Mathematical Methods in Engineering and Science
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Module - I
Solution of Linear Systems
Lecture - 02
Basic Ideas of Applied Linear Algebra

Welcome, in this lecture, we will be discussing operational fundamentals of linear algebra.

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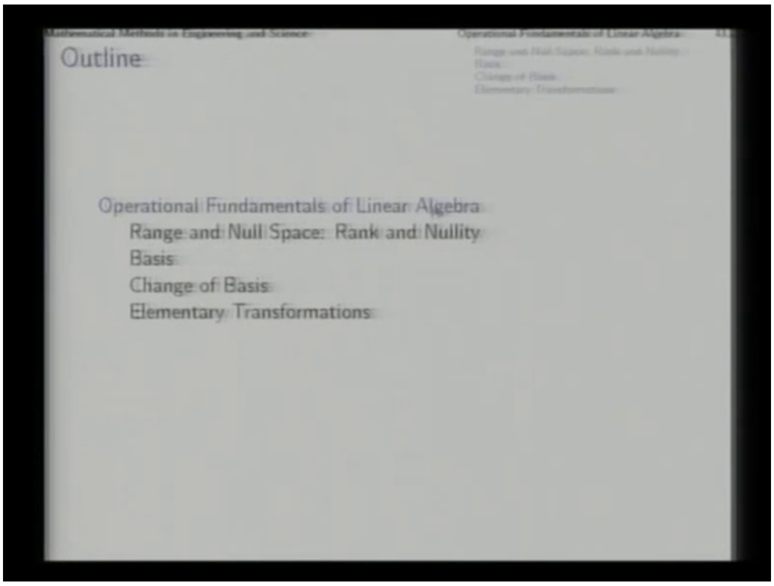

In particular; we will be talking about range and null space the dimensions rank and nullity respectively basis for representing vectors in a space and change of basis and introduction to the technique of elementary transformations.

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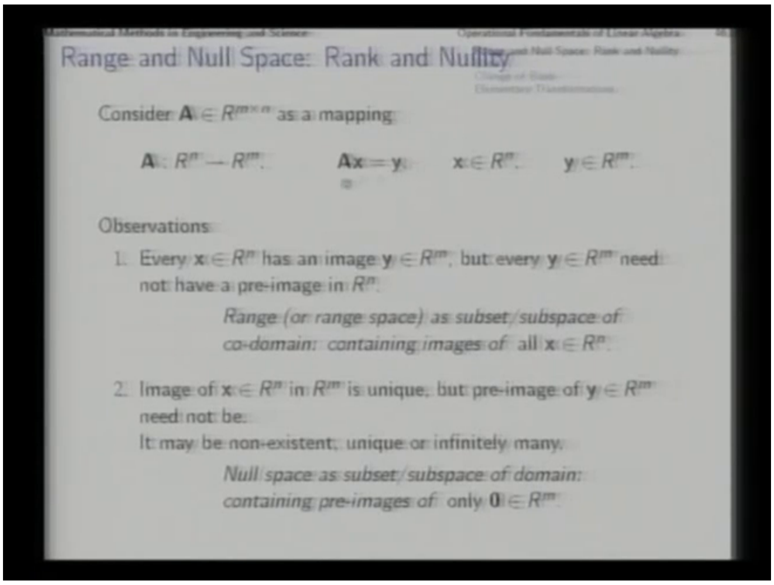

To start with consider a mapping A from R n to R m, the matrix a will be of m by n size which will be something like this.

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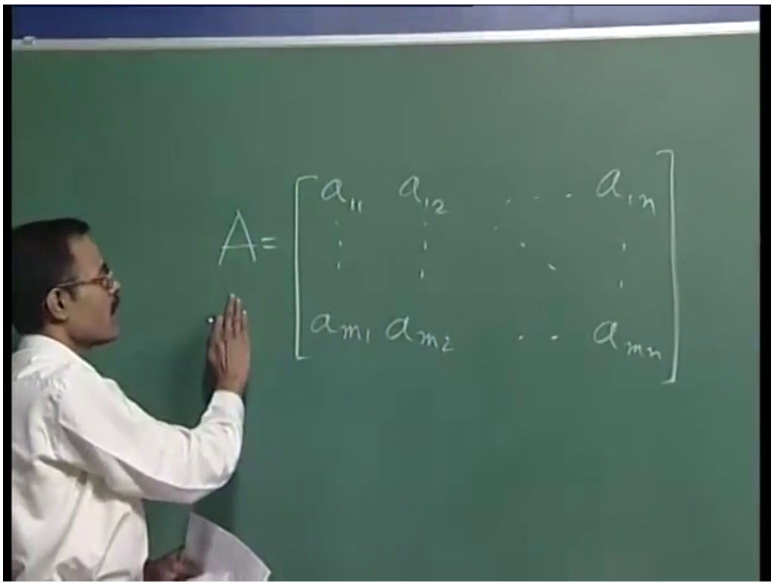

It will have m rows and n columns and therefore, it will multiply a vector of size n and give a vector of size m and therefore, we say that the mapping is from R n to R m, x is in R n and y is in R m m dimensional vector. We observe the same 2 make the we make the observations as we made in the last lecture that every x in R n when multiplied like this will suddenly result into a y that is every x in R n has an image y in r m, but on the other hand every y in R m does not have to have a pre image x, to which multiplying with a we will get y such is not necessary.

Now, then we talk of those vectors in R m for which there is some x and the set of all these vectors is called the range space or the range. So, we find that the range space is a subset also a subspace of the co-domain containing images of all x; that means, R m could be a larger set and then we can single out all those members in R m, which come as a result of multiplying the matrix a to some vectors or to all the vectors in the domain which is R m and this subset of the co-domain is called the range there is a range of linear transformation in the co-domain there may be larger space available, but the entire space available in the co-domain is not used by the linear transformation a. It only a subset of it which is the range of a is being utilized for the purpose of mapping.

The second observation that we made is that the mapping from x to y in order to be a mapping must be unique that is for a given x we have a unique y that is obvious because in this matrix multiplication the result is always unique, but the reverse does not have to be true that is for a given y, it is not necessary that the x should be unique and no other x should map to y that is not necessary. So, the second observation says that image of x in R n to R m is unique, but the pre image of y does not have to be; that means, for the same y in the co-domain or in the range for that matter there could be several x which map to the same y why several? There could be infinite vectors in R n which map to the same vector in R m.

Now, if such is the scenario with matrix a, that it maps infinite vectors from the domain to the same vector in the range then that will in particular be true for the zero vector in R m as well and then we find another definition from this observation and that is the definition of null space. Those vectors in the domain in R n which map to the zero vector in the co-domain or in the range they form a subset of the domain and that subset is called the null space. So, we find that null space is a subset or subspace of the domain which containing contains all pre images of only 0 pictorially or schematically we can see it like.

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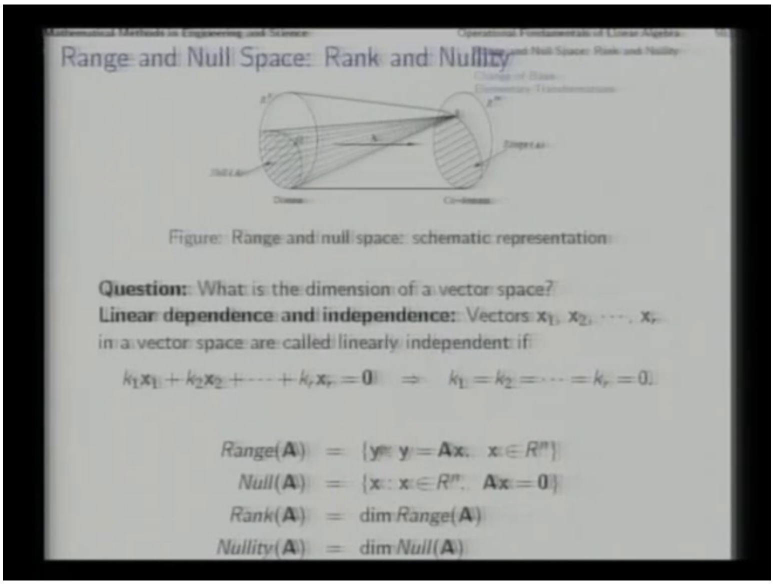

That is this entire space is the domain R n, this entire space is the co-domain R m. Now the entire R n domain must get mapped and, but then it may get map to a subspace of the co-domain that subspace is the range space. On the other side in the domain there is a subspace here has differently which all gets map to the 0 vector in the co-domain this subspace here is called the null space that is it does not have any information content in it all that goes to 0.

Now, we introduce two terms 2 more definitions that is rank and nullity before doing that we need to ask what is the dimension of a vector space note that here we are talking about 4 vector spaces. The domain co-domain the subspace of the domain which is a null space which is also a vector space and the subspace of the co-domain, that is the range space which is also a vector space. So, we are talking about all these 4 vector spaces. So, we ask what is the dimension of domain and that is n; what is the dimension of the co-domain that is m then we introduce rank and nullity we ask another question which we should have ask by now and that is what is the dimension of a vector space what is the dimension of a vector space.

So, for that we need to talk about linear independence of vectors linear dependence and linear independence. In a vector space suppose we take several vectors x 1, x 2 up to x r, r vectors we have taken.

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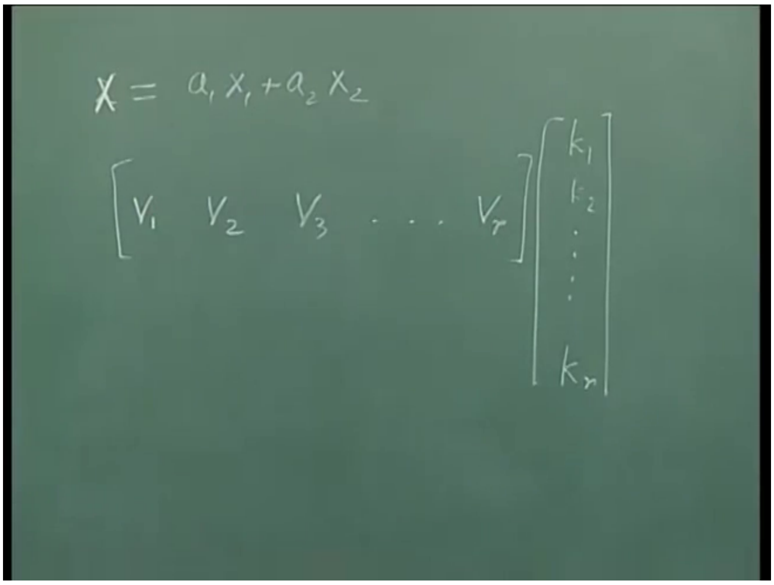

Now, when do we call these vectors as linearly dependent or linearly independent? So, the simple straight forward idea of linear dependence and independence is that if x a vector x can be expressed as a linear combination of several vectors like this then in such a situation we say x is linearly dependent on x 1 and x 2, which means that x can be formed by a linear combination of x 1 and x 2. In other words we can say x 1, x 2 and x are linearly dependent together. So, this is these 3, then will form a linear dependent linearly dependent set.

So, there is a linear dependence among these vectors; when we talk about the linear dependence of several vectors together then typically in the most general sense we say it in this manner that is if k 1 x 1 plus k 2 x 2 up to k r x r if the whole sum being 0 necessarily implies that all these contributions are independently 0 that will mean that these vectors are linearly independent. In the sense that from one place on earth if you need to go to another place on the earth on the surface, then you think of 2 directions how much east west I have to go and how much north south I have to go.

Now this east west and north south they are 2 independent directions, then you say that wherever I need to go I can go by moving a little east west or a little north south then basically you will be talking about a little east west plus a little north south and this together gives you a movement on earth and that movement is then linearly dependent on these 2 individual movements east west movement north south movement.

Now, these 3 directions that you have got the east west direction north south direction and the direction which you already travelled these 3 are linearly dependent now this statement that these vectors are linearly dependent if the whole sum equal to 0 necessarily implies this entire set of k 1, k 2, k 3 being totally 0; that means, that a moment on ground is 0 is no movement if its east west component is 0 as well as its north south component is 0 that shows these are linearly dependent vectors so; that means, any change from 0 in any of these will not be compensated by any suitable change in any other. That means, the moment you change any of these coefficients this sum will turn out to be non zero, no other compensation from any other component will be possible. So, in that true sense these vectors x 1 x 2 x 3 etcetera are independent of each other is any change in 1 cannot be compensated by any suitable change in others.

Now, we say that if in a vector space we are hunting for vectors, which are linearly independent among themselves. So, we find one vector, the moment we find one vector in that vector space many other vectors in the vector space become linearly dependent on it we hunt for another which is linearly independent of it. So, we get 2 such vectors like this as we go on looking for vectors which are linearly independent of the vectors which are already collected finally, we might reach a situation where the process ends, where no more vectors are found; which are linear independent of the vectors, which are already collected.

Now, we will see how many vectors we already collected which are linearly independent among themselves that number is the dimension of this vector space. The earth surface as a dimension 2, because after you exhaust east west direction and north south direction no more direction is necessary with these 2 directions movements in these 2 directions you can reach any place on the earth. In this 3 dimensional world in which we live; why we say 3 dimensional world because here there are 3 linearly independent directions, you go east west and you go north south and you go up down is with these 3 independent movements you can reach any point in this physical space, that is why physical space is 3 dimensional.

With this idea of the dimension of a vector space to be the number of linearly independent vectors contained in it, we define 2 more terms. As we have already discussed range of A is the set of all images of all x’s in the domain, null space of A is the set of all those x’s in the domain which map to 0 in the range. Now the dimension of the range space is called the rank, similarly dimension of the null space is called the nullity. These definitions are extremely important and should be remembered in all the discussion that we conduct subsequently.

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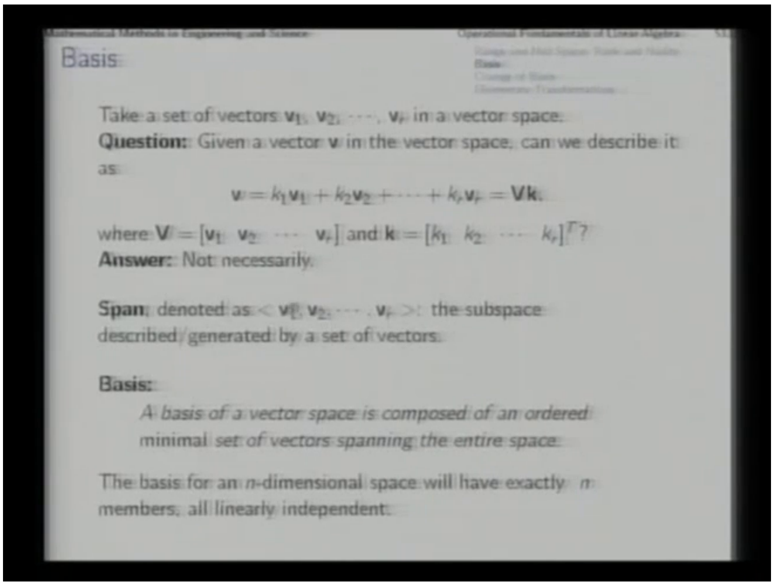

Now, to describe vectors in a vector space, we talk of basis that is in terms of which vectors we can describe all the vectors in a vector space in this manner. We want a handful of vector x 1 x 2 etcetera as linear combinations of which we would like to represent all vectors in that vector space, and this is the idea behind looking for a basis these members x 1 2 etcetera will be called the basis members to define a basis we ask this question given a vector v in the vector space, question is can we describe it in this manner as a linear combination of r chose vectors v 1, v 2, v 3 up to v r, r chose vector can we do this.

As if we have chosen r vector to form the basis to describe all that vectors in that vector space, and then in that vector space a candidate vector v appears and we are trying to express this vector v as a linear combination of the chosen vectors. Is it possible or not where this can be written in shorten in this manner because these vector collected as column vectors and multiply with this k’s it actually gives us k 1 v 1 plus k 2 v 2 plus k 3 v 3 and so on up to k r v r the way it has been shown here and that is why this matrix this large matrix we can call as capital V and this vector as k.

Now the question is that with v 1 v 2 v 3 up to v r known we essentially know this matrix and the vector v small v that has been given to us we know this and we are interested in finding all suitable k 1, k 2 up to k r such that this turns out to be right that is the new vector v gets represented as a linear combination of the vectors which are there in our collection.

Then the question has been raised and the answer to this question is that it is not necessarily always possible. Now if it is not possible always then we say that then these vectors what these vectors are doing. These vectors among themselves in this kind of a linear combination in this kind of a linear combination will generate a subspace in the vector space that subspace will have lesser dimension then the entire vector space and that subspace sometimes we denote as the span of these vectors. So, this is the span of this vectors and we say that the subspace described or generated by these vectors is called the span of this vectors, denoted by these angle brackets and in other words we say that these vectors span this particular subspace. So, span is used either as a verb or as a noun.

Then when we look for the complete basis of a vector space we define it in this manner a basis of a vector space is composed of an ordered minimal set of vectors spanning the entire space; that means that after we collect so many members v 1, v 2, v 3, v 4 etcetera such that all vectors in the vector space can be expressed in this manner when finally, the answer to this question the answer to be yes after collecting enough then we say that can we do with less number of vectors in this collection.

If we can then whichever vectors are not required we through them out that is the meaning of the word minimal here and then we need an ordering among them this is our first basis member, this is our second basis member, this is our third basis member and so on, but remains gives us the basis of a vector space for example.

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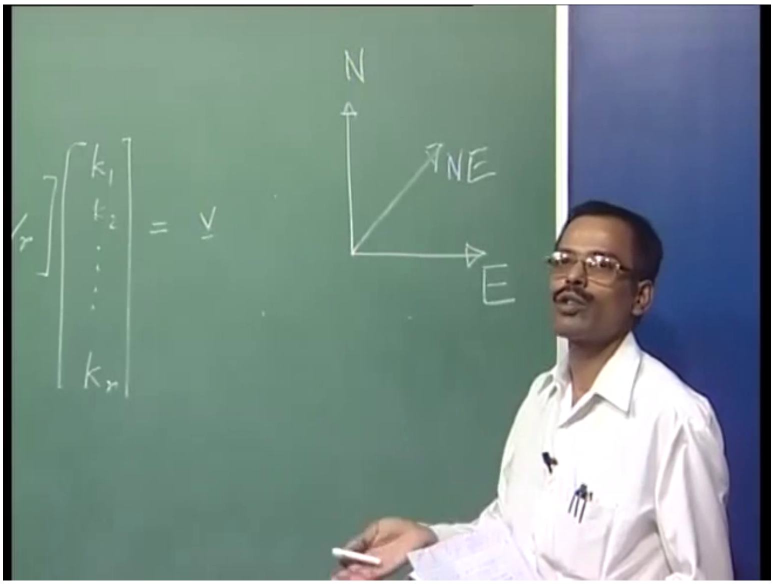

When we want to describe the points or vectors in this volume which we leave, we say we took we take one direction that is north. Many vectors many movements in this world can be represented by this all northward movements to be précised, then we need one more direction then we say east, note that west is same as east just the negative sign.

Then we say that all places on this plane consisting of north and east directions are covered including here in the other coordinates right, then we say that still we remain on the ground only what we need to do is to go up also then suppose we try this direction North East and it does not help it does not help us to go upward by going north is we do not go upward. So, then by going North East which are the directions which we can go we will find by going North East we go a little east and little north; that means, this direction of North East is actually linearly dependent on east and north. So, it is not needed and we will look for a minimal set. So, this is not needed. So, the direction of north is not needed.

So, which is the direction that is needed any direction which has an upward or downward component will help.

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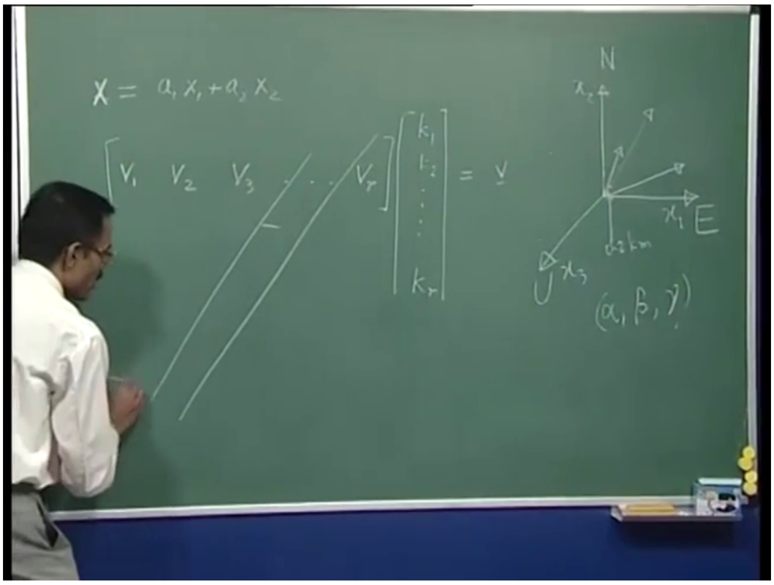

So, we take a direction which is upward or downward it can be perpendicular to this plane, but it need not be. So, let us call it up away from the black board as I draw it. Now we have got 3 directions east north and up and with these 3 directions we have formed a triad like this. So, with these 3 directions at our disposal we will be able to represent any other direction any other movement. So, there is no need of going for afford and we keep only 3 you noticed that 3 were barely minimum which were needed and more than 3 we will not give.

So, why 3 were barely needed because the dimension of this space which we are trying to describe is 3 and therefore, we also say that the basis for an n dimensional space will have exactly n members all linearly independent. Exactly n members to keep the set minimal and all linearly independent to be able to describe all vectors in the vector space finally, we need to order them. We have to say east is our first coordinate north is our second coordinate and up is our third coordinate. As we do this our job performing basis is complete because after doing this now when we say the coordinate of the particular point in the atmosphere happens to be alpha beta gamma, then the description of this is completely known we have to go alpha distance in the east direction, then beta distance in the north direction and gamma distance in the upward direction.

So, only when the ordering is also prescribed then the statement of the basis gets completed; sometimes we talk of a particular kind basis which is called orthogonal basis.

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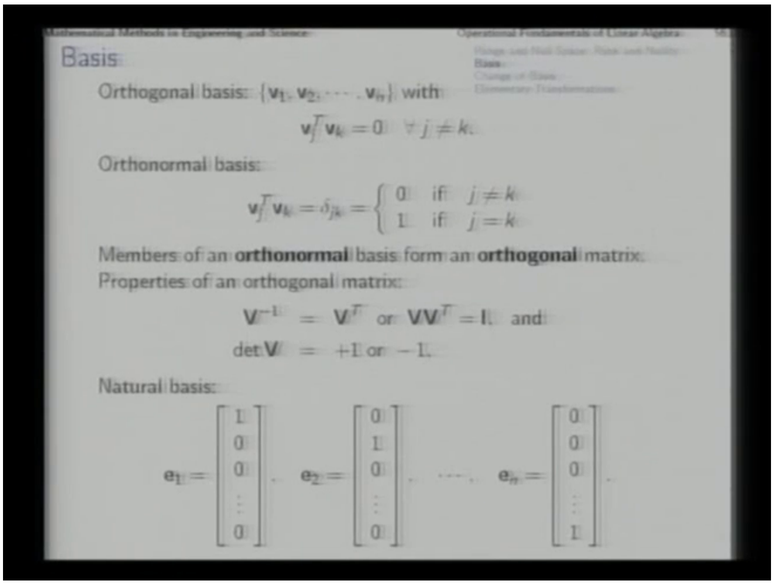

In which these vectors are perpendicular to each other, in this particular situation we have actually taken the example of a basis which happens to be orthogonal because in the directions between the directions east and north, there is a right angle and between the direction of east and up also there is a right angle and so on. So, mathematically we can say v j t, v j transport, v k is equal to 0 when this holds for all j and k which are not equal there is v 1 transport v 2, v 1 transpose v 3, v 2 transpose v 3 when all these 3 are 0 that is when this angle this angle and this angle all are right angles like this then we say that this basis is orthogonal right angled.

Similarly we take another special case in which all 3 have unit vectors of the same size, that is if we consider 1 kilometer in the east, and 1 kilometer in the north and 1 kilometer in the upward direction as of the same size in the description, then we will say that this is orthonormal basis orthonormal right sized not only right angle, but also of right sized. In that case we say that this v j transpose v k is not just zero for j and k different, but it is 1 for j and k same.

You might note that members of an orthonormal basis they will have several unit vectors all perpendicular to one another. So, these members these vectors together written like this will form a matrix which is orthogonal matrix and of course, you already know that these are the properties of an orthogonal matrix is inverse is the same as transpose or you can say VV transpose is identity this is the test for a matrix being orthogonal. In particular is determinant also is known it will be either plus 1 or minus 1.

Sometimes we describe vectors without mentioning the basis the way still now we have doing by just writing x 1, x 2, x 3, x 4 in a column what we are doing what basis we are using then? Then we are actually without mentioning using a particular basis which comes naturally to us and that is called the natural basis natural basis numbers are just this 1 0 0 0 0, then 0 1 0 0 0 1 0 0 1 0 0 and so on. So, these are the natural basis members and they are quite often represented with these symbols e 1 e 2 e 3 e 4 up to e n this n shows the dimension of that vector space.

Now, after describing the vectors of a vector space for quite some time in this basis, the natural basis or say some other basis we want to change the basis that is we do not want suppose we do not want these directions and we do not want this lengths also we want to describe the vectors in this 3 dimensional world, in terms of 5 kilometer distance in this direction which is a little north of east, then 3 kilometer movement in this direction which is a little east of north and the third direction is not directly upward, but a little upward (Refer Time: 26:37) like this. In particular we could do that and for example, in this we say we will take a very small unit this is just 200 meters.

We can do it, we can describe the 3 dimensional world in terms of 3 coordinates which are the locations in which are the distances along this direction, along this direction and along this direction which the units being different that kind of a thing is possible and sometimes it makes sense to do that. For example, if you want to prescribe locations on a road how you do it? Suppose this is the road and you want to describe locations points on this road, then it will not help to describe the distances along north and east for this road if you want to specify points on this road, it will not be a great idea to describe.

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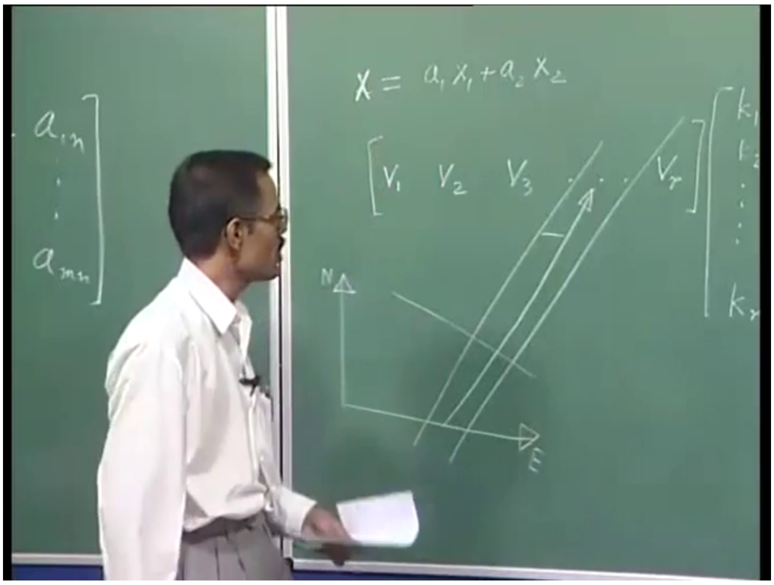

In this manner that is this much is movement in North East, this much is the movement in north that will not be a good idea. Besides using the same units on both sides also will not be a good idea, because in that case the descriptions will be always quite log sided the correct description would be movement along this direction which is in kilometers and movement along this direction which is in meters.

Because otherwise if you choose the same unit say kilometers, then for the direction along the road you will be talking about 10 kilometers, 100 kilometers and so on and for this you will end up telling 0.3 kilometer, 0.15 kilometer and so on which will not be very nice. On the other hand if you choose meter then also you will have fine description for the breath of road, but not for the length of the road. So, therefore, it is quite often possible to require to need the change of basis.

Let us take an example.

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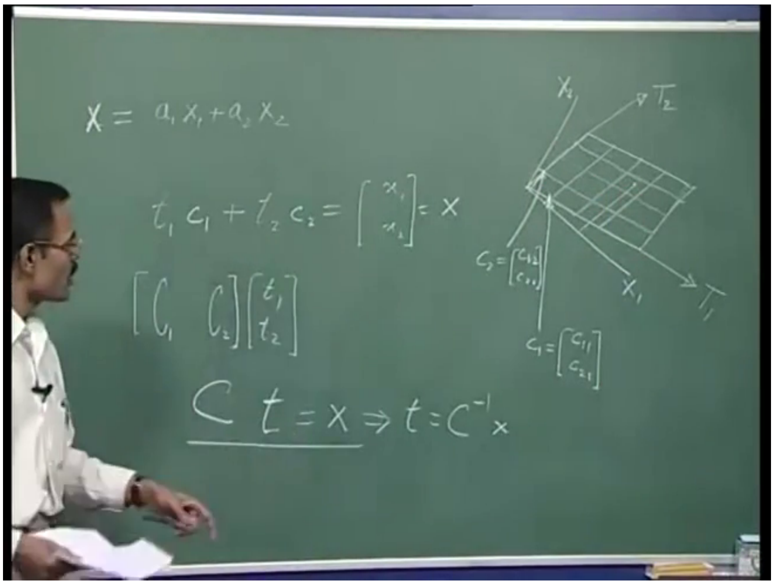

Suppose you have a table which is not rectangular, but which is of this shape and in this rectangular in this table I think it is paralog ram parallelogram shape and on this table on the surface of the table there are marks like this and the sizes of these small parallelograms are 3 centimeters in this direction, 2 centimeter in this direction. So, 3 2, 3 2 like this; right. So, this is suppose the top of the table in which this kind of marks are made it is a design.

Now, suppose this particular table is kept and you want to know the distances along this and along this edge of the table, which will take you here. Distances not in centimeters, but in terms of the steps how many steps in this direction and how many step in this direction we will take here. So, for that suppose our original frame of reference in terms of North East or whatever in the rectangular Cartesian frame, suppose that Cartesian frame happens to be this let us call it x 1 and x 2.

Now, when we say we want to know how many steps along this edge and how many steps parallel to this edge we will take up here, we are essentially asking for the coordinates of this point in the table system and table system has its axis like this and like this right. So, then first thing we need to know; what is the description of the units of the table system, in the original x 1 x 2 system. So, for that we take this unit and say how much it is from here to here. So, this point in the original system suppose it is described by a vector c 1 which is this much along x 1 this much along x 2 like this.

Similarly, this unit which is 2 centimeters in this case, we are want to find out if this vector description in the original x 1, x 2 system and call it c 2 like this. Now we have this movement as a vector c 1, this movement as another vector c 2 and now we want to find out how many c 2, c 1 movements and how may c 2 movements will take us here right. So, that is why we say how may c 1 movements plus how many c 2 movement will take as here, x 1 2, right.

Now, this left side we can write like this, these are column vectors this vector c 1 and this vector c 2 those basis members for the table system are sitting here into t 1 t 2 as a column vector right. Note that this matrix vector multiplication will give us c 1 t 1 plus c 2 t 2 what is here right. So, or we can concisely write it as the square matrix C into this vector t is equal to the actual location of this in the x 1 x 2 frame that is x this gives us an equation relating the number of steps along the edges of the table c 1 t 2 to the location of this point which we know in the old x 1 x 2 system, and if we want to find out t then we need to invert the matrix c and get it like this right. So, that will tell us how many steps along this will go here and here to find this point right. So, this tells us the conversion from the old coordinates to the new coordinates, and this tells us the conversion from the new coordinates to the old coordinates.

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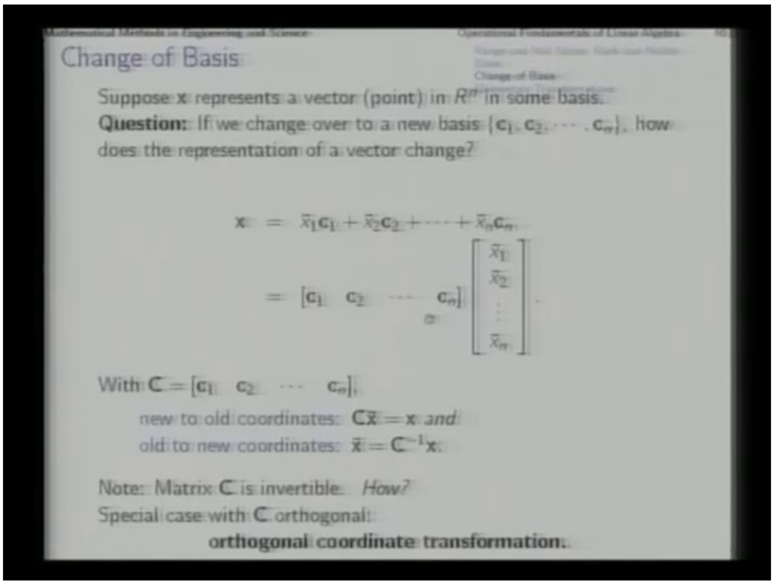

As we formulize this methodology we say that suppose x represents a vector or point in R n in some basis. Now if we change to a new basis consisting of these as the basis members then how does the representation of the vector change? We say that if with x 1 bar x 2 bar x 3 bar movements along c 1 c 2 c 3 basis members, we can achieve the complete vector x, then will be representing x as a linear combination of these individual components along c 1 c 2 c 3 etcetera this entire sum can be shown in this manner. Note that matrix vector mollification will be given x 1 bar c 1 plus x 2 bar c 2 and so on right.

Now, these column vectors sitting here will have n column vectors each of size can and that is a square matrix we can write this in this manner with c 1 c 2 c 3 etcetera to c n in this manner then will have x is equal to c into x bar vector and that gives us the conversion system from the new coordinates to the old coordinates. By inverting this matrix c we get the transformation from the old coordinates to the new coordinates and make note that this inversion will also be possible that is this matrix is invertible because we already know that to form a basis c 1 c 2 c 3 up to c n and that gives us a methodology for conversion from the new system to the old system and when we want the reverse that is from the old to the new when we want the conversion, we have to invert this matrix c and we get the formula as x bar is equal to C inverse x as we found in this case.

Now, in a special case we have c matrix C as orthogonal; that means, in that case the basis members will be all unit vectors and mutually orthogonal and; that means, it will be an orthonormal basis and in that case this type of basis change we call as orthogonal coordinate transformation. Now we have seen that how the change of basis affects the representation of vectors, the vector x in the old system get represented as vector x bar and the relationship between the old description and the new description is given by this matrix consisting of the basis numbers as its columns. Now we say that how does such a change of basis will affect the description of a linear transformation how does this basis change affect the representation of a linear transformation?

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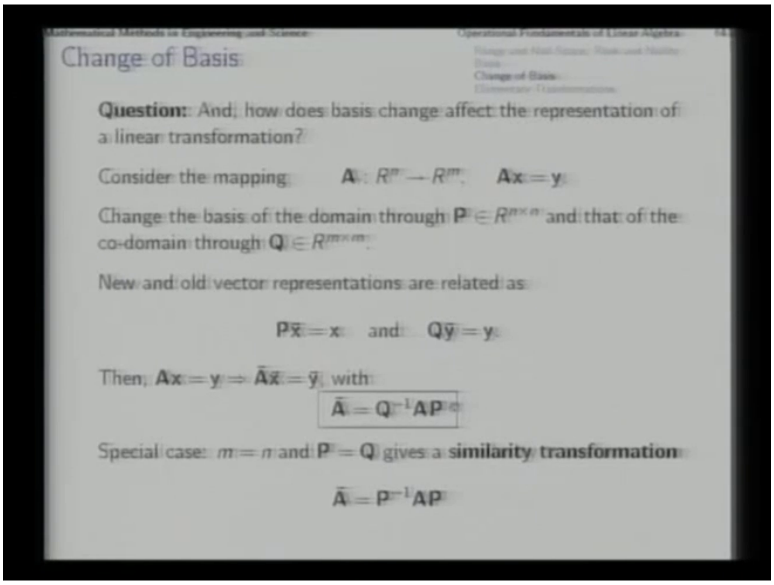

Consider this mapping A from R n to R m in this manner; A multiplying with x produces y. So, that gives the description of a linear transformation.

Now, we change the basis of the domain through a matrix P; that means, P will be an n by l matrix with its columns as the new basis members for the domain. Similarly Q will be another square matrix of m by m size in which the columns will be the basis vectors for the co-domain that is for y. In such a situation based on the discussion that we just now had the new vector x bar in the domain and the old vector x in the domain will be related like this to the matrix p, similarly the new and old vectors in the co-domain corresponding vectors in the co-domain will be y bar and y representing the same point geometrically and they will be related through this matrix Q which contains the basis members for the co-domain.

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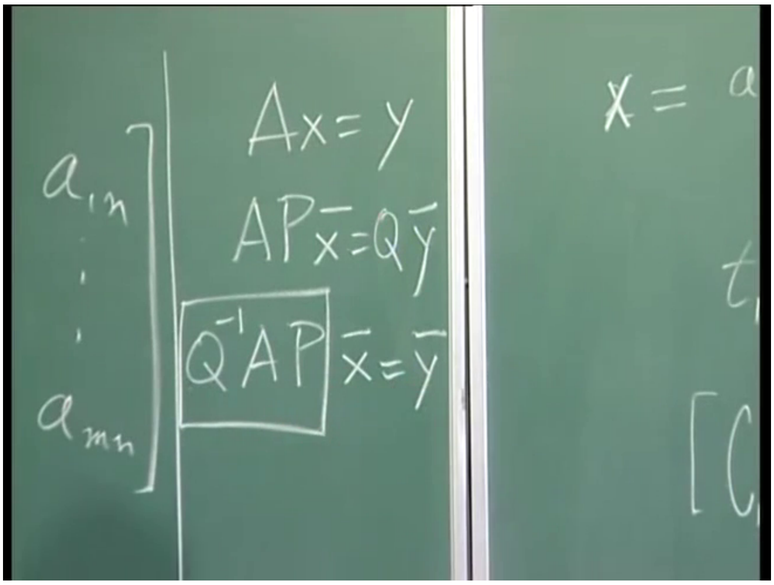

Now, originally the transformation the mapping represented with matrix A between x and y. Now the same thing we write here A x equal to y and as we do that A x equal to y. Now for x we use this and say x is nothing, but P x bar x is nothing, but P x bar and what is y? Y we get from here y is Q y bar now remember what we want to find out, we wanted to find out the matrix representation of the same linear transformation in the new basis as a result of the basis change that is we wanted to find out the matrix representation of the linear transformation which will map x bar to y bar; that means, we wanted something such that something into x bar gives us y bar. Look at that we can pre multiply with Q inverse and get that as Q inverse A P x bar is equal to y bar. This matrix will now map x bar to y bar; that means, it will describe the same linear transformation as the basis in the domain and in the co-domain have been changed to p and Q respectively.

And that formally we say that the new matrix representation of that same old linear transformation will be Q inverse A P. In the special case where the mapping is from R n to R m itself we have m is equal to n and P is equal to Q this gives us what is called a similarity transformation. This is an issue this is a topic to which we will have ample amount of devoted later when we discuss the matrix Eigen value problem or the algebraic Eigen value problem. Before that in this lecture we will consider one more important topic which will be very crucial in our discussion of systems of linear equations and that topic is elementary transformations.

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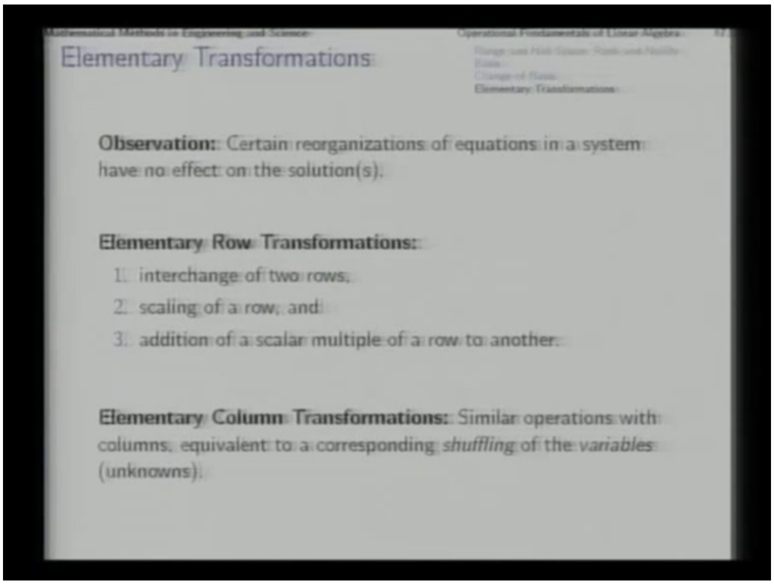

Now, suppose you have a number of linear equations among a number of unknowns. Now we need to recognize that certain reorganizations of the equations in the system have no effect in the solutions or for that matter on the description of the system itself it does not have too much effect, it has the basic defense of expressing the same relationships in a different manner. For example, these 3 we will find that if we interchange 2 equations the first equation we write at the third location and the third equation we write in the first location then the system does not change and in the way of writing the equation in this manner A x equal to b, that will amount to the interchange of 2 rows of A and B; is if we interchange the first equation with the third equation that will basically mean that in this matrix sector presentation of the system of equations that is equivalent to changing of 2 rows this is called one of the 3 transformations that are called elementary row transformations. So, this is one elementary row transformation.

Next if we multiply the seventh row with 2 or one-third does it change the system no it does not. So, that is equivalent to changing the scaling of a particular equation. So, that is scaling of a row. So, if we multiply one of the equations in the system by a scalar all over on the left side as well as on the right side then that does not change the solutions of the system of equations. So, scaling of a row is another elementary row transformation. The third one is addition of a scalar multiple of one row to another that is suppose earlier we had one equation and a second equation.

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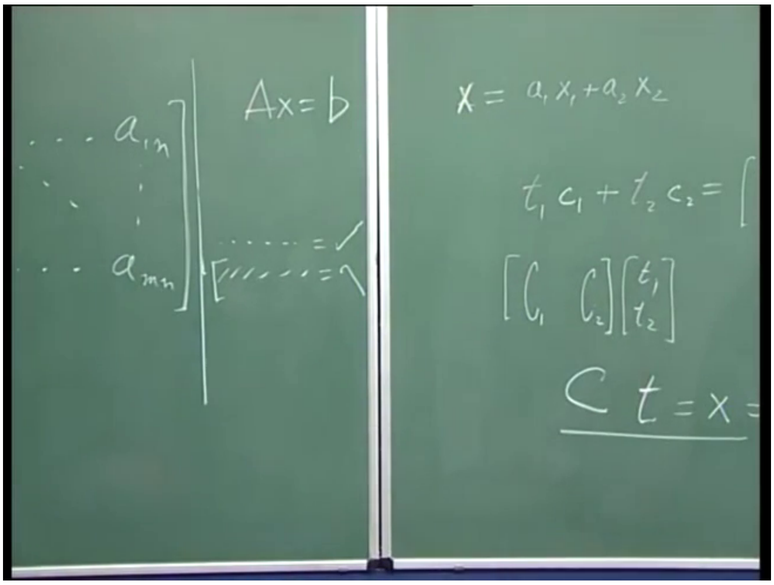

Now, how does it change the system if in place of the second equation we considered a sum of that 2 equation. It will change the solutions of the system of equation one individual equation will be basically replace by another equation which is equivalent to the old one in relationships with the rest of the equation. So, this gives us the third elementary row transformation additional of a Scaler multiplication of a row to another.

Corresponding to these 3 elementary row transformations, we have elementary column transformations also similar operations with columns equivalent to corresponding shuffling of not the equations, but of the variables. In that case interchanging of the first and second column will mean the equivalent of writing x 1 and x 2 in the reverse manner.

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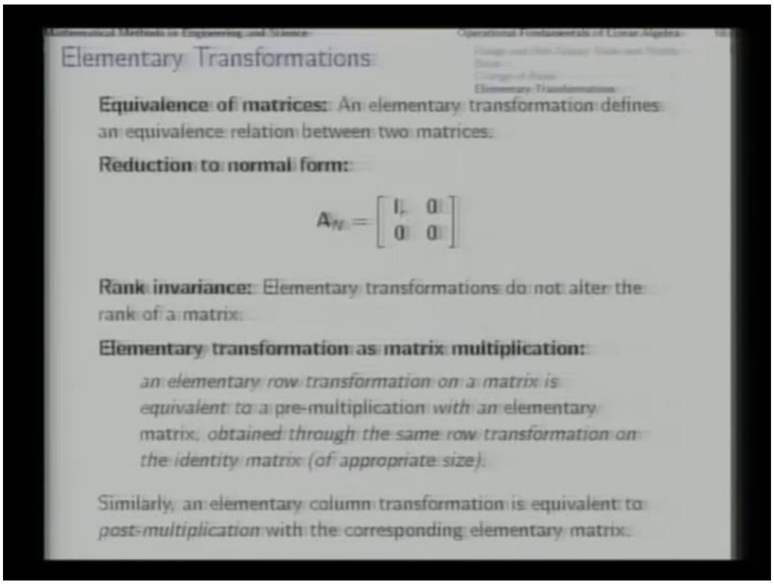

Now, the 2 matrices are called equivalent, if you can change from one of the matrices to the other through a series of elementary transformations. So, these matrices which can be shifted from 1 to other through elementary transformations are called equivalent matrices. They are equivalent matrices they are equivalent in the sense that they satisfy the requirements of equivalence relations.

Now through elementary transformations you can reduce any matrix to this normal form in which there will be a leading block leading square block of r by r size of identity matrix and an everything else is 0. Any matrix you can reduce up to this form to elementary row transformations and elementary column transformations. You can note that in this other than dimension and rank of the matrix nothing else survive, the rank shows up here in r to this identity matrix is of size r by r.

Now, up to this much reduction is possible, but not of with significance it does not have unanimous amount of applied used, most of the time in applications we use only row transformations or only column transformations for desired ends. This is an important issue that after we keep on using the matrix through equivalent row transformation elementary row transformations, we cannot get rid of this r that shows that elementary transformations do not alter the rank of a matrix. There is one interesting way of looking at an elementary transformation.

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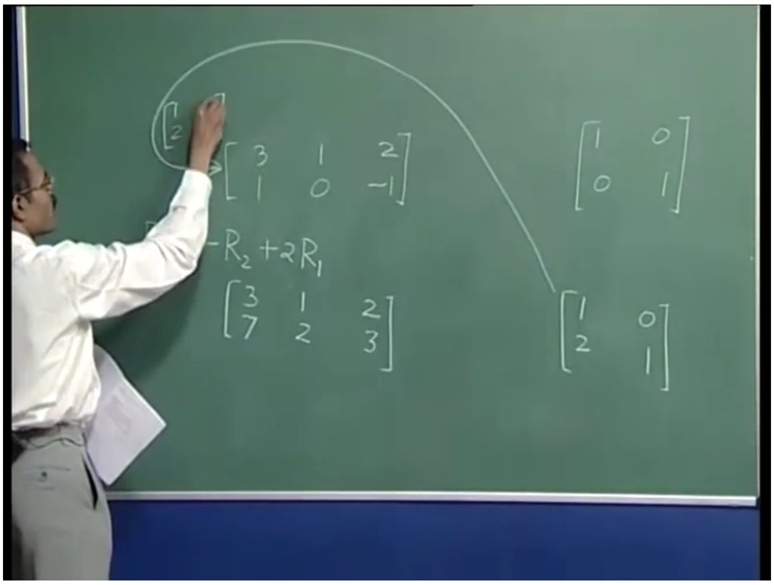

Let us do that with a small example what we do here is that we take this matrix 3 1 2 1 0 minus 1 and apply a linear transformation elementary row transformation to it and the same elementary row transformation we also apply to an identity matrix. Suppose elementary row transformation that we decide to apply is this and 2 times the first row to the second row. If we do that then what we get here first row remains unchanged the second row gets twice the first row added to it look at this. When you apply the same transformation same elementary row transformation to the identity matrix, we get twice the first row gets added to the second row we get this matrix.

Now, if we have applied the elementary row transformation to the identity matrix of the appropriate size and got this, then it would also be possible to get this same matrix not through the direct application of the elementary transformation on this matrix, but by pre multiplying this matrix here, I am reproducing it here for convenience.

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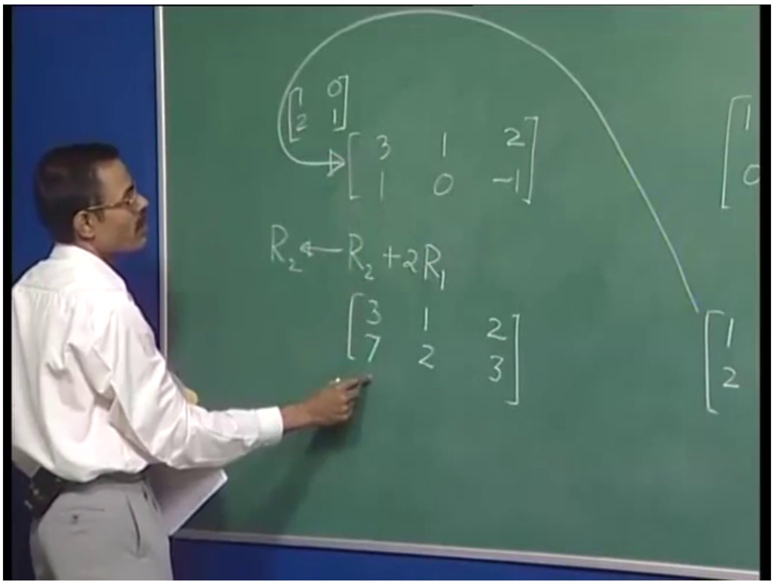

Now, if we multiply this matrix to this then see 3 plus 0 3, 1 plus 0 1 2 plus 0 2 and the next row 2 into 3 plus 1 7 2 plus 0 2, then 4 minus 1 3. So, that shows that to get this matrix we actually had 2 ways available, either applied elementary row transformation on this matrix directly or pre multiply this matrix with this square matrix which was found through the application of the same elementary row transformation on an identity matrix and this matrix in that case is called an elementary matrix.

So, we get this situation and elementary row transformation on a matrix is equivalent to a pre multiplication with an elementary matrix and that elementary matrix is the one which is obtained through the same row transformation on the identity matrix of appropriate size. Similarly an elementary column transformation is equivalent to corresponding post multiplications. These issues will be found extremely important when we try to apply elementary transformations for the solution of systems of linear equations.

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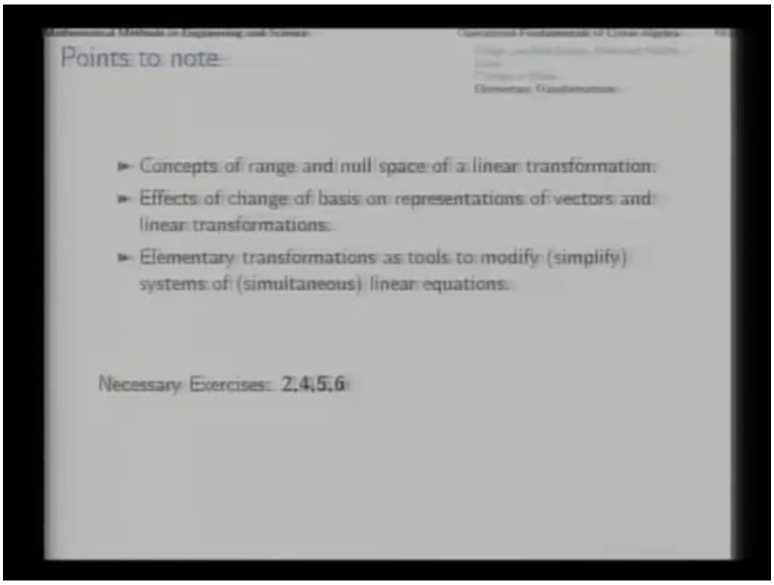

In the next lecture for the time being we summarize the important points that must be noted from this lesson. The important issues are first the concepts of the range and null space of a linear transformation; second effects of change of basis on representation of vectors and representations of linear transformations from one vector space to another vector space. Third important issue from this lesson is the idea of elementary transformations as tools to modify essentially simplify systems of simultaneous linear equations. These concepts will be extremely useful for our subsequent lessons on the systems of linear equations.

Thank you.