**Mathematical Methods in Engineering and Science**
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**Module - I**
**Solution of Linear Systems**
**Lecture - 03**
**Systems of Linear Equations**

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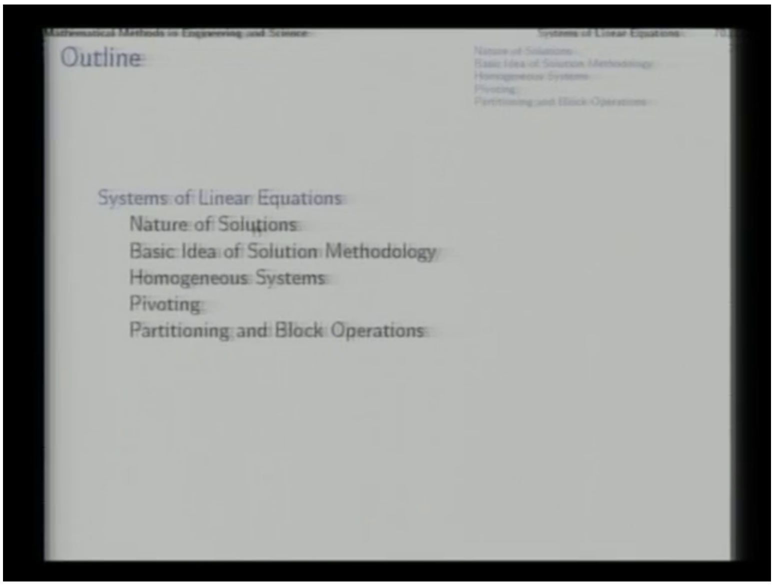


In this lecture we will be discussing solutions of systems of linear equations, first we will discuss the abstract ideas regarding nature of solutions. Then we will go into the basic idea of the methodology by which we solve the systems of linear equations and then we will consider the particular case of homogeneous systems of equations in the which right hand is 0. The other case when the right hand side is not 0, we will be taken up later which is the more regular and routine case of such systems. After this we will consider 2 important issues in such systems one is pivoting and other is partitioning.

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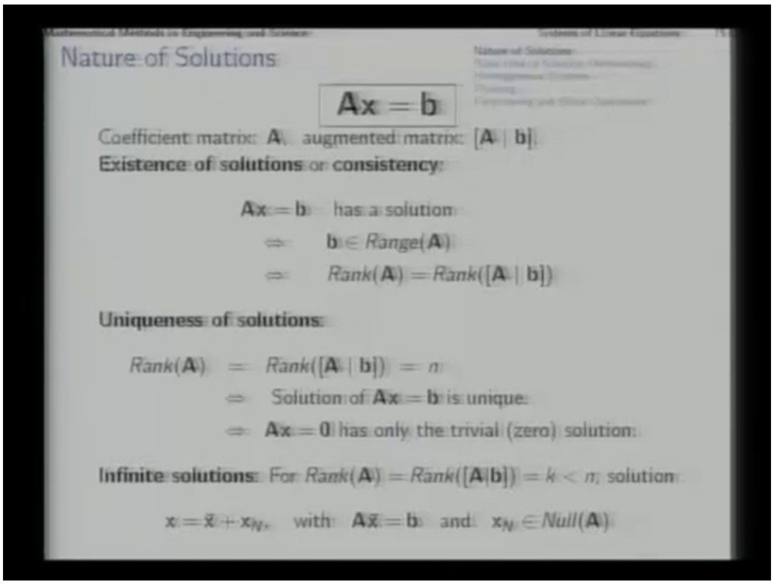


The first nature of solutions of systems of linear equations the systems of linear equations in the general can be expressed in this manner A x equal to b in which the matrix A is the coefficient matrix and when we enhance it or augment it with the right side vector on this side then we construct this larger matrix or augmented matrix which is A and b together we will refer to this matrix, this larger matrix as the augmented matrix.

The first question that arises is does this system have a solution that is the question of existence of solutions or consistency of the system of equations if the solution does not exist then we say that this particular system is inconsistent that is there is some internal conflict among the different equations in the system.

Now when we try to answer this first question that is whether this system is consistent or whether solution exists then we have this following important theoretical result that is A x equal to b has a solution this statement is equivalent to this second statement which is at b is in the range of A and that is again equivalent to the third-one which is at Rank of A is the same as the Rank of the augmented matrix this is very easy to visualize.

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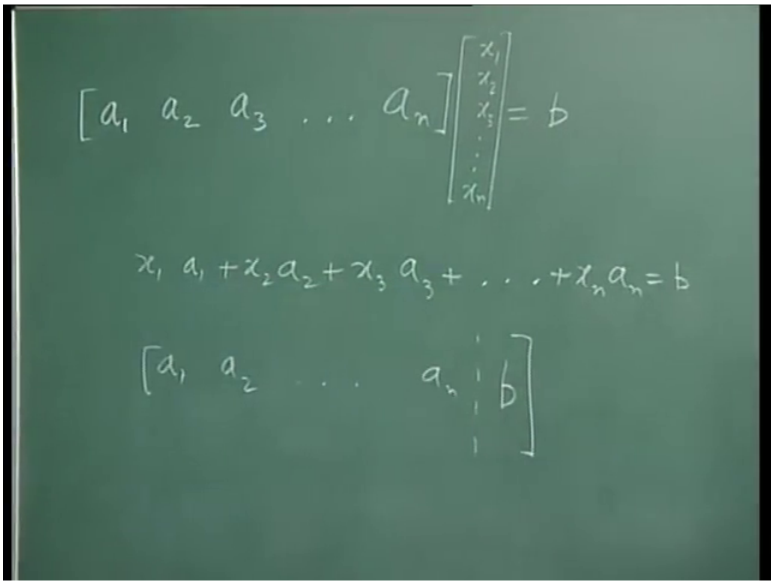


Because if we write this system A x equal to b in this manner in which the columns of a are written as a 1, a 2 and so on then the entries of x will appear in this manner this shows that what we are talking about is actually a linear combination of these columns on the left hand side because a 1 into x 1 gives us this plus column a 2 multiplied with x 2 gives us this and so on.

Now when we say A x equal to b has a solution; that means that there is a set of x 1, x 2, x 3 values which when put in this linear combination will result in a total sum which is equal to b and that is as good as saying that b is in the range of A that is as a linear combination of columns of A as A x A multiplied with x we can find b that is directly from definition and this is also equivalent to say that since as a linear combination of these columns a 1, a 2, a 3, a 4, etcetera, we can find b that means, when we enhance these columns with one more column then this is a linear combination of all these and that means this is linearly dependent on these, then that means, that whatever rank these vector together have whatever rank this matrix has this will not add anything new in terms of linear independence that is why the augmented matrix also has the same rank.

So, when the rank of the coefficient matrix and the rank of the augmented matrix are same in that case b will be in the range of A and that means that system is consistent there is a solution.

The question is still open whether there will be only one solution or many solutions and then we come to the second question which is that of uniqueness of solutions for that the important result is that when these 2 ranks are same and also equal to the number of columns of the matrix A that is number of variables then that will imply and will be implied by this statement which is there is a solution of A x equal to b is unique which is again equivalent as saying that the corresponding homogeneous equation homogeneous system A x equal to 0 has only the trivial or 0 solution. Now, to verify the established the result as shown here, we consider this situation that is suppose p and q are 2 different solutions of this system A x equal to b p and q are 2 different solutions.

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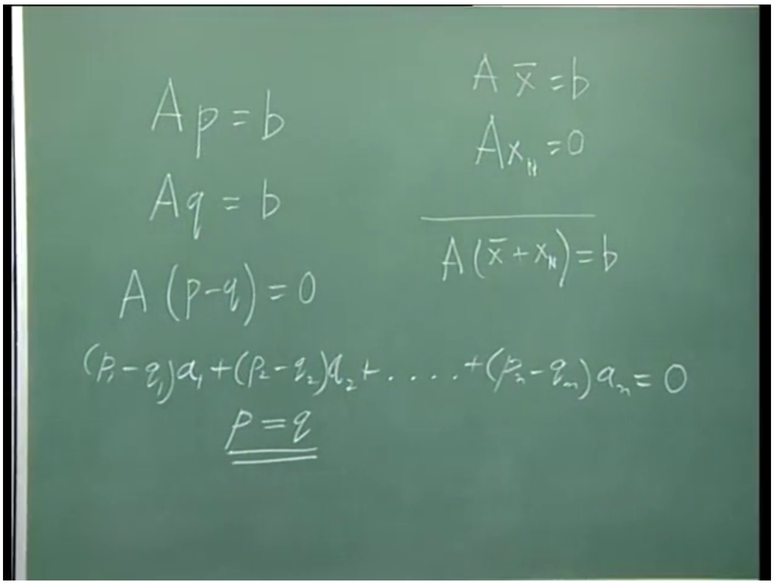

Then we can say A p is b as well as A q is also b right and in that case we can subtract and find A into p minus q is 0 and what is A it is a matrix with a 1, a 2, a 3 etcetera as columns and what is p minus q p minus q is a vector with entries p 1 minus q 1; p 2 minus q 2 and so on. So when we open this in this manner then we find the first entry of p minus q in to the first column plus the second entry in to the second column of matrix A and so on, this equal to the 0 vector.

Now, we know that since the matrix A has full n rank and it has only n column; that means, all these n vectors are linearly independent. Now if a 1, a 2, a 3 etcetera are all linearly independent then from the definition of linear independence we know that this sum equal to 0, will mean that individually these coefficients are all 0 that means, p 1 minus q 1 is 0, p 2 minus q 2 equal to 0 and so on that means p 1 equal to q 1 p 2 equal to q 2 and so on which will mean that the full vector p and the full vector q are same that means, these 2 solutions that we picked up they cannot be different they have to be same if a 1, a 2, a 3, a 4, etcetera are all n linearly independent vectors that shows that if these have the same rank and that rank is equal to n that will mean that this system has a unique solution.

It has a unique solution for all b in particular that is true for b equal to 0 as well and that shows that A x equal to 0 also has a unique solution and we already know that x equal to 0 certainly satisfies this equation system whatever may be the A matrix and therefore, in that case it will mean that it has it is unique solution is 0 solution or the trivial solution. Again from here we can show this in a similar manner and that will that again from the definition of linear independence that if this has a trivial solution only then that will mean that rank A is equal to n and so on so that way we find that this result regarding uniqueness of solutions and this and earlier result regarding existence together gives us the complete theoretical background for the nature of solutions.

When it happens that this rank and this rank are same, but not equal to n but somewhat less, then the solution will exists but it will not be unique in that case we will have infinite solutions for the system.

And then we need to characterize those infinite solutions. How do we characterize that suppose in a situation rank of A and rank of augmented matrix is same and is equal to k that is less than n in that case if x bar is 1 solution and x n is a member of the null space which means A x n is 0 that is if x n is a member of the null space then x n get mapped to 0 and x bar is some solution of the original system given. Now if we add these 2 then we find at this also considered to be a solution on this side we have added these 2 on this side this will be b because A x n will contribute nothing that is 0 only and A x bar contribution will remain that means, for every member of the null space x n if we add that to another to a solution of the original system then this sum is also going to be a solution of the original system and since the null space has infinite members, so adding any of them will get another solution so that will characterize the complete solution of the linear system A x equal to b.

So, now we have 2 sub problems one is to make a description of null space to describe all vectors which consist in this place and to find one solution of the system A x equal to b and then when we can we add that one solution one particular solution with this null space member arbitrary null space member we have got a complete description of the solutions of A x equal to b.

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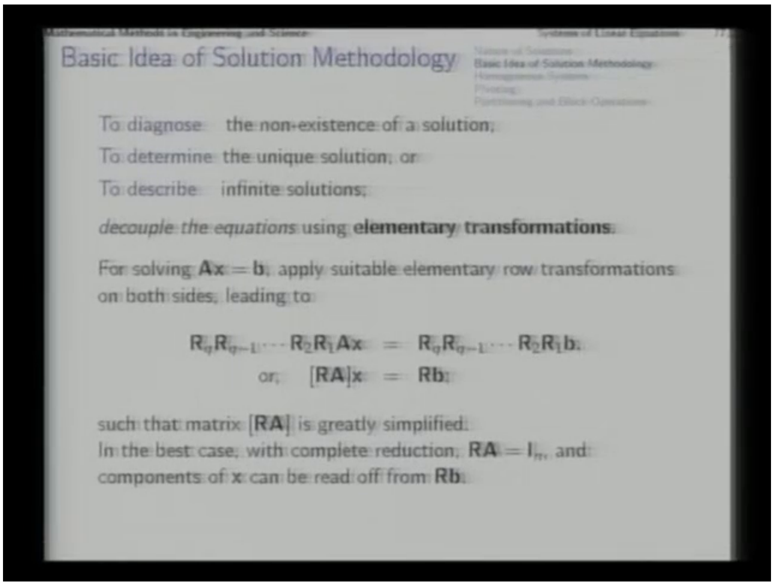


So in the basic idea for solving the system we have 3 small pieces of works. First to diagnose if the system does not have a solution we have to diagnose a non existence of a solution, if the solution exists then we have and in that case if it is unique then we the problem of determining that unique solution that is an easy job compared to the difficult will be the third piece of work which is if the system has infinite solutions then our task will be to describe this infinite solutions, to diagnose the non existence, to determine the unique solution and to describe infinite solutions these are the 3 pieces of work.

In all this cases it will help or doing all of these a priori you do not know which will be the case to do any of these or all of these we take the part to decouple the equations to the extent possible using elementary transformations which we discussed in the previous lectures.

What we do to solve this or to simplify this is that which is we take A x equal to b and apply suitable elementary row transformations on both sides on this side it will be applied on left side it will be applied on A, on the right side it will be applied on b; that means, every elementary row transformation that we apply on both sides is equivalent to the pre multiplication by the corresponding elementary matrix say first with the R 1 simplification to an extent.

Next we apply another row transformation in order to simplify this situation further say R 2 both sides, then R 3, then R 4 and so on. Finally, lot of elementary row transformations that applied that means a lot of pre multiplications take place with the corresponding elementary matrices. Finally, we can consider this entire product R q, R q minus 1 R q minus 2 up to R 2 R 1 A this whole thing together is written here A is written as it is and the product of all these elementary matrices is getting as R, the same R will be sitting on the right side also as R b. Now after this point we examine what happens to this matrix here R A we want R A to be as simple a matrix as possible has close to an identity matrix.

Accordingly we decide our strategy and finding the suitable row transformations that we apply on both sides. In the best case with complete reduction R A become identity in identity into x is simply x and in that case whatever is the vector on the right hand side the entries of that will give us a values of x 1, x 2, x 3 etcetera if it cannot be reduces up to identity then we try to make as much simplification as possible in the matrix R A so that finally, a little bit of processing is sufficient to find out the values of x, x 1, x 2, x 3, etcetera entries of x or to describe them in terms of a few variables in case of infinite solutions.

The process as we discussed earlier has 2 issues, one is to describe the infinity of infinite solutions in which case we need to find a null space of A. On the other hand we have to find one solution the problem of finding one solution we will consider in the next lecture the immediate task is to characterize or describe the case of infinite solutions in which the matrix A has a non trivial null space. So, for that purpose let us first see the elaboration of the schemes which we utilize and then we will be talking about the formal structure of the method.

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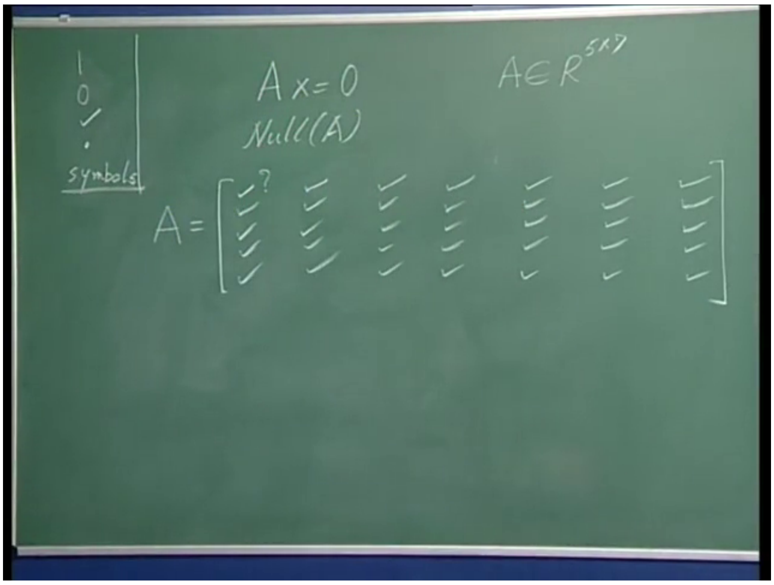


For that we take this problem of finding a solution of this determinant in which A is of 5 by 7 it is a rectangular matrix. Now when we say that we want to find the solution of this system A x equal to 0 in which the right hand is 0 that is equivalent at saying that we want to find out the null space of A right because we looking for all those vectors x which in this product A x give us 0.

So, we are basically trying to the null space of A, now note that we apply an elementary row transformation on A on this side, then on that side we will be applying the same elementary row transformation on 0 and whatever elementary row transformation we apply on the 0 vector it will still result in 0 that is way it is not necessary to go on writing the 0 on this side we will just keep on applying elementary row transformations on a with a view to reduce it significantly and here what I will do is at every step whichever values in that 5 by 7 matrix get changed I will show that with a tick mark like this and whichever values it will remain same I will shows as dots and whenever it is needed for us to use a particular value a particular entry then we will give it a name otherwise if that particular entry becomes 1 or 0 then we will write that 1 and 0 that means we will be using symbols which are 1 when it is known that it is a 1 there 0 when it is known to be 0 there tick mark when it is changed from the previous step and dot when it remains unchanged from previous step and we will use a symbol when that is going to be used.

So these 5 notations we will be using in elaborating in this method. So first we have all say tick mark 5 by 7 matrix, our first step to simplify this matrix is to find a 1 here for that we examine whether this number is 0 or not if it is 0 then we would like to interchange the rows to get a non zero number here either from here or here or here or here.

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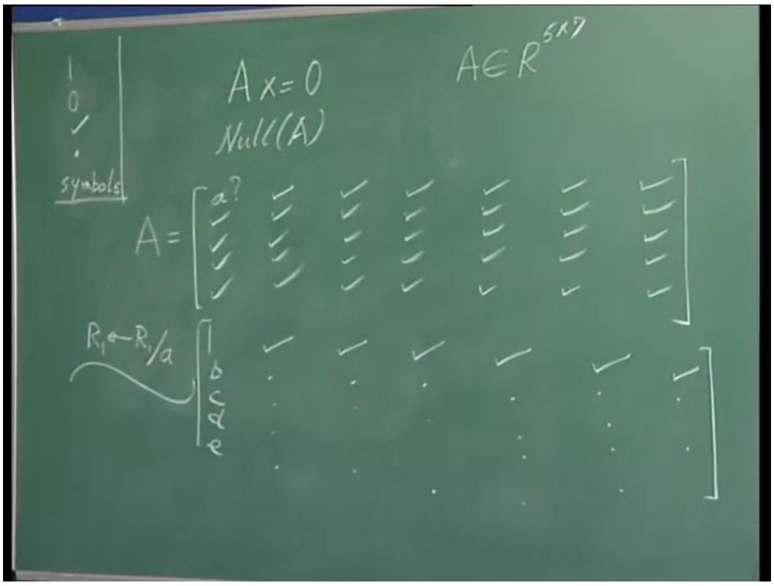


So, first we see whether this is 0 or not in the first case suppose we find that it is not 0, so give it a name for this entry for the time being we give a name a and then we wanted a 1 here we apply here elementary row transformation which is divide the first row by a that is this number, if we divide then what we get we get 1 here something here is whatever is old entry by a something these get changed, the other 4 rows do not get changed so right so these do not get changed.

Our next intention will be to get 0's here right now what are there, so there are 4 numbers sitting here lets us give them some names. How do we get 0's here from elementary row transformations what we do is that we take the first row multiply it with b and subtract from the second row then what ever happens in the later columns later entries 1 thing is sure that here we will get 0, right.

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So, next elementary row transformation will be a set of 4 transformations and those 4 transformations will be from second row multiply subtract from the second row subtract b times the first the row so b minus b into 1 will give 0 here and other entries will appear here. In the mean while the first row remains unchanged and here we will get 0, so second row will become second row minus b times the first row and so on.

Similar situation for the third row, fourth row, fifth row so all these rows will get changed here we will have 0 and here whatever 0 0 0 these will be changed currently we are not interested what are those changed numbers, right after processing the first column like this and getting 1 0 0 0 0, now we shift our attention to the second column we want a 1 here we cannot do anything because right now we cannot do anything here because 1 is here so we preserve this 1 below that the 0's that we have found we preserve we want a 1 here if possible, then we examine what is this is it is 0 or is it non zero so suppose in this case we find that it is non zero call it x.

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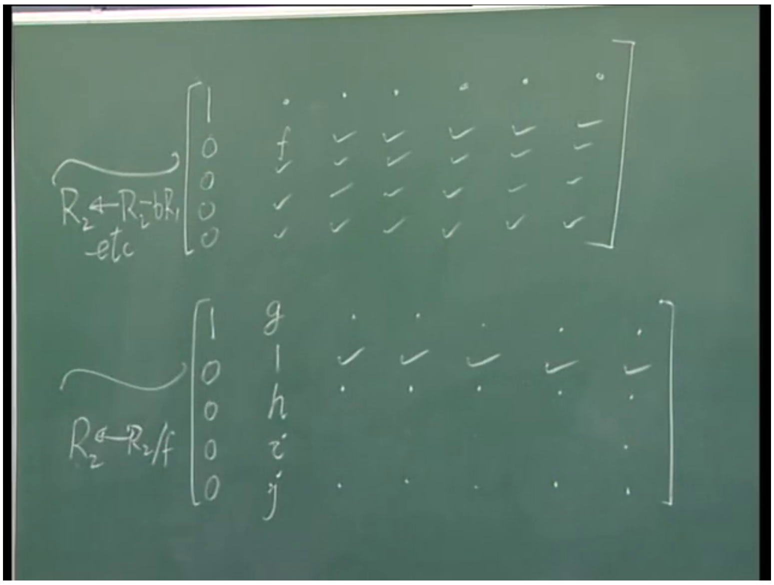


In a later situation, we will consider 0 there and then we will examine what to do if 0 appears in such a situation currently suppose it is f right we wanted a 1 here see if it is 0 then we cannot get a 1 here by just dividing because 0 which whatever you divide you will get 0 only so if it is a non zero number, then we can divide by this number to get a 1 here right.

So, the next elementary row transformation will be divide the second row with f as a result the first row will remain unchanged the second row will change 1 will come here and then other stuff will come here and the lower rows will remain unchanged after getting the 1 here we will try to get 0's in the other positions of this column that means, here, here, here and here what are they are currently, currently some number which have been copied from the last step so give them some names g h i j. Next what will be our steps to get 0's in the these locations in place of g h i j we will multiply row 2 with g h i j and subtract from row 1 3 4 5 respectively in doing that we will not be spoiling this 1 and these 0's because this 0 multiplied with g h i j and subtracted from the respective rows we will not lead to anything new because this is 0 here.

So as we multiply the second row with g h i j then we will get 0's here right and now onwards to save time I will not be writing all this detail I will be just discussing the steps and carrying out the appropriate operations in the matrix.

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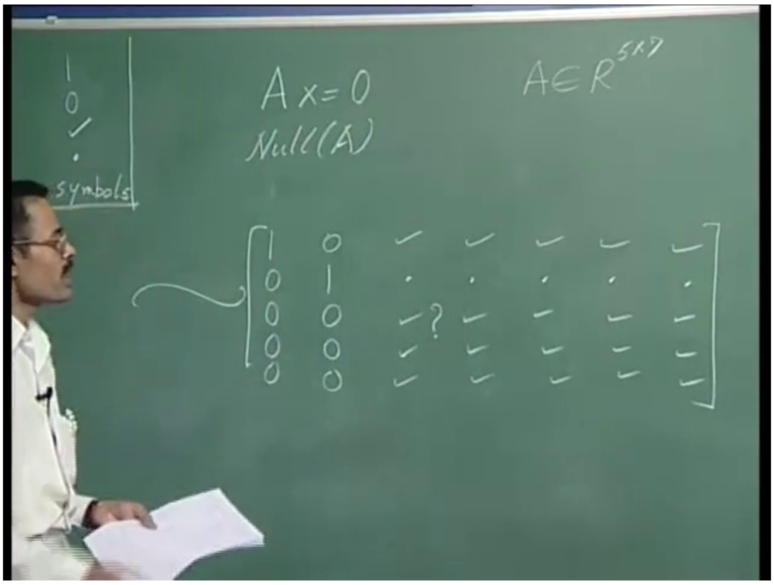


In the first column there will be no change because this 0 will fail to make any changes here that have been part of the strategy right in the first column these entries will become 0 by R 2 equal to R 2 minus R 1 equal to R 1 minus g into R 2 and so on.

So we will have 0 then some certain changed entries this will remain unchanged in this step this will change right. Now in the same process what we will we looking forward next after we have set these 2 columns we will asking for a 1 here so for that we need to first see what it is what it is this can be 0 or non zero. What to do if this is non zero that we have already seen we divide this row with this number.

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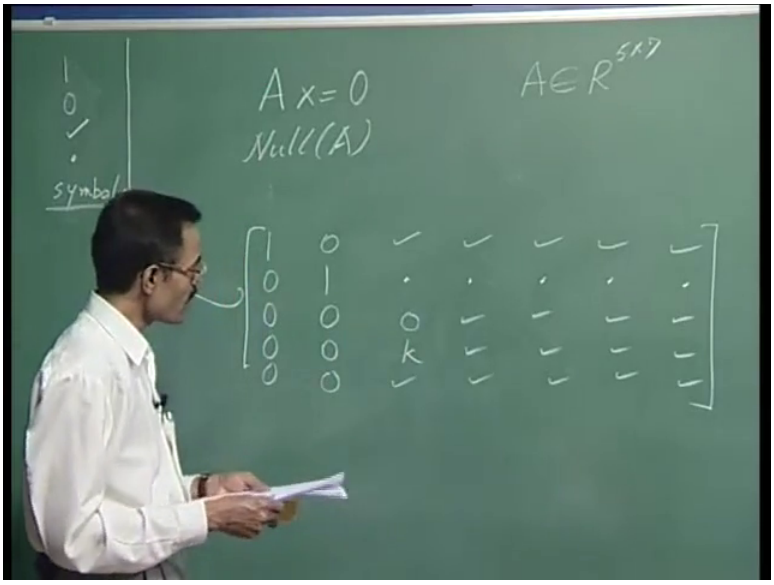


Now let us suppose that in this particular case this turns out to be 0, if this turns out to be 0 then we examine what is below that what is here suppose this is non zero.

Suppose this is non zero say k, then what we do is that this third row and the fourth row we interchange as we interchange the third row and fourth row then these are the 0's nothing very special these entries and these entries interchange their locations.

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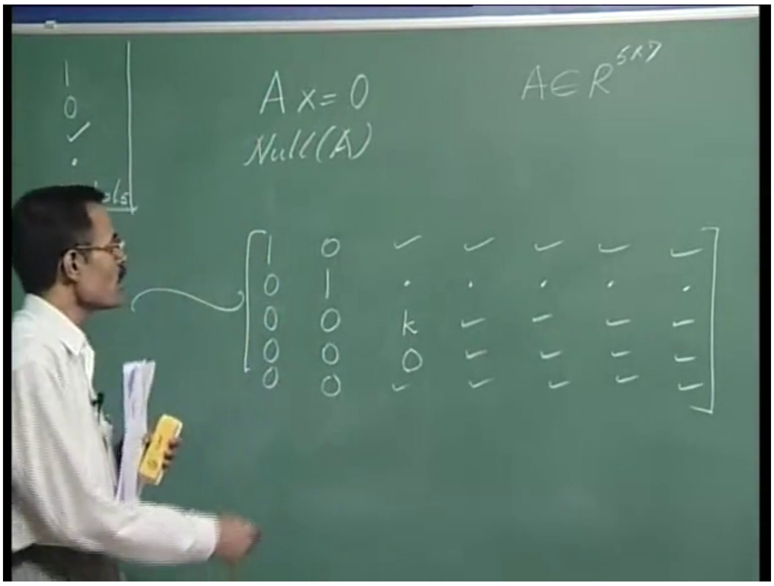

And this k comes here this 0 goes there these will be changed they will be sort now this is non zero and we wanted a 1 here, so what we do we divide the third row with k whatever is this number that changes these entries and here we get 1.

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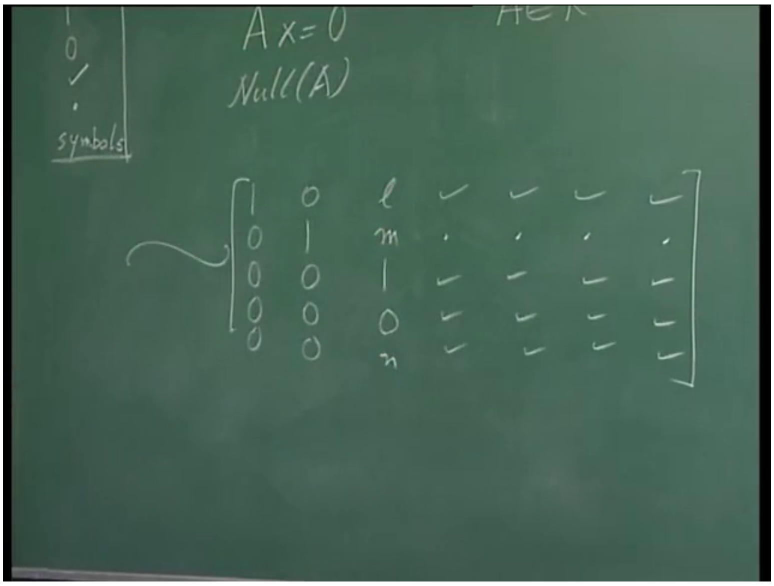


Next in this column other than this one we want everything has to 0, this column is already 0 other 3 we want to be 0, so what are these numbers by the way suppose they are l m n for entry these are the entries l m n. Then what we do we subtract from the first row l times the third row subtract from the second row m times the third row and subtract from the last row n times the third row that way we get these 3 also as 0's first, second and fifth are changed.

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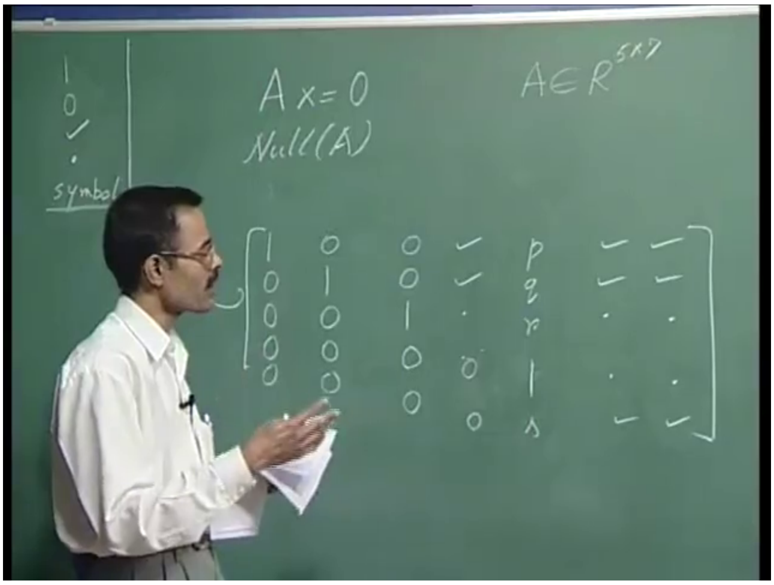


So, in this step these have changed and these have remained same, after coming this point then we examine what is this number right suppose this is found to be 0 then we examine what is this number in this case suppose this also found is found to be 0 that means that in this column we could not get a 1 the way we found till now in this case we could not find that.

Now we want further next column what is here we examine this suppose this is found to be p then we want this to be 1 just like earlier cases here we could not find unfortunately here we then we are found a non zero number we want a 1 here and as such we will be dividing this row with p and that will mean that we will get a 1 here these we will change and then we want a 0 here, if we have got 1 here we want 0 below it and 0 above it all these locations we want to be 0’s.

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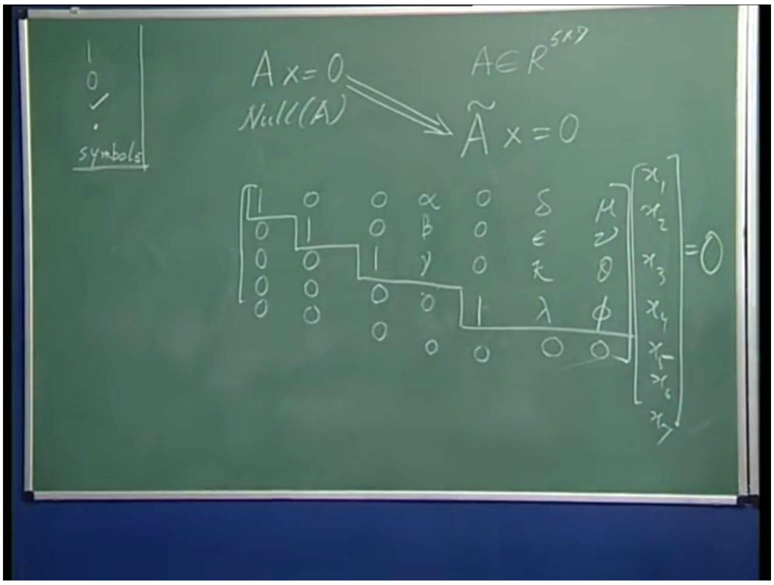


So, currently suppose we have got here p q r s then fourth row multiplied with p q r and s is subtracted from row 1, row 2, row 3 and row five right then that will give us these 4 entries as 0 0 0 and 0 and these corresponding terms change or whatever these will remain same in this step. Now after getting this till this point we examine what is this and what is this.

What is this when we examine if it is not 1 say so suppose it is 0 and suppose this is also found to be 0, then we cannot do anything about these numbers because above the 0 and below the 0 below the 0 is to be 0 only above the 0 whatever is sitting here we cannot do anything regarding these.

So, now the non trivial entries in this matrix remain in these locations in that column in which there is no 1 in the leading position, so now let us give them names the non trivial entry suppose we call them alpha, beta, gamma. Then next round of non trivial entries are in these columns so let us give them also certain names say delta, epsilon, kappa, lambda, mu, nu, theta, phi these are the non trivial entries which remain in the matrix and now let as call this reduced matrix simplified matrix as A tilde from a to this step has been arrived at through all elementary row transformations and now in this reduced matrix see this step there like this like this like this like this like this like this like this below which everything else is 0 this form is known as the row reduced echelon form.

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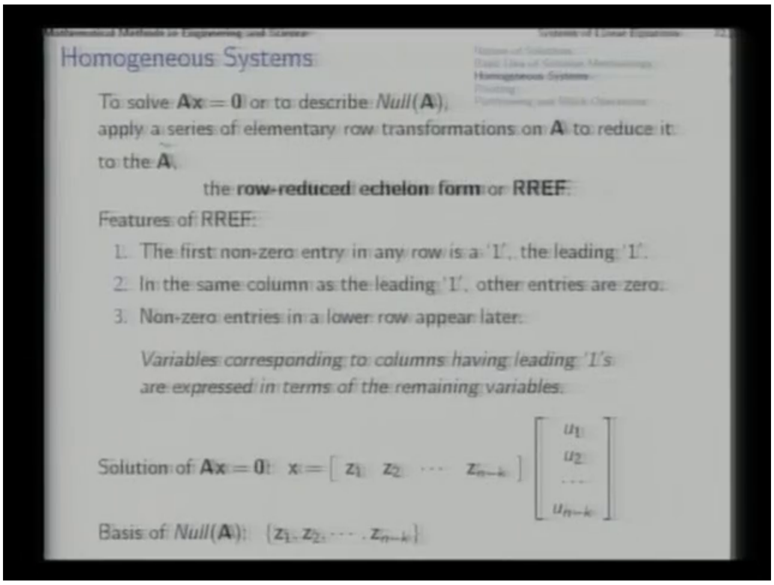


Echelon is a French word which means steps so this step wise formation that we have got through row reduction this form is called the row-reduced echelon form or RREF. So through a series of elementary row transformations we have reduced the original matrix A to this form A tilde and this form is the row reduced echelon form.

The features of this form are that the first non zero entry in any row is a 1, in this row there is no non zero entry if it where there it would be a 1 wherever it appears right this particular entry is called the leading 1. So the first non zero entry in any row is a 1 which is quite often referred to as the leading 1, the second feature is that in the same column as the leading 1 other entries are 0 whenever that leading 1 appears in that column other entries are all 0 above and below above and below above and below.

The third feature is that non-zero entries in a lower row appear later the non zero entries in this row will appear later than in this row and so on. So, with these 3 features we have got the row reduced echelon form. Now again go back to the original thing that if A x is equal to 0 is the system then the solutions of the system remain unaltered through all these elementary row transformations that we have tried to that means, if this is 0 then that will also mean that A tilde x is also 0 which means that we can say the equations are now greatly simplified from the first equation we find that x 1 plus alpha x 4 plus delta x 6 plus mu x 7 equal to 0.

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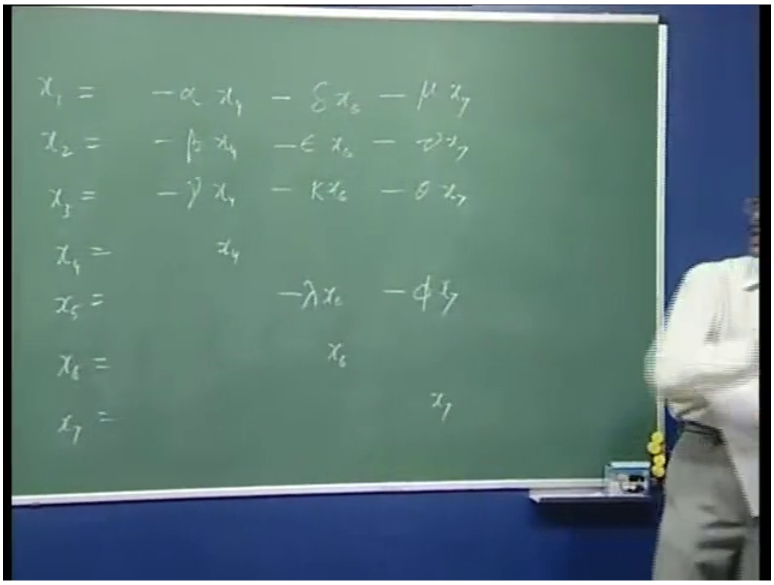


And therefore we can express x 1 as minus alpha x 4 minus delta x 6 minus mu x 7. Next the second equation tells us 0 into x 1 plus 1 into x 2 plus 0 into x 3 plus beta into x 4 and so on. So, keeping x 2 on one side we take everything else on the other side and then we get minus beta x 4 minus epsilon x 6 minus nu x 7 like this. From the third row we similarly get x 3 as minus gamma x 4 minus kappa x 6 minus theta x 7, x 4 we write simply as x 4 because the fourth column does not have a leading 1 right so that we leave.

Then x 5 that is coming from here this is the leading 1 of the fifth column so x 5 we get as minus lambda x 6 minus phi x 7, then x 6 we do not have to write like that because for the sixth column the leading 1 is missing this column does not have a leading 1, So x 6 we write as x 6 only x 7 we write as x 7 only.

What do you notice here all the variables in particular variables x 1, x 2, x 3 and x 5 have been expressed in terms of 3 variables x 4, x 6 and x 7 that means, that the variables x 1, x 2, x 3 and x 5 those with a leading 1 in the corresponding column had being expressed in terms of those columns which those variables the corresponding columns of which do not have a leading 1 right and that is what we do as a rule.

Variables corresponding to columns having leading ‘1’s are expressed in terms of the remaining variables.

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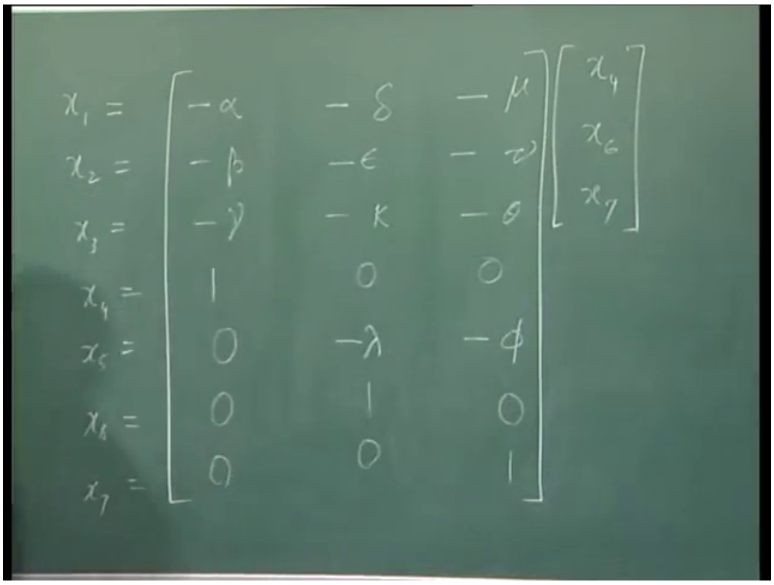

And this whole thing can be written nicely in terms of this matrix vector multiplication which will look like this right x 4 is simply x 4 1 into x 4 plus 0 into everything else x 6 and x 7 are similarly 1 into x 6 1 into x 7 nothing else. The other variables I express in terms of x 4, x 6, x 7 right that means, that this system will have a solution as any vector x in which these 3 variables can be chosen at will and after the choice of these 3 variables rest of the variables get defined by this.

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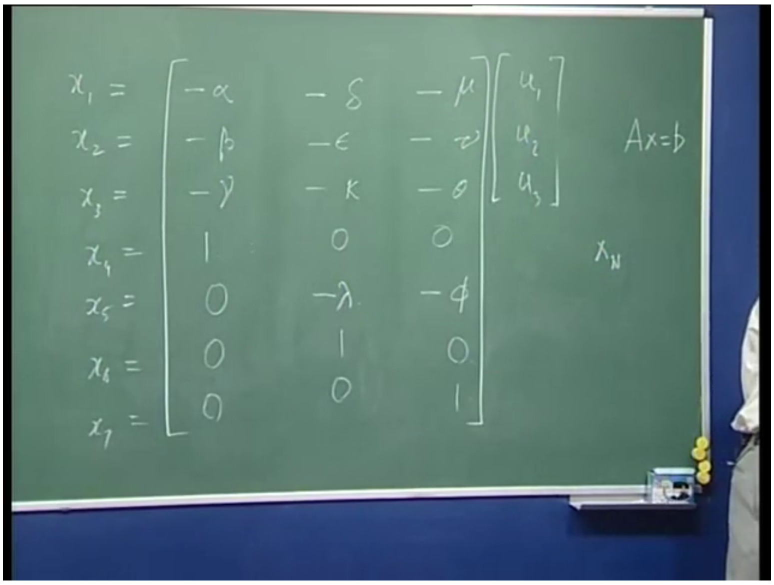

So, for that matter there is no more any reason to call them x 4, x 6, x 7 we can call it simply u 1, u 2, u 3 or anything any name and this matrix this 7 by 3 matrix inside itself actually codes the null space of the matrix A which is the same as a null space of the matrix A tilde, this first column is 1 null space member this second column is another null space member, the third column is the third null space member and these 3 are linearly independent that linearly independence shows here in the third one there is a 1 in the 7th location which cannot be found by any linearly combination of these two.

Similarly, here a 1 is found in the second one which cannot be found by the any linear combination of the others because others are 0's here similarly here this cannot be found by any linearly combination of the other two rows that shows that these are indeed linearly independent any of these is a solution of these and arbitrary linearly combination of the 3 by giving arbitrary values here will also the solutions of the equation A x equal to 0 and this describes the null space of the matrix. So, these 3 column are actually 3 linearly independent null space members any of them or any linear combination of them can sit in the place of X N in the general solutions called system A x is equal to b for which the same A is used.

So, solution of A x equal to 0 will be then written as this matrix having the null space member z 1, z 2, z 3, etcetera, here these 3 vectors are z 1, z 2, z 3 a null space members and how many of them will be there in this case we have shown 3 that means, this matrix as rank 4 this as 7 columns and if rank is 4 that we can see here that is after the reduction these 4 rows are linearly independent in terms of columns you can say first second third and fifth columns are linearly independent. So, 4 columns are linearly independent or 4 rows and not the fifth one so that means that rank is 4 and rank plus nullity will be the number of columns here.

So, in this case the nullity is 3 that means, the null space will have 3 linearly independent members, so in this case a is 4 n minus k is 3 and so you have 3 arbitrary values u 1, u 2, u 3 we choose and these vectors sitting here will give us a basis for the null space because null space has 3 independent 3 linearly independent members and these are 3 such linearly independent null space members so they together form a basis called the null space. Any linear combination of these 3 through these coefficients arbitrary values that will gives us a null space member so this is how we solve homogeneous systems of linear equations by reducing them to the row reduced echelon form.

Now, in this entire process as well as the process of solving square systems for a unique solution when that is the case we you another important technique quite often which is called Pivoting, in this example I have not discussed that this is of pivoting any non zero entry we were trying to get in the leading position in order to later divide it with that number to get a 1 here. But typically in numerical word what happens is that if that number is very small then division with that number may lead to numerical errors which may turn out to be quite unacceptable at times therefore, there is a non trivial exercise called Pivoting.

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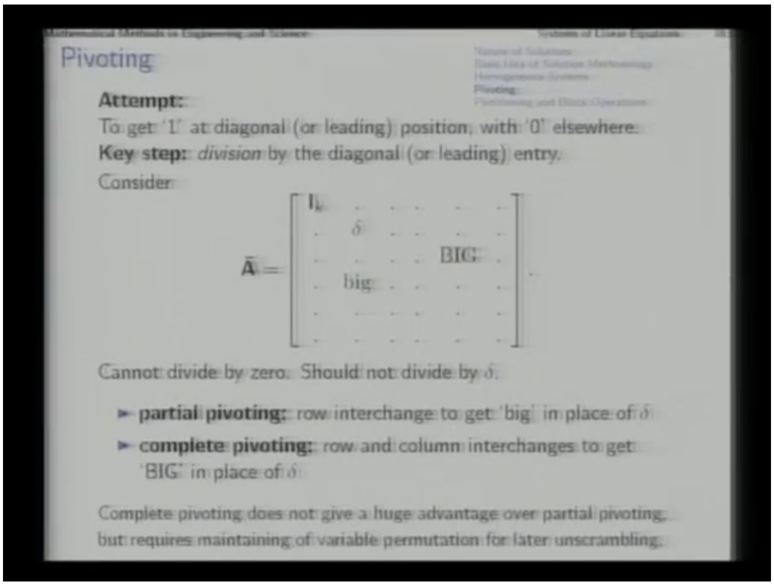


Note that this kind of steps typically try to get '1' at the diagonal or leading position with '0' elsewhere; a key step is division by the diagonal or leading entry.

Now consider a matrix in which suppose the leading k by k part has been already processed and something is sitting here which is say delta now is that delta is 0 then we cannot divide by in that case I have already discussed that we can exchange rows in order to get a non zero number sitting here so we cannot divide by 0, but even if it is not 0 but if is a number with very small magnitude say 0.0000003 or minus 0.00005 something of third.

So, in that case we should not divide by delta why is so we should not divide by delta because division by very small number may lead to numerical errors in a large computational program so cannot divide by 0 should not divide by delta that means by small number. Now we can still apply row interchange or for that matter column interchange if we want to get a larger number here.

So, what we do is that we try to apply row interchanges to get the largest here whatever is a largest in this column say big, so to get that number here we apply a row interchange that means, if we do pivoting then whenever we were faced with a situation of dividing by this leading term rather than dividing directly with it we could haunt out the largest number from this point onwards downwards and put that number here through an a appropriate row interchange that would be the pivoting step.

So, if we only apply row interchange to get this big in this location which will send this delta here then that kind of a pivoting is called partial pivoting on the other hand there is also a technique called complete pivoting in which we first apply a row interchanges and then column interchanges in order to get the largest possible number from this entire block to come here that is called complete pivoting.

Now, quite often in computational programs we will find that complete pivoting is actually a waste full exercise we might do it but it does not achieve a lot of advantage quite often almost often partial pivoting helps to sort out the issue much more easily. Though complete pivoting can be done so complete pivoting does not give a huge advantage or partial pivoting but maintain requires maintaining of variable permutation because column interchange will mean that the variables x 1, x 2, x 3, x 4 actually get permuted.

So that extra book keeping you have perform if you divide if you to do complete pivoting on the other hand partial pivoting is must simpler to handle and it accomplishes the task to a satisfactory extent. On the other hand if the problem appear that this is a extremely small and below that everything else is also extremely small all are 0's that will mean that you have a singular matrix and all of these you consider as 0 and the case is actually one of this type of situation in which you should consider them as 0's and proceed to the next column so this step of pivoting we will come across quite often in the next lecture as well in the current one one more small topic we need to discuss which in later theoretical developments will be quite often encountered and that is the issues of partitioning and block operations.

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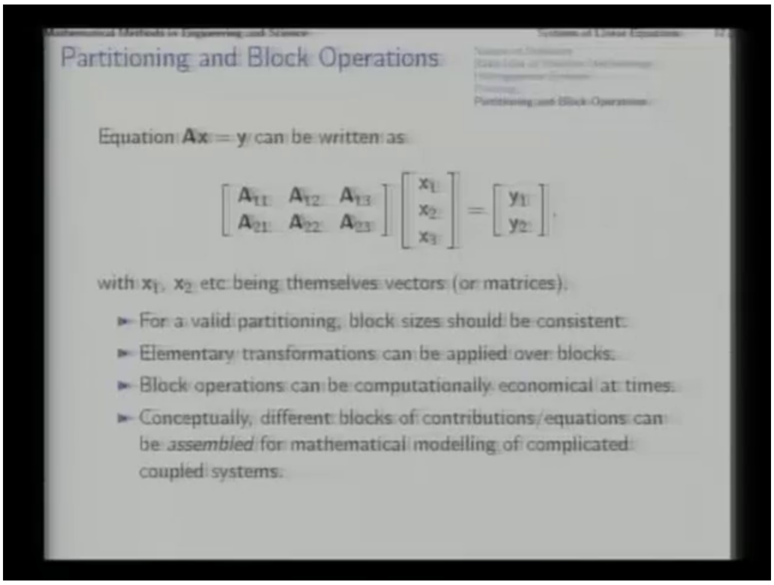


Quite often we will find that a matrix can be written partitioned in this manner and along with that the variable vector also gets partitioned in this manner and the right hand side vector also gets partitioned in this manner and this is fine as long as the associated steps of matrix multiplication, matrix vector additions and the equalities make sense what I mean is the following:- a large matrix x a and a large long vector x and another long vector y may be forming a system A x is equal to y in this a manner and in this manner if we write it in token form then it will mean A 11 x 1 plus A 1 2 x 2 plus A 1 3 x 3 equal to y 1 and so on.

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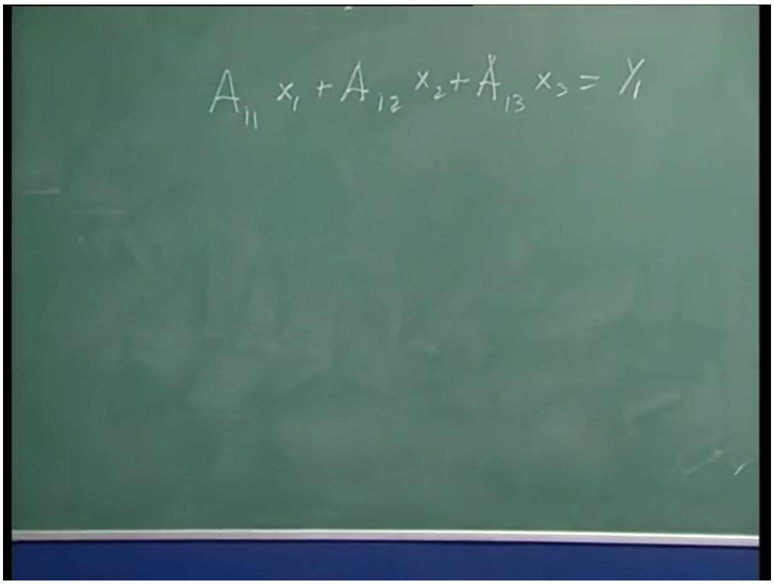

So, we will have the first row block that will appear will be like this and this kind of a partitioning is valid as long as these individual matrix vector multiplications make sense their dimensions are compatible. These additions make sense their dimensions are compatible this equality make sense on both sides we have vectors and matrices of the appropriate correct sizes. Similarly there well be a second row block from here this kind of partitionings and block operations in that manner can be carried out over complete blocks and these operations help a lot in theoretical developments and selective solution of certain unknowns through elimination of others.

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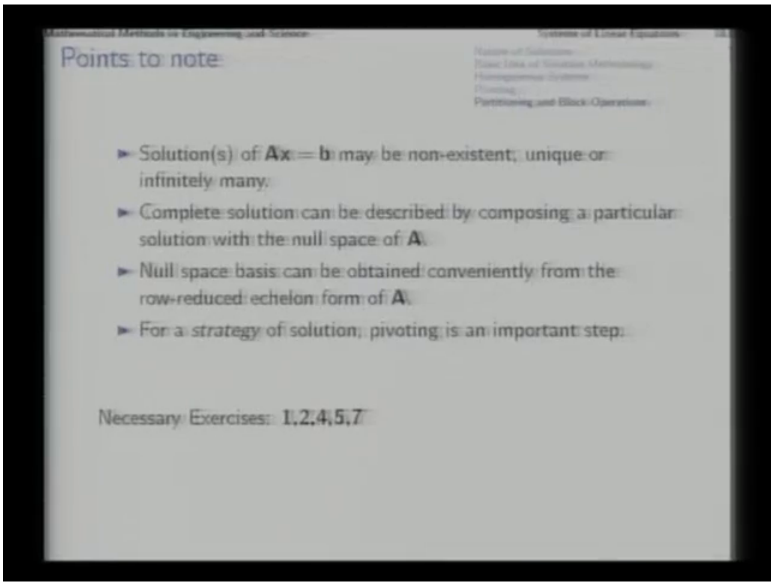


Some of the exercise problems here we will elaborate the idea more and I suggest from the at that stage the lessons given in chapter 3 and the chapter 4 the corresponding exercises should be attempted to from at part with the lectures going.

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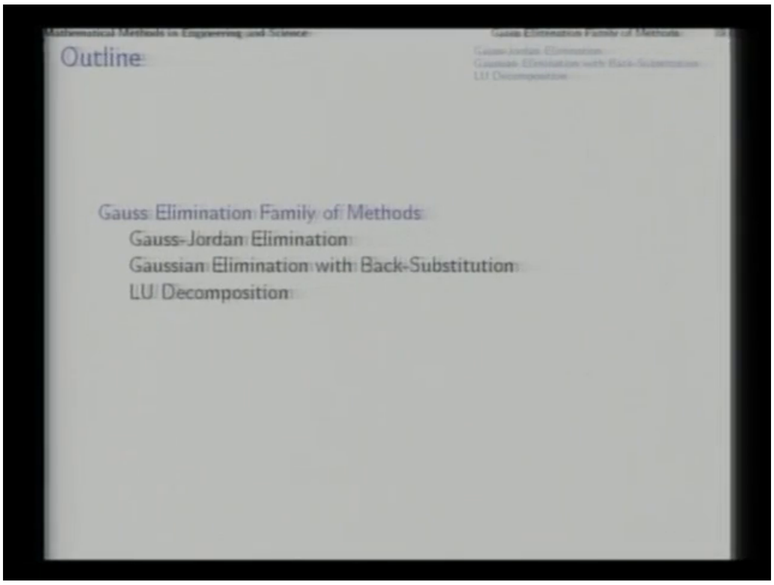


In the next lecture, we will be discussing the Gauss-Elimination family of methods to solve square system of equations which are most common in application and most important also.

Thank you.