know that our objective is to find out the effect of forces on a machine or the kind of forces that are generated by motions of machines and machines parts and such things. Therefore, let us start the effect of force or a system of force on a rigid body, which is in plane motion. We again take our rigid body and say this is being acted upon by a large number of forces, which can be all ultimately combined to make this resultant of all the external forces F, which is acting like this (Refer Slide Time:15:10).

Now, this rigid body is acted upon by this force. Our objective is to find out that what is the motion of this? We will not go back to the basic fundamentals because that is the subject which we have studied in the dynamics or this kind of topics or problems we have solved in dynamics. We will not go into the basic fundamentals of that but certain important points we will recapture it a little bit for going further.

Now, you have seen in basic dynamics courses that motion of a rigid body due to a force can be found out in the following manner. First what we do? We identify a particular point, which you consider as the center of mass. What is the center of mass? The definition of a center of mass of a rigid body, I will show you here this is C. If you take any particle whose mass is a delta m_i and which is located at a position rho_i, then if you sum up this product for all the particles; it will result in zero, if C is the center of mass, this we have to keep in mind, that is the definition of center of mass or if you consider

that if you take the moment of all the masses about any other point, it can be considered to be the moment of all the particles about an axis passing through this plus the whole mass considered to be concentrated here and its moment about that. We will look into it later.

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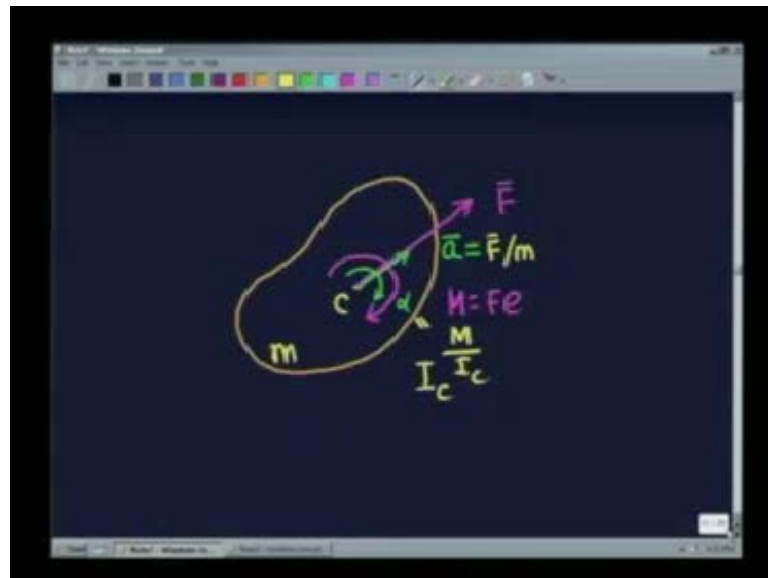
Now will go back to our problem, this force which is acting at a distance e from the center of mass, what we can do now is, we can apply an equal and opposite force minus F and plus F here, then this becomes equivalent to a force passing through this and a moment acting on this, which is given by F into e. Now, you will notice that I have not put a moment vector here because in plane motion all angular activities like whether angular velocity, angular displacement, moment they can be represented by algebraic quantities. We may consider that if it is in the clockwise direction, they are positive. If they are in the anticlockwise direction, they are negative. So, they are summing up or subtraction becomes or it is just like an algebraic quantity, whereas force, its directions are different and so they need to be represented by vector.

If the total mass of this be m and second moment of the mass, that is moment of inertia of the body about an axis passing through the center of mass is, moment of inertia about the center of mass I_c. This force will cause an acceleration in this direction of the center of mass which is a and it will cause an angular acceleration alpha due to this and their quantitative dependence will be equal to F divided by mass and this will be M divided by

10

I_c. Therefore, any set of forces on a rigid body in plane motion can be considered to be the effect of a single resultant force on that rigid body, which in turn is equivalent to that force acting through the center of mass of that body and a moment acting about the center of mass which is given by the force and the product of the force and the eccentricity of the force which causes the motion of the rigid body in this following form that the center of mass is accelerated, which is given by this and the body gets an angular acceleration alpha, which is given by this, about the axis passing through this (Refer Slide Time: 20:09). So that means what we are doing? That this rigid body motion is being described as the acceleration of this center of mass and angular acceleration about an axis passing through the center of mass which we call C.

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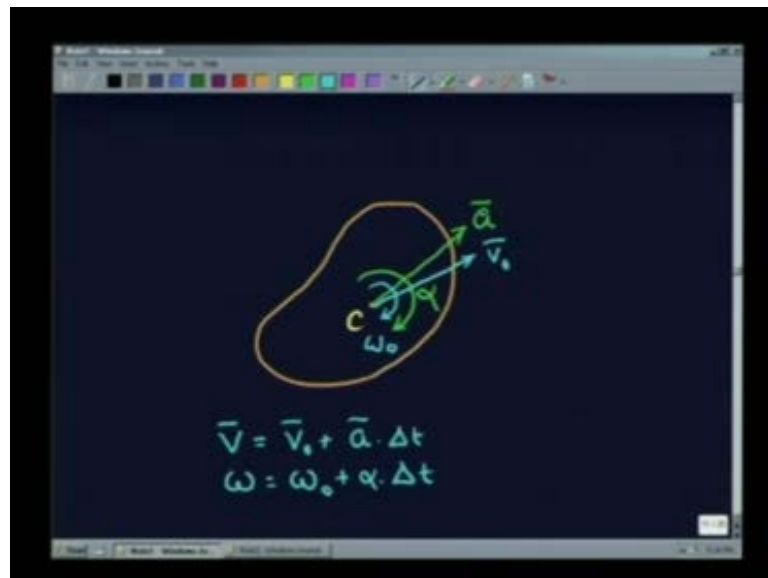
As you know from your basic courses in dynamics that a rigid body, if a particular point is the center of mass, which plays a central role, its velocity (or) its acceleration and acceleration angular acceleration about an axis passing through is given that completely describes the motion of this rigid body, if the initial conditions are known, we can find out the initial or final values. For example, if we say that initially its velocity was V_0 and its angular velocity was omega$_0$, then at any instant of time the velocity after a time dt, V will be V_0 plus a delta t and omega will be omega$_0$ plus alpha into delta t. Therefore, we can proceed ahead and we can get the velocity at any instant of time, if we know the initial condition because of this linear and angular accelerations. This brings to the end of

the case of a single rigid body under the action of a set of forces and what accelerations are developed. We find out like this.

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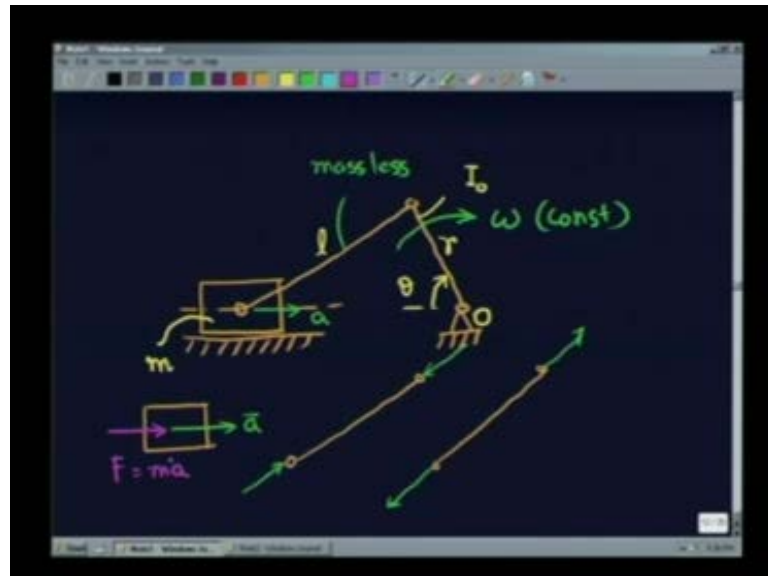
Let us find out, how we consider this particular aspect that we have just discussed in studying the forces which are developed in various machines. As I mentioned in the introductory lecture that a body when is in motion that means when it is accelerated, then the system is acted upon by certain dynamics forces which are because of the inertia forces of the system. To explain this little bit more, let us consider this case.

The simplest case will be again our slider crank and then we will go to more complicated examples. This is a very useful mechanism that is why we are using it so many problems the same thing. Now, I tell that this is a pure rotating body about its fixed hinge O and let its moment of inertia about this fixed hinge be I_o.

Let us consider this is a pure reciprocating or rectilinear motion body, so I have to give only mass of this. If the length of this rod is l, radius of this crank is R and at any particular instant which is shown here like this is theta and say it is being rotated at a constant angle of velocity omega (Refer Slide Time: 24:16). If you have to design this particular rod, say our interest is that how do you do it? Now, first what we will do is that as follows. Let this particular rod, if it is mass less for simplicity, then it will be subjected to either pure tension or pure compression, which means that resultant force on

this will be zero, because it is a mass less system it cannot sustain any resultant force or moment.

When this is weakened from kinematics, you have already found out that the acceleration of this body at this instant will be a. If that be the situation, then you know that this particular point that means this rigid body must be subjected to a force in this direction, which is mass into acceleration. That means, this body its acceleration is a so resultant force on the body must be F, which is simply mass into acceleration, but we know that this body is acted upon by these forces. One is, a force from the support and either from this side, in this particular case it will be from this side, let me correct my drawing. So, it can be from this and a force which must be acting along this because this rod cannot sustain any force here which is not in the line of this (Refer Slide Time: 27:00) so, if this angle is theta or phi, then this angle as to be also phi.

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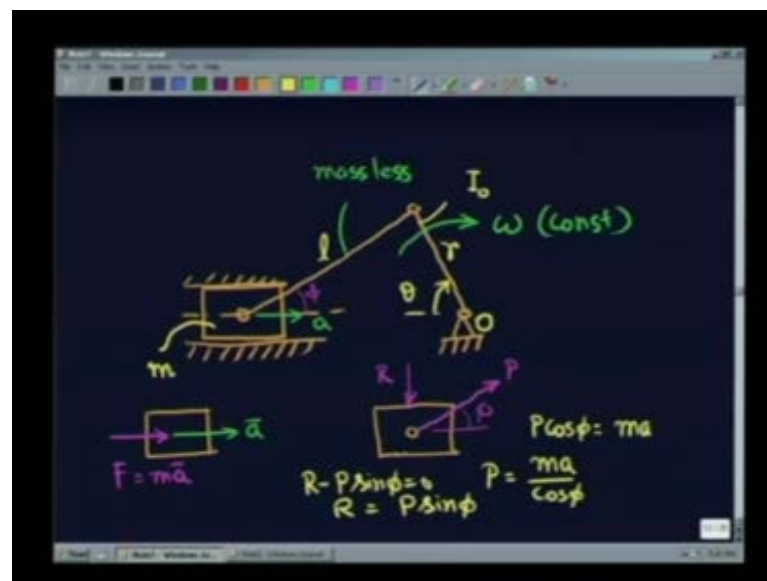
How much will be this force? That we can find out from this equation P cosine phi must be equal to mass into acceleration or this force P will be mass into acceleration by cosine phi. We can find out this reaction because there is no movement along this direction which is perpendicular to the slide or surfaces, so total force in this direction will be zero. You can then write that R minus P sine phi is 0. That is, R will be equal to simply P sine phi. Therefore, we know that this connecting rod which is mass less in this example

has a force P in this direction. So, obviously here also it must be acted upon by an equal and opposite force P. So, that this system or this rod is under no resultant force.

This is a particular example and we solved the loading on the connecting rod as just a simple case of example, but I think this is just from the basic fundamentals we try to develop the procedure by which you could find out the force, but while dealing mechanisms with many members more complex motion, a better more generalized approach of analyzing forces on various members, at various contacts, at various hinges it will be necessary to develop that methodology and we will next try to develop that methodology. Before taking up the problem of general mechanism and the problem of finding out the various forces and moments at various surfaces then the contact forces at the hinges and so on. We will see that it will be convenient to consider the dynamic problem after you convert the dynamic problem into a ((strategically)) problem, what I mean to say is this. Let any system under general motion, if it is in dynamic equilibrium it means, that it is running under the influence of some forces, then each and every member of the mechanism will be subjected to some forces which are acting on it and which are producing the resulting motion parameters like accelerations; both angular and linear.

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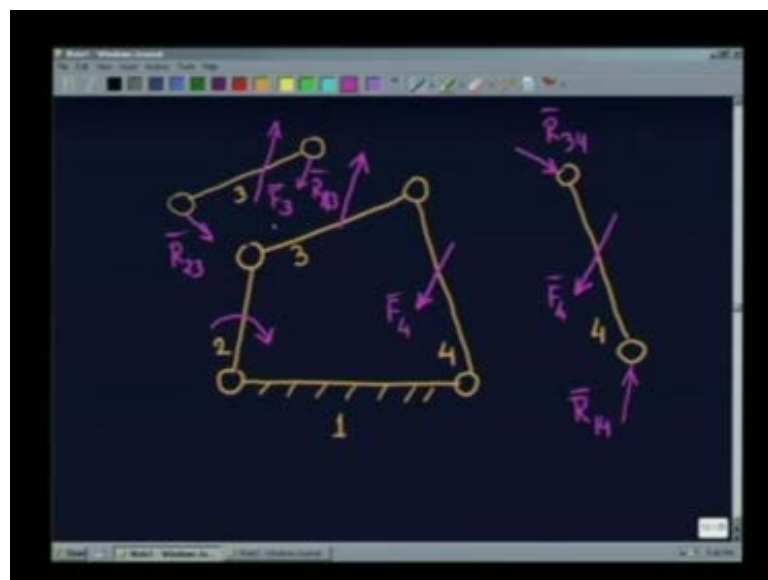
 If I say that is a system or a mechanism or a machine is under static equilibrium under the influence of certain forces. What does that mean? Say for example, if you consider

this case, a four link mechanism with the ground link fixed, I state that under the action of this moment, this force and this force, this system is in static equilibrium, if I tell that. It means that no member will have any motion when this moments and forces are acting. If the whole system is motion less; it implies that each and every member of this mechanism or system is in equilibrium that means this link, which I think in your terminology you have termed it as link four. So, fourth link which is subjected to this external force we call it say F_4 and there will be two hinge forces and reaction forces where one reaction force is from here and another reaction force; we call it R_{14} and this I call R_{34}. Then the combine action of this reaction force, this reaction force and the externally applied force ease to keep the system under static equilibrium (Refer Slide Time: 32:26).

Similarly, this particular link which you call the connect coupler or it is under the influence of this force, which you call F_3 and the reaction force is like this it will remain under static equilibrium, that is each and every member of this mechanism will be in static equilibrium under the action of the external forces acting on that particular member. So, this principle we can use in solving the dynamics problem.

In the next lecture, I will explain the technique of converting a dynamics problem into a static equilibrium problem and illustrate it with help of an example.