Mathematical Methods in Engineering and Science
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Module – II
The Algebraic Eigenvalue Problem
Lecture – 03
Methods of Plane Rotations

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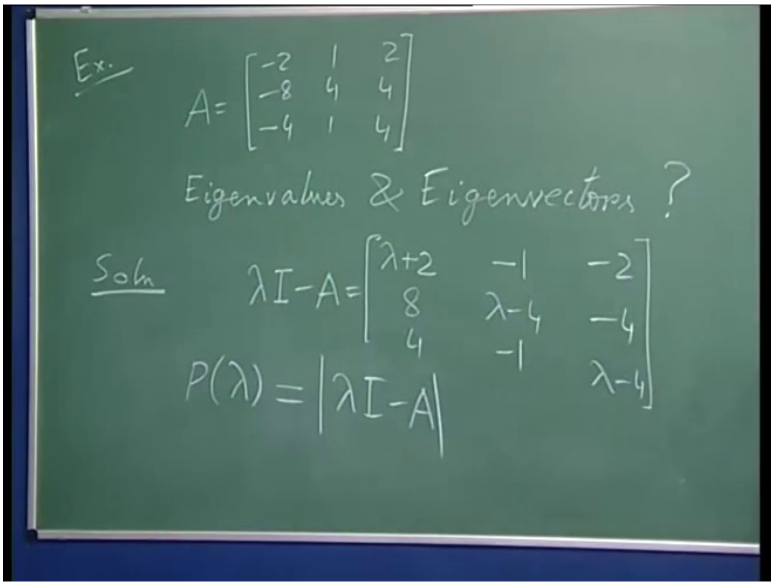

Good morning, let us take this example, usually have this 3 by 3 matrix A for which we want to find out the Eigenvalues and Eigenvectors. So, this is our problem. So, first what we should do we should find out the matrix lambda I minus a and set that equal to 0 to get the characteristic equation and you will find its solutions or you can say that we find the characteristic polynomial and try to find its roots, right.

So, lambda I minus a that is matrix will be lambda minus 2 that is lambda plus 2, then lambda minus 4, then lambda minus 4 and on the off diagonal element, there will be no effect of this these will just become negative like this, right. Now the characteristic polynomial is the determinant of this matrix, right. So, you will know the characteristics polynomial will be the determinant of this matrix, right which you can expand from here; that means, you will take lambda plus 2 into this-this minus this-this, right.

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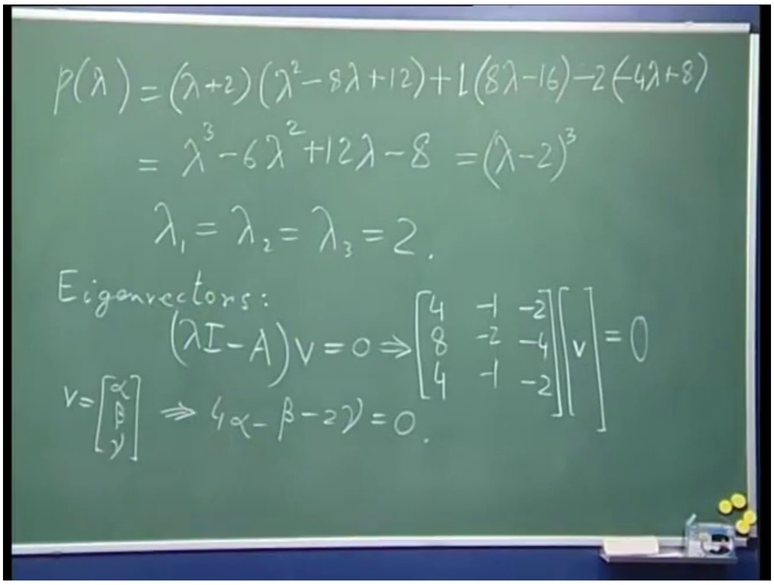

Lambda plus 2 into from here, you will get lambda square minus 8 lambda plus 12, then minus this minus 1 which will give you plus 1 into the determinant that you find from these 4 elements, right that will give you 8 lambda minus 16 plus this minus 2 into the determinant that you get out of these right so that you find as this as you simplify this you will get this.

Now, this particular polynomial is very easy to factorize because you can see that it is exact cube of lambda minus two. So, factorization in this case is actually very simple which will not be. So, in the case of a general polynomial or general even cubic polynomial and this tells you what this tells that the characteristic polynomial p lambda has 3 roots which it must have, but in this case all 3 roots are coincident that is there is a single Eigenvalue and that Eigenvalue is 2; that means, a single Eigenvalue appearing thrice. So, you have got lambda 1, lambda 2, lambda 3, all equal to 2; this is the very special case.

Now, we say that for this Eigenvalue, then we would like to find out the Eigenvectors it will be nice if you can find 3 linearly independent Eigenvectors, but it may be possible that we find only 2 or we find only one. So, for doing that what we need to do we need to write the full equation from here lambda I minus a into the vector v equal to 0. So, as we do that for finding Eigenvectors we will take lambda I minus a into v equal to 0 or that same lambda I minus a we can use by putting lambda equal to 2 here right.

so that will give us 4 8 4 minus 1 N 2 minus 4; that means, minus 2 minus 1 and finally, in the third column minus 2 minus 4 and then 2 minus 4; that means, minus 2 into this Eigenvector, see now how to solve this, you will say that will use the same old method, we will apply elementary row transformations and in this case, it is very case because it is found that the second row is exactly the same as the first row exactly the same as twice the first row and the third row is same as the first row.

That means actually the second row and the third row 2 elementary row operation will immediately go out and only the first is something which is of use, right. So, to an elementary row operation in the second and third rows we will get all 0s. So, they are anyway useless it is only one equation which is of use. So, we take that equation and say; if we represent v as this vector then that equation will mean 4 alpha minus beta minus 2 gamma equal to 0.

Now, what alpha beta gamma will satisfy these; any set of alpha beta gamma that satisfies this is an Eigenvector, now we will say that the scale has no importance of an Eigenvector that is if you have decided alpha beta gamma as some values then making them twice or thrice or halving them will have no difference that will not be as a different Eigenvector. So, one of these 3; we can set.

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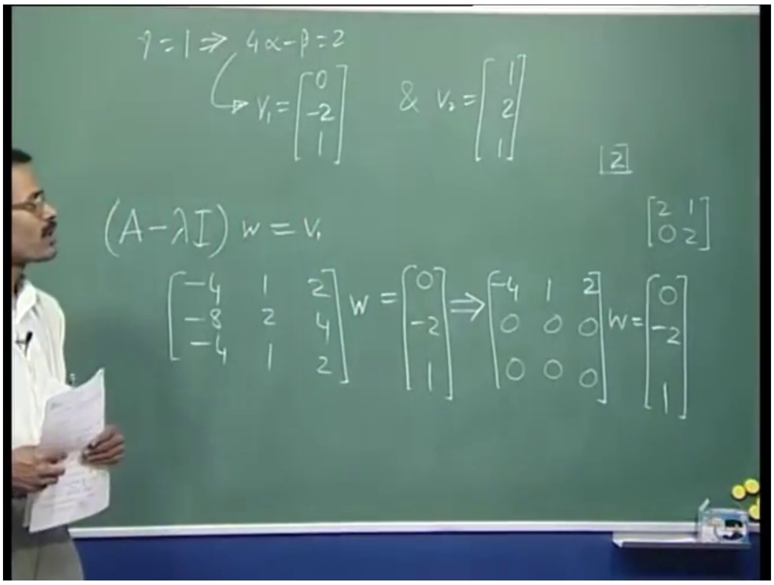

So, suppose, we will set gamma equal to one that will give us 4 alpha minus beta equal to two; that means, that still we can choose alpha and then we will get beta right.

So, different choice of alpha we will give us different beta. So, for example, suppose we choose alpha to be 0. So, then we get one Eigenvector in which alpha is 0 and beta will then turn out to be minus 2 and gamma is anyway chosen as one in another; suppose, we choose alpha to be 1. If you choose alpha to be one then we will get beta as 2 right then we will have this satisfied. So, one 2 and gamma is already chosen as one.

So, these are the 2 linearly dependant Eigenvectors linearly independent Eigenvectors you can see that there linearly independent. So, 2 linearly independent Eigenvectors we have got and only 2, you will get because no other independent choice is possible any third choice of alpha we will give you something which will be only linearly dependent on these 2 that is it will be a linear combination of these two. So, in this case we have got only 2 Eigenvectors not a full set of 3 Eigenvectors; that means, that this particular matrix is not diagonalizable it is defective.

And since it has got 2 linearly independent Eigenvectors; that means, that it will have 2 Jordon blocks in its Jordon canonical form one Jordon block will be a one by one Jordon block which will have only 2 the Eigenvalue and the other Jordon block will be a 2 by 2 Jordon block of this kind. So, now, in order to get up to that Jordon block we will need to find out a generalized Eigenvector apart from these 2 Eigenvectors, right. So, this is what we expect to be the look of the Jordon blocks that we will get, right.

But we will actually find that after we get the complete basis that is other than the 2 Eigenvectors after we find out the generalized Eigenvector also. So, to begin with suppose we want this fellow to admit a generalized Eigenvector and try to find put the generalized Eigenvector. So, the generalized Eigenvectors w will satisfy this and A minus lambda I. So, whatever just now we wrote as lambda I minus A, this matrix is actually its negative. So, that negative matrix I am reproducing here this. So, we are looking for the generalized Eigenvector corresponding to this Eigenvector v 1 right and then what we can do again the same elementary operations we will mean that these 2 rows become completely 0.

Let us apply those 2 elementary operations R 2 minus twice R one to be put in R 2 and R 3 minus R 1 to be in R 3; that means, from the second row twice the first row first row remains unchanged from the second row on this side also first row remains unchanged from the second row twice the first row is subtracted 0 0 0 minus 2 from the third row the second the first row is subtracted. So, 0 0 0 one can you see that this system of equations is actually in consistent because whatever is w the second and third row will give you 0 on the left side, but non 0 numbers on the right side.

That means this system equations is actually inconsistent; that means, that v 1 does not admit a generalized Eigenvector, right; that means, the generalized Eigenvector has to come from some other Eigenvector.

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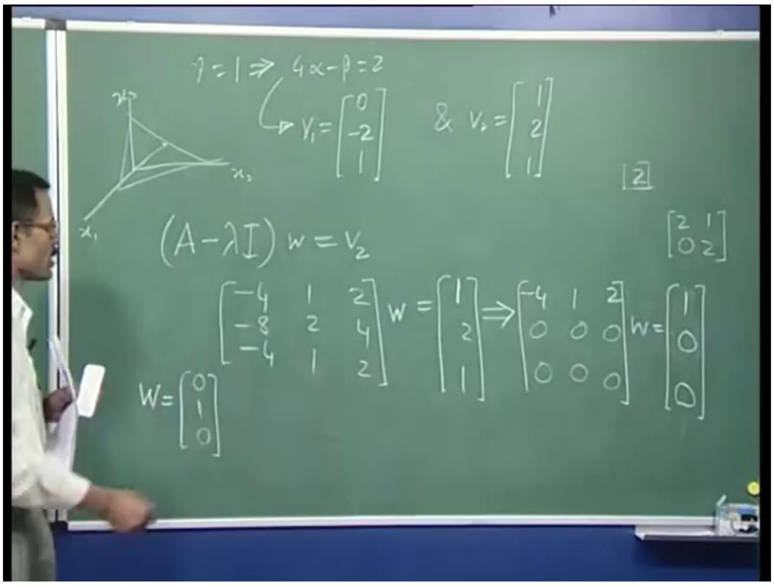

So, we try with v 2; v 2 1, 2, 1 like this and when you do that in these row operations as we subtract twice the first row from the second row and the first row from the third row here from the second row twice the first row is to be subtracted and here you get 0 and then when you subtract the first row from the third row also here also you will get 1, minus 1, 0; now this is consistent sorry the first row remains unchanged.

First row remains unchanged to get this. So, first row remains unchanged and the second and third rows get 0 now this is consistent. So, as this is consistent now then you will find that w can be determined minus 4 first element of w into plus 1 into second element of w plus 2 into 13 element of w is 1. So, from there any w vector which satisfies that equation is a variant generalized Eigenvector to be used with the second Eigenvector. So, suppose we can take w to be 0 1 0.

So, you see 0 1 0 will give you 0 plus 1 plus 0 which is satisfied. So, in the 3 d plane 3 d space this one single equation is actually acting like a plane. So, any vector which is from the origin to this plane defined by this first row is actually a valid generalized Eigenvector in this case now we address 2 points; one is that the similarity transformation matrix S that we get out of this whole thing will then be.

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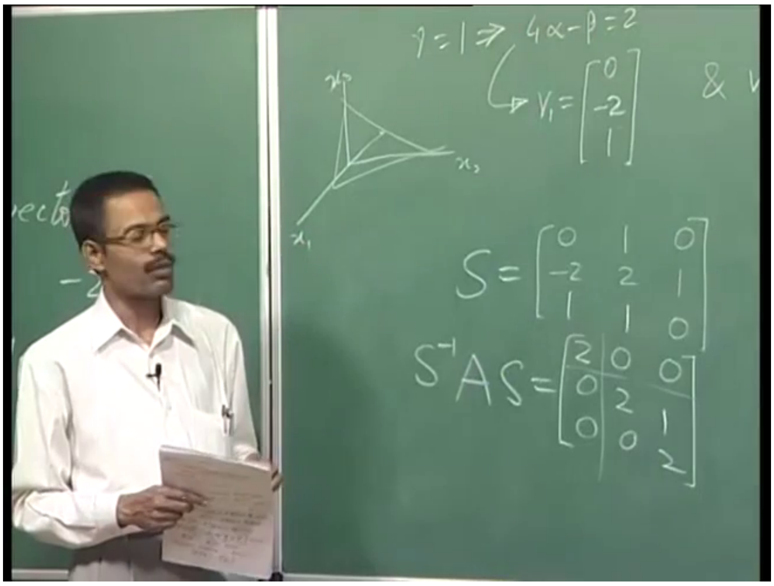

The first Eigenvector the second Eigenvector and this generalized Eigenvector which is associated with the second Eigenvector and that is why it is placed after the second Eigenvector.

We find out its inverse and then try to calculate this and I leave it for your verification to find out that the resulting matrix turns out to be this which is the Jordon canonical form in which there are 2 Jordan blocks one is this the other is this and in this particular case both the Jordan blocks are corresponding to the same Eigen value which is repeated here 2. So, 2 has this Jordan block as well as this Jordon block that is more the Jordon block are actually corresponding to the same Eigenvalue in this particular kind of situations the problem is actually a little more tricky, then it seems till now in this case corresponding to this Eigenvector we tried for a generalized Eigenvector we said for the second Eigenvector we tried and we succeeded in finding these are Eigenvector, but that was not necessary we might not have succeeded.

For example when we choose when we chose alpha to be 0 and alpha to be 1, then we got these 2 Eigenvectors in another situation we could have chosen a different value if we had rather chosen v 1.

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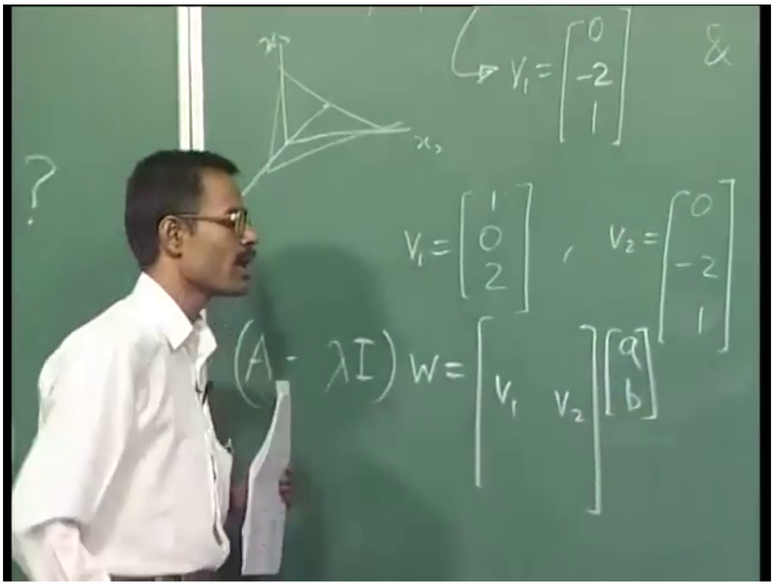

As 1 0 2 and v 2 as this we could have chosen that we can verify 2 things one is that these 2 are also bonafide the Eigenvectors for this matrix and second is that if you take this and try to find w you will fail if you take this and find try to find w we have already fail that we have seen then why do you get that generalized Eigenvector w for that you can argue that these 2 Eigenvectors belong to the Eigenspace of Eigenvalue 2; that means, their Eigenvectors corresponding to the same Eigenvvalue; that means, in that Eigenspace any linear combination of these 2 is also an Eigenvector and that Eigenvector if you take in terms of a v 1 plus a v 2, then you can find out that for which value of a and b we get an Eigenvector which admits an Eigen generalized Eigenvector.

I read this for you as an exercise in which you can say that you want to solve the generalized Eigenvector from this; this is basically a v 1 plus b v 2 a linear combination of these 2 Eigenvectors and if you try to do that then you will find that only for a specific combination of values of a and b you will find that a generalized Eigenvector is admitted and that turns out to give you this Eigenvector; that means, in the Jordan canonical form in the basis matrix this vector this Eigenvector must come that this much I leave you leave for you as an exercise and right now we proceed with the next lesson in our study which is the lesson based on plane notations.

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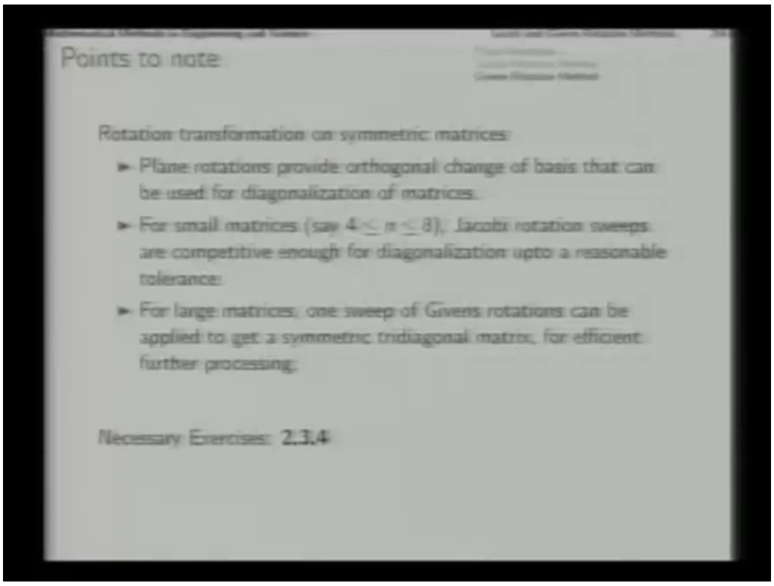

As I told you in the previous lecture that in the next 4 lessons, we will be studying 4 methods for finding suitable similarly transformation to bring about diagonalization of a A matrix.

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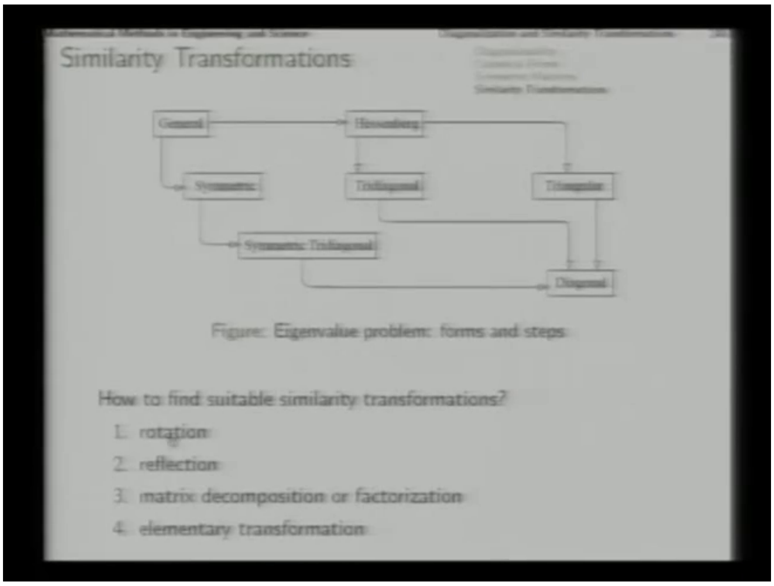

And these are the different ways by which we workout suitable similarity transformation. So, the first way first method to find suitable similarity transformation is based on rotations 10 rotations and make note that.

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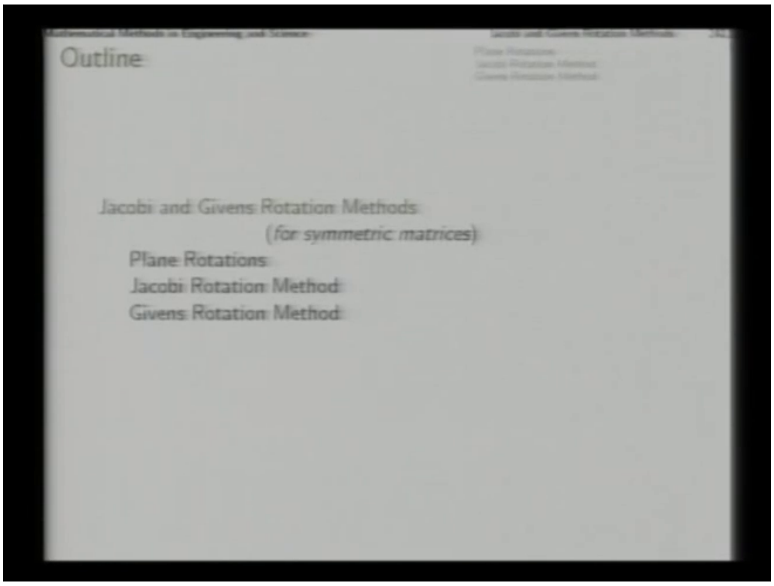

Now, onwards most of our discussion will be focused on symmetric matrices which have a lot of interesting properties which we have seen in the previous lecture. So, in this topic we will first try to see the geometric implication of plane rotations and how they give us suitable basis change and suitable similarity transformation matrices and then based on plane rotations we concentrate on 2 methods Jacobi rotation method and givens rotation method.

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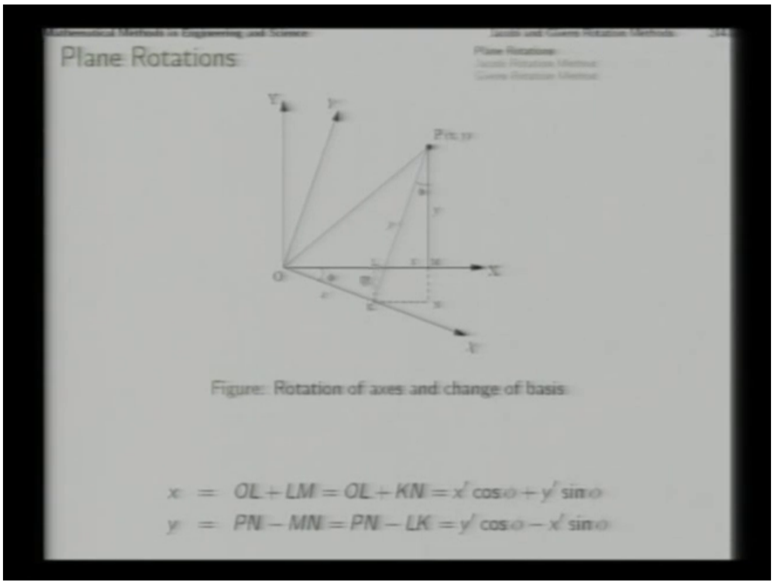

First we study the plane rotation in a simple 2 d plane x y plane suppose we have got a point P with coordinates x y and we want to make a basis change that is we want to change the frames of reference through a pure rotation such that the x and y axis undergo a rotation of i. So, the new x axis is along x prime and the new y axis is along y prime through a rotation of angle phi. Now in the new x prime, y prime axis, the new coordinators are x prime and y prime that is OK and PK. Now if you want to express the old coordinates x and y in terms of these new coordinates x prime and y prime, then this x OM is a sum of OL plus LM, right. So, x is OL plus LM and OL from this triangle OK L you find that OL is x prime into cos phi O L is x prime into cos phi.

And this lm is same as the parallel k M which from this large triangle P and K; define that kN is the same as y prime into sin phi. So, you get this similarly when you try to find out y you find that y can be conveniently written as a difference of PN and MN. So, PN minus MN. So, PN is y prime cos phi and MN is the same MN is the same as LK which from this small triangle is given as x prime sin phi. So, you have got this now this shows that the old coordinates can be easily expressed in terms of the linear combinations of the new coordinates and the coefficients are cos phi sin phi minus sin phi cos phi coefficients of x prime and y prime in x and y.

When we write this in terms of a matrix that a product in which the matrix houses, these coefficients and the vector houses x prime and y prime then we find x y vector as a matrix into the vector x prime y prime the coefficients are put in the matrix.

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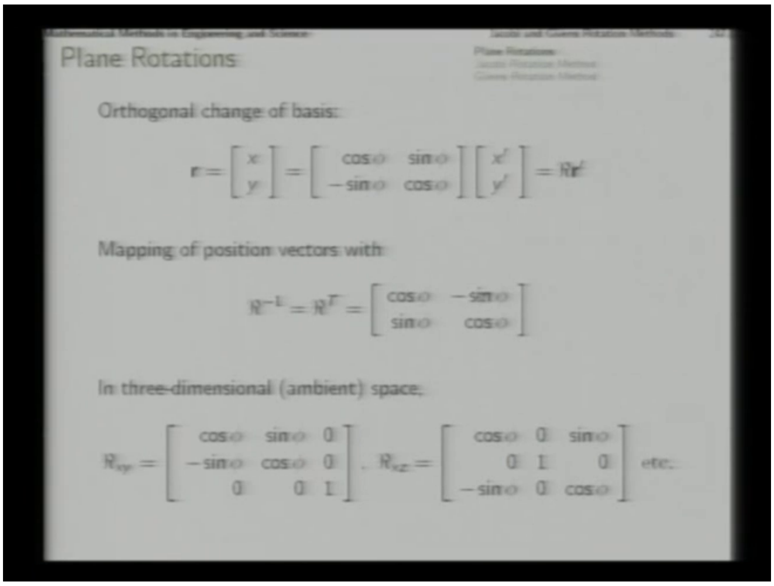

So, that matrix is this; this matrix will be represented as the rotation matrix and with this big r. So, the old coordinates R old position vector R is found as this rotation matrix regard into the new position vector R prime, right.

Now, when we want to find out the new position vector in terms of the old position vector then what we need to do where to find out R prime is equal to R inverse into R that is capital big R inverse into small r and R inverse is same as R transpose. So, this matrix finally, gives us the mapping from the old coordinates to the new coordinates in 3 dimensional space this particular matrix will be augmented with a 0 0 1 row in the bottom and 0 0 1 column in the right side, right.

So, corresponding to this; this matrix in the x y plane, we will have this matrix in which the third column and the third row is the same as identity which basically means that in this rotation in the x y plane, the z coordinate does not change and the z coordinate does not affect the x and y coordinates at all. So, that fact is obtained through this column this row and this column similarly a rotation in the x z plane, we will get represented like this in which y axis the second axis will have a similar situation. So, far we are talking about ordinary physical space.

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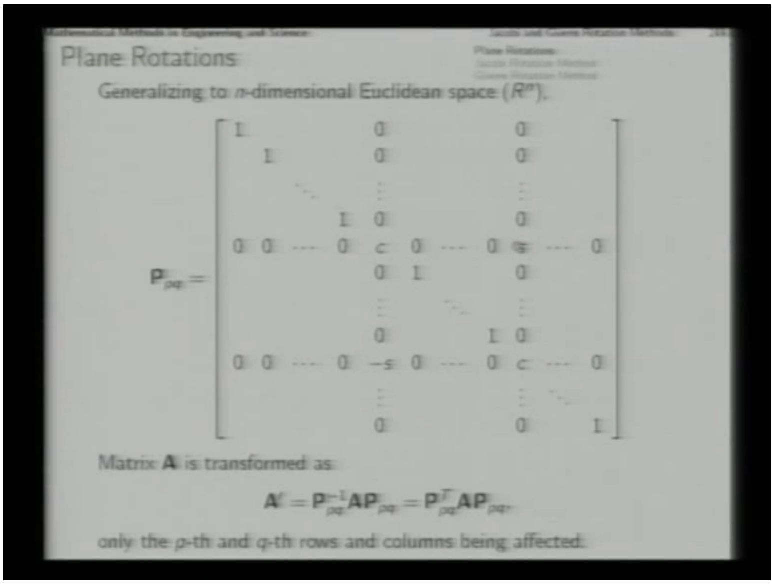

Now, when we go to the algebraic conception of space in which the dimension could be much larger than the corresponding N dimensional analogue of this kind of plane rotation matrices will have this kind of a structure.

In which the p q plane rotation will be represented like this with this large matrix in which all the entries are the same as an identity matrix except for the p p, q q, p q and q p elements. So, this is the rotation matrix cos phi 7 phi minus sin phi cos phi the same 4 elements which were appearing here the same elements are appearing again, here in those corner locations those 4 corners are the points are the locations where there is any entry other than what is found in the identity matrix all other entries with ones and 0s are equivalent to the identity matrix. Now this matrix P pq is the plane rotation matrix in an N dimensional space representing a rotation in the plane of p th and q th axis.

Now, when we apply this rotation on vectors we get relationships like this that is new coordinates to old coordinates by this transformation R equal to big R; R prime and the opposite that is R equal R prime equal to R transpose small r. So, when we apply this message change on a matrix on a linear transformation then that will operate like this that is the new representation of the same linear transformation it is representation of a in the new basis will be a prime which will be this basis matrix inverse a into this basis matrix as we have been seeing all the time and now since this matrix is orthogonal a fact which you can establish very easily. So, you can replace this inverse with transpose, right.

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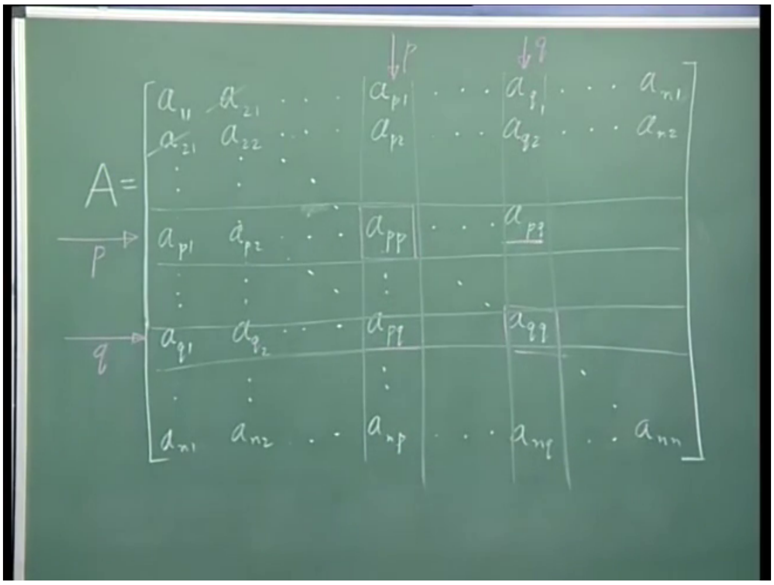

Now, consider this matrix A which is symmetric out of symmetric we are representing this as a p 1 same as this otherwise actually it is a 1 p, but because a symmetry within represent it as a p 1 itself . So, now, onwards will be discussing most the symmetric matrices now when a matrix of this kind is multiplied on the right side of a then that a p the product will not change any element in this huge matrix A except for entries in the p th column and q th column because only the p th column and q th column of this matrix has anything other than what is found in the identity matrix of this size similarly when this matrix transpose is multiplied on the left side of a then no element of a, we will change except for elements in the p th row and the q th row; that means, through this entire transformation only those elements only those members entries of a get changed which fall either on the p th and q th row or the p th and q th columns.

So, these are the entries these are the elements of a which are going to change through this entire transformation, right; now how they are changed this is a matter of pure algebraic calculations.

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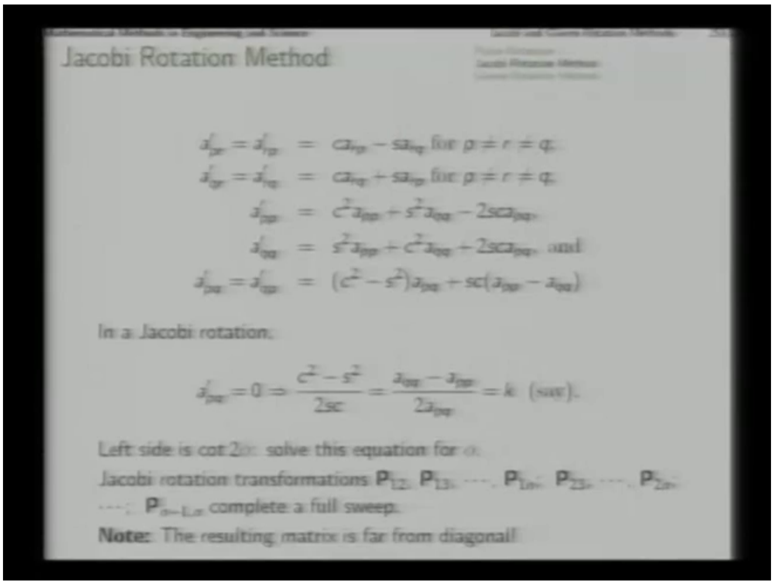

And if you conduct those small calculations, then you will find that the new elements of the matrix after transformation that is elements of a prime will have these expressions in terms of the old coordinates old members that is a r p, a r q are the r p and r q elements of the old matrix A and corresponding a p r prime is the a p r element in the new p r element in the new matrix A prime.

So, these are the changes in the p th row and column except the corner points similarly the elements in the qr row or column same because it is symmetric the transformation is symmetric the original matrix A is symmetric and you are multiplying p on the right side and p transpose on the left, right. So, the resulting matrix is also symmetric. Now the corner points change in a quadratic manner not in the linear manner why because they get changed once as part of these 2 columns and then once again as part of these 2 rows.

So, this cos phi sin phi elements enter into the mapping twice among these 4 corner entries and these quadratic expressions in sin and cos turn out to be like this now we say that if this is the transformation, then what gives us a method to find a suitable; similarity transformation in order to reduce the off diagonal elements and at their cost consolidate the diagonal elements that is what we want to do when we want to diagonalize the matrix right. So, there are various choices here one choice is very straight forward that is try to make these 2 corner elements 0 and in that whatever is the consideration of these corner elements is fine is welcome. So, when you ask for this p q term of the transformed matrix to be 0, then you are actually trying to apply what is known as Jacobi rotation and here you can see that the p q element of the new transfer matrix is given by this.

If you want this to be 0 then you can transpose this on the other side of the equality and then you can divide by twice sin cos c here represents cos phi S represents sin phi then if you divide both sides by twice S c, then you get this equal to this fellow taken on the other side divided by a p q twice a p q because of this 2 and now note what is this this is cos square phi minus sin square phi which is cos twice phi and this is twice sin phi cos phi which is sin 2 phi. So, this is cos 2 phi by sin 2 phi that is cot 2 phi. So, cot 2 phi is equal to an expression of the old elements of a which is known so; that means, we can solves for phi once phi is solved we have got cos phi sin phi and. So, we have got the complete rotation matrix in hand and using that cos phi and sin phi here we find out all the changes in the p th row p th column q th row and q th column.

This will certainly turn out to be 0 because that is the condition which we have used in order to find out the angle phi through that you would have set these 2 values as 0 and other values in the 2 rows and 2 columns would have appropriately and constantly changed; now what do we next. So, we choose p q 1 by 1 in order to annihilate the entries which we want to be reduced to 0 annihilate means kill to reduce to 0. So, for that we take p q first as 1 2 that will mean that we will be working with this corner and that will mean that these 2 will turn out to be 0 through the process of p 1 2.

Next we will apply P 1 3 in order to make the 3 1 and 1 3 element 0 and so on. So, like that as we go on applying the rotations P 1 2, P 1 3, P 1 4, P 1 5, up to P 1 n; that means, one by one we will be trying to set this as 0 then this as 0 then this as 0 up to this which due to symmetry at a same time we will set these as 0 next we move to the second column and second row and below a 2 2 below the diagonal entry we would try to make these as 0 0 0 0 0 0 at the same time these also will become 0 intern so; that means, that if we continue like this then we will have a complete sequence of operations P 1 2 to P 1 N, then P 2 3 2 to P 2 [noise], then P 3 4 2, P 3 N and so on.

Finally PN minus 1 N when we will be operating at this corner we will make this as 0 and that will mean that we have undergone the full sweep the matrix as undergone through the full sweep of such Jacobi rotations, but then what does it mean at the end of it shall we get the matrix as diagonal because we set these as 0s one by one then these as

0s one by one and so on and according you have found the rotation matrices and applied those rotations. So, as a result shall we get a complete diagonal matrix with all sub diagonal entries and all super diagonal entries 0 that is not right that will not happen the resulting matrix in general is far from being diagonal the reason is that after we have set this as 0 by applying P 1 2 operating on this 4 has the corner points.

After this has become 0 next when we apply P 1 3; 1 3 like this applying using this as corner points, then that transformation we will change these 2 columns which will mean that it has the potential of changing these entries also and in general it will change that will mean that as we apply P 1 3, P 1 4, P 1 5 the old ones old 0s that is in a 2 1, a 3 1 locations, they might get changed and stayed no longer to be 0 then the question arises that what was then the necessity of applying Jacobi rotations in a full sweep or rather what is the advantage of doing it if older 0s are spoiled in later operations, then what was the great advantage of setting the 0s in the first place.

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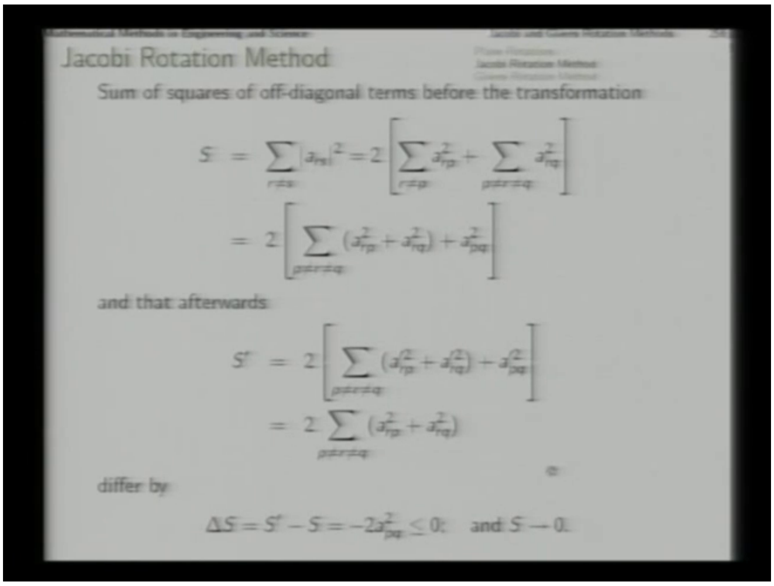

In order to notice that we need to make a little calculations and there we define this sum of the squares of all the off diagonal elements off diagonal that is why r is not equal to S right. So, sum of all these when you try to find out then that will mean that we take a r p square that is r not equal to p and then a r q square in which r is not equal to p in neither equal to q because the r equal to q term has been already taken here. So, what you are doing is it take this a r p square. So, there you will get squares of these except for this corner point take this and similarly take this and this.

Now, but then when you take this the same will be this because of symmetry; that means, this has been covered actually. So, when you considered the rows when you considered the columns the twice of that we will give you the some that of the columns as well as rows. So, 40 you have taken this and make you to make it twice. So, you get this also; that means, this has been actually covered similarly when you then considered q then you need to emit these emission is represented here.

Now, what this means this means that sum up all those entries from here to here and below and here to here and below except these 2 corner points and these identified separately. So, this is the complete sum of the off diagonal terms before the transformation and after the transformation we have kept only the p th and q th term entries here because others anyway do not undergo any change. Now when you try to calculate the same sum for the matrix A prime, then you get this and now note that through this Jacobi rotation ppq you have actually set this as 0. So, you can write simply this right this has been thrown up because this has been set 0 by this particular rotation.

Now, you compare these 2 and find out delta S, if you do that then you will notice that this sum and this sum is actually same this sum and this sum is actually same that is very easy to notice because it is this square plus this square and these 2 actually do not change because as you square these 2 term and add, then you will find that with c square you will find a r p square plus a r q square with S square also you will find a r p square plus a r q square and c square plus S square is 1.

So, square of this plus square of this plus square of this plus square of this is the same as a r p square plus a r q square and the 2 a beta in the square here and 2 a b in the square here with cancel each other so; that means, a r p prime square plus a r q prime square will turn out to be the same as a r p square plus a r q square. So, that tells you that this sum and this sum remains same for every r and this earlier was something which has been now set to be 0 and this much has been reduced so; that means, that even if old 0s are over written in the new Jacobi rotation transformations yet at every Jacobi rotation transformation; there is a net decrease in the sum of squares of off diagonal terms; that means, over every Jacobi rotation the off diagonal terms go on becoming poorer and poorer in their magnitudes.

And finally, after a large number of such rotations take place this sum will converge to 0, but that may not happen in one sweep. So, therefore, the there are several strategies to use Jacobi rotation method to diagonalize a symmetric matrix one is that after one complete sweep you start all over again from this and complete another sweep and then another sweep and then other sweep one strategy is to go on applying these sweep in iteration, you see it is not a fixed operations process it is an iterative process.

So, sweep after sweep you will be reducing the off diagonal entry magnitudes overall; this is one strategy in another strategy what people do is that after if you initial speech then later at the time of writing the new entries you can keep track of what is the largest magnitude entry off diagonal entry in the matrix and then if it turns out that the 4 7 entry turns out to be the largest magnitude at a after the completion of a sweep then after that you can say that now onwards we will now we will try to make this 0 and you apply P 4 7 selectively.

Next if P 5 9 turns out to be the largest magnitude entry then you say we will apply P 5 9 and so on. So, after a few initial sweeps after reducing the off diagonal entries to some extent then you can check for the largest entry largest magnitude entry and that way you try to annihilate the largest positive entry first and that way you can expedite the process expedite the iterations and make the process faster this is one way, I am applying plane rotations to diagonalize a matrix there could be another choice that is rather than asking for the corner values to become 0 through the transformation you could have chosen any other value that is not necessarily this you could have chosen some other entry to become 0s.

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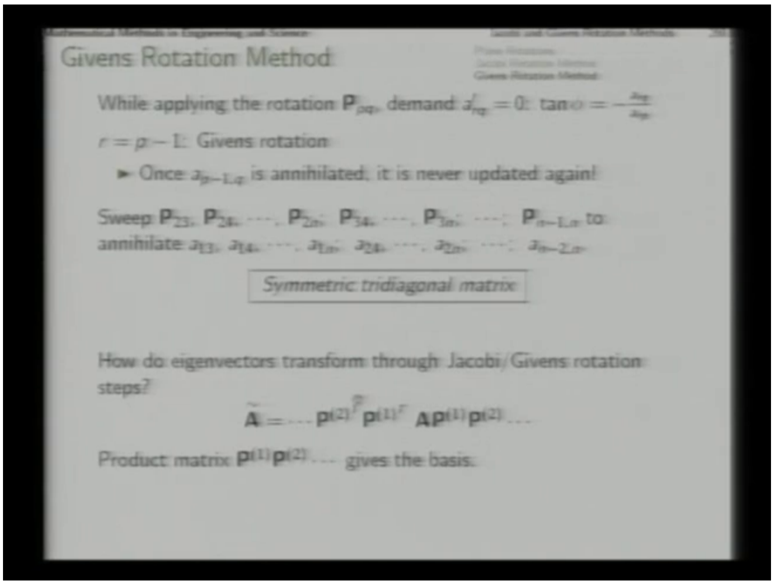

For example while applying the rotation P p q rather than asking for a p q prime to be 0 you could have chosen a r q prime to be equal to 0 that is rather than this you could have chosen this, this, this, this; whichever accordingly it would mean one of these. So, you have put have chosen that and that if you say a r q prime equal to 0 for any r you can chose in principle yes. So, a r q, a r q or A; a q r is the same thing. So, one of these you want to be set equal to 0 that will mean that you want sin by cos is minus a r q, a r p minus a r q by a r p that is tan phi. So, that will give you another value of phi which we will set given chosen element in this row and in this column to become 0 through the transformation.

A particular choice gives you givens rotation method in which r is taken as p minus one; that means, you do not try to make this corner element 0, but you just want to make this element 0 just left of the this corner and just above this corner. So, this choice of element for the annihilation process gives you the method known as the givens rotation method which means that AP minus 1 q is annihilated and the advantage of this particular annihilation is that in the subsequent rotation transformations this is never updated again because if you then apply start the sweep then you will not be starting the sweep from here.

But you will be starting the sweep from here. So, you will apply the transformations in this order first we have apply a P 2 3; that means, you have be first operating on this corner 4 corner blocks and then you will not be trying to make this 0, but you will be trying to make this 0 this and this. So, after they at least accomplished, then you will ask for P 2 4; that means, with these 4 as the corner points in order to make this 0 and in that you will find that if you are applying P 2 4, then only the second row and the fourth row will be updated third row will not be updated and the 0 that we have said we have said in the previous case is sitting actually in the third row.

That way the successive givens rotation transformations will not at all update those locations which we are said to be 0 in the previous givens rotation and this has an advantage. So, with these givens rotations P 2 3, P 2 4, etcetera, we would have made all these 0s which will never be updated again as you then go to the next givens rotation which is P 3 4 to P 3 N then you completely move from here and operator here. So, in that in P 3 4 2 P 3 N you will be setting these as 0 and so on and the symmetric nature will ensure that above the super diagonal similar 0s are getting established.

So, at the end of a givens rotation sweep which is P 2 3 2, P 2 N, P 3 4 to P 3 N and finally, PN minus 1 N adjacent you will get all these 0s because the old 0s will not be updated in the new givens rotations and the result of this whole thing is a symmetric tridiagonal matrix and this we this givens rotations sweep has to be applied only ones and after transforming the matrix into this form no further givens rotation will contribute anything that is why the givens rotation sweep is applied only once not in iterations now in this whole process whether you apply Jacobi rotation or given rotation givens rotation how do the Eigenvectors transform as you know that in the new final matrix you have got all the transformations P 1, P 2, P 3, etcetera all those rotation transformations those matrices similar transformation matrices sitting like this.

Here the corresponding transposes which are same as inverses. So, then in the entire product if you consider P 1 to PN all of them together as a big matrix P then you will say that that matrix P gives you the basis through which the transformation has finally, taken place here the transpose of that entire thing is actually sitting right now computationally when you want to apply these transformations and you want to at find out at the end of the process not only the Eigenvalues, but Eigenvectors also in those situations it is important to keep track of this.

You do not want to save all this matrices. So, what you do in the beginning you say that before we have applied any transformation.

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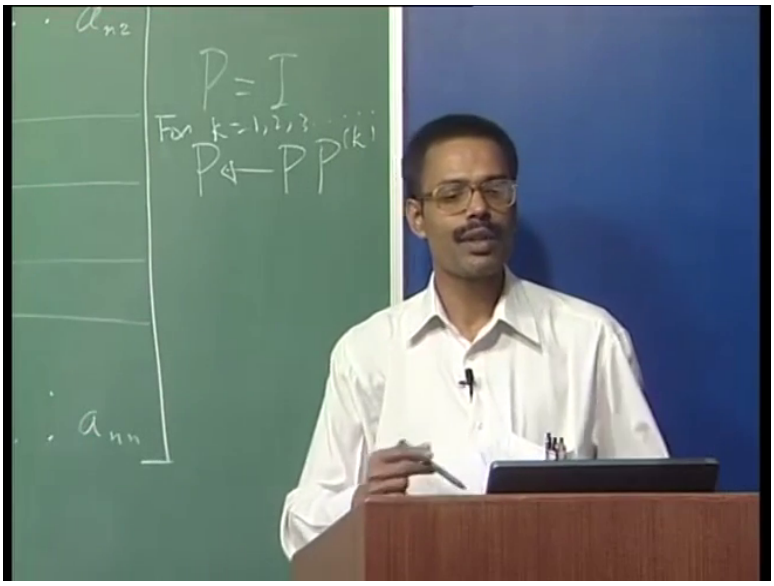

We have considered P S identity and as a transformation has been applied, we will keep on multiplying the new transformation on this side that will mean that in place of P, we will stored the product whatever transformation has been taken place whatever rotation has been applied that rotation matrix will be multiplied to this. So, first time this initialization with identity is actually dummy, but next the moment P 1 has been multiplied on this side to it in this we have P 1.

Next when P 2 is multiplied like this we will have P 1, P 2 the product next P 3, then we will have P 1, P 2, P 3 and so on. So, the iteration we will actually go like this so; that means, for K equal to 1, 2, 3, 4 as many rotation transformations are applied all of those get multiplied from the right side and finally, you will have the P storing all the Eigenvectors by the time the matrix has been diagonalized if the matrix has been processed only up to symmetric tridiagonal form in the givens rotation method, then that P matrix resulting P matrix will relate the Eigenvectors of the original matrix and the matrix A prime which we have in our hand now which can be processed further through some other method.

Now, if your questions arise because on based plane rotations, we have considered 2 methods one is givens rotation method and the other is Jacobi rotations method.

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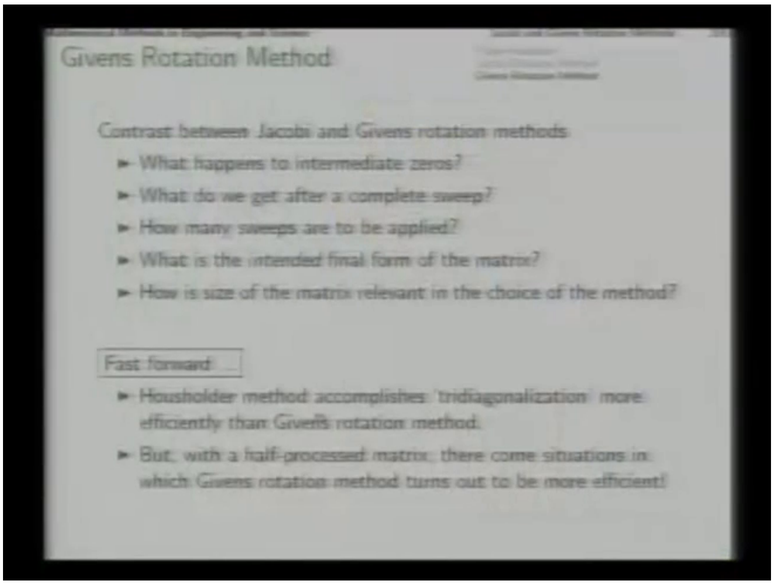

These are the points of contrast which it would be interesting to summarize ones first question is what happens to intermediate 0s in the case of Jacobi rotations, they get spoiled in the case of givens rotation, they are preserved second question is what do we get after a complete sweep in the case of Jacobi rotation we get another matrix which is also perhaps full matrix, but with the off diagonal terms a bit reduced in magnitude compared to the old matrix.

In the case of givens rotation, after a complete sweep, we get a completely symmetric tri diagonal matrix as long as the original matrix is symmetric third question is how may sweeps are we suppose to apply in the case of Jacobi rotations, we have to apply sweeps through iterations several sweeps till the off diagonal, it turns get reduced to sufficiently small magnitude, in the case of givens rotation method, we have to apply only one sweep resulting in one symmetric tri diagonal form after which there will be no further advantage what is the intended final form of the matrix in the case of Jacobi rotation after a sweeps; after the necessary number of sweeps whatever is required for convergence the intended final form is actually diagonal how many sweeps will be required for that we do not know.

In the case of givens rotation method, actually half way processing is intended only half way processing is intended further than that givens rotation method does not require to go at all final question which is of practical relevance is how is the size of the matrix relevant in the choice of the method typically for small matrices say 5 to 7 a Jacobi rotation method is good enough, but for much larger matrices 9 by 9 or 12 by 12 Jacobi rotation method may be computationally very expensive.

So, there the strategy should be to apply givens rotation method and then reprocess the tri diagonal matrix through some other method you will later find that householder method which will consider in the next lecture also accomplishes tridiagonalization a little more efficiently than givens rotation method; however, for a off process matrix sometimes givens rotation method turns out to be more efficient we will come across one or 2 such situations in the exercises.