Mathematical Methods in Engineering and Science
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Module - I
Solution of Linear Systems
Lecture - 01
Introduction

Welcome to this course on mathematical methods in engineering and science. I am Bhaskar Dasgupta from IIT, Kanpur instructor of the course.

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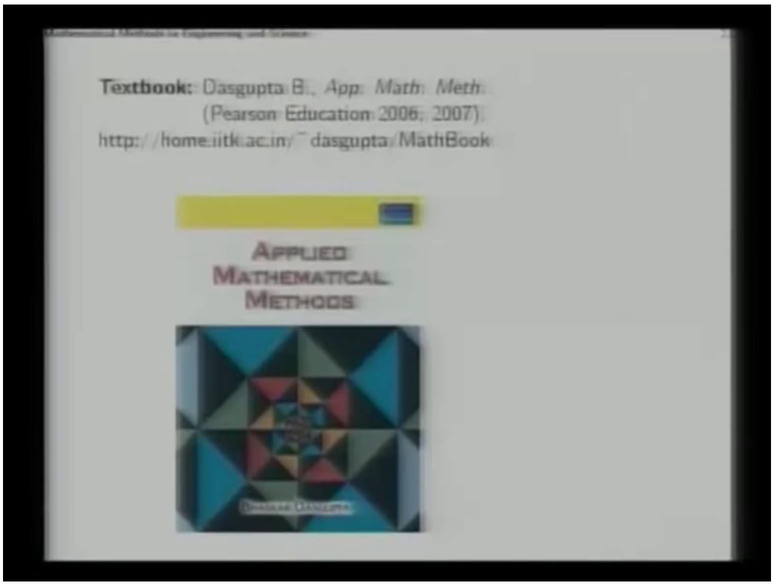

This is a course in order to bridge gap between the undergraduate mathematics and the level of applied mathematics that is needed for serious analytical and computational research. For this course we will be using this book applied mathematical methods for as the text book, the course content will have a large number of topics.

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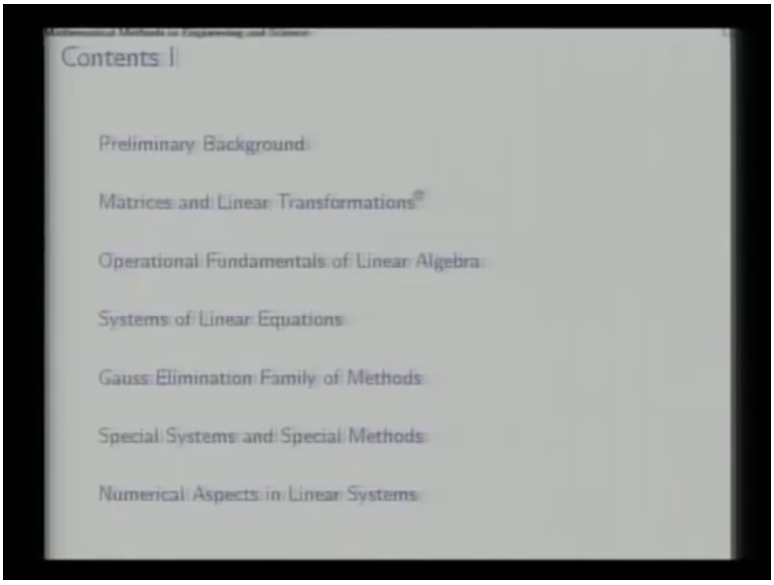

The first module of linear algebra that is systems of linear equations will be covered in these lessons.

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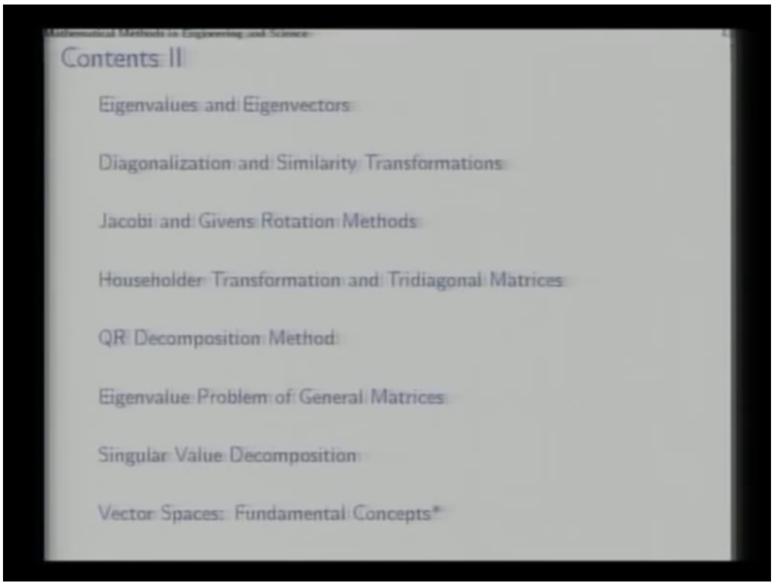

Next the second important module of linear algebra will be taken up among these chapters, which are the chapters on eigenvalue problem. Finally, in singular value composition, we will be connecting the 2 different important problems of applied linear algebra and we will briefly see the abstractions and fundamental ideas behind all these analysis.

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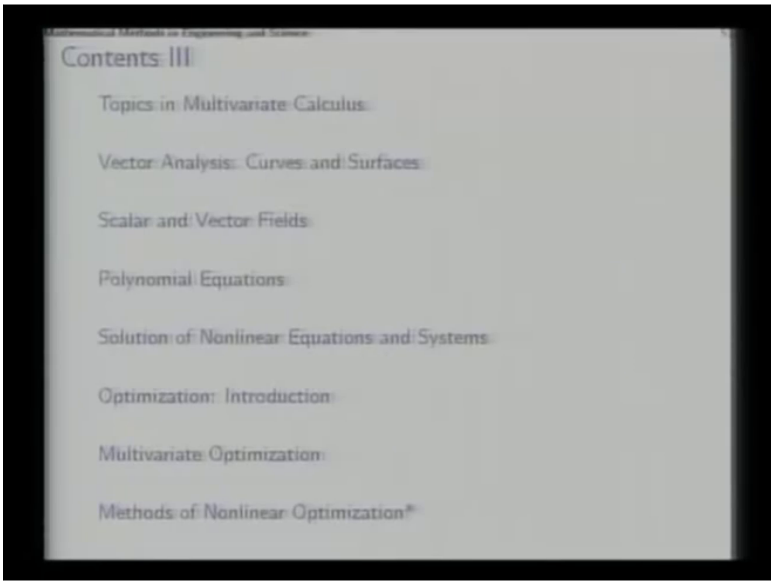

After that we will take up a small module on the crucial topics of analysis or calculus which includes multivariate calculus and also vector calculus in these 3 lessons. Next from here our module for numerical analysis starts the 2 chapters; 2 lessons on non-linear equation solving and then 2 further lessons on introduction to optimization we will cater to the first sub module of this part. Next there will be a very brief introduction on the frame work of constrained optimization methods.

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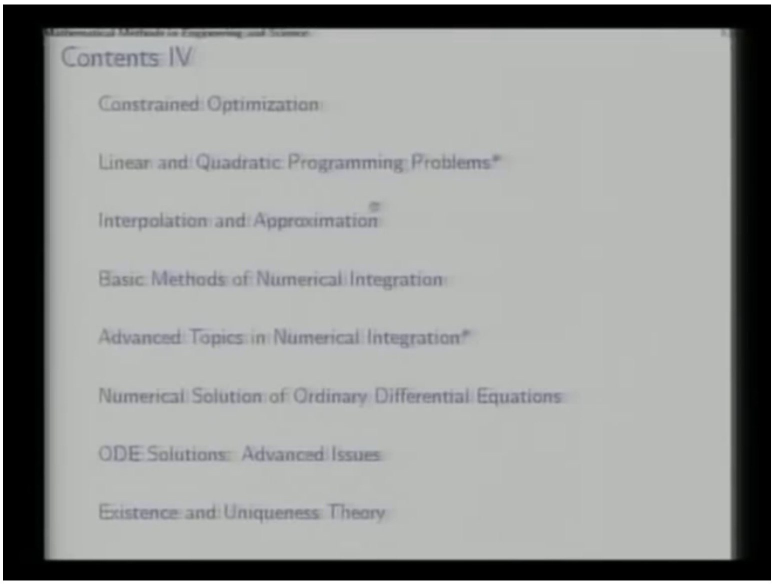

After that the usual topics of numerical analysis which are extremely important in applied science and engineering will be taken up which include interpolation approximation numerical integration and solution of ordinary solution differential equations.

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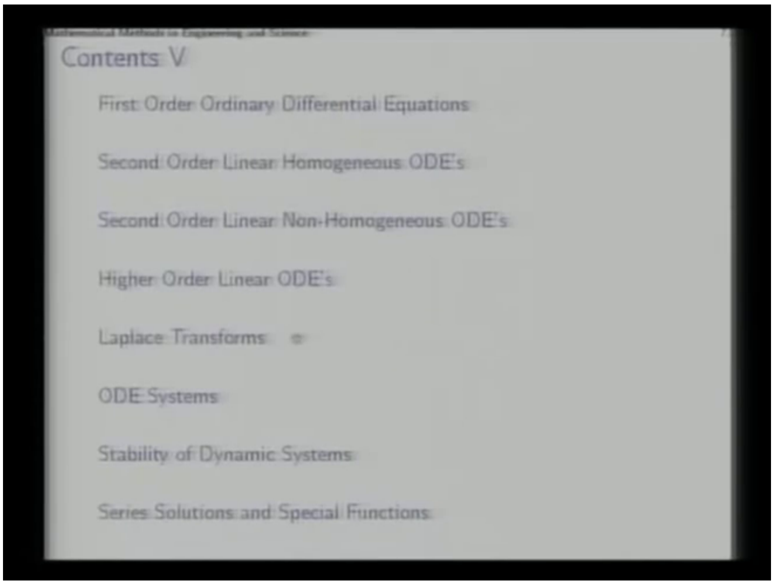

Next our ordinary differential equation module, in itself we will start with these few chapters after which we will go to the systems of ordinary differential equation.

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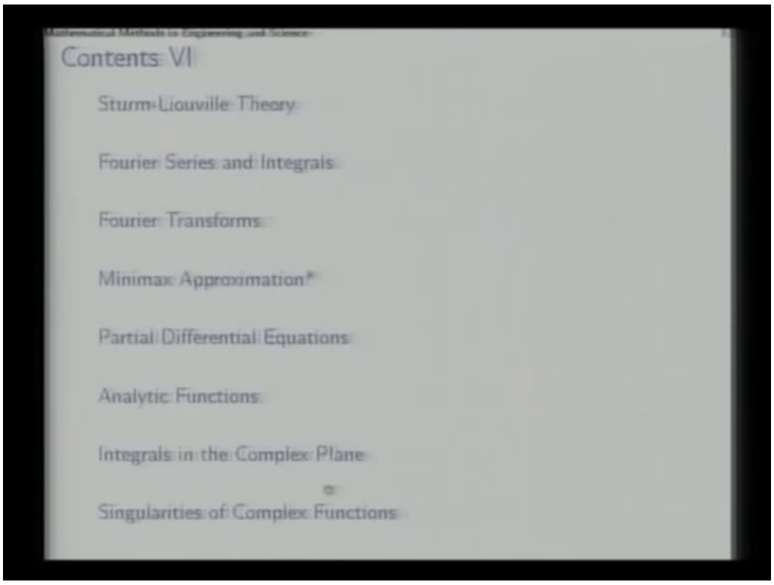

Next from the chapter from the lesson on series solutions through the lesson on Sturm Liouville theory; we will be proceeding from the topic of ordinary differential equations to the approximation theory, the tools developed in the ordinary differential equation will be then applied into approximation theory in these few chapters.

Finally after at the end of this differential equation module, we will have one important lesson on partial differential equations. Now partial differential equations are actually a very large topic and within a course on applied mathematics complete justice cannot be done to this topic. Therefore, we will only briefly recapitulate the most important aspects of partial differential equations which are applicable in various branches of science and engineering. After this our module on complex analysis will be taken up in these few lectures. All through the course in order to bridge the gap between undergraduate mathematics and serious (Refer Time: 03:59) research computational requirements we will be going through all these topics and emphasizing on the important inter connections between several branches several areas of applied mathematics.

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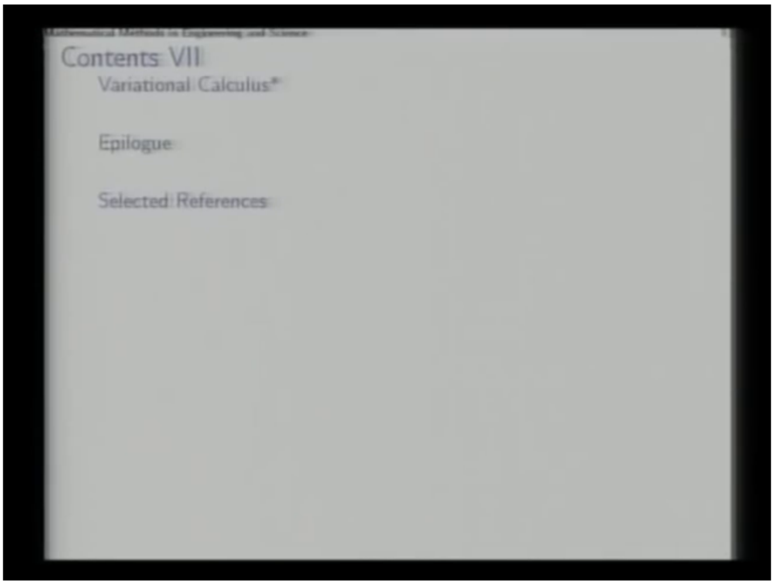

These interconnections will be most apparent in the concluding lesson on variation calculus, which we will one go connect linear algebra optimization differential equation and many other areas of applied science.

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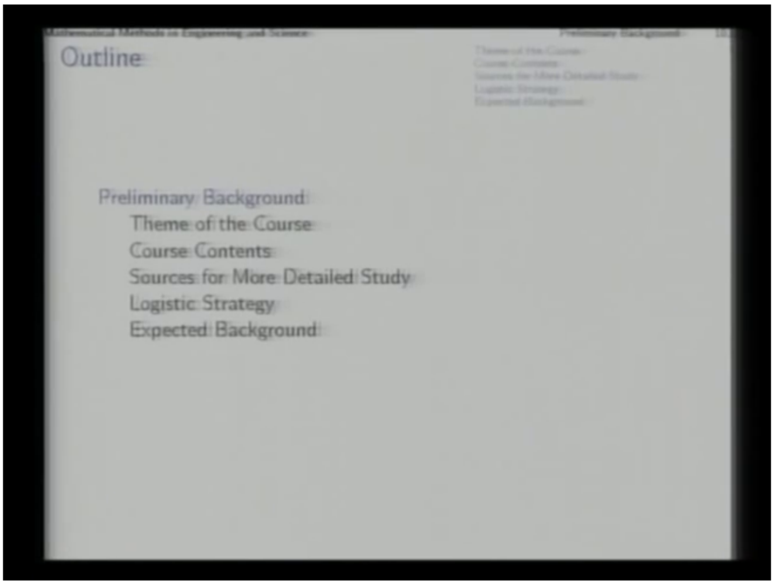

So, before proceeding into the important topics of the course, let me briefly give the preliminary background that will be needed throughout the course.

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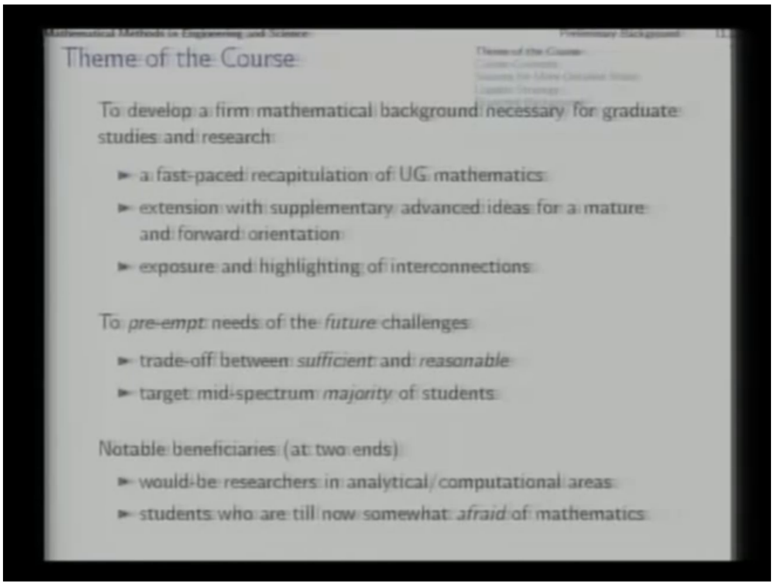

The theme of this course is to develop a firm mathematical background necessary for graduate studies and research and therefore, it is important that we conduct a fast paced recapitulation of the undergraduate mathematics. Here and there with extensions with supplementary advanced ideas for a matured and forward orientation of the subject matters covered and all through we will expose the students to the several interconnections between different areas of mathematics and the highlighting of this interconnection will be one of the major focuses of this course. The purpose of this course is to preempt needs of future challenges that students might face when they take up serious computational and analytical research.

Therefore we will make a tradeoff between what is sufficient subject matter for coverage of the applied mathematics area and what is reasonable within the scope of a single course. Typically the level and pace of the instruction will target the mid spectrum majority of students who were who are not extremely fast in understanding, but neither lacking background by too much. The notable beneficiaries therefore, will be at the 2 ends those who would be efficient researchers in computational and analytical areas later and those students who still now have somehow been afraid of mathematics, one of the purposes of this course is to make them free of their fear of this subject.

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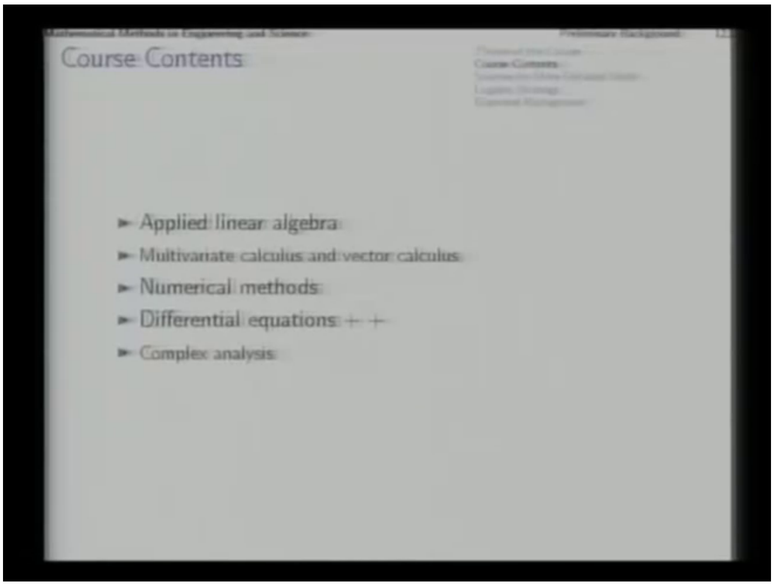

In the course content as I outlined earlier we will have 3 large modules, applied linear algebra numerical methods and differential equations with applications, interferes with 2 small topics on multivariate calculus and vector calculus and complex analysis.

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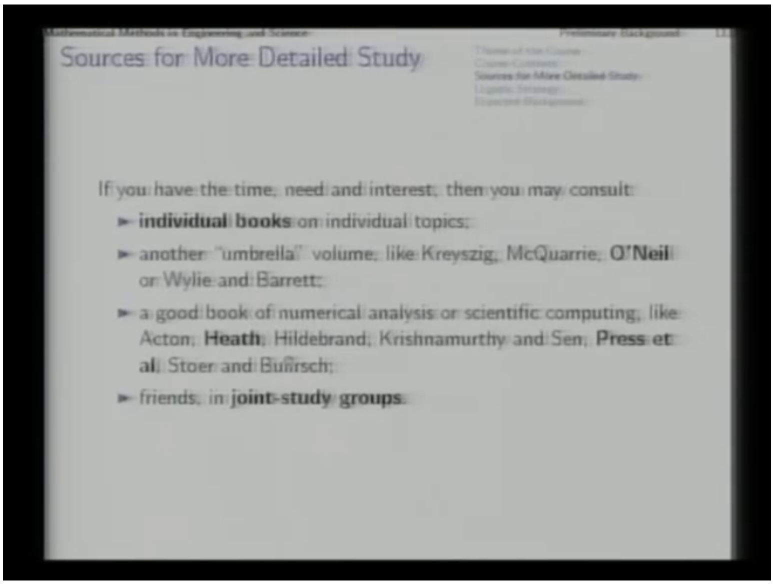

Now, apart from this text book that we will be covering, those students who can afford to cover more material they can be they can consider consulting one of these books, which now these slides will be available in the website given further. So, these references can be consulted at any time.

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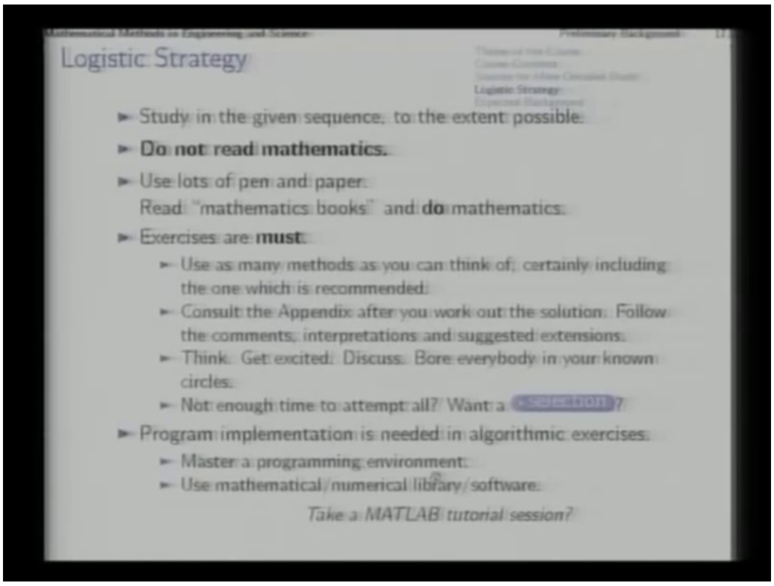

Now the logistic strategy that will be following and we advise the students to follow during the course, is to the extent possible study in the given sequence of lessons the lesson sequence that we follow in the course and one another important advice is do not read mathematics there is a habit among many students to read through mathematics and that way the understanding of mathematics typically does not become firm. Our advice to students is not to read mathematics, but rather to use lots of pen and paper. Read mathematics books, but when it comes to mathematics you do not read mathematics you do mathematics. So, whatever we develop here the student should on his own try to develop the same theme parallely with his own hand on his own paper that way the understandings becomes really deep.

As we proceed through exercises, as we proceed through the several lessons which gives out exercises it is important to go through those exercises in the structure of this course since it is a fast based recapitalization, many of the important theoretical developments are also framed or an tailored in the form of exercises as a student goes through them and as the students attempt exercises many of the theoretical background issues are actually covered there.

Now, in many of the exercises that I will be giving there are prescriptions of which method to use. It is not necessary that the student uses only that method the; my advice is use as many methods as you can think of and certainly including that method which has been advised. After you complete the exercises of the book you can consult the appendix to check the solution and also see through the several interpretations given in the comments in the solution there.

Important issue in following this kind of a course is to think and get excited beyond what is there in the text and what is there beyond in the lecture and beyond the exercises for this matter.

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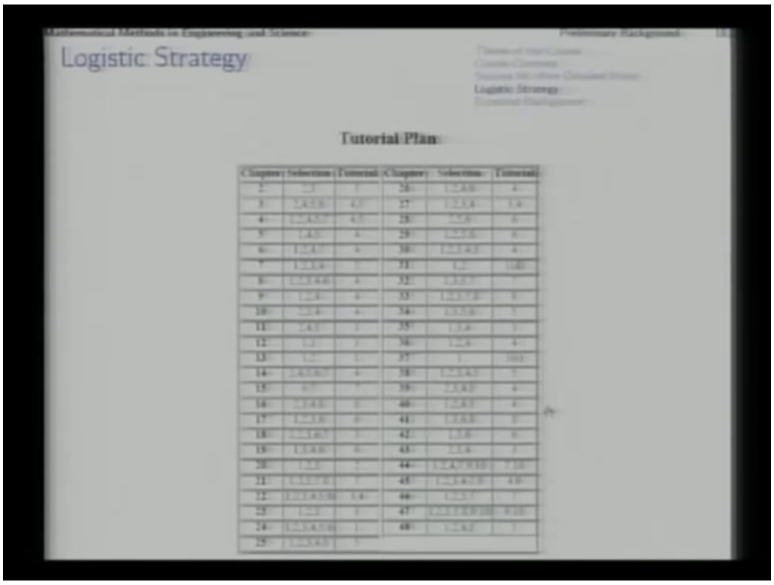

If you find that not enough time is there to attempt all the exercise given and if you want to make a selection then this will be my advice in which for every chapter out of the problems given at the end of the chapter in the text book there are some problems which are marked here as selection. In my understandings in my opinion, if you follow through these numbers and complete these problems in every chapter then your understanding of the basic material covered in the course will be adequate ideally you should try to attempt all problems, but if time is lacking, then the selection will help you in carrying forward without losing too much on the subject matter.

In many of the exercises you will find that you will need to write a small program because there may be some repetitive calculations, which it is not very easy to complete without a computer program. Therefore, it will be what when to master a programming environment you can choose your language you can use c program c Pascal Fortran whatever language is your favorite and in that choose a programing environment, in which you will be writing those small programs which will be needed to solve a few of the problems; quite often you may use mathematical numerical library or software like nag or (Refer Time: 11:40) both of these things you can get if you use the matlab programming environment in which you can write small programs very easily.

And therefore, if you feel that you do not have a very good matlab background right now you can take a tutorial session in that itself and prepare yourself for taking up the problems later in the book and in the course.

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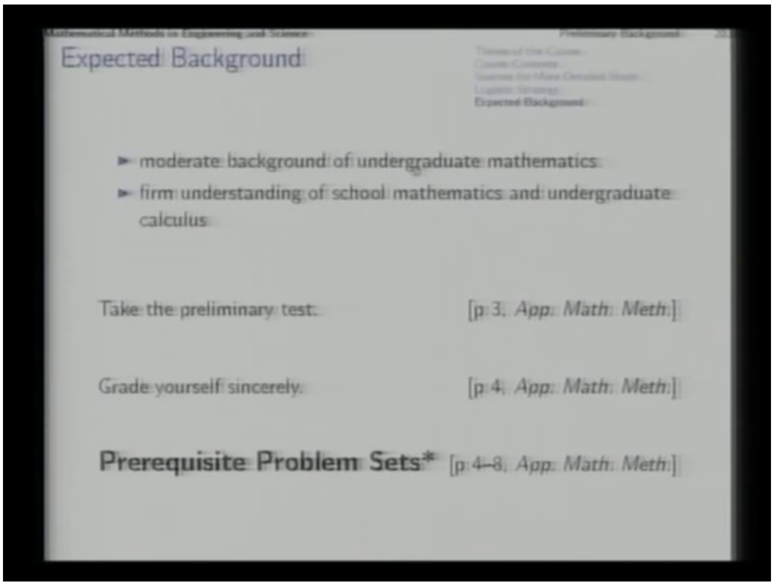

Now, the background that is expected for this course is only a moderate background of undergraduate mathematics, either in either from engineering or from science. However, the background of school mathematics and undergraduate calculus will be assumed firm in order to know whether you would have the necessary background already you can take a small preliminary test and that test appears in the page 3 of the test book and after you take the preparatory test, then in the page 4 in next page there is a grading scheme given based on which you can grade yourself as you grade yourself you will be classified in one of the 3 classes.

In the first it will be if you score too high in the preliminary test, then that will mean that you are thoroughly equipped to take up the course right now on the other hand if you fall too below in this preliminary test then that may mean that you need a little further preparation before you take up this course formally or seriously because otherwise it to may be a little difficult to follow the pace of the course.

If your score is in between; that means, that if you can take up the course right now you perhaps have the necessary background, but it will be still advisable that you take up that little preparation that little preparation that is needed in order to prepare yourself to take up the course directly is given in the form of 6 problem sets, which is on the undergraduate calculus and analytic geometry and all these topics. So, as you complete these six small assignment sets small problem sets, then we understand that you will have covered or recapitulated that undergraduate mathematics background which is taken as the firm necessary background for this course.

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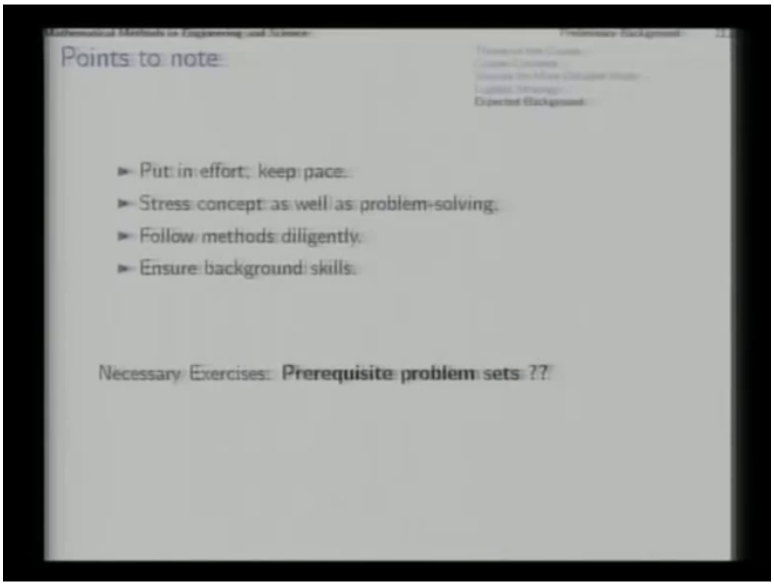

So, the points to note from this little introduction is that the important steps to follows with the course is to put in effort keep pace and the stress will be on both the concept as well as on problem solving, and the methods that are studied in the course they should be followed diligently along with other methods that you may find expeditious at time to solve certain exercise problems and the overall purpose of this course is to ensure background skills which may be called upon to utilize at any step of your future research carrier. Necessary exercises at this stage will be only prerequisites problem sets those six problem sets, for those who find that their background is not yet enough to take up the challenges of this course.

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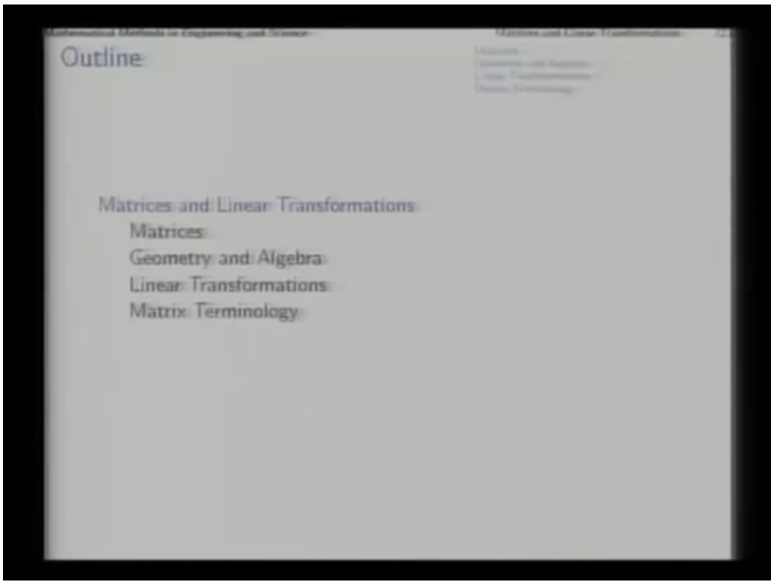

Next we proceed to the first important lesson of this course and that is on matrices and linear transformations.

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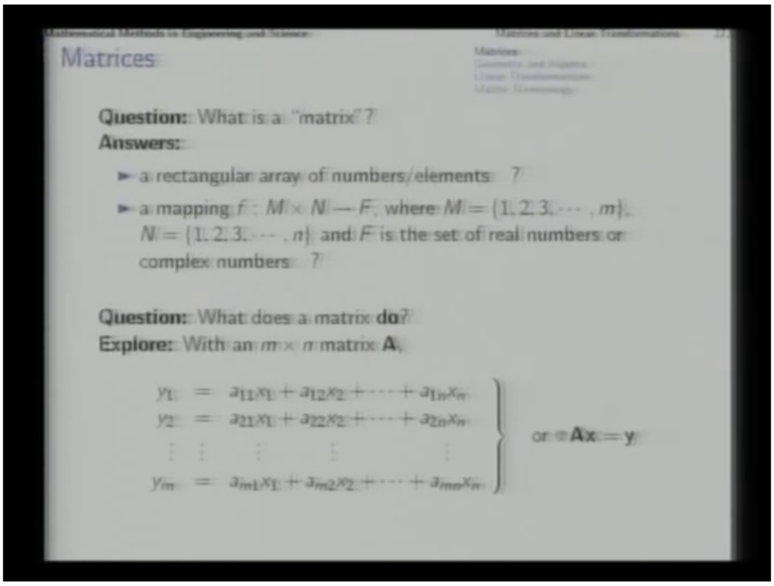

When you ask when I ask this question what is a matrix to a classroom? Typically the answer that I get is it is a rectangular array of numbers or elements something like this.

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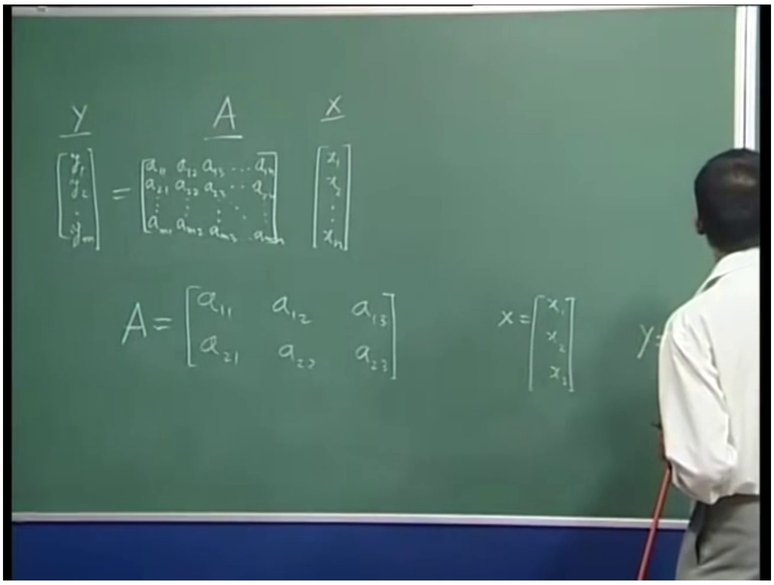

Matrix A is given like this it is a rectangular array of elements. A small variation a slight variation of this answer is that it is a mapping f from the set M cross N to the set capital F where mM is 1 2 3 up to m and varies from 1 to N and F is the set of real numbers or complex numbers.

Now, this technically detailed answer is essentially the same as this answer because here as you vary capital M from 1 to small m you essentially get the row numbers capital M equal to 1 2 3 4 up to small m similarly as you vary the other index capital N from 1 to n you basically change the column numbers. So, what this particular description of matrix essentially says is that if you tell me the row number and column number, then I can tell you a unique number of this array which is actually the same as a rectangular array of numbers or elements.

But then the question arises if this is a matrix then what do we do with the matrix therefore, a very good understanding of matrix does not come from this question directly what is a matrix rather if we try to explore the answer to a slightly different question which is what does a matrix do then as the answer unfolds, we can figure out a better way to answer the original question that is what is a matrix.

So, for example, in this particular case, we want to explore what does this matrix do. So, anybody who gone through undergraduate mathematics, knows that when a matrix a operates on or multiplies or multiply is multiplied with a vectors in this manner then as a result you get another vector. So, the matrix operates on a vector of n dimension and as a result you get another vector of m dimension that is the job that is the primary job of this matrix of size m by n.

Therefore, when you ask this question what does a matrix do the trivial answer that we get is at with an m by n matrix A when the matrix A multiplies a vector x then as a result we get a vector y and the matrix vector mollification rules will directly give us the expressions for y 1, y 2, y 3, etcetera from here y 1 is a 11, x 1 plus a 1 x 2 plus a 13 x 3 and up to a 1 n x n. So, this complete description in this manner or concise the in this manner tells us what does a matrix do.

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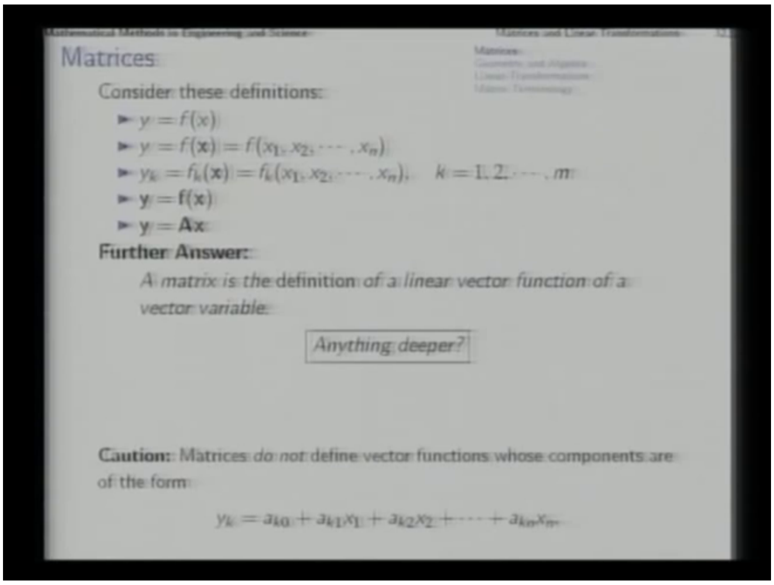

As we take this and proceed on this theme, then we will see what are the other things that the matrix will be able to do before that we will consider these small definitions of several functions as a small diversion, and try to understand what is actually taking place here when the matrix multiplies with a vector consider this small diversion. Y equal to f x this is the description is the statement of a scalar function f of a scalar variable x.

Now, we have heard of multivariate functions, functions of several variables. So, you can say that it there can be another definition of a function in which y is a function of x, but here x is not a scalar, but it is a vector. Then we say that y is a function of a vector variable x or we can say that it is function of several variables x 1 to x n. If we can define one such function then we can define many such functions.

So, suppose we define y 1 as 1 of x, y 2 as 2 of x and so on k running from one to m now if we have these y 1, y 2, y 3, y 4, then it is also possible to organize them in a vector like this and in that case the function representation will be y equal to f of x. In which y and x both are vectors then we say it is a vector function of a vector variable, right and a special case of such vector functions of vector variables are actually represented by this kind of a relationship y equal to A x.

What is a special situation of these functional requirement, you note that here also you can consider this mathematical relationship this equation as merely the description of a function. So, this gives us y as a function of x, y is a vector variable x is another vector variable and how y depends on x the dependence is stated here now what is so, special? In this dependence in this dependence we will see that in the description in the expression for every y x 1, x 2, x 3 up to x n are appearing in a particular fashion there is no sin x 1 no log x 2 no e to the power x 3 no x 2 square no x to x 4 such terms are absent and this defines a particular kind of function which is also stated like this.

Now, in the statement y equal to f of x the function could be anything as we say y equal a x we are talking about a particular kind of function and that kind of function is a linear vector function therefore, when we ask that question back what is a matrix the detailed or more informative answer would be a matrix is the definition of a linear vector function of a vector variable. The point to note or the caution to be exercised is that matrices do not define vector functions whose components are of this form why because in the matrix sector modification that we saw just now in that this term will not be present.

Therefore we will we will say that this kind of relationships this kind of functional forms in which there is a constant term x 3 term is there that kind of mappings that kind of functions matrices will not defined. Other than that if you remove this term then whatever is remaining that kind of functional relationships matrices this time matrices explain matrices express directly. Now this gives us one definition that is one meaning of the word matrix in applied mathematics.

A matrix is the definition of a linear vector function of a vector variable the vector variable is x and y is a function of that what kind of function? Linear vector function vector function because y itself is a vector and linear because of this particular relationship. Anything deeper than this we can think of other than there description of a linear vector function other than the description of a function something more revealing something more deep we can find out when we try to connect this algebraic entities with geometric entities consider this equation.

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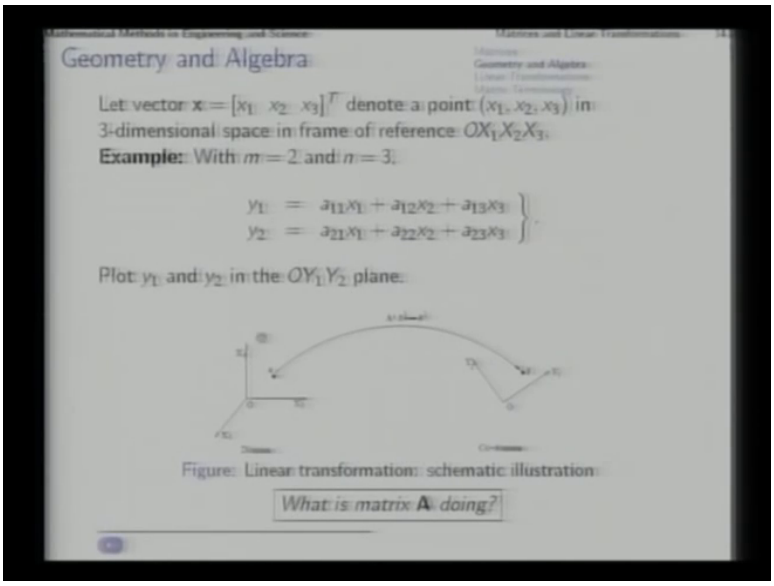

Let vector x, x 1, x 2, x 3 transpose; that means, it is a finally, column vector x is a column vector denote a point x 1, x 2, x 3 in a 3 dimensional space with frame of reference o x 1, x 2, x 3 which will be which will be look like this o, x 1, x 2, x 3 is the frame of reference in which x the x vector x with coordinates x 1, x 2, x 3 will denote a point x 1 in this direction then x 2 in this direction and x 3 in this direction. So, that denotes a point. So, this vector x will denote a point in this 3 dimensional space.

Now, with m equal to 2 and n equal to 3 we will have a matrix A which will look like.

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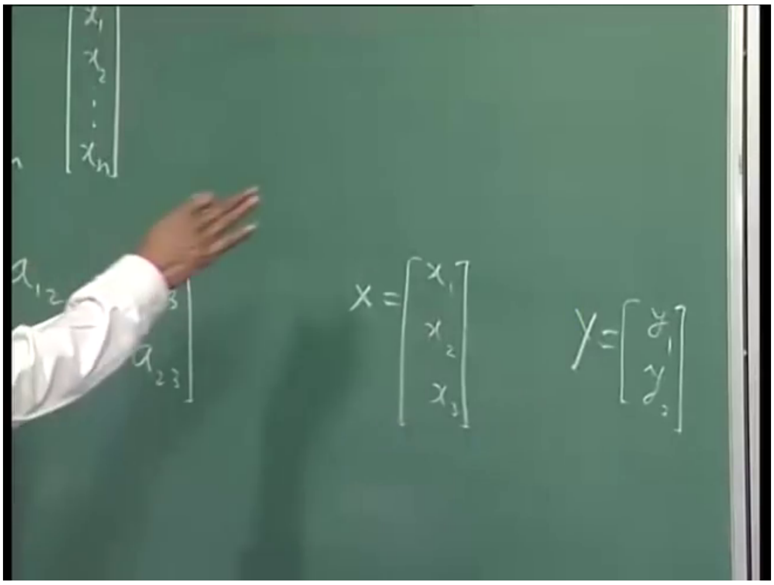

Since this has 3 columns, it can multiply a vector which is like this with 3 elements and as it multiplies a x then we will get another vector which we will call y and which will have 2 elements, right the detailed expressions for y 1 and y 2 will be like this.

Now, we take this y 1 and y 2 these values and consider them y 1 and y 2 as the coordinates of a point in another space a plane in this in this case in the directions oi one and o I 2 axis and plot them; that means, every point that we get here and call it x, it has 3 coordinates x 1, x 2, x 3 which from here will give us 2 coordinates y 1, y 2 for plotting in this plane and accordingly, we get this point y and that essentially is arising out of this linear transformation.

So, now we ask this question again what is matrix a doing in this particular instance in this particular schematic.

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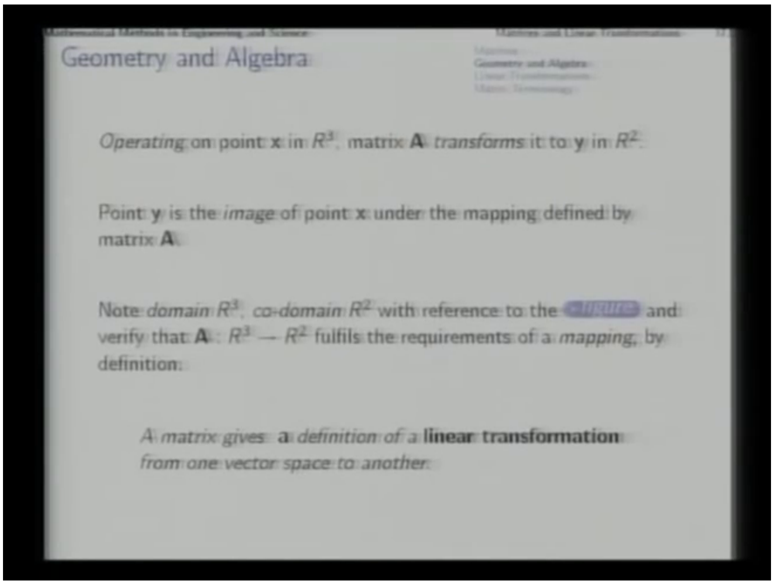

It is operating on point x in R 3 that is 3 dimensional space with real coordinates real refers to this R and the dimension 3 is shown here. So, here the matrix A is operating on point x in the 3 dimensional real space and transforming it to a point y in a 2 dimensional space, that is a plane with real coordinates and we say that point y is the image of point x under the mapping defined by matrix A. Note that this domain of this mapping R 3 codomain of this mapping R 2 can be immediately seen from the figure and we can verify that this matrix A this transformation from R 3 to R 2 does fulfill all the requirements of a mapping by definition.

For example for definition of a mapping for the successful definition of a mapping you will require that every point in the domain gets uniquely mapped to the codomain. So, every point that you take here for that you get the coordinates and these coordinates we certainly give you 2 real numbers which can be plotted here and you can get point here the definition is always there that is one important requirement for a mapping and the other important requirement for the mapping is that it should be unique.

Now, since this is a linear relationship with the same x 1, x 2, x 3 you can never find different sets of y 1, y 2 that means; this definition is unique. So, that way this matrix a does fulfill the basic requirements for a mapping. Note here that this necessity of definition and necessity of uniqueness are only in the forward direction as it is in the case of a mapping it is not necessary that for every point here there must be a point here that is not necessary in the mapping and that is not necessarily given in this kind of a matrix multiplication either.

Similarly, it is not necessary that for a point here there must be a single pre imagine, there could be infinite pre images and that is not necessity in mapping neither a necessity in the matrix multiplication here. Then we arrive at another important definition of the matrix or another important action of a matrix that is a matrix gives a definition of a linear transformation from one vector space to another. In this particular example we saw a linear transformation from R 3, 3 dimensional real space to R2 2 dimensional real space that is a plane.

Now, note that we are talking about a definition and not the definition; that means that preempts the issue which will be discussing later that there could be different definitions or different matrices defining the same linear transformation.

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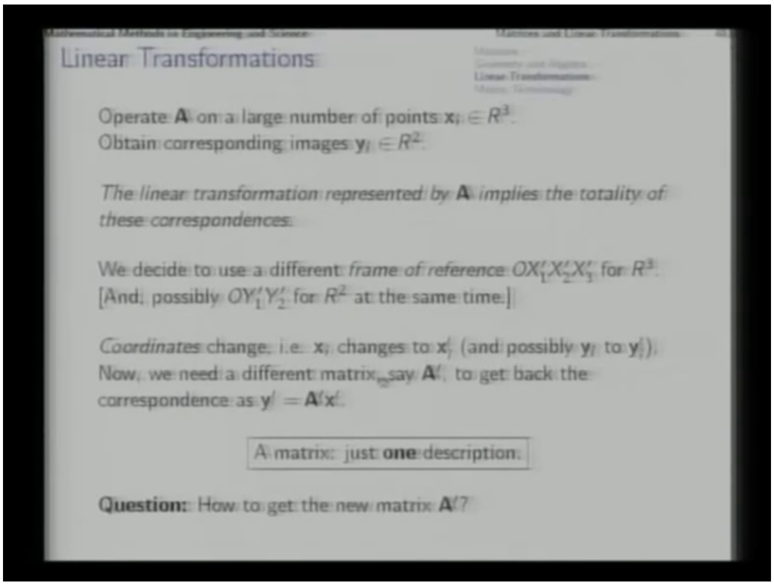

Now, as we have started discussing the issue of linear transformations; consider this, suppose we take this matrix A and we operate it on a large number of points in the domain R 3; that means, x 1. Now this x 1 is a vector x 1, we are not referring to the coordinates anymore we are referring to the vector.

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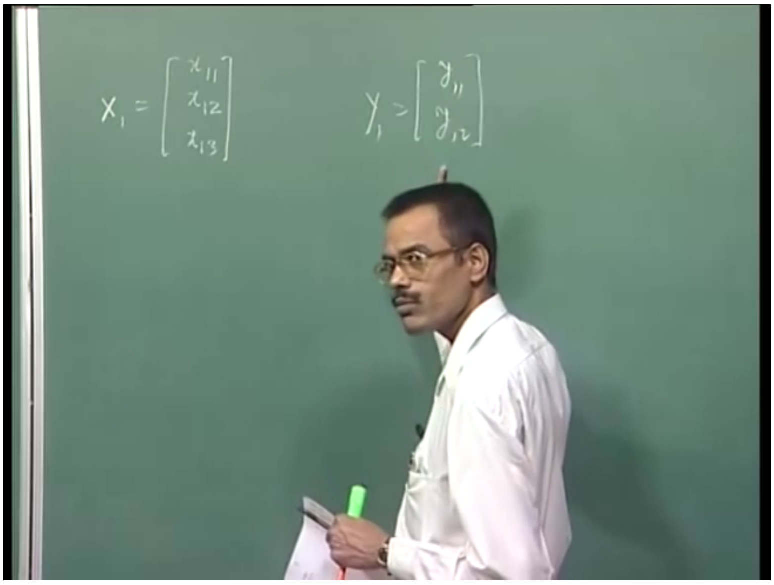

One of the such vector and referring it as x 1 which will have several coordinates 3 coordinates in this particular case which could refer to that as like this.

So, one such vector we take and multiply that vector with a, then we get another vector now like this if we consider a large number of vectors then here we will have for each of this vectors, we will have images right. So, if we denote with x i and y i the i th vector in the codomain and the corresponding image i th vector in the domain and the corresponding image in the codomain then we find that for every in R 3 we can associate one vector in R 2 and the linear transformation that the matrix a represents actually implies the entirety of all these correspondences. X 1 is corresponding to y 1, x 2 corresponds with y 2, x 3 corresponds to y 3 and so on. All these correspondences together are represented by the linear transformation which here we are representing with this matrix a.

Now, consider the situation suppose we decided to use a different frame of reference not o X 1, X 2, X3, but some other axis X 1 prime and X 2 prime and X 3 prime. If we take a difference frame of reference like this in the domain and possibly a different frame of a reference in the codomain also at the same time then what happens? For the same old points due to the change in the frame of reference the coordinates will change and accordingly the numbers in these vectors x i will change suppose the change coordinates are represented with x i prime.

Similarly, in the codomain if there is a change in frame of reference in the codomain also in the y plane also then possibly y i those vectors will now be represented by different new number given in y i prime that will mean that we will need at different matrix. Let us call a prime to get back the same correspondences that is we have associated with a linear transformation points on the domain with their counter parts in the codomain.

Now, if we change the frame of reference in the domain or in the codomain or in both, then the same linear transformation that is the same correspondences between the old points will be represented not by the old matrix, but by a new matrix and therefore, we say that the matrix a matrix is a definition of a linear transformation and not the definition of the linear transformation because a matrix is just one description the moment frame of reference changes the description will change the matrix will change.

Now, in this particular scenario if we now ask, that we suppose know that how the frame of reference is changed in the domain and in the codomain and in that case we want to find out what is the new matrix A prime which mapping to this relationship will lead to the same original set of correspondences between points in the domain and points in the codomain this problem in linear algebra is known as basis change. This is one important issue which will be taking up in the next lesson before that it is important to point out that.

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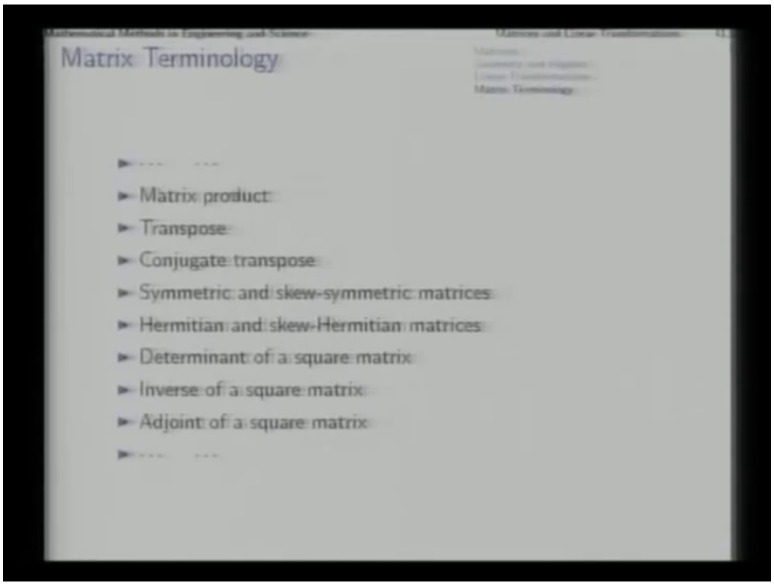

In this entire course we will be assuming a large number of matrix terminology as matter which is firmly there with the student.

Now, if these and many such basic matrix terminologies are out of practice then it is important that before proceeding further, you take up a matrix algebra book and brush up on these issues. These are the topics which will be these are the definitions or concepts which will be taking as background all the time.

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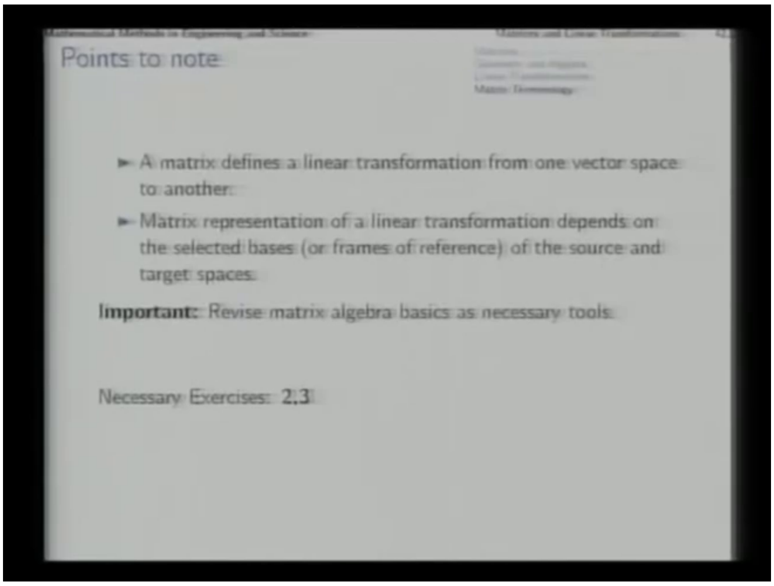

In this lesson the important issues that we came across are these 2 1 a matrix defines a linear transformation from one vector space to another, and second that matrix representation of a linear transformation depends upon the selected bases or the frame of reference of the source space and the target space.

In these particular lesson problems 2 and 3 are in the classification of necessary exercises which will cover the appropriate background. As we have time in hand we take up an example of linear transformation and matrices relevant to this particular lesson here.

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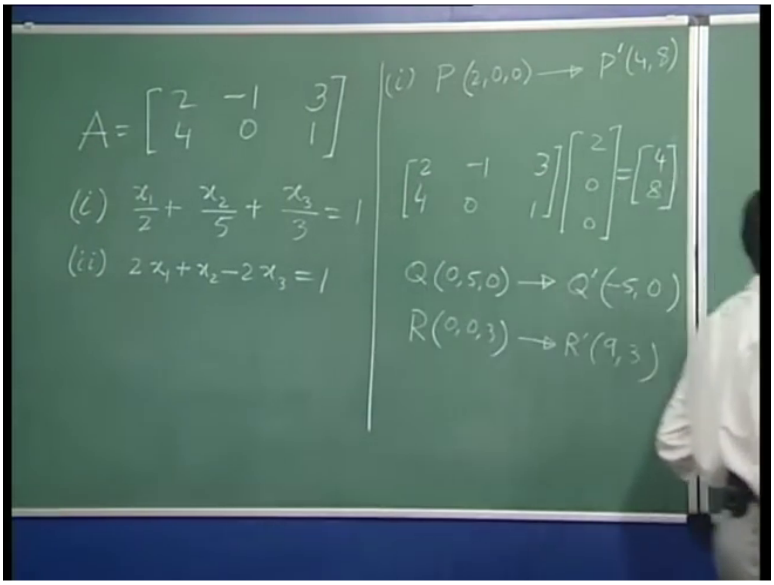

We have this matrix a 2 by 3 matrix A; obviously, its domain is R 3 its domain is 3 dimensional and therefore, it will multiply with vectors which have 3 components and its codomain is R 2 that is as a result we will get 2 dimensional vectors. So, what we do? We sketch here.

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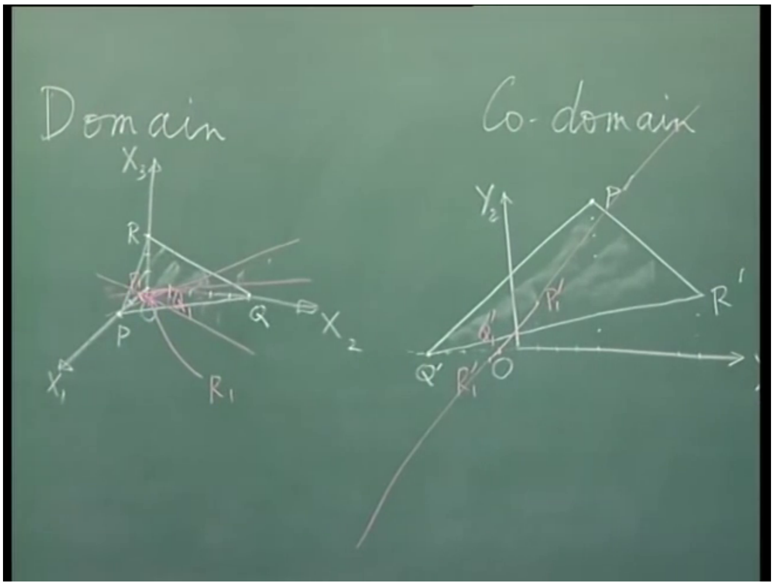

The way the domain and the codomain will look like. The domain will have 3 axis we will show this as x 1, this as x 2 and this as x 3 origin form the same origin, but shown separately is here and we have this as o, y 1, y 2, right.

Now, as we take several points in this domain and we multiply the vector x 1, x 2, x 3 with this matrix then we get coordinates y 1, y 2 which we can plot there now we will examine in this example the mapping of these 2 planes from the x space to the y space from the domain to the codomain. This is one plane that we will consider and this is the other plane that we will take up. In this case we can see that the way the equation of the plane is written the intercepts are 2 5 3 on the x 1, x 2, x 3 axis. So, that we plot here 2 5 3 3 here. So, this is a point.

So, let us denote this point as P this is P and its coordinates 2 0 0 and that get mapped to p prime in this y space and how we get that mapping that is we take this matrix a and multiply with the vector which denotes this point; obviously, the result is twice this column because 2 is here, and here we have 0. So, this gives a 4 8; that means, the point p prime which is the map position of this point p here is 4 8. So, we plot that point here 1 2 3 4; 1, 2, 3, 4, 5, 6, 7, 8. So, this is point p prime the map point from P.

Similarly, the other 2 points we marked here point Q is taken as the x 2 intercepts here. So, we can say that the x 2 intercepts of this plane is 0 5 0 and that goes to Q prime which similarly will be given as 5 times this column is minus 5 0. So, here we try to plot minus 5; 1 2 3 4 this is point Q prime finally, point R here the x 3 intercept is 0 0 3 so, that point is here and it get gets mapped to 9 3 from here it get mapped to 9, 3, 1, 2, 3, 4, 8, 9 and then 1, 2, 3. So, this is r prime so; that means, this plane this triangle to begin with this triangle P Q R gets mapped to this triangle P prime Q prime R prime. So, this triangle gets mapped to this triangle and the plane of this triangle gets mapped to the plane of this triangle in y 1 y 2. Now what is the plane of this triangle in y 1 y 2? It is entire plane therefore, we can say that this triangle P Q R gets mapped to this triangle P prime Q prime rR prime and the plane of this triangle in this 3 dimensional space, gets mapped to the entire y 1 y 2 plane here complete plane is covered.

Now, let us take the in the other case in this case 2, we have got this as the equation of the plane.

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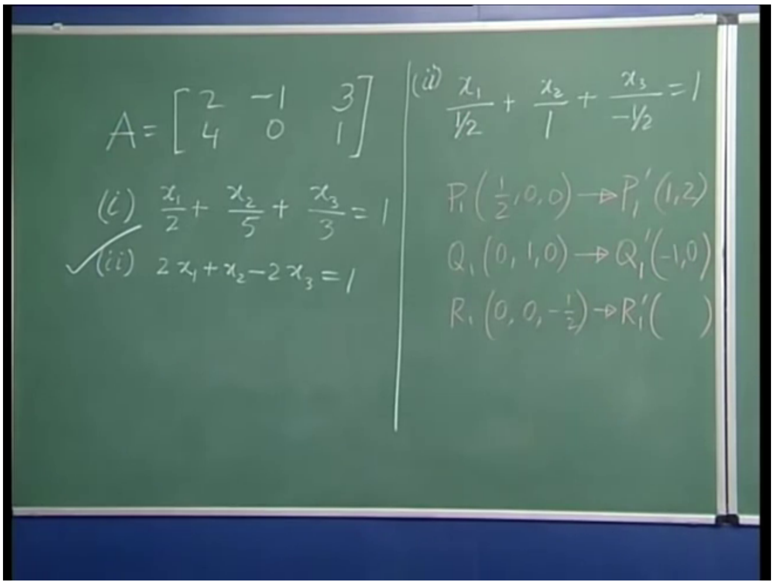

If we want to describe the plane in this case also with the help of the intercepts then we need to slightly reorganize this equation and we do that reorganizations here 2 x 1 is x 1 by half plus x 2 by 1 plus x 3 by minus half and in that case, we have got those 3 intercepts here, this point is at the intercept half let us call it P 1.

Next this point 010 here that is here, let us call it Q 1 and this third is at minus half. So, that will go somewhere here and this will be the plane this will be the triangle P 1 Q 1 and R 1 this red triangle is the triangle under question and we try to see mappings here. So, P 1 half 00 will map to a point P 1 prime which will be basically half of this column that is 12

So, we map it here 12 this is P 1 prime in the y plane, next 010 that is Q 1 and that will be map to Q 1 prime and that will be simply this vector minus 1 0 and we map it here minus o1 0 finally, the third point that is at minus half this will be mapped at minus 3 by 2 minus 1 by 2. So, let us map it minus 3 by 2 minus 1 by 2 here. So, wherever this triangle P 1 Q 1 R 1 go, P 1 prime P 1 prime R 1 prime they are coordinates here and therefore, this entire plane of P 1, Q 1, R 1 from here we will get mapped to this line the triangle will get mapped to this line segment the triangle here is regenerate because all 3 vertices are in the same line. And plane of that those 3 points the plane there represented by this equation on the y space regenerates to not a plane, but a line now why this discrepancy in the 2 cases.

In order to see that we need to conduct a little analysis into how this matrix is mapping several vectors.

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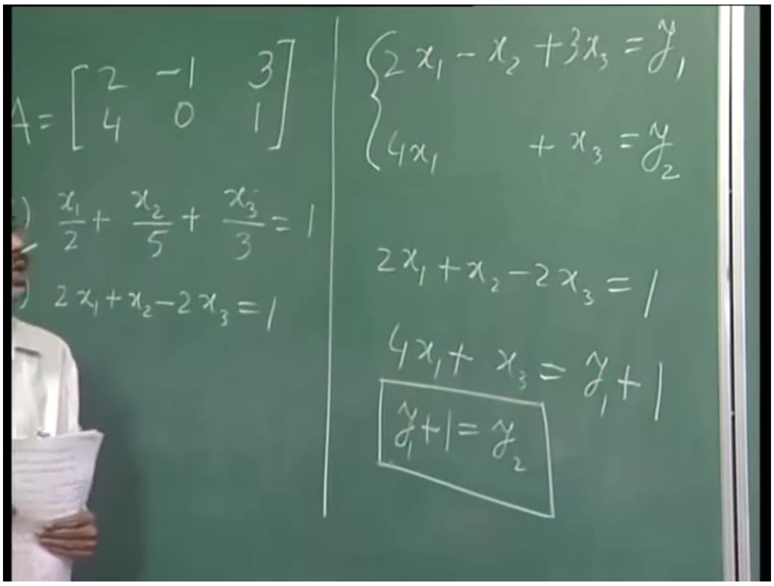

In this plane and in this plane from this matrix giving the linear transformation we can directly see that x 1 x 2 x 3 and y 1 y 2 are related in this manner. This particular plane the second one which is degenerate which got mapped to a particular line that equation we now write here.

And now we find that in the second equation coming from the linear transformation x 2 is already missing now we want to get rid of this x 2 from here and that we can do by adding the first equation from here to this as we do that we find, but then we find that 4 x 1 plus x 3 is y 1 plus one, but the second equation here says that 4 x 1 plus x 3 is actually y 2; that means, that y 1 plus one equal to y 2 gives us a special relationship between the coordinates which are on this plane whatever points here we take on this particular plane on the other side on the codomain side the coordinates y 1 and y 2 must respect this relationship and in a plane o y 1, y 2 this is the relationship which describes a line this particular line and therefore, in this case we find that the entire plane reduces to a line.

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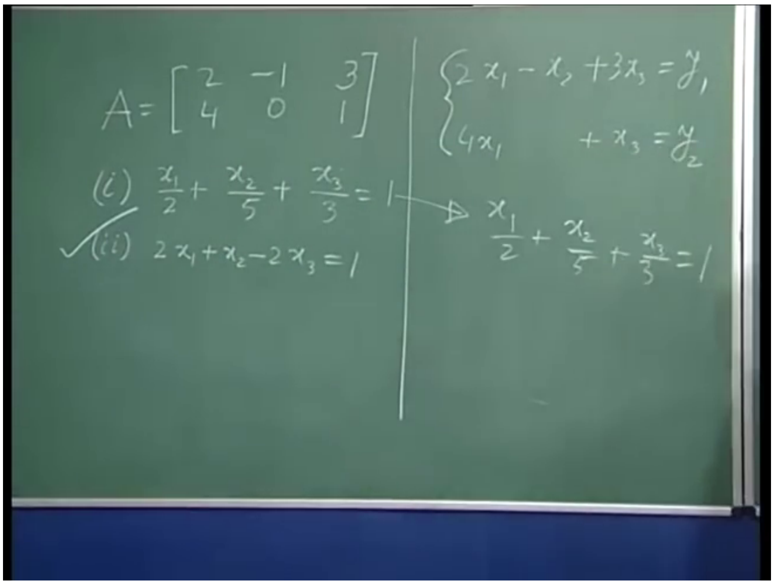

If on the other hand here we had taken up the other plane this one, this plane if we had taken up then among these 3 equations no organization no deduction would give us a relationship between y 1 and y 2 which is independent of x 1, x 2, x 3 and therefore, in the case of this plane as we vary all coordinates x 1, x 2, x 3, we get all possible combinations of coordinates y 1 and y 2 here and therefore, it covers the entire plane. In the second case we find that the null space of this matrix this transformation falls along a direction which is contain in this plane therefore, this plane while mapping operates like a projection and the complete plane gets mapped to this.

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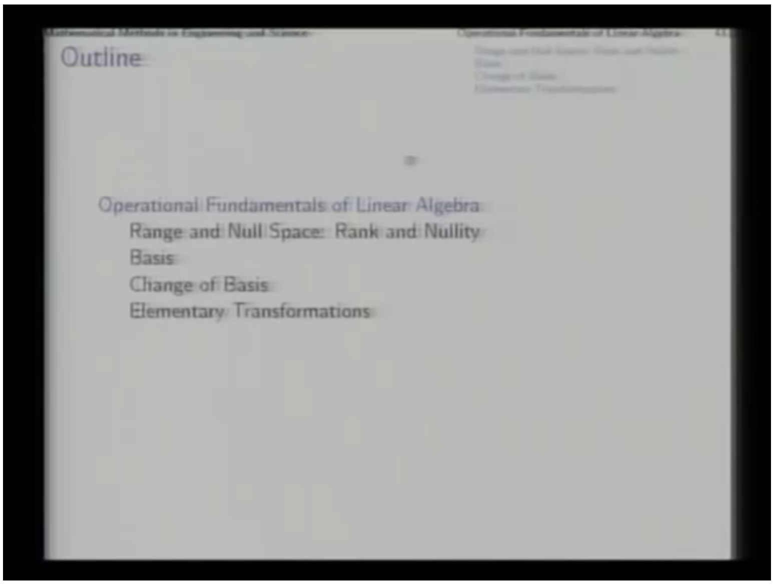

Line such issues will be taken up in more detail in the next lesson which is the operational fundamentals of linear algebra.

Thank you.